SC 651: Estimation on Lie Groups

Assignment-1

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Contents

1 Problem 1

1.1 Spring Mass Damper

1.1.1 Theory

Given the Spring Mass Damper's differential equation:

$$\ddot{x} + \dot{x} + x = 0, \tag{1}$$

the analytical solution can be expressed as:

$$x(t) = e^{\alpha t}(C_1 \cos(\beta t) + C_2 \sin(\beta t)), \tag{2}$$

where $\alpha = -\frac{1}{2}$ and $\beta = -\frac{\sqrt{3}}{2}$ are the real and imaginary parts of the roots r_1 and r_2 of the characteristic equation $r^2 + r + 1 = 0$.

Given initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$, the specific solution is determined as:

$$x(t) = e^{-\frac{1}{2}t}\left(C_1 \cos\left(-\frac{\sqrt{3}}{2}t\right) + C_2 \sin\left(-\frac{\sqrt{3}}{2}t\right)\right), \tag{3}$$

where C_1 and C_2 are constants calculated based on the initial conditions.

For example, with initial conditions $x_0 = 4$ and $v_0 = 4\sqrt{3}$, the constants C_1 and C_2 can be determined, leading to a specific form of the solution.

1.1.2 Methods

The Spring-Mass-Damper system, governed by the differential equation

$$\ddot{x} + \dot{x} + x = 0, \tag{4}$$

was simulated using three different numerical methods: Explicit Euler, Implicit Euler, and Symplectic Euler. The parameters for the simulations were as follows:

- Time step (Δt): 0.001 seconds.
- Total simulation time: 25 seconds.
- Initial conditions: $x(0) = 1$, $\dot{x}(0) = 0$.

The analytical solution of the system is given by

$$x(t) = e^{-\frac{1}{2}t}\left(x_0 \cos\left(\frac{\sqrt{3}}{2}t\right) + \left(\frac{v_0 + \frac{1}{2}x_0}{\frac{\sqrt{3}}{2}}\right) \sin\left(\frac{\sqrt{3}}{2}t\right)\right). \tag{5}$$

1.1.3 Results

The results of these simulations, alongside the analytical solution, are presented in Figure 1. The Figure contains the simulation output and error analysis for the Explicit Euler method, Implicit Euler method, and Symplectic Euler method. These error plots highlight the deviation of each numerical method from the analytical solution.

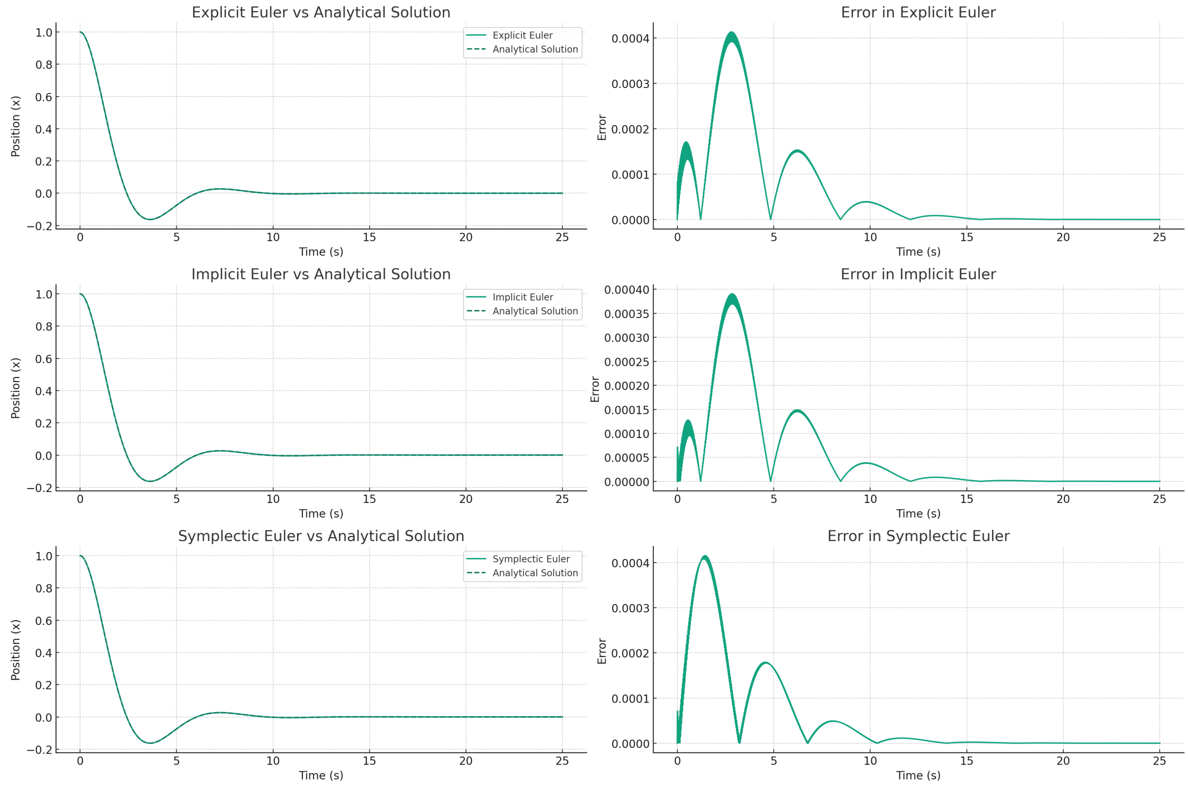

Figure 1: Spring-Mass-Damper system Plots

1.1.4 Conclusion

The Explicit Euler method, although simple to implement, shows a significant deviation from the analytical solution over time, particularly in stiffer systems. The Implicit Euler method offers better stability but requires more computational effort. The Symplectic Euler method, known for its energy-preserving characteristics in conservative systems, exhibits a unique behavior in the damped Spring-Mass-Damper system.

1.2 Planar Pendulum

1.2.1 Model

We analyzed the dynamics of a Planar Pendulum, governed by the non-linear differential equation

$$\ddot{\theta} + \sin(\theta) = 0. \tag{6}$$

Due to the non-linear nature of this equation, finding analytical solutions is not feasible. Therefore, the pendulum's motion was simulated using numerical methods exclusively. The details of the simulations are as follows:

- The time step was varied between 0.01 and 0.0001 to observe the impact on the simulation results.
- Four different numerical methods were employed: Euler, Explicit Euler, Implicit Euler, and Symplectic Euler.

1.2.2 Results

1.2.3 Conclusion

2 Problem 3

2.1 Simulation of Kalman Filter

2.2 Results/Plots

2.3 Conclusions

3 Problem 4