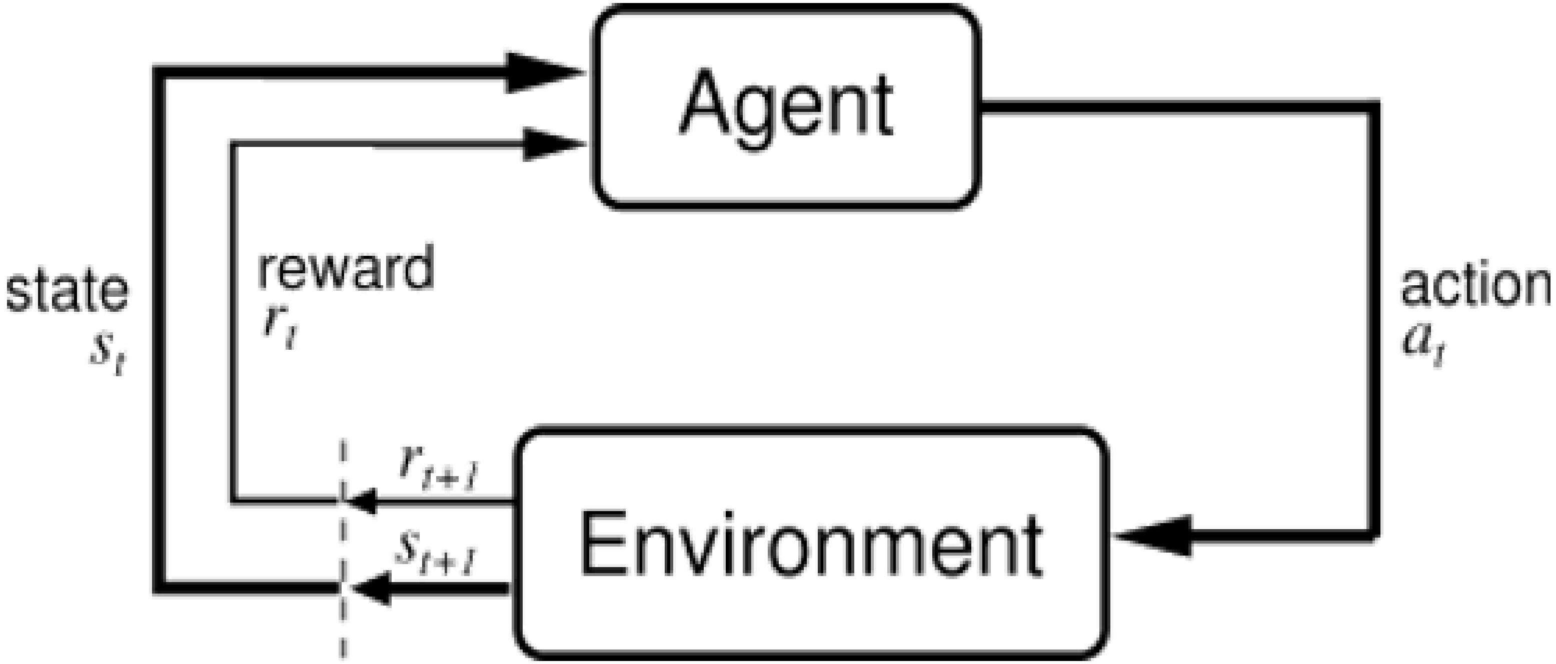
Reinforcement learning



(Sutton and Barto, 1998)

Why GAN and $a = f(s)$ are difficult to learn?

	Supervised Learning	Self-supervised Learning	Reinforcement Learning
1. Function to be learned	MLP, CNN families	AE/VAE, GAN	Actor learned by PPO-AC
	$y = f(x)$	$\hat{x} = f(x)$	$a = f(s)$
2. Loss function $\mathcal{L}(f)$	MSE, CE	MSE, CE, KLD, JSD	MSE, KLD
3. Minimize $\mathcal{L}(f)$	Gradient decent, Maximum Likelihood		

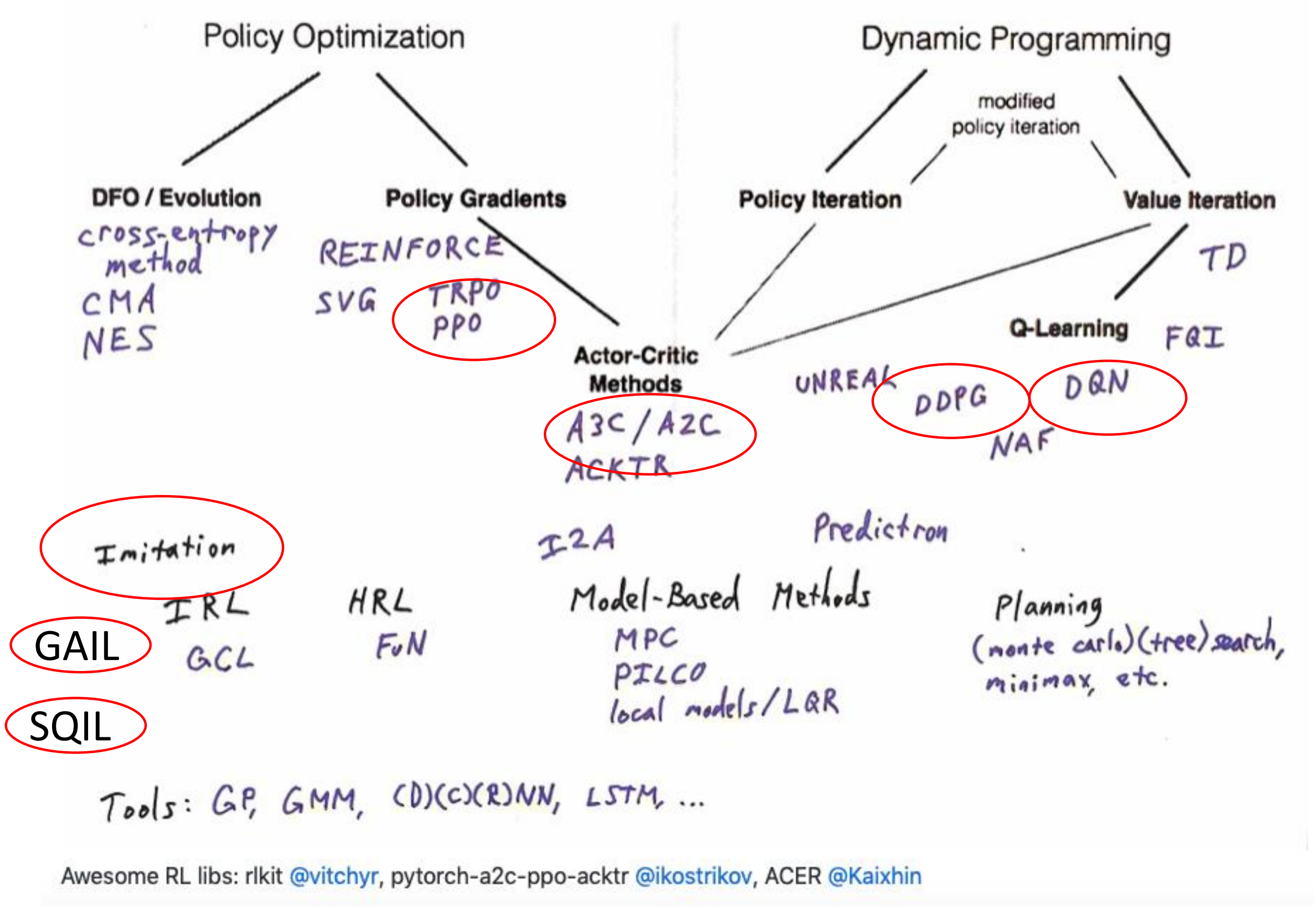
Why $a=f(s)$ is difficult to train?

- Define a function to be learned: $a = f(s)$
- Define a loss function $\mathcal{L}(f)$ to describe the error between y^n and $\hat{y}^n$

  If at time *t* we perform action a_t under state s_t, we only know the immediate reward r_t and there is a time delay between the action a_t performed at time *t* and the total accumulated reward $\hat{y}$. Besides, after preforming a_t at state s_t, there are infinite number of possibilities for following state- actions $s_{t+1}, a_{t+1}, s_{t+2}, a_{t+2}, \cdots s_{t+T}, a_{t+T}$. It is difficult to estimate the true answer *y* (the true final reward).

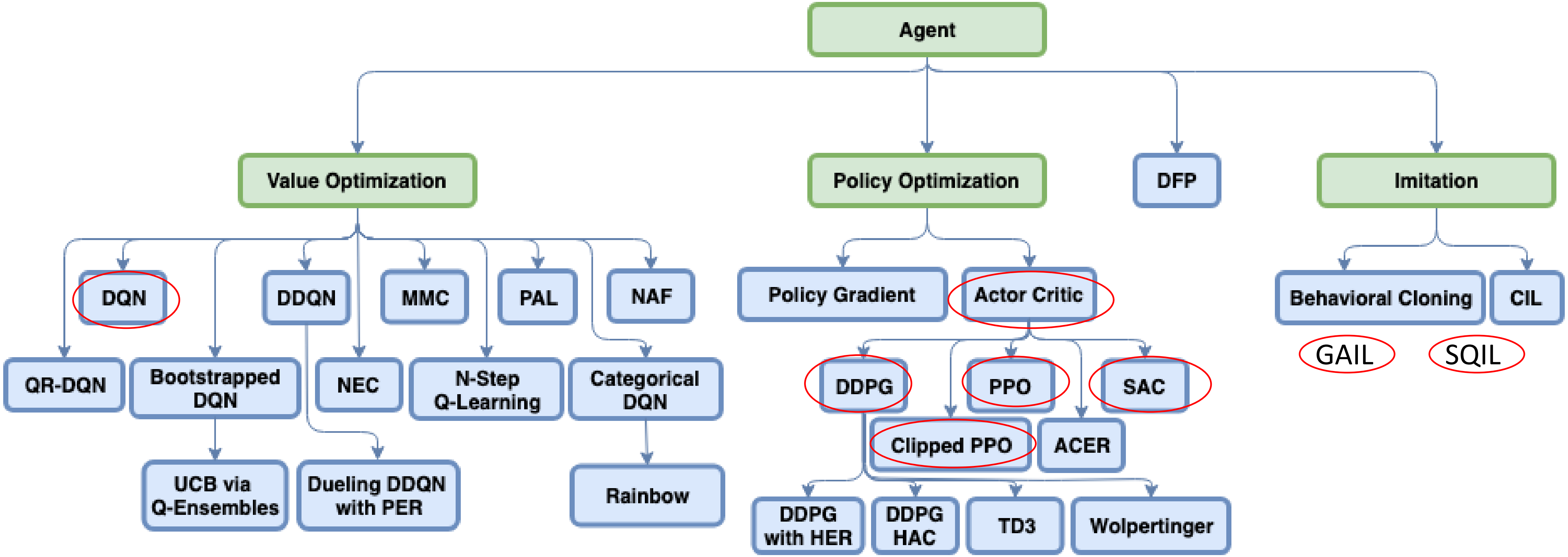
- Find the optimal parameters that minimize $\mathcal{L}(f)$

Policy optimization vs dynamic programming approach to learn $a = f(s)$ under time delay



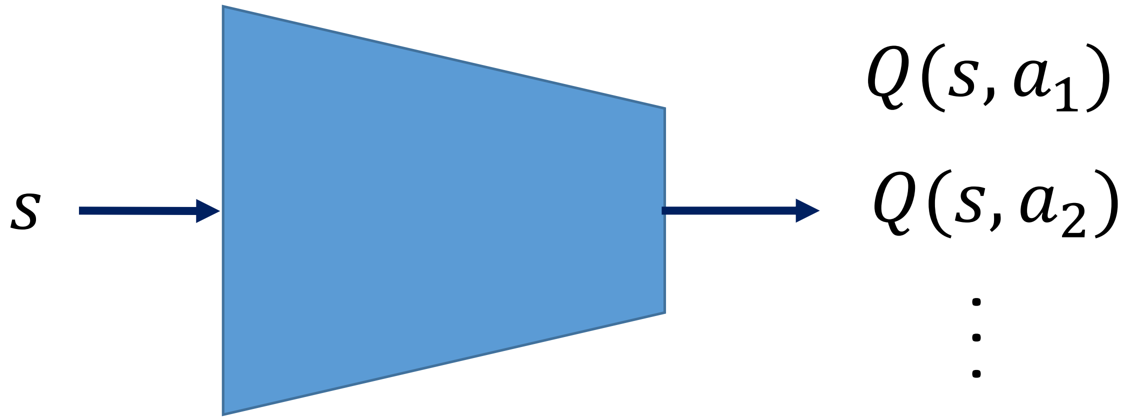
圖片來源: https://images.app.goo.gl/nPJftvi3CChYmyFDA

Policy optimization vs value optimization (DP)



圖片來源: https://nervanasystems.github.io/coach/selecting_an_algorithm.html

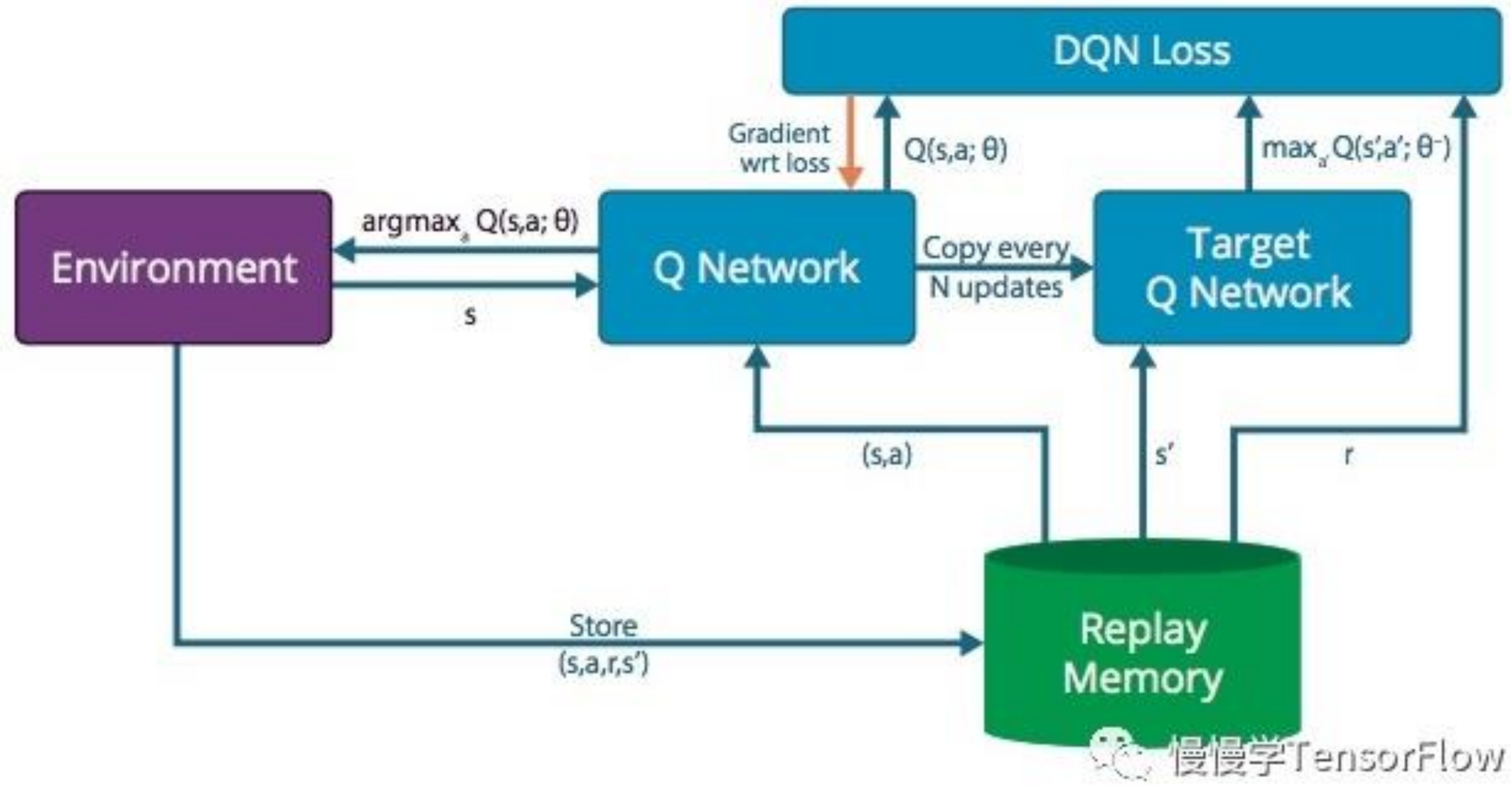
Deep Q-Network (DQN)



Bellman Equation:

$$Q^*(s, a) = \sum_{s'} P(s'|s, a)\left[R(s, a, s') + \gamma \max_{a'} Q^*(s', a')\right]$$

Deep Q-Network (DQN)



圖片來源: https://zhuanlan.zhihu.com/p/25546213?from_voters_page=true

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., ... & Petersen, S. (2015). Human-level control through deep reinforcement learning. Nature, 518(7540), 529.

Policy gradient

$$\tau = \left(s_1, a_1, r_1, s_2, a_2, r_{2,} \cdots s_{\mathrm{T}}, a_{\mathrm{T}}\right)$$

$$p_\theta(\tau) = p(s_1)p_\theta(a_1|s_1)p(s_2|s_1, a_1)p_\theta(a_2|s_2)p(s_3|s_2, a_2)\cdots$$

$$\mathrm{R}(\tau) = \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{r_t}$$

$$\bar{R}_\theta = \sum R(\tau)\, p_\theta(\tau) = E_{\tau\sim p_\theta(\tau)}[R(\tau)]$$

Max $E[\bar{R}_\theta]$

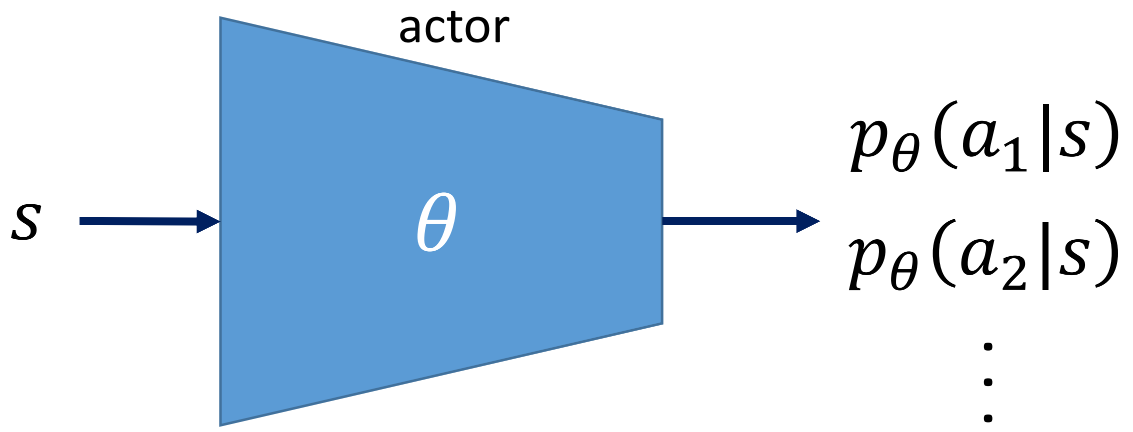
$$\max_\theta E[\bar{R}_\theta]$$

Gradient of the expected value

$$\nabla\bar{R}_\theta = \sum R(\tau)\,\nabla p_\theta(\tau) = E_{\tau\sim p_\theta(\tau)}[R(\tau)\nabla \log p_\theta(\tau)] \approx \frac{1}{N}\sum_{n=1}^{N} R(\tau^n)\nabla \log p_\theta(\tau^n)$$

$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n)\nabla \log p_\theta(a_t^n|s_t^n)$$

Use $\nabla \bar{R}_\theta$ to update policy network



$$\theta^{\pi'} \leftarrow \theta^{\pi} + \eta \nabla \bar{R}_\theta$$

$$\nabla \bar{R}_\theta = \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p_\theta(a_t^n | s_t^n)$$

Tips to improve bias and reduce variance of $\nabla\bar{R}_\theta$

$$\nabla\bar{R}_\theta = \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} R(\tau^n)\nabla\log p_\theta(a_t^n|s_t^n)$$

Add a baseline to calculate the reward

$$\nabla\bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} (R(\tau^n) - b)\nabla\log p_\theta(a_t^n|s_t^n), \qquad b \approx E[R(\tau)]$$

$$\nabla\bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} \left(\sum\nolimits_{t'}^{T_n} r_{t'}^n - b\right)\nabla\log p_\theta(a_t^n|s_t^n)$$

Assign suitable time delayed credit

$$\nabla\bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} \left(\sum\nolimits_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b\right)\nabla\log p_\theta(a_t^n|s_t^n), \gamma < 1$$

$$A^\theta(s_t, a_t) = \left(\sum\nolimits_{t'}^{T_n} \gamma^{t'-t} r_{t'}^n - b\right)$$

Off-policy to improve efficiency of calculating $\nabla\bar{R}_\theta$

On-policy

$$\nabla\bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n} A^\theta(s_t,a_t)\nabla\log p_\theta(a_t^n|s_t^n), \gamma < 1 \qquad A^\theta(s_t,a_t) = \left(\sum\nolimits_{t'}^{T_n}\gamma^{t'-t}r_{t'}^n - b\right)$$

Importance sampling

$$E_{x\sim p}[f(x)] = E_{x\sim q}\left[f(x)\frac{p(x)}{q(x)}\right]$$

$$Var_{x\sim q}\left[f(x)\frac{p(x)}{q(x)}\right] = E_{x\sim q}\left[\left(f(x)\frac{p(x)}{q(x)}\right)^2\right] - \left(E_{x\sim q}\left[f(x)\frac{p(x)}{q(x)}\right]\right)^2$$

$$= E_{x\sim p}\left[f(x)^2\frac{p(x)}{q(x)}\right] - \left(E_{x\sim p}[f(x)]\right)^2$$

Off-policy

$$\nabla\bar{R}_\theta = E_{(s_t,a_t)\sim\pi_{\theta'}}\left[\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}A^{\theta'}(s_t,a_t)\nabla\log p_\theta(a_t^n|s_t^n)\right]$$

From $\nabla \bar{R}_\theta$ to loss function

Off-policy

$$\nabla \bar{R}_\theta = E_{(s_t,a_t)\sim\pi_{\theta'}}\left[\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}A^{\theta'}(s_t,a_t)\nabla \log p_\theta(a_t^n|s_t^n)\right]$$

Sampling efficiency

Loss function

$$J^{\theta'}(\theta) = E_{(s_t,a_t)\sim\pi_{\theta'}}\left[\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}A^{\theta'}(s_t,a_t)\right]$$

Proximal policy optimization (PPO)

$$J_{PPO}^{\theta'}(\theta) = J^{\theta'}(\theta) - \beta KL(\theta,\theta')$$

$$J_{PPO2}^{\theta'}(\theta) = \sum_{(s_t,a_t)} min\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}A^{\theta'}(s_t,a_t), clip\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon\right)A^{\theta'}(s_t,a_t)\right)$$

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347.

Actor-critic strategy to calculate $\nabla \bar{R}_\theta$

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}\left(\sum_{t'}^{T_n}\gamma^{t'-t}r_{t'}^n - b\right)\nabla \log p_\theta(a_t^n|s_t^n)$$

$$G_t^n = \sum_{t'}^{T_n}\gamma^{t'-t}r_{t'}^n$$

unstable when sampling amount is not large enough

$\boxed{V^{\pi_\theta}(s_t^n)}$ Expected value of b

Use expected value to reduce sampling variance

$$\nabla \bar{R}_\theta \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{T_n}\left(\boxed{\sum_{t'}^{T_n}\gamma^{t'-t}r_{t'}^n} - \boxed{b}\right)\nabla \log p_\theta(a_t^n|s_t^n)$$

$\boxed{E[G_t^n] = Q^{\pi_\theta}(s_t^n, a_t^n)}$ Expected value of G_t^n

Use one neural network that estimates V

$$Q^{\pi_\theta}(s_t^n, a_t^n) = \mathrm{E}[r_t^n + V^{\pi_\theta}(s_{t+1}^n)] = r_t^n + V^{\pi_\theta}(s_{t+1}^n)$$

$$Q^{\pi_\theta}(s_t^n, a_t^n) - V^{\pi_\theta}(s_t^n) = r_t^n + V^{\pi_\theta}(s_{t+1}^n) - V^{\pi_\theta}(s_t^n)$$

$$\boxed{A^\theta(s_t, a_t) = \left(r_t^n + V^{\pi_\theta}(s_{t+1}^n) - V^{\pi_\theta}(s_t^n)\right)}$$

Use temporal difference to calculate V

$$A^{\theta}(s_t, a_t) = \left(r_t^n + V^{\pi_\theta}(s_{t+1}^n) - V^{\pi_\theta}(s_t^n)\right)$$

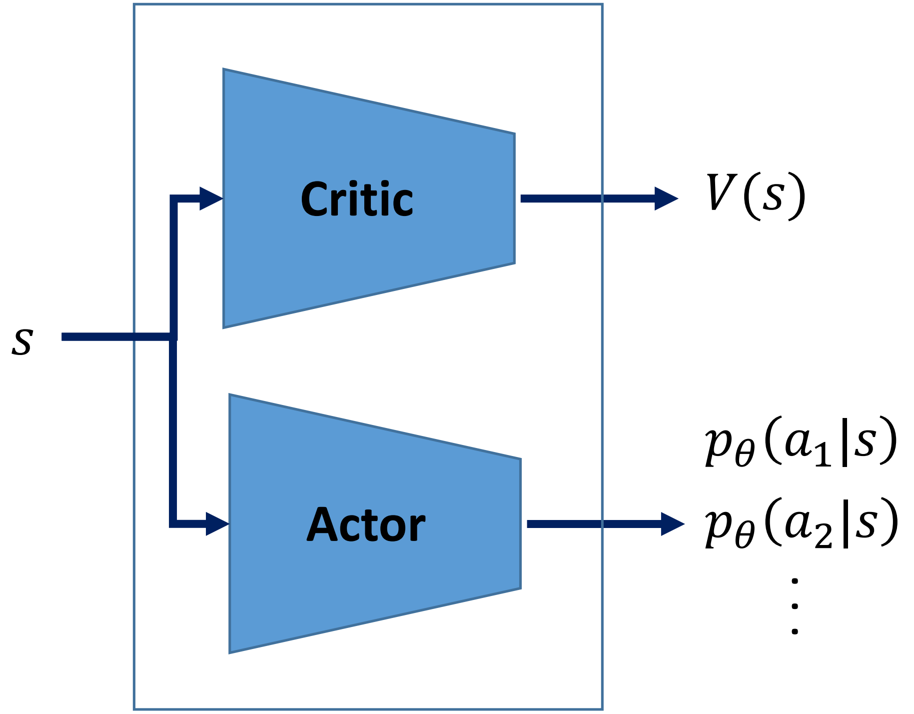
Monte-Carlo approach $V^{\pi_\theta}(\, s_a) \leftrightarrow G_a$ Until the end of the episode, the cumulated reward is G_a

Temporal-difference approach

$$V^{\pi_\theta}(\, s_t) + r_t = V^{\pi_\theta}(s_{t+1})$$

$$V^{\pi_\theta}(\, s_t) - V^{\pi_\theta}(s_{t+1}) \ \leftrightarrow r_t$$

Train the network



TD Error

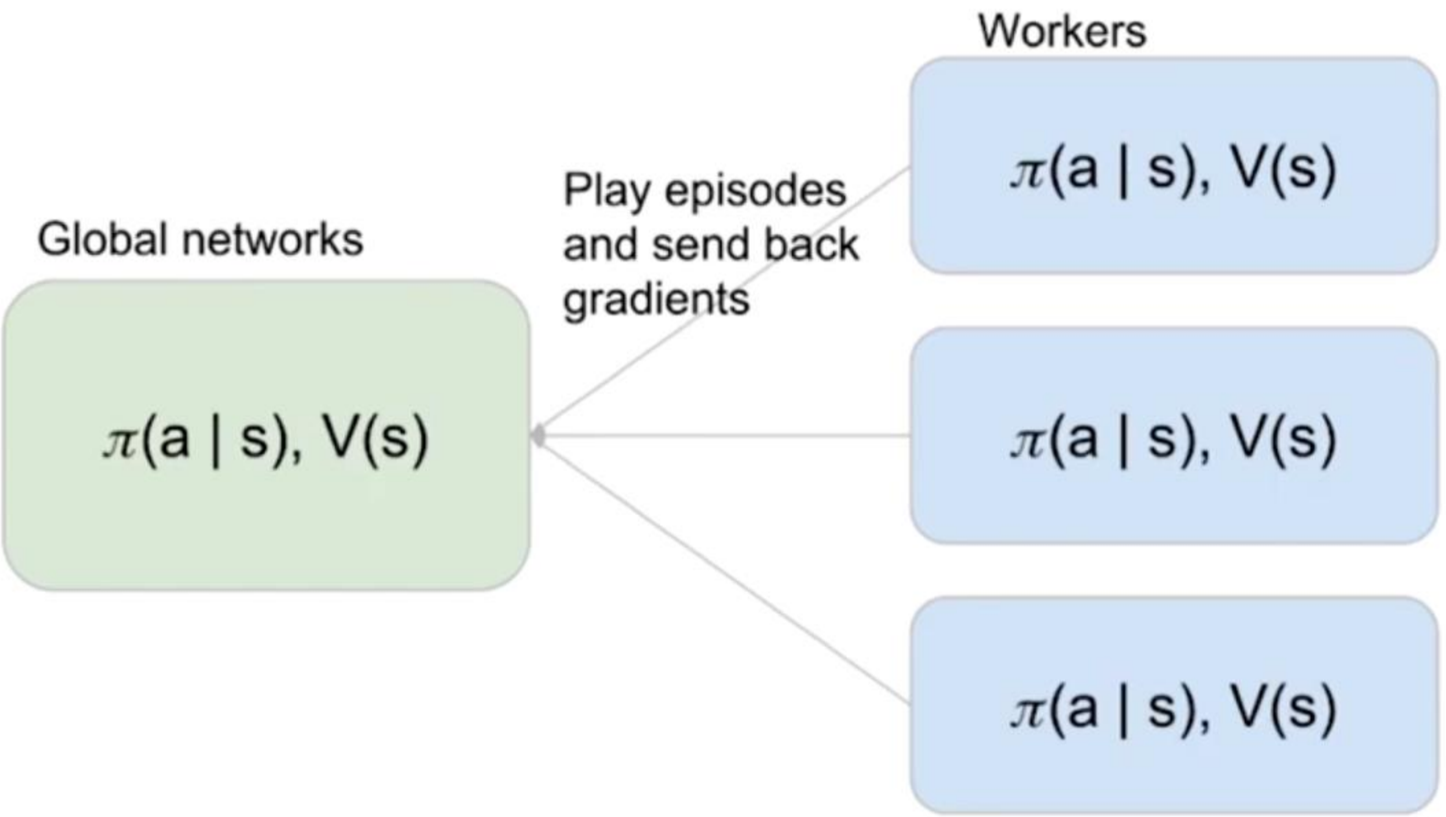
$$L = L_\pi + c_v L_v + c_{reg} L_{reg}$$

$$A^\theta(s_t, a_t) = G_t^n - V^{\pi_\theta}(s_t^n) = Q^{\pi_\theta}(s_t^n, a_t^n) - V^{\pi_\theta}(s_t^n) = r_t^n + \gamma V^{\pi_\theta}(s_{t+1}^n) - V^{\pi_\theta}(s_t^n)$$

$$L_v = (G_t^n - V^{\pi_\theta}(s_t^n))^2 = \left(r_t^n + \gamma V^{\pi_\theta}(s_{t+1}^n) - V^{\pi_\theta}(s_t^n)\right)^2$$

$$L_\pi = \sum_{(s_t, a_t)} min\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t, a_t), clip\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon\right) A^{\theta'}(s_t, a_t)\right)$$

A3C



Reference: https://youtu.be/iCV3vOl8IMk

Mnih, V., Badia, A. P., Mirza, M., Graves, A., Lillicrap, T., Harley, T., ... & Kavukcuoglu, K. (2016, June). Asynchronous methods for deep reinforcement learning. In International conference on machine learning (pp. 1928-1937).

A3C

Stability

- Each episode will progress randomly
- Each action is sampled probabilistically
- Occasionally, performance of agent can drop off due to bad update
  - Well, this can still happen with A3C so don't think you are immune

Reference: https://youtu.be/iCV3vOl8IMk

A3C

- DQN is also interested in stabilizing learning
- Techniques:
  - Freezing target network
  - Experience replay buffer
- Use experience replay to look at multiple examples per training step
- A3C simply achieves stability using a different method (parallel agents)
- Both solve the problem: how to make neural networks work as function approximators in classic RL algorithms?

Reference: https://youtu.be/iCV3vOl8IMk

A3C

- Remember: the theory part is not new, just need to create multiple parallel agents and asynchronously update/copy parameters
- 3 files:
  - main.py (master file; global policy and value networks)
    - Create and coordinate workers
  - worker.py (contains local policy and value networks)
    - Copy weights from global nets
    - Play episodes
    - Send gradients back to master
  - nets.py
    - Definition of policy and value networks

Reference: https://youtu.be/iCV3vOl8IMk

main.py

Instantiate global policy and value networks

Check # CPUs available, create threads and workers

Initialize global thread-safe counter, so every worker knows when to quit (when # of total steps reaches a max.)

Reference: https://youtu.be/iCV3vOl8IMk

A3C

worker.py

```
def run():
  in a loop:
    copy params from global nets to local nets
    run N steps of game (and store the data - s, a, r, s')
    using gradients wrt local net, update the global net
```

Conceptually, it's like:

1) $$g_{local} = \frac{\partial L(\theta_{local})}{\partial \theta_{local}}$$

2) $$\theta_{global} = \theta_{global} - \eta g_{local}$$

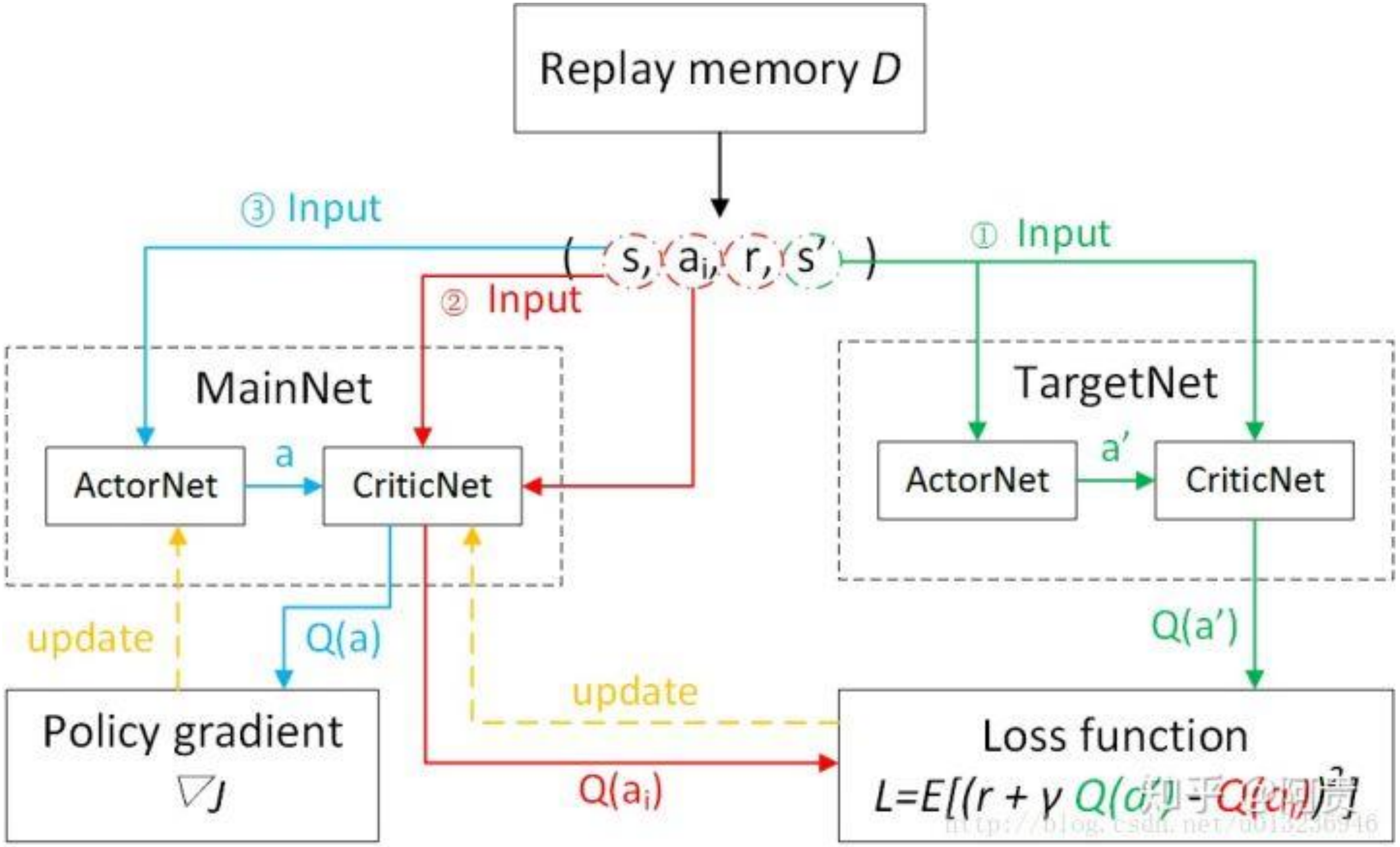
But in reality, we'll use RMSprop

Reference: https://youtu.be/iCV3vOl8IMk

A3C

Reference: https://youtu.be/O5BlozCJBSE

Deep deterministic policy gradient (DDPG)



圖片來源: https://zhuanlan.zhihu.com/p/47873624

Silver, D., Schrittwieser, J., Simonyan, K., Antonoglou, I., Huang, A., Guez, A., ... & Chen, Y. (2017). Mastering the game of go without human knowledge. Nature, 550(7676), 354.