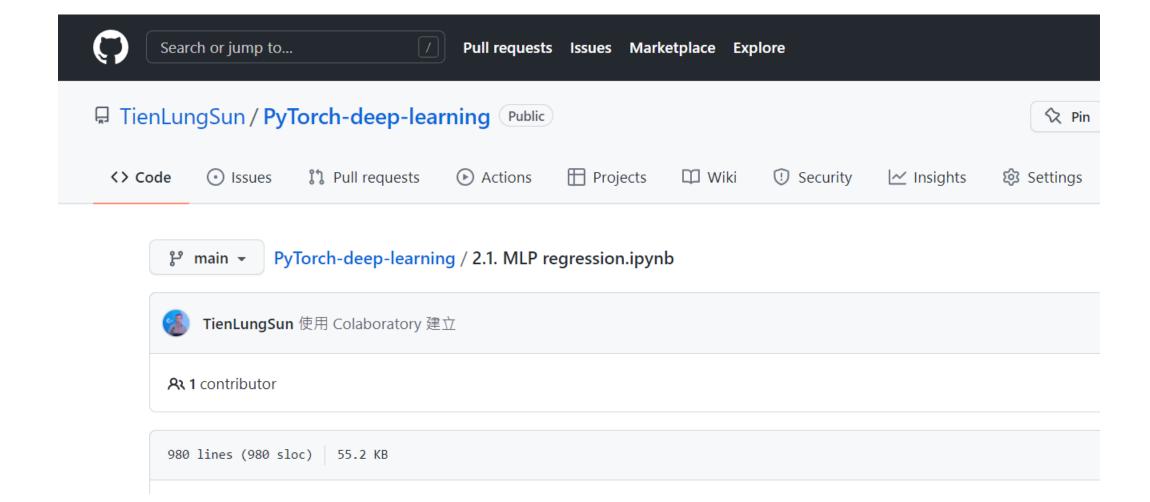
Regression

2.1 MLP regression.ipynb



How machine learns from data?

• Regression
$$y = f(x)$$
• Classification

- Define a function to be learned: y = f(x)
- Define a loss function \mathcal{L} to describe the error between $\hat{y} = f(x)$ and y
- Find the optimal parameters of f that minimize \mathcal{L}

Statistics vs Machine Learning

Statistics versus machine learning

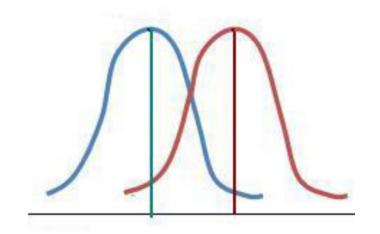
Statistics draws population inferences from a sample, and machine learning finds generalizable predictive patterns.

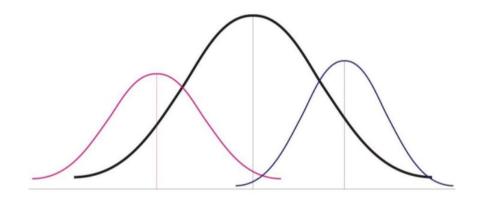
$$y = f(x)$$

Ij, H. (2018). Statistics versus machine learning. Nat Methods, 15(4), 233.

Statistics draw population inferences

動作	range (delta D)	
A1	8.20	
A1	6.42	
A1	8.26	
A1	14.55	
A1	11.70	
A1	11.75	
A1	12.72	
A1	11.96	
A1	6.03	
A1	9.20	
A1	14.25	
A1	13.93	
A2	14.17	
A2	19.91	
A2	9.40	
A2	8.48	
A2	10.61	
A2	13.86	
A2	7.15	
A2	7.66	
A2	6.09	
A2	13.18	
A2	9.15	
A2	13.29	





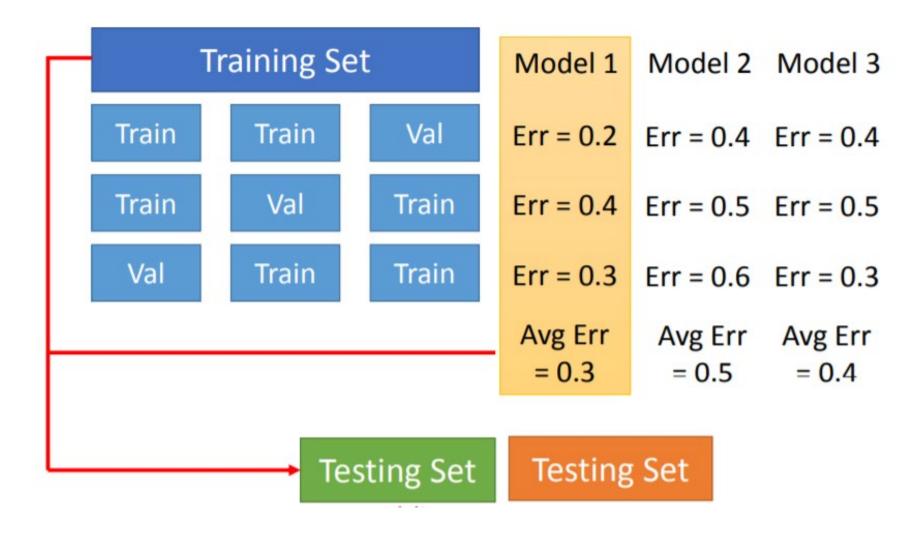
Evaluation of generalizable prediction performance with test data

Split input data to train and test data

```
from sklearn.model_selection import train_test_split
    trainX, testX, trainY, testY = train_test_split(numpyX, numpyY, test_size=0.20, random_state=0)
    print(trainX.shape, testX.shape, trainY.shape, testY.shape)

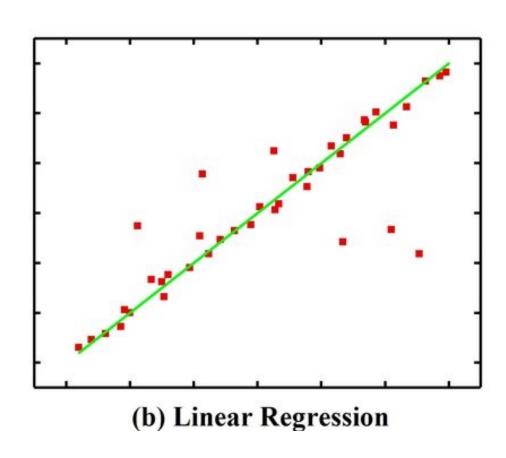
(1600, 7) (400, 7) (1600, 1) (400, 1)
```

Cross validation



Reference: 李弘毅 ML Lecture 2 https://youtu.be/D_S6y0Jm6dQ

Statistics vs Machine Learning



Statistics – R² of the variance explained ML – MSE, MAE of test data which are unseen by the model

Input data

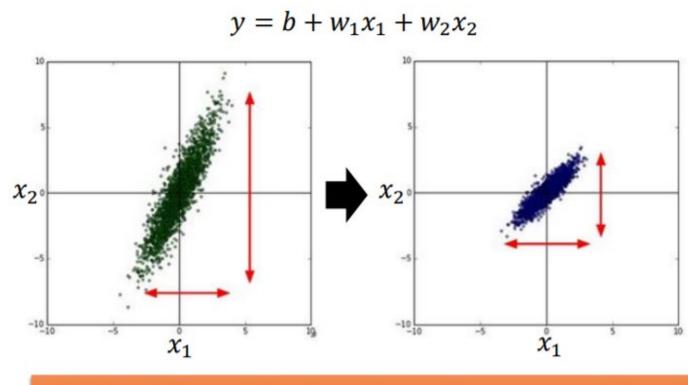
у	x1	x2	•••
0.578			
0.64			
-0.96			
0.23			

$$y = 0.323x_1^2 + 0.586x_1x_2 + 0.4x_3 + 0.8972x_5^3 + 0.267x_3^2x_5x_6 + 0.78x_7^2$$
$$x_1 \sim \mathcal{N}(0.5, 0.3)$$

$$x_2 \sim \mathcal{N}(0.8, 0.5)$$

• • •

Feature scaling

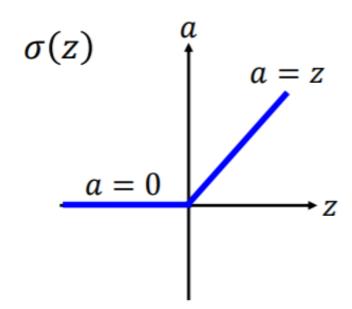


Make different features have the same scaling

Reference: 李弘毅 ML Lecture 3-1 https://youtu.be/yKKNr-QKz2Q

ReLU activation function

Rectified Linear Unit (ReLU)

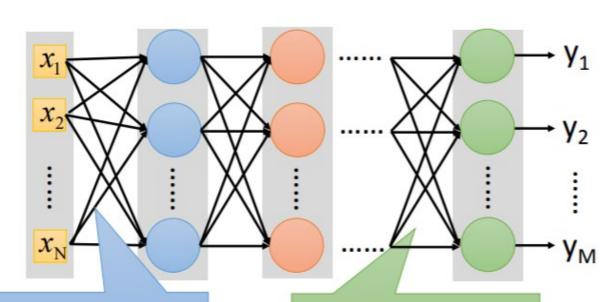


[Xavier Glorot, AISTATS'11] [Andrew L. Maas, ICML'13] [Kaiming He, arXiv'15]

Reason:

- 1. Fast to compute
- 2. Biological reason
- 3. Infinite sigmoid with different biases
- 4. Vanishing gradient problem

Vanishing gradient problem



 w_{13} w_{36} w_{14} w_{46} w_{47} w_{7y} w_{7y} w_{15} w_{23} w_{24} w_{56} w_{57} w_{57}

Smaller gradients

Learn very slow

Almost random

Larger gradients

Learn very fast

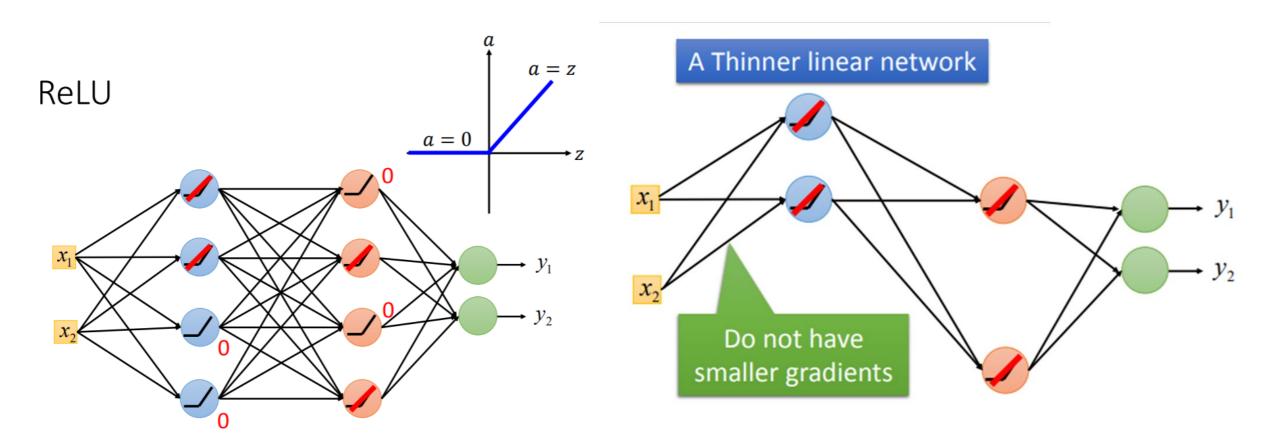
Already converge

 $\frac{\partial L}{\partial \mathbf{w}_{6y}} = \frac{\partial L}{\partial \mathbf{y}_1} \frac{\partial \mathbf{y}_1}{\partial \mathbf{w}_{6y}}$

$$\frac{\partial L}{\partial w_{57}} = \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial n_7} \frac{\partial n_7}{\partial w_{57}}$$

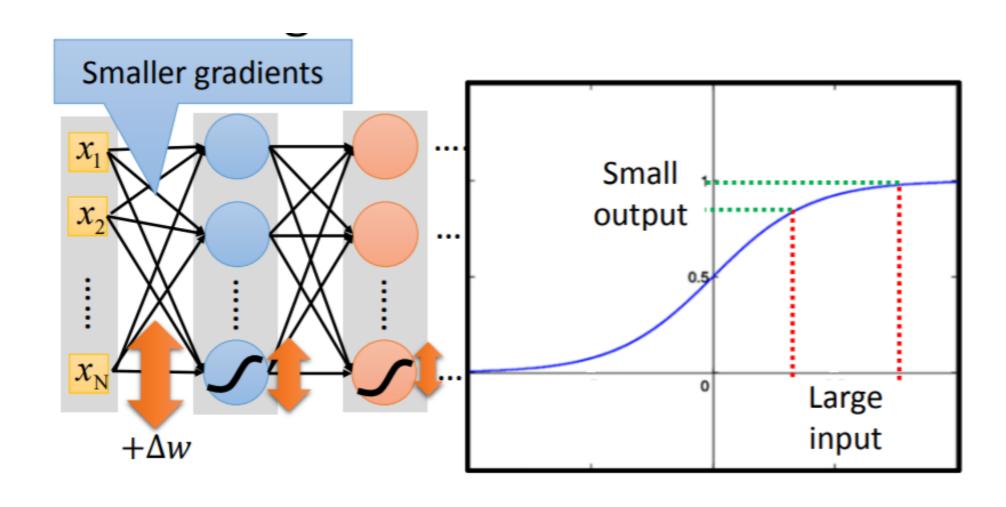
based on random!?

ReLU results in a thinner linear network

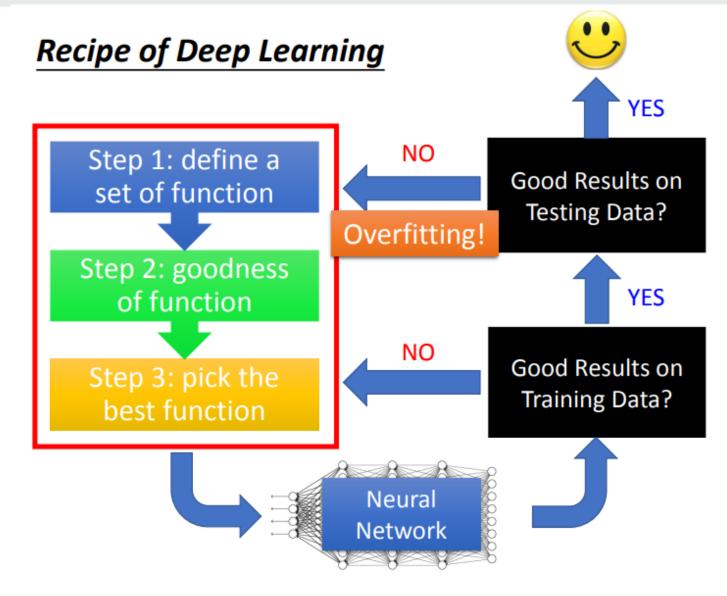


Reference: 李弘毅 ML Lecture 9-1 https://youtu.be/xki61j7z-30

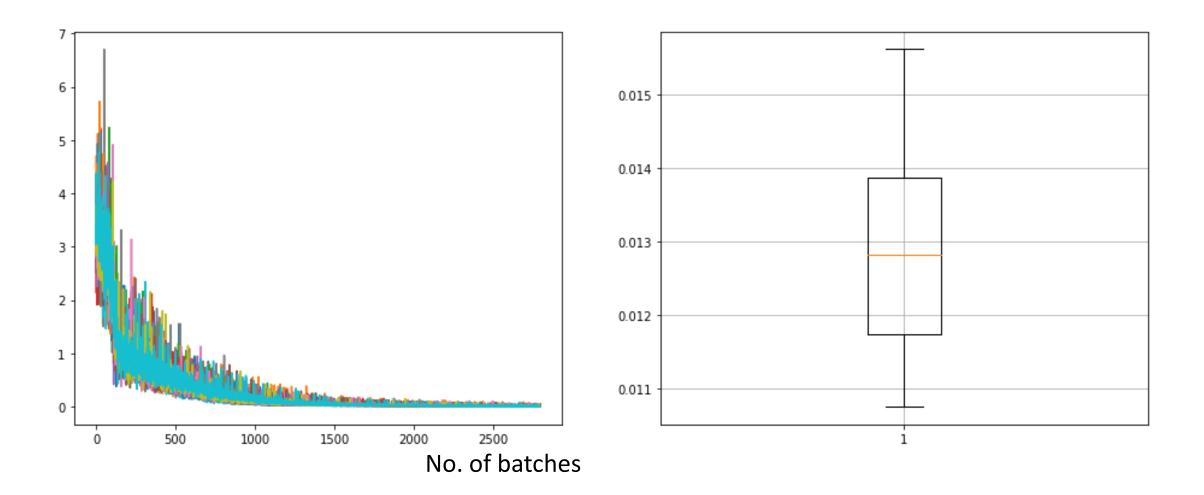
Sigmoid is hard to get the power of deep



ML finds generalizable predictive patterns



ML finds generalizable predictive patterns



Good results on training data?

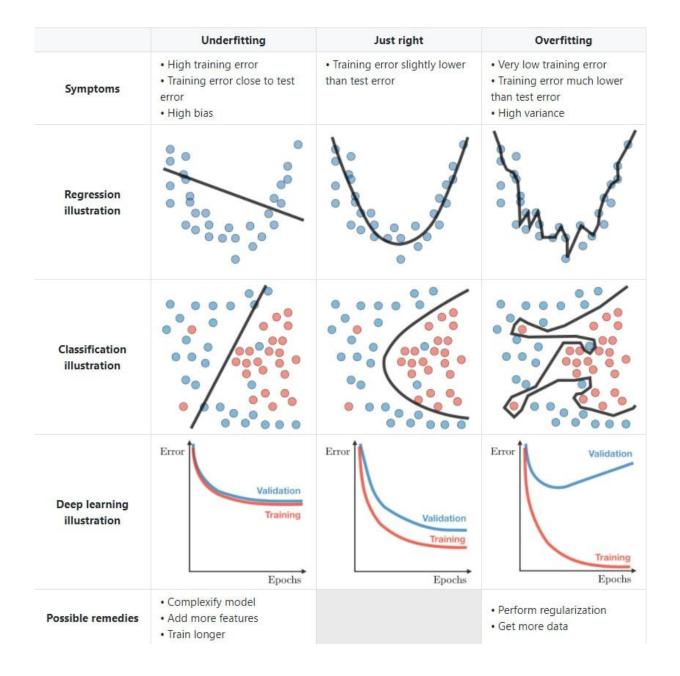
Good results on test data?

- 7-56-56-1
- 7-256-256-256-1
- 7-512-512-512-1
- 7-1024-1024-1024-1024-1

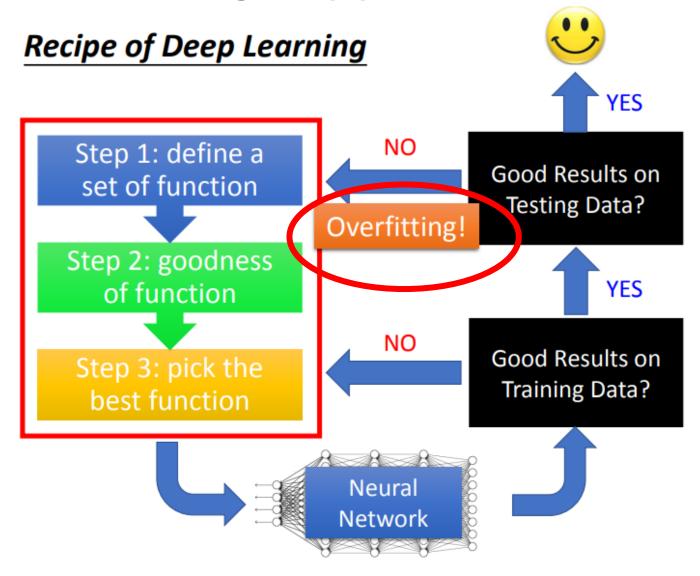


- Good results on training data?
- Good results on test data?

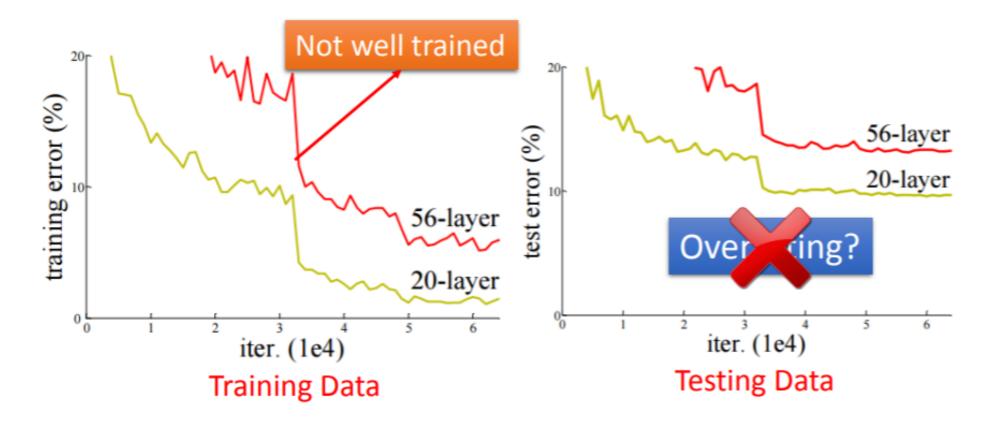
Overfitting



How to know overfitting happens?



But be careful, do not always blame overfitting



Deep Residual Learning for Image Recognition http://arxiv.org/abs/1512.03385

Overfitting

2.2. Overfitting.ipynb

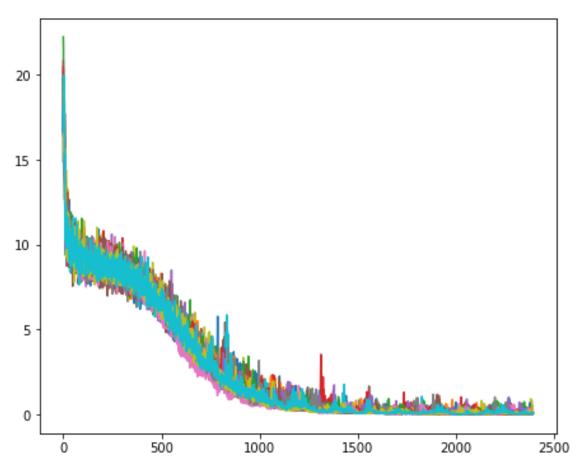
Add noises to y

$$y = 0.323x_1^2 + 0.586x_1x_2 + 0.4x_3 + 0.8972x_5^3 + 0.267x_3^2x_5x_6 + 0.78x_7^2 + \mathcal{N}(2,3)$$

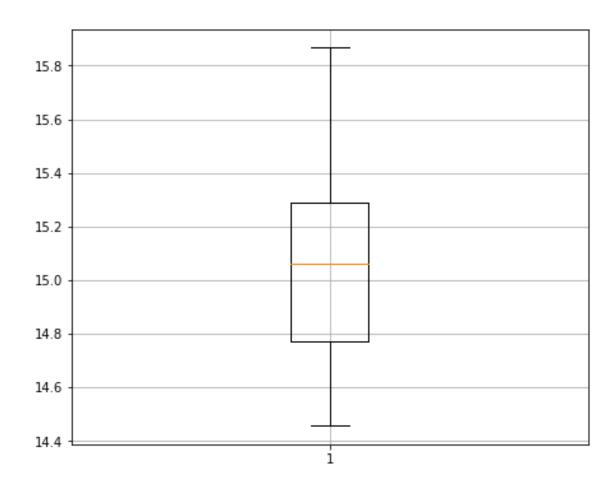
Use a complicated NN

7-1024-1024-1024-1024-1

Overfitting

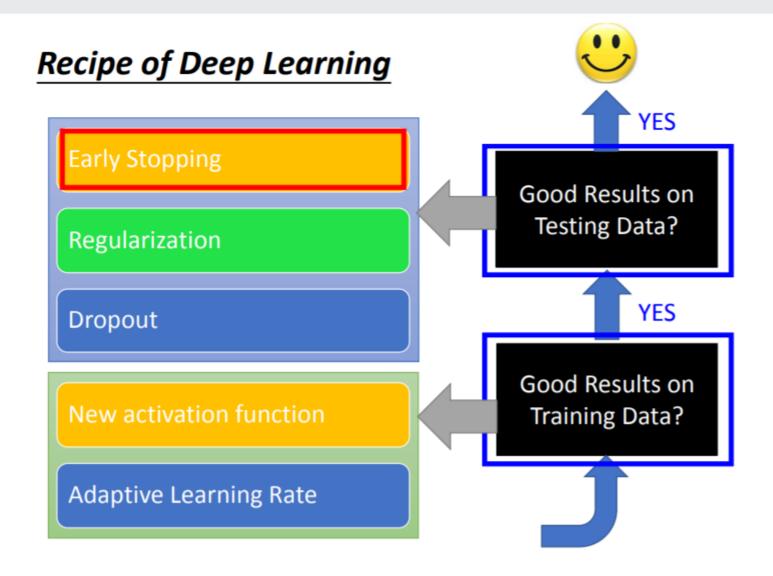


Good results on training data

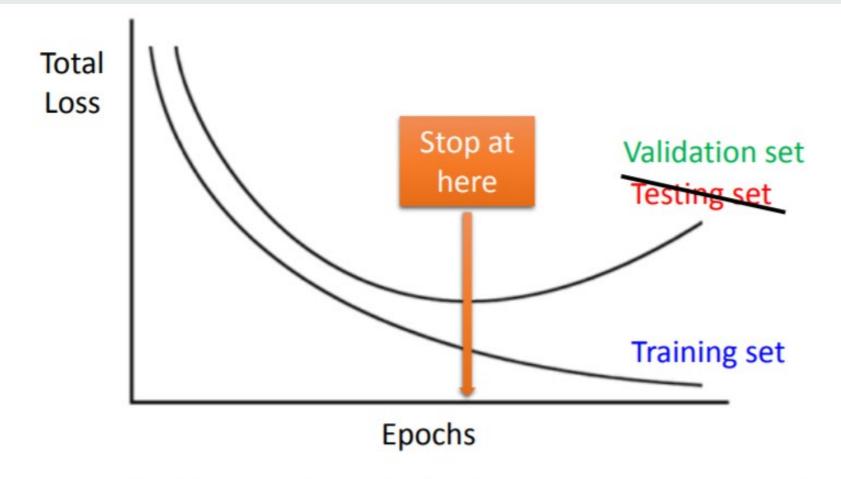


Bad results on test data

How to solve overfitting?



(1) Early stopping

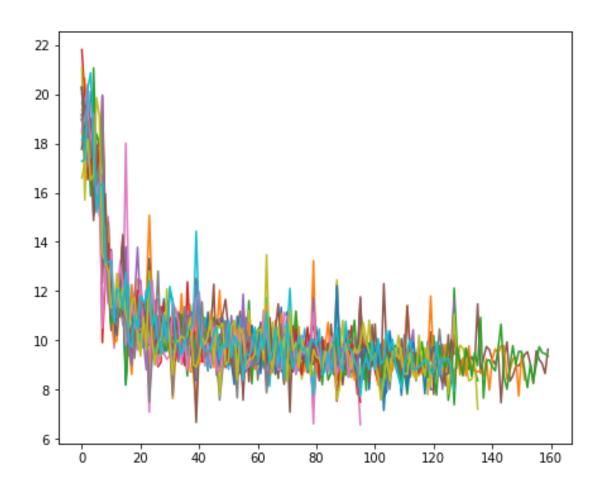


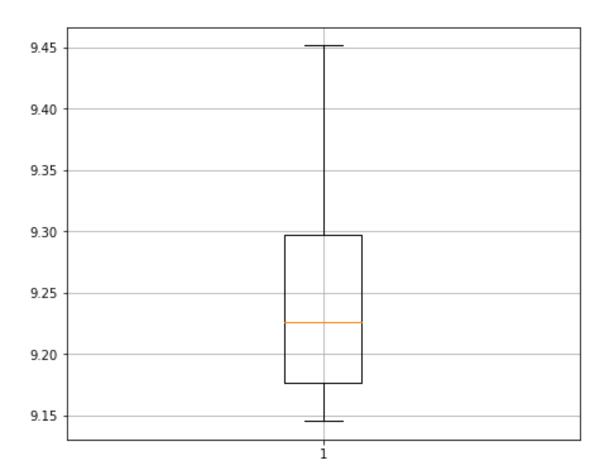
Keras: http://keras.io/getting-started/faq/#how-can-i-interrupt-training-when-the-validation-loss-isnt-decreasing-anymore

(1) Early stopping

```
tensor_validationX = torch.FloatTensor(validationX).to(device)
 tensor_validationY = torch.FloatTensor(validationY).to(device)
 tensor_validationY_hat = MyNet(tensor_validationX)
 validatiion_loss = loss_func(tensor_validationY, tensor_validationY_hat)
 #early stop
 the_current_loss = float(validatiion_loss)
 if(the current loss > the last loss):
   trigger times += 1
   print('trigger times:', trigger_times)
   if(trigger_times >= patience):
      print('Early stopping!')
      break # early stop when validation loss increases 2 consective times
 elif(trigger_times >0):
   print('trigger times reset to 0')
   trigger times = 0
 the last loss = the current loss
train_lossLst.append(epoch_lossLst)
```

(1) Early stopping





Use your own
$$y = f(\vec{x})$$
 data

- 7-512-512-512-1
- 7-256-256-256-1
- 7-56-56-1



Is the overfitting problem solved by early stop?

 Find a set of weight not only minimizing original cost but also close to zero

$$L'(\theta) = L(\theta) + \lambda \frac{1}{2} \|\theta\|_{2} \longrightarrow \text{Regularization term}$$

$$\theta = \{w_{1}, w_{2}, \ldots\}$$

Original loss

$$L(\theta) = \sum_{n=1}^{N} (\hat{y}^n - y^n)^2$$

L2 regularization:

$$\|\theta\|_{2} = (w_{1})^{2} + (w_{2})^{2} + \dots$$

(usually not consider biases)

$$L'(\theta) = L(\theta) + \lambda \frac{1}{2} \|\theta\|_{2} \quad \text{Gradient:} \quad \frac{\partial L'}{\partial w} = \frac{\partial L}{\partial w} + \lambda w$$

$$\text{Update:} \quad w^{t+1} \to w^{t} - \eta \frac{\partial L'}{\partial w} = w^{t} - \eta \left(\frac{\partial L}{\partial w} + \lambda w^{t} \right)$$

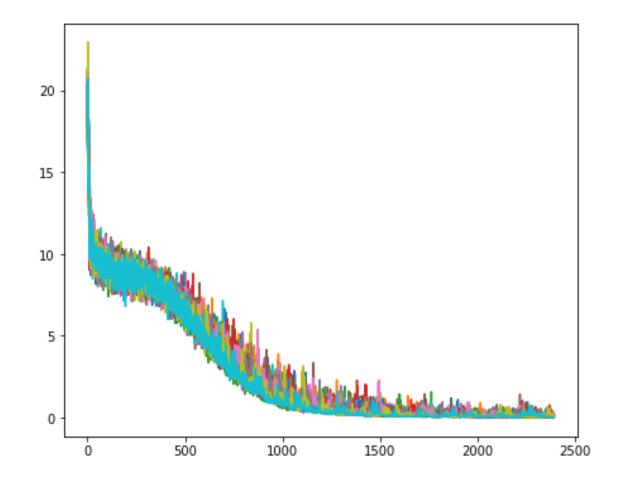
$$= \underbrace{(1 - \eta \lambda) w^{t}}_{\text{Closer to zero}} - \eta \frac{\partial L}{\partial w} \quad \text{Weight Decay}$$

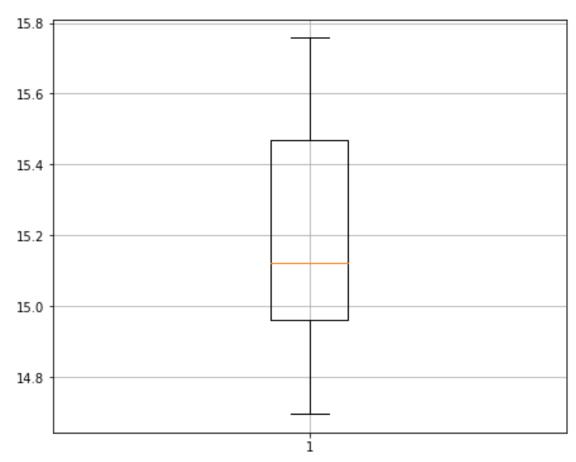
Vanilla gradient decent update $w^{t+1} o w^t - \eta \, \frac{\partial L}{\partial w}$

Regularization may not be that important in deep learning as in other machine learning algorithms like SVM.

```
# initialize NN weights
for name, param in MyNet.named_parameters():
    if(param.requires_grad):
        torch.nn.init.normal_(param, mean=0.0, std=0.02)
loss_func = torch.nn.MSELoss()
optimizer = torch.optim.Adam(MyNet.parameters(), lr=0.0003, weight_decay=0.0001)
```

Regularization may not be that important in deep learning as in other machine learning algorithms like SVM.





L1 regularization:

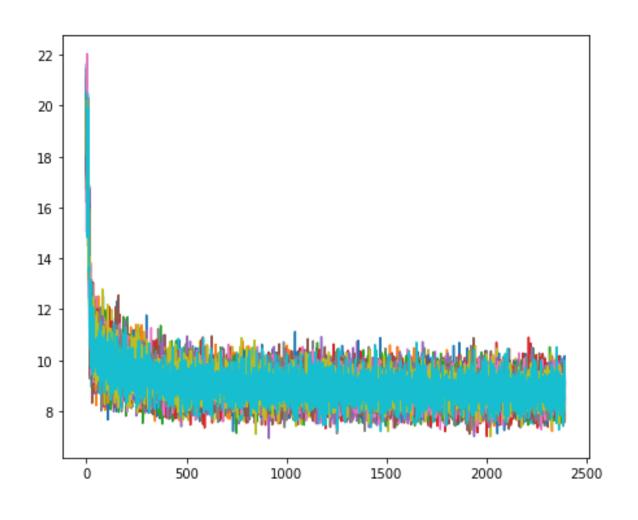
$$\|\theta\|_1 = |w_1| + |w_2| + \dots$$

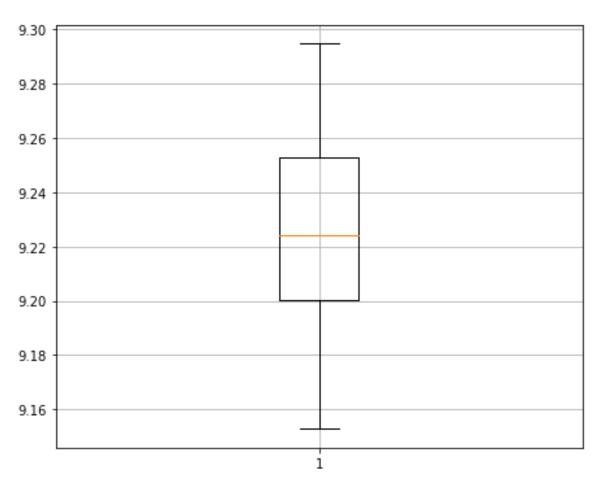
$$L'(\theta) = L(\theta) + \lambda \frac{1}{2} \|\theta\|_{1} \qquad \frac{\partial L'}{\partial w} = \frac{\partial L}{\partial w} + \lambda \operatorname{sgn}(w)$$
Update:
$$w^{t+1} \to w^{t} - \eta \frac{\partial L'}{\partial w} = w^{t} - \eta \left(\frac{\partial L}{\partial w} + \lambda \operatorname{sgn}(w^{t}) \right)$$

$$= w^{t} - \eta \frac{\partial L}{\partial w} - \underline{\eta} \lambda \operatorname{sgn}(w^{t}) \text{ Always delete}$$

$$= (1 - \eta \lambda) w^{t} - \eta \frac{\partial L}{\partial w} \quad \dots \quad \text{L2}$$

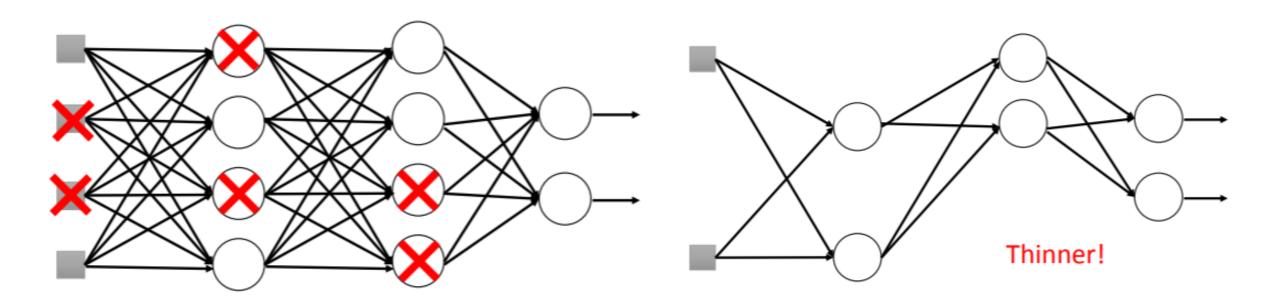
```
for (batchX, batchY) in loader:
  batchY_hat = MyNet(batchX)
  loss = loss_func(batchY_hat, batchY)
  epoch_lossLst.append(float(loss))
 # add L1 regularization
  regularization_loss = 0
  for name, param in MyNet.named_parameters():
    if('weight' in name):
      regularization_loss += torch.sum(abs(param))
  loss = loss + lamda * regularization_loss
  optimizer.zero_grad()
  loss.backward()
  optimizer.step()
```





(3) Drop out

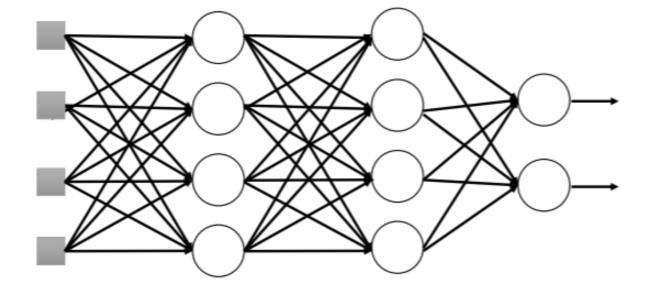
Each time before updating θ , each neuron has p% to dropout. So the *NN* structure is changed (become thinner). That is, for each mini-batch, we resample the dropout neurons.



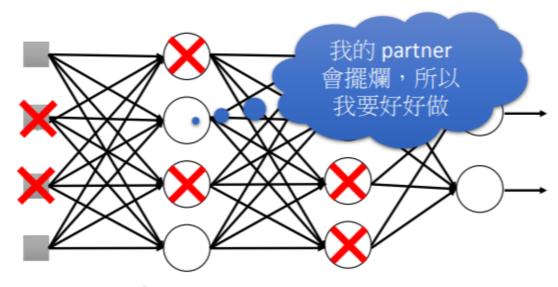
(3) Drop out

No neuron drop out at test stage. All weights time 1 - p%

Testing:



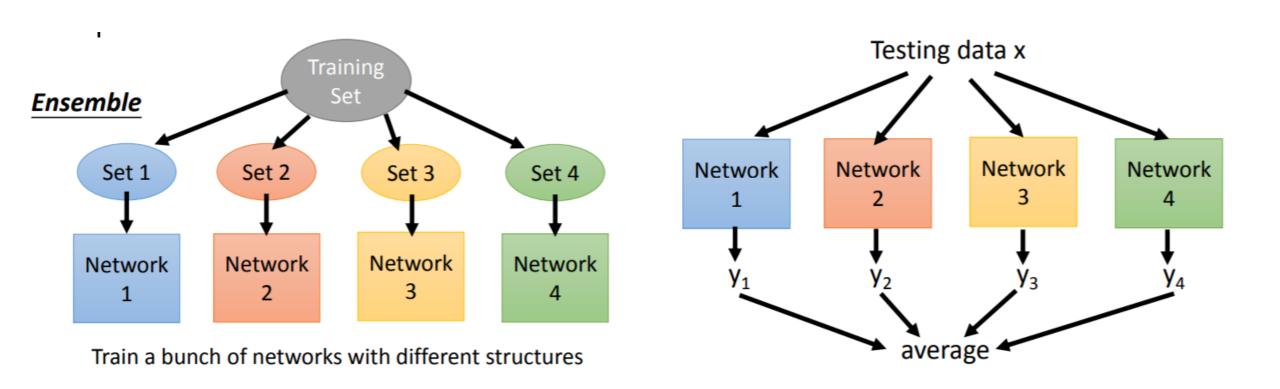
Why drop out makes NN perform better?



- When teams up, if everyone expect the partner will do the work, nothing will be done finally.
- However, if you know your partner will dropout, you will do better.
- When testing, no one dropout actually, so obtaining good results eventually.

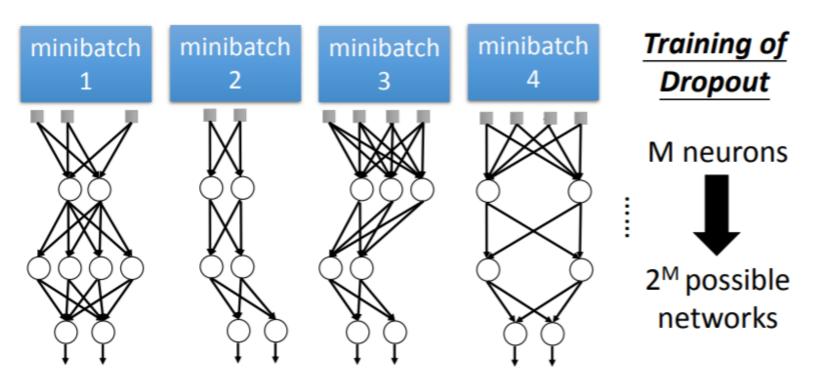
Why drop out makes NN perform better?

Drop out can be seen as an ensemble method.



Why drop out makes NN perform better?

Drop out can be seen as an ensemble method.



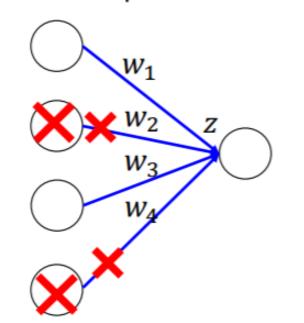
- ➤ Using one mini-batch to train one network
- ➤ Some parameters in the network are shared

Why multiply weights by (1-p)% during testing?

 Why the weights should multiply (1-p)% (dropout rate) when testing?

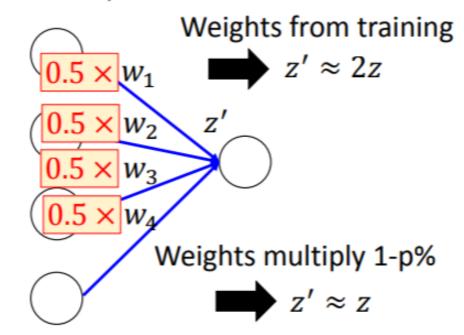
Training of Dropout

Assume dropout rate is 50%

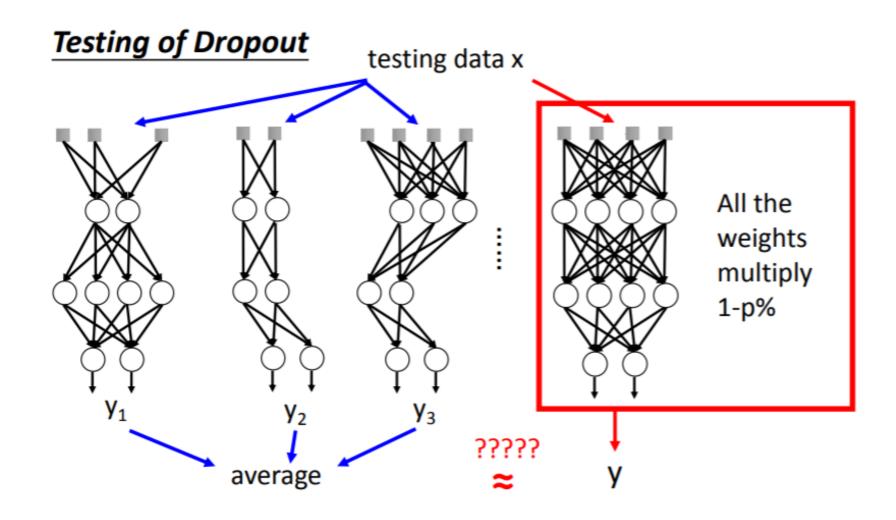


Testing of Dropout

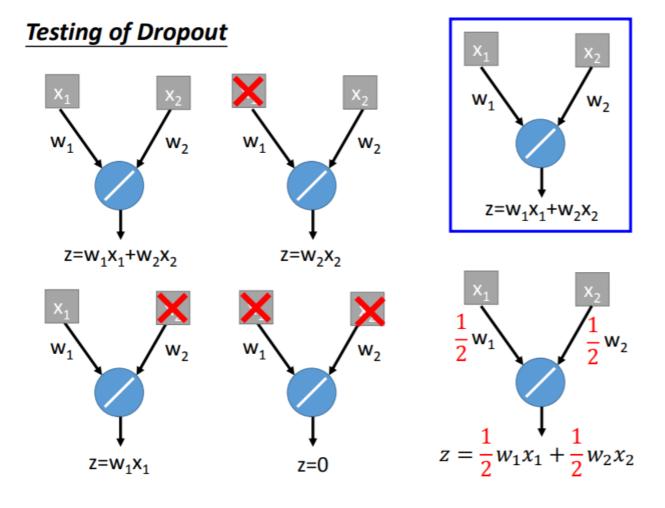
No dropout



Why multiply weights by (1-p)% during testing?



Why multiply weights by (1-p)% during testing?

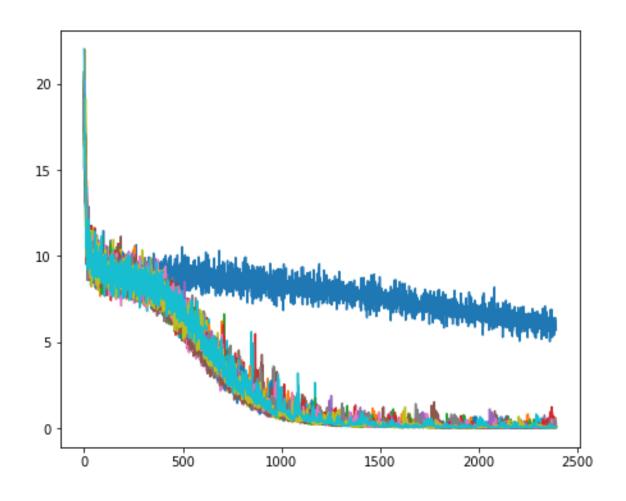


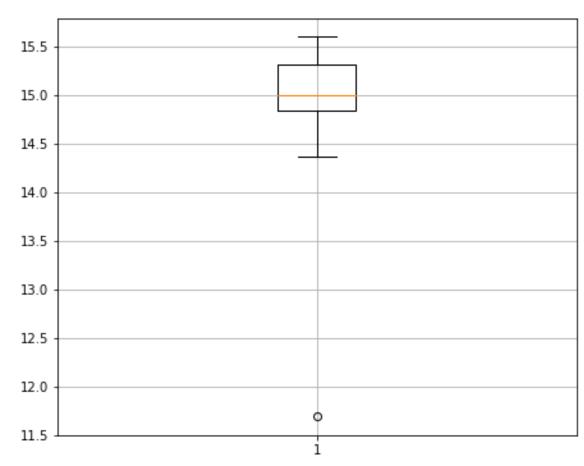
(3) Drop out

```
MyNet = nn.Sequential(
    nn.Linear(7, 1024),
    nn.ReLU(),
    nn.Dropout(p=0.5),
    nn.Linear(1024, 1024),
    nn.ReLU(),
    nn.Dropout(p=0.5),
    nn.Linear(1024, 1024),
    nn.ReLU(),
    nn.Dropout(p=0.5),
    nn.Linear(1024, 1024),
    nn.ReLU(),
    nn.Dropout(p=0.5),
    nn.Linear(1024, 1024),
    nn.ReLU(),
    nn.Dropout(p=0.2),
    nn.Linear(1024, 1),
MyNet.to(device)
print(MyNet)
```

```
#model.eval() will turn model to test mode and PyTorch will
# automatically handle weight scaling of the dropout layer
MyNet.eval()
tensorX = torch.FloatTensor(testX).to(device)
tensorY = torch.FloatTensor(testY).to(device)
tensorY_hat = MyNet(tensorX)
loss = loss_func(tensorY, tensorY_hat)
test_lossLst.append(float(loss))
```

(3) Drop out





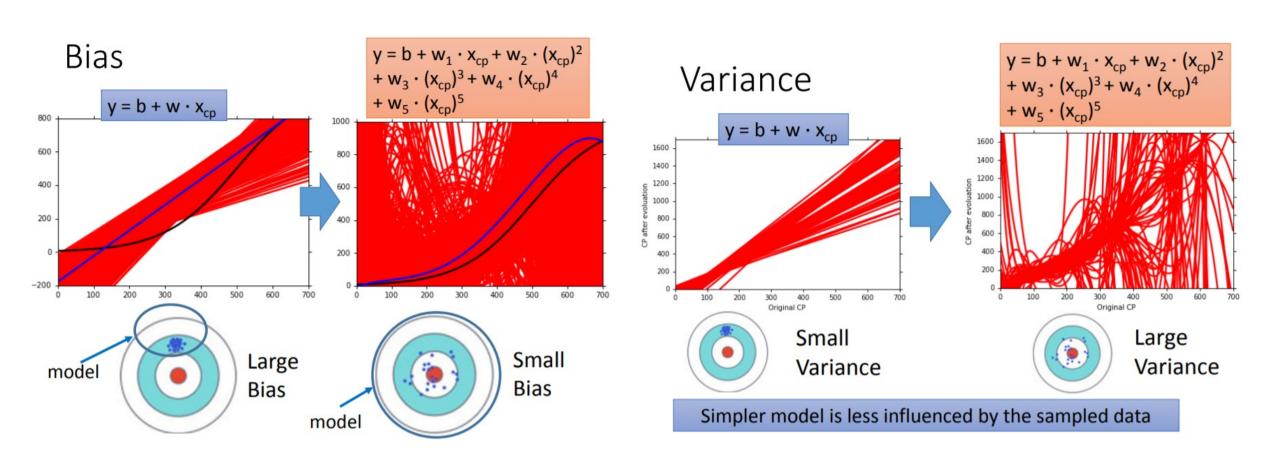
Use your own
$$y = f(\vec{x})$$
 data

- 7-512-512-512-1
- 7-256-256-256-1
- 7-56-56-1



Is the overfitting problem solved by drop out?

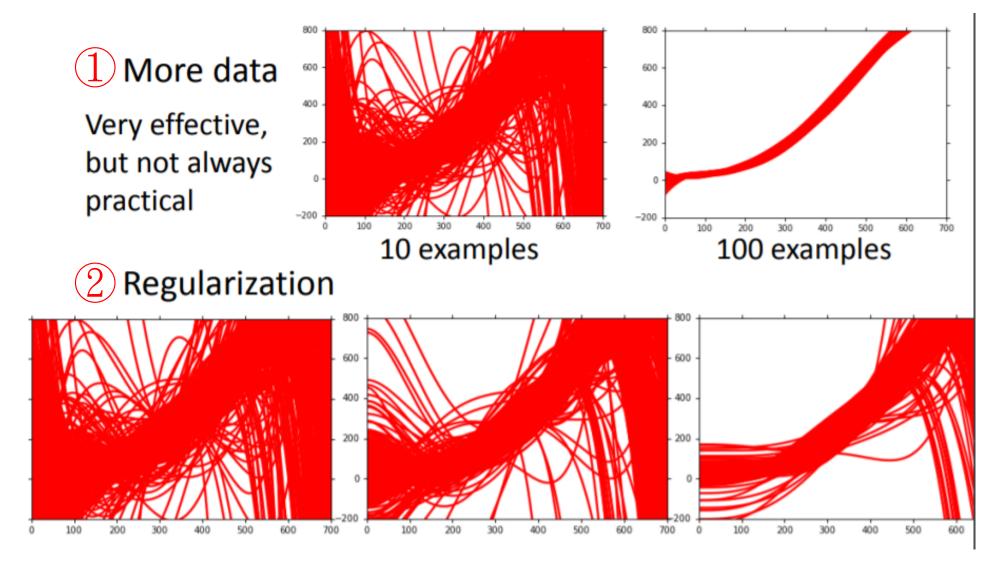
Simpler model is less influenced by the sampled data and has smaller variance



Reference: 李弘毅 ML Lecture 2 https://youtu.be/D_S6y0Jm6dQ



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