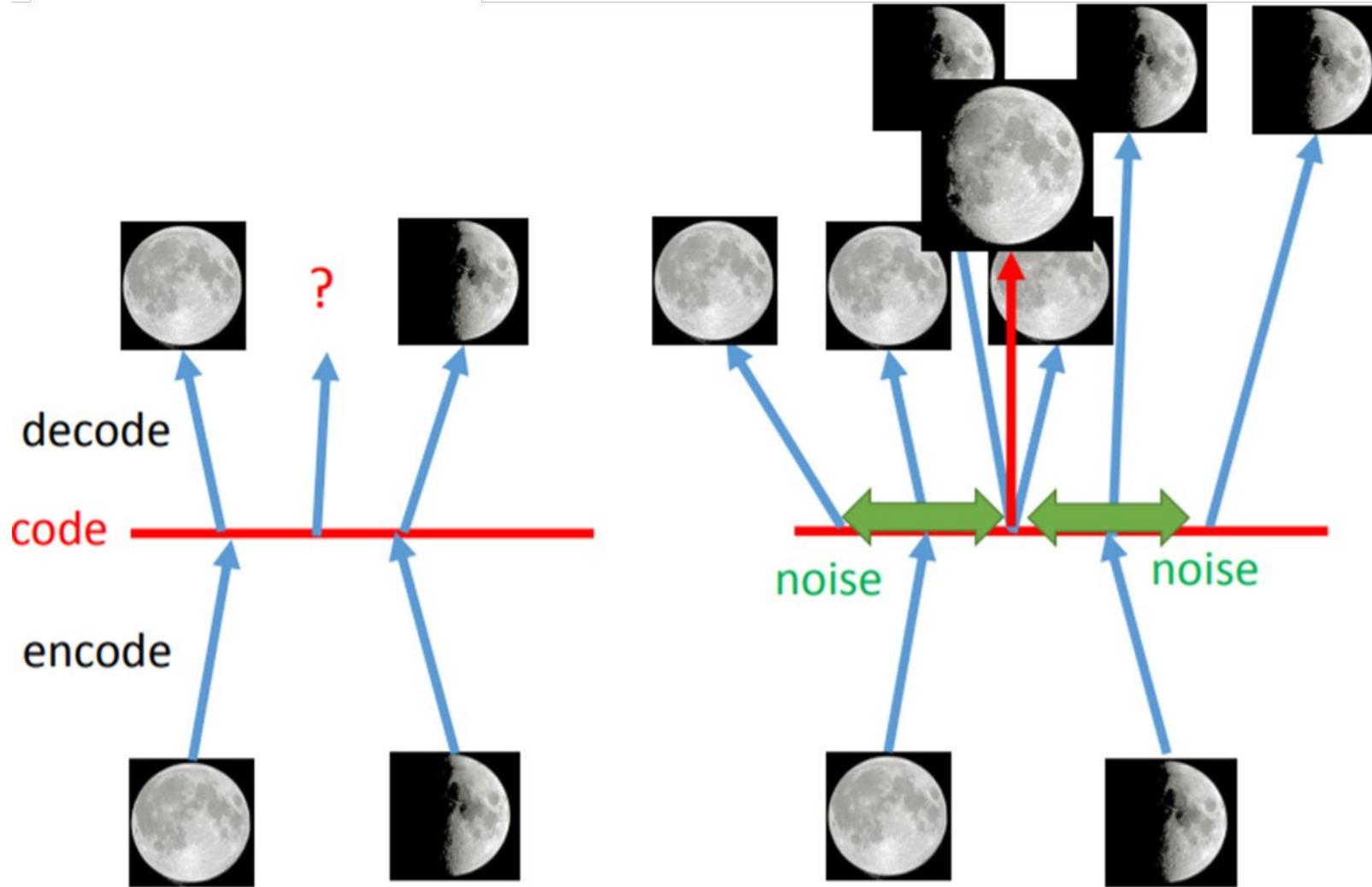


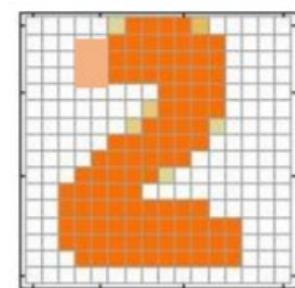
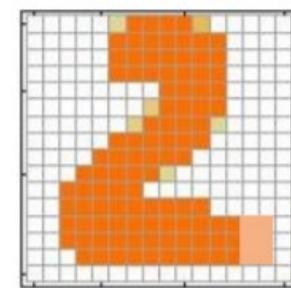
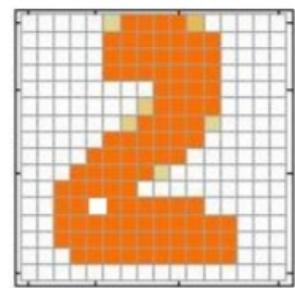
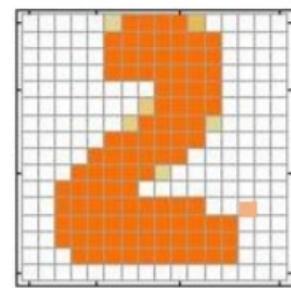
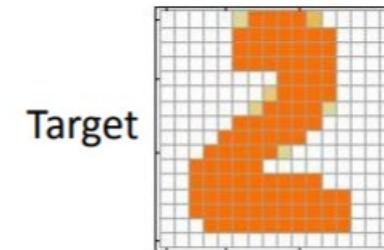
Generative Adversarial Network (GAN)

Images generated by AE/VAE are blur



Decoder in AE or VAE generates images that minimize MSE + KL. It does not consider the context information

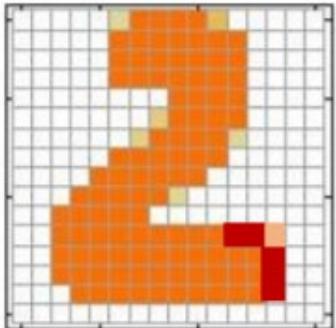
What do we miss?



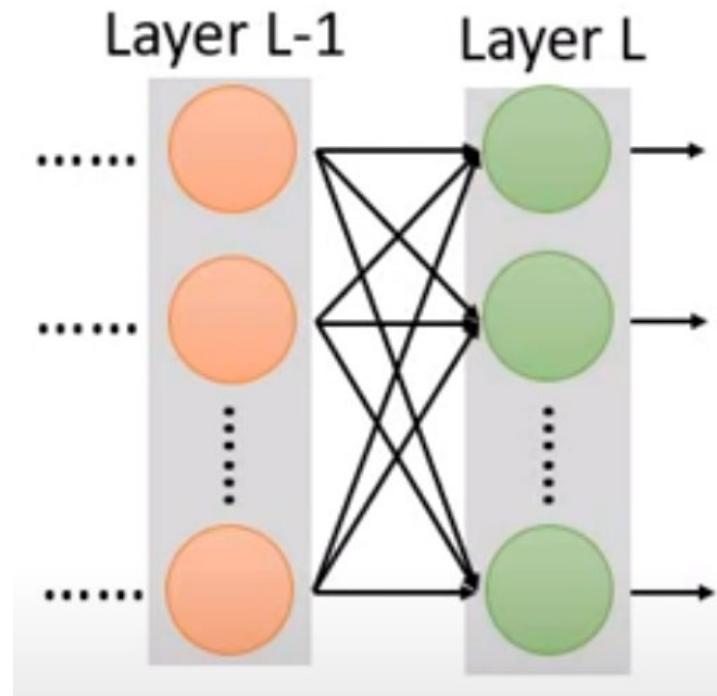
The decoder does not consider the structured relationships between pixels



我覺得不行



我覺得其實可以



The relation between the components are critical.

Although highly correlated, they cannot influence each other.

Need deep structure to catch the relation between components.

Advantages of GAN

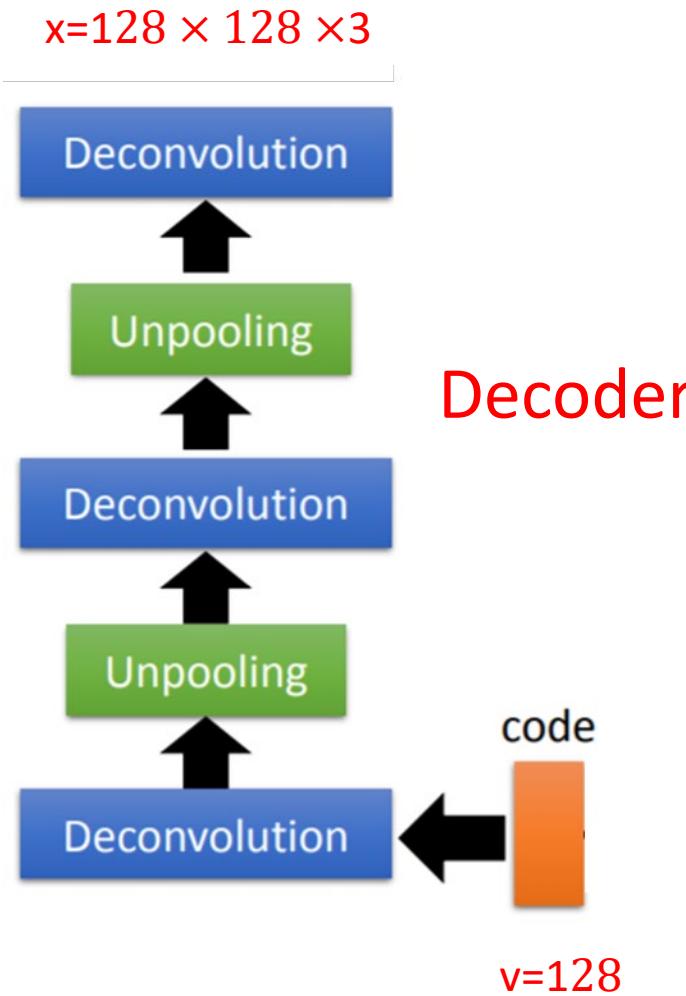
- Still generate the object component-by-component
- But it is learned from the discriminator with global view.

Practice

- Open "8.1. GAN.ipynb"

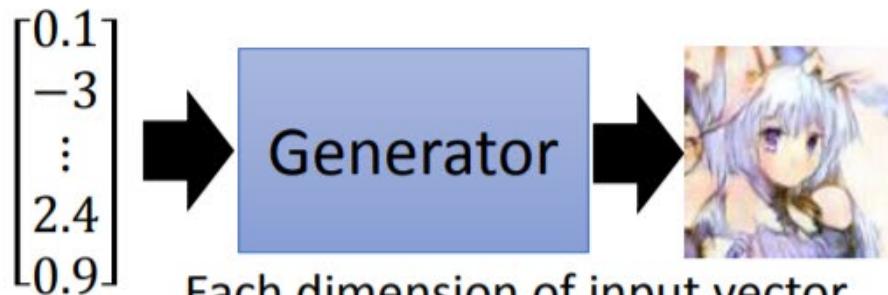


Generator

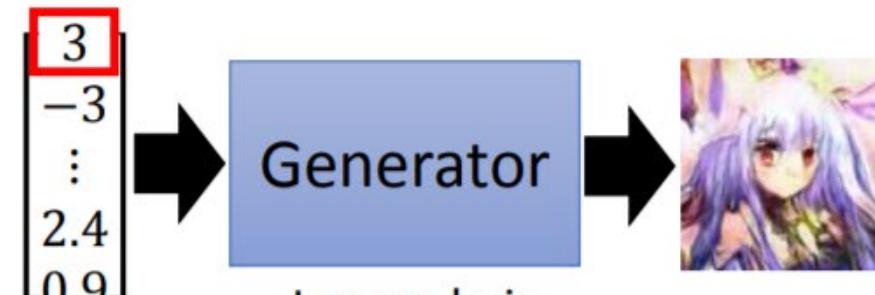


```
[10]: latent_size= 128  
[11]: generator = nn.Sequential(  
# in: latent_size x 1 x 1  
  
nn.ConvTranspose2d(latent_size, 512, kernel_size=4, stride=1,  
nn.BatchNorm2d(512),  
nn.ReLU(True),  
# out: 512 x 4 x 4  
  
nn.ConvTranspose2d(512, 256, kernel_size=4, stride=2, padding=  
nn.BatchNorm2d(256),  
nn.ReLU(True),  
# out: 256 x 8 x 8  
  
nn.ConvTranspose2d(256, 128, kernel_size=4, stride=2, padding=  
nn.BatchNorm2d(128),  
nn.ReLU(True),  
# out: 128 x 16 x 16  
  
nn.ConvTranspose2d(128, 64, kernel_size=4, stride=2, padding=1  
nn.BatchNorm2d(64),  
nn.ReLU(True),  
# out: 64 x 32 x 32  
  
nn.ConvTranspose2d(64, 32, kernel_size=4, stride=2, padding=1,  
nn.BatchNorm2d(32),  
nn.ReLU(True),  
# out: 32 x 64 x 64  
  
nn.ConvTranspose2d(32, 3, kernel_size=4, stride=2, padding=1,  
nn.Tanh()  
# out: 3 x 128 x 128  
)
```

Each dimension in the feature vector represent a property of the output image



Each dimension of input vector represents some characteristics.



Longer hair

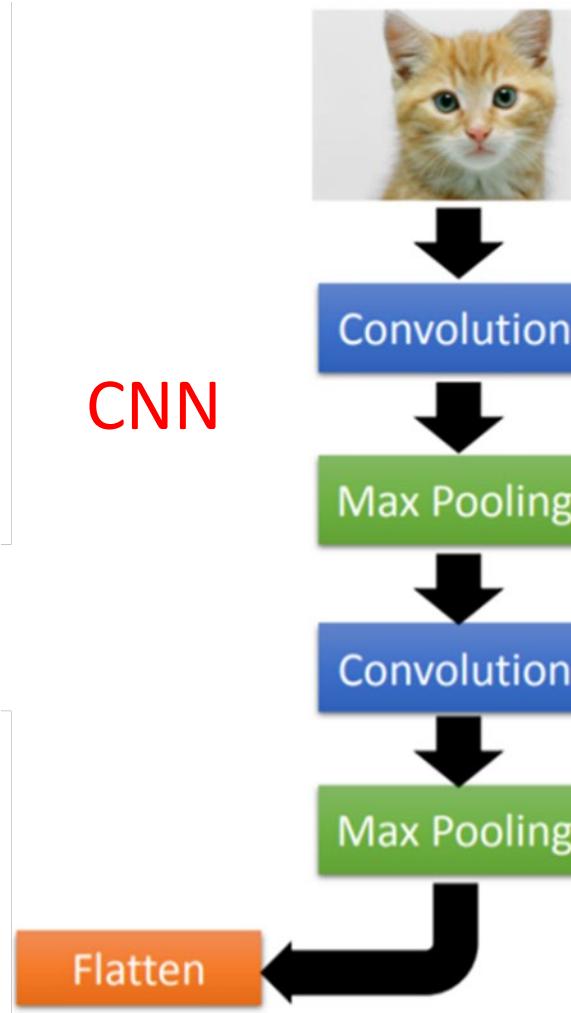
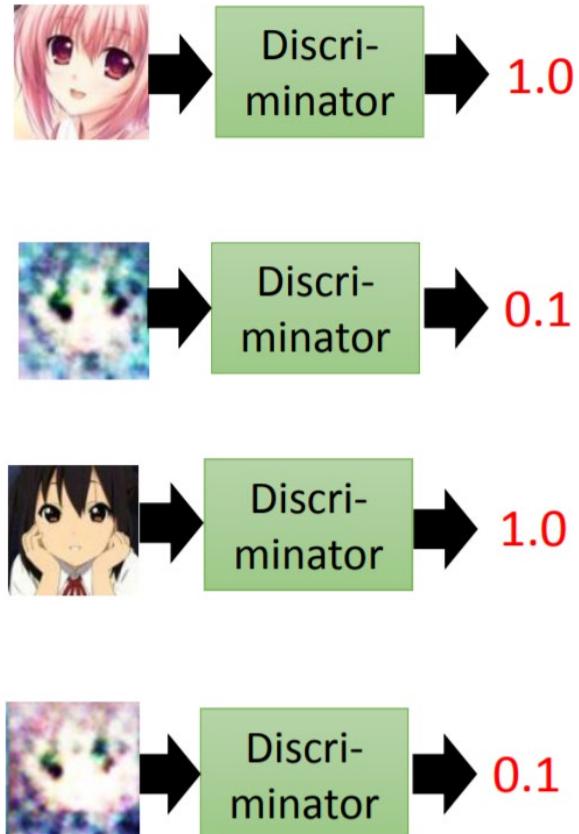


blue hair



Open mouth

Discriminator



[15]:

```
discriminator = nn.Sequential(  
    # in: 3 x 128 x 128  
    nn.Conv2d(3, 64, kernel_size=4, stride=2, p:  
    nn.BatchNorm2d(64),  
    nn.LeakyReLU(0.2, inplace=True),  
    # out: 64 x 64 x 64  
  
    nn.Conv2d(64, 128, kernel_size=4, stride=2,  
    nn.BatchNorm2d(128),  
    nn.LeakyReLU(0.2, inplace=True),  
    # out: 128 x 32 x 32  
  
    nn.Conv2d(128, 256, kernel_size=4, stride=2,  
    nn.BatchNorm2d(256),  
    nn.LeakyReLU(0.2, inplace=True),  
    # out: 256 x 16 x 16  
  
    nn.Conv2d(256, 512, kernel_size=4, stride=2,  
    nn.BatchNorm2d(512),  
    nn.LeakyReLU(0.2, inplace=True),  
    # out: 512 x 8 x 8  
  
    nn.Conv2d(512, 1024, kernel_size=4, stride=2,  
    nn.BatchNorm2d(1024),  
    nn.LeakyReLU(0.2, inplace=True),  
    # out: 1024 x 4 x 4  
  
    nn.Conv2d(1024, 1, kernel_size=4, stride=1,  
    # out: 1 x 1 x 1  
    nn.Flatten(),  
    nn.Sigmoid())
```

Reference: 李弘毅 GAN Lecture 1 (2018)

Step1 – Fix G and train D

- Initialize generator and discriminator



- In each training iteration:

[12]: `generator.to(device)`

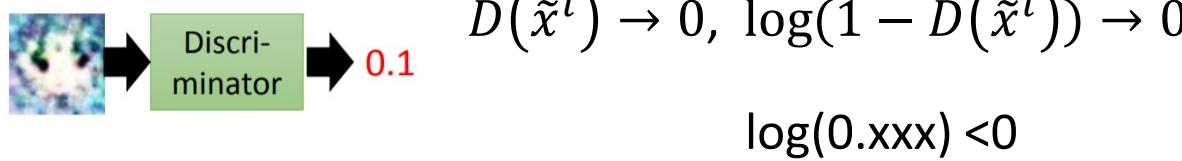
[16]: `discriminator.to(device)`

Step 1: Fix generator G, and update discriminator D

```
[35]: for epoch in range(epochs):
    if(epoch % 10 ==0):
        print(epoch, end=",")
    for real_images, _ in train_dl:
        # Train discriminator
        loss_d, real_score, fake_score = train_discriminator(real_images.
        # Train generator
        loss_g = train_generator(opt_g)
```

Train discriminator D

- Sample m examples $\{x^1, x^2, \dots, x^m\}$ from database
- Sample m noise samples $\{z^1, z^2, \dots, z^m\}$ from a distribution
- Obtaining generated data $\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}, \tilde{x}^i = G(z^i)$



Update discriminator parameters θ_d to maximize

- $\tilde{V} = \frac{1}{m} \sum_{i=1}^m \underline{\log D(x^i)} + \frac{1}{m} \sum_{i=1}^m \underline{\log (1 - D(\tilde{x}^i))}$
- $\theta_d \leftarrow \theta_d + \eta \nabla \tilde{V}(\theta_d)$

[19]: `def train_discriminator(real_images, opt_d):`

```
# Clear discriminator gradients  
opt_d.zero_grad()
```

```
# Pass real images through discriminator
```

```
real_preds = discriminator(real_images)  
real_targets = torch.ones(real_images.size(0))  
real_loss = F.binary_cross_entropy(real_preds,  
real_score = torch.mean(real_preds).item()
```

```
# Generate fake images
```

```
latent = torch.randn(batch_size, latent_size)  
fake_images = generator(latent.to(device))
```

```
# Pass fake images through discriminator
```

```
fake_targets = torch.zeros(fake_images.size(0))  
fake_preds = discriminator(fake_images)  
fake_loss = F.binary_cross_entropy(fake_preds,  
fake_score = torch.mean(fake_preds).item()
```

```
# Update discriminator weights
```

```
loss = real_loss + fake_loss  
loss.backward()  
opt_d.step()  
return loss.item(), real_score, fake_score
```

Review – Cross entropy

Measures the differences between the true probability p_i and the predicted probability q_i

$$H(p, q) = - \sum_i p_i \ln(q_i)$$

A	B	C	D	E	F	G	H	I	J	K
z1	z2	y-hat	p1	p2	EXP(z1)	EXP(z2)	F+G	q1	q2	-(P1*LN(Q1)+P2*LN(Q2))
-0.018	0.0855	0	1	0	0.982	1.089	2.071	0.474	0.526	0.74624
-0.0244	0.0741	0	1	0	0.976	1.077	2.053	0.475	0.525	0.74361
-0.0187	0.085	0	1	0	0.981	1.089	2.070	0.474	0.526	0.74634
-0.0258	0.0687	1	0	1	0.975	1.071	2.046	0.476	0.524	0.64701
-0.0267	0.0617	1	0	1	0.974	1.064	2.037	0.478	0.522	0.64992
										0.70662

Step2 – Fix D and train G

Step 2: Fix discriminator D, and update generator G

Generator learns to “fool” the discriminator

```
[35]: for epoch in range(epochs):
    if(epoch % 10 ==0):
        print(epoch, end=",")
    for real_images, _ in train_dl:
        # Train discriminator
        loss_d, real_score, fake_score =
        # Train generator
        loss_g = train_generator(opt_g)
```

- Sample m noise samples{ z^1, z^2, \dots, z^m } from a distribution
- Update generator parameters θ_g to maximize
 - $\tilde{V} = \frac{1}{m} \sum_{i=1}^m \log(D(G(z^i)))$
 - $\theta_g \leftarrow \theta_g - \eta \nabla \tilde{V}(\theta_g)$

```
[28]: def train_generator(opt_g):
    # Clear generator gradients
    opt_g.zero_grad()

    # Generate fake images
    latent = torch.randn(batch_size, latent_size,
    fake_images = generator(latent)

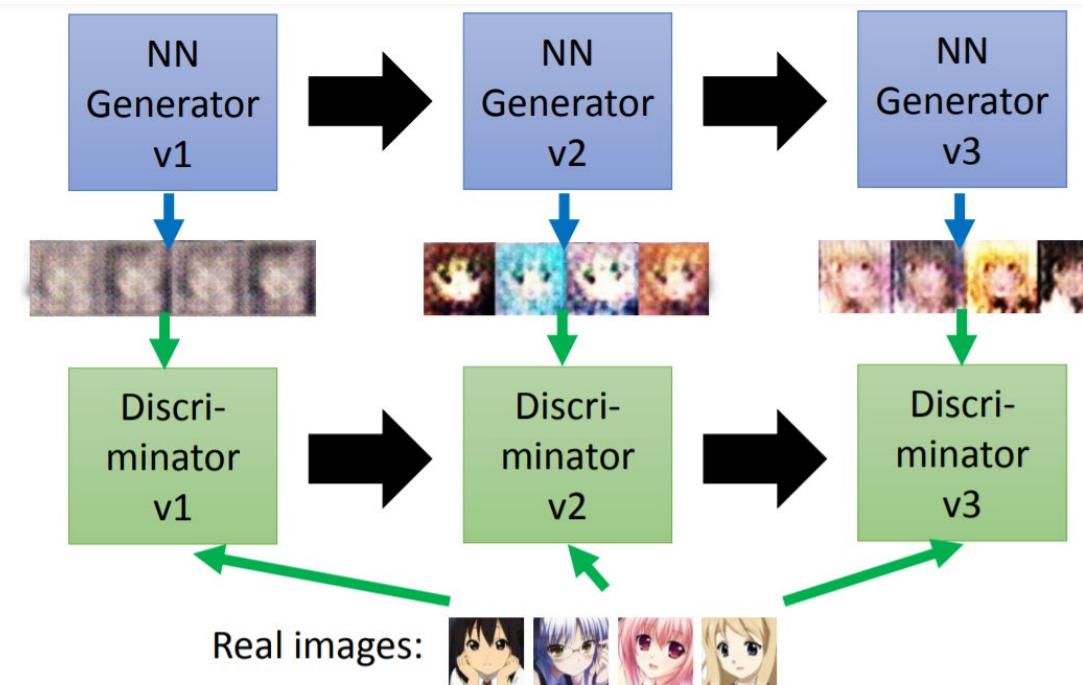
    # Try to fool the discriminator
    preds = discriminator(fake_images)
    targets = torch.ones(batch_size, 1, device=device)
    loss = F.binary_cross_entropy(preds, targets)

    # Update generator weights
    loss.backward()
    opt_g.step()

    return loss.item()
```

An evolution process

```
[35]: for epoch in range(epochs):  
    if(epoch % 10 ==0):  
        print(epoch, end=",")  
    for real_images, _ in train_dl:  
        # Train discriminator  
        loss_d, real_score, fake_score = train_discriminator(real_images)  
        # Train generator  
        loss_g = train_generator(opt_g)
```



Visualize fake images generated by G during the evolution process

```
[17]: sample_dir = 'generated'  
os.makedirs(sample_dir, exist_ok=True)
```

```
[34]: fixed_latent = torch.randn(64, latent_size,  
# used to generate saved images
```

```
if(epoch % 50 ==0):  
    # Log Losses & scores (Last batch)  
    print("Epoch [{}/{}], loss_g: {:.4f}, loss_d: {:.4f}, real_scores: {:.4f}, fake_scores: {:.4f} ".format(epoch+1, epochs, loss_g, loss_d, real_scores, fake_scores))  
    # Save generated images  
    save_samples(epoch+start_idx, fixed_latent, sample_dir)
```

```
[18]: def save_samples(index, latent_tensors, show=True):  
    fake_images = generator(latent_tensors)  
    fake_fname = 'generated-images-{:0=4d}.png'.format(index)  
    save_image(denorm(fake_images), os.path.join(sample_dir, fake_fname))  
    print('Saving', fake_fname)  
    if show:  
        fig, ax = plt.subplots(figsize=(8, 8))  
        ax.set_xticks([]); ax.set_yticks([])  
        ax.imshow(make_grid(fake_images.cpu().detach(), nrow=8).permute(1, 2, 0))
```



8.1. GAN.ipynb

檔案 編輯 檢視畫面 插入 執行階段 工具 說明 無法儲存變更

共用



檔案



<>
..
gdrive

generated

generated-images-0001.png
generated-images-0051.png
generated-images-0101.png
generated-images-0151.png
generated-images-0201.png

sample_data

+ 程式碼 + 文字

複製到雲端硬碟

```
# Log losses & scores ()  
print("Epoch [{}/{}], loss_ epoch+1, epochs, 1c  
# Save generated images  
latent = torch.randn(batch_ save_samples(epoch+start_idx,  
#save_samples(epoch+start_id
```

```
0, Epoch [1/1200], loss_g: 5.6803, los Saving generated-images-0001.png  
10, 20, 30, 40, 50, Epoch [51/1200], loss_ Saving generated-images-0051.png  
60, 70, 80, 90, 100, Epoch [101/1200], los Saving generated-images-0101.png  
110, 120, 130, 140, 150, Epoch [151/1200], Saving generated-images-0151.png  
160, 170, 180, 190, 200, Epoch [201/1200], Saving generated-images-0201.png  
210, 220,
```

RAM
磁碟

編輯

generated-images-0201.png X

generated-images-0001.png X





8.1. GAN.ipynb

共用



檔案 編輯 檢視畫面 插入 執行階段 工具 說明 無法儲存變更

檔案



..

gdrive

generated

generated-images-0001.png

generated-images-0051.png

generated-images-0101.png

generated-images-0151.png

generated-images-0201.png

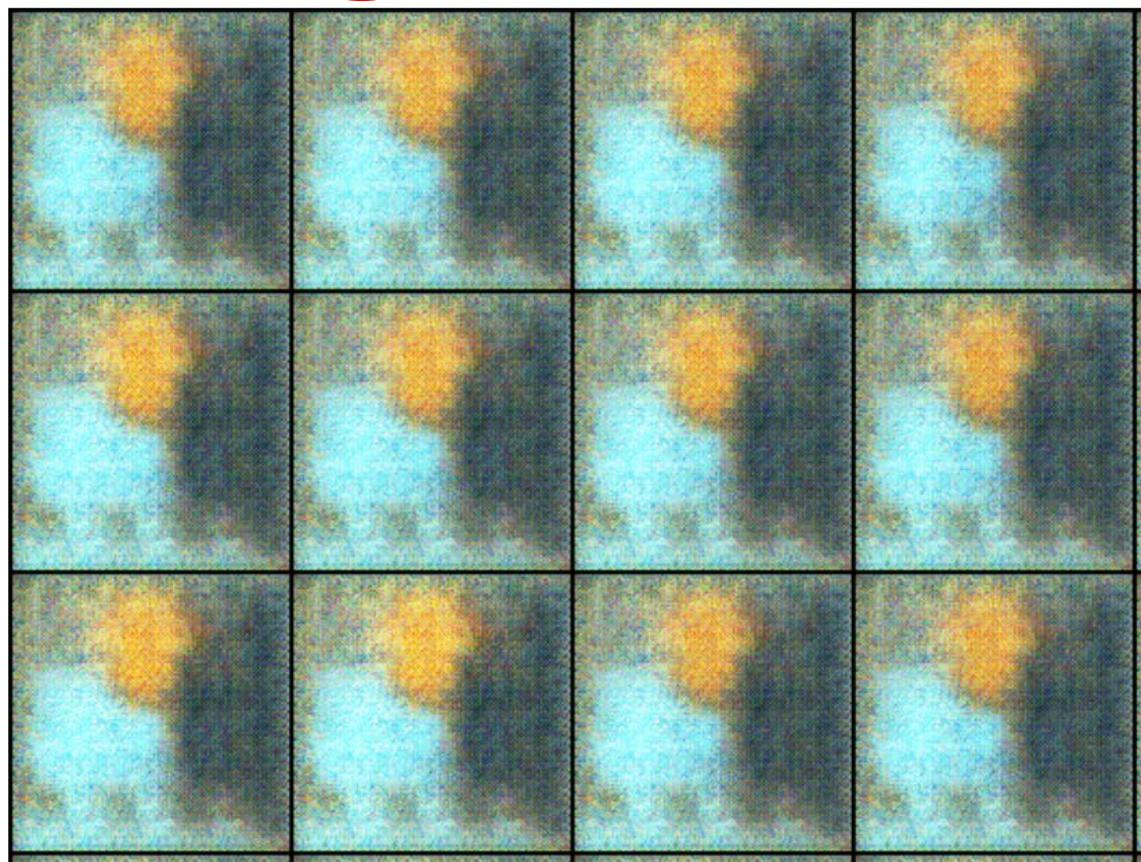
generated-images-0251.png

sample_data

+ 程式碼 + 文字 | 複製到雲端硬碟

```
Saving generated-images-0001.png  
10, 20, 30, 40, 50, Epoch [51/1200], loss_  
Saving generated-images-0051.png  
60, 70, 80, 90, 100, Epoch [101/1200], los  
Saving generated-images-0101.png  
110, 120, 130, 140, 150, Epoch [151/1200],  
Saving generated-images-0151.png  
160, 170, 180, 190, 200, Epoch [201/1200],  
Saving generated-images-0201.png  
210, 220, 230, 240, 250, Epoch [251/1200],  
Saving generated-images-0251.png  
260, 270, 280,
```

generated-images-0251.png X



磁碟 29.48 GB 可用



8.1. GAN.ipynb

共用



T

檔案 編輯 檢視畫面 插入 執行階段 工具 說明 無法儲存變更

檔案



..

gdrive

generated

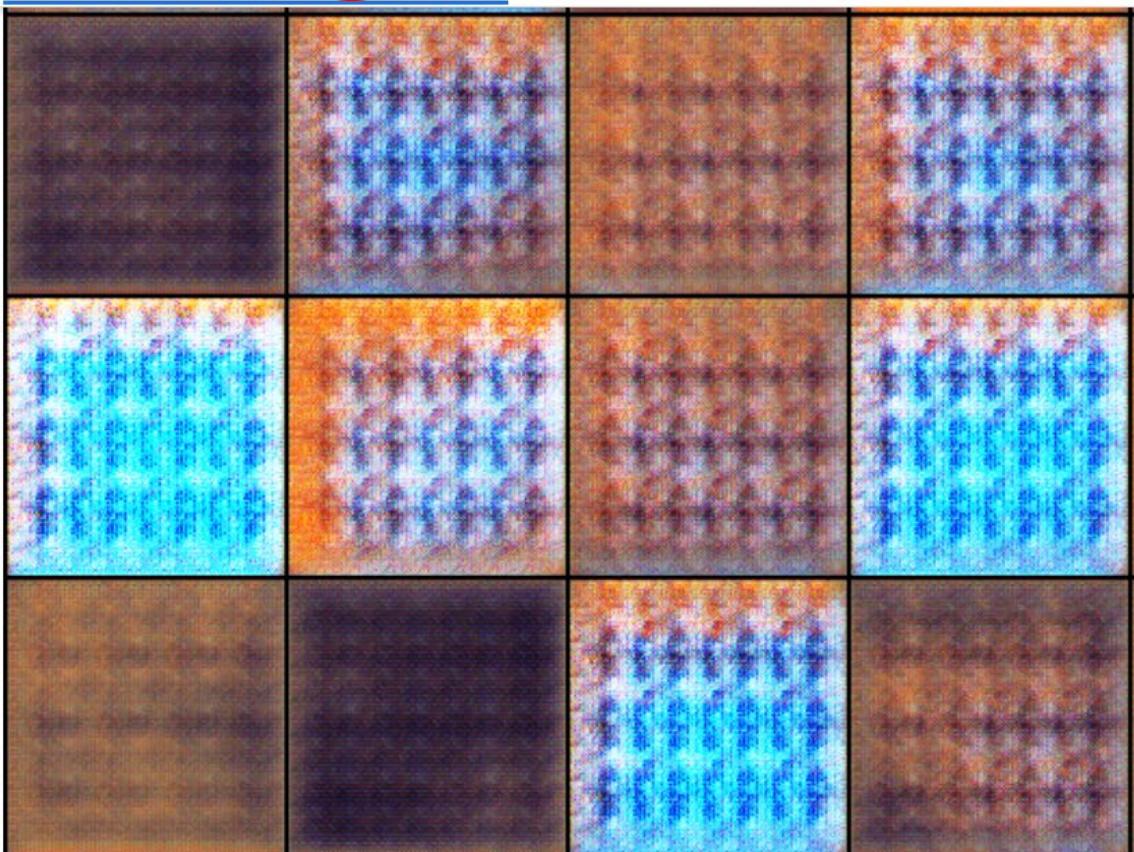
- generated-images-0001.png
- generated-images-0051.png
- generated-images-0101.png
- generated-images-0151.png
- generated-images-0201.png
- generated-images-0251.png
- generated-images-0301.png

sample_data

+ 程式碼 + 文字 複製到雲端硬碟

```
Saving generated-images-0001.png  
10, 20, 30, 40, 50, Epoch [51/1200], loss_  
Saving generated-images-0051.png  
60, 70, 80, 90, 100, Epoch [101/1200], los  
Saving generated-images-0101.png  
110, 120, 130, 140, 150, Epoch [151/1200],  
Saving generated-images-0151.png  
160, 170, 180, 190, 200, Epoch [201/1200],  
Saving generated-images-0201.png  
210, 220, 230, 240, 250, Epoch [251/1200],  
Saving generated-images-0251.png  
260, 270, 280, 290, 300, Epoch [301/1200],  
Saving generated-images-0301.png  
310, 320, 330, 340,
```

generated-images-0301.png X



Epoch =41201



34151



28651



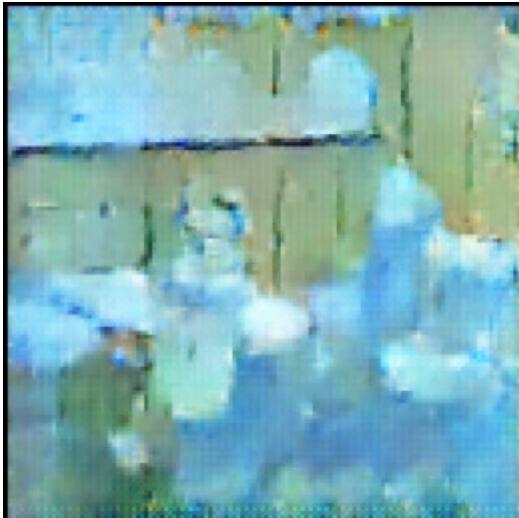
25901



23201



16401



15551



14301



Save the generated images as a video

```
[37]: img_array = []
for filename in files:
    img = cv2.imread(filename)
height, width, layers = img.shape
size = (width,height)
img_array.append(img)

out = cv2.VideoWriter('GANTrainingVideo.avi',cv2.VideoWriter_fourcc(*'DIVX'), 15, size)

for i in range(len(img_array)):
    out.write(img_array[i])
out.release()
```

Tom & Jerry video <https://youtu.be/uDEGITFmhiQ>

HW7 (1)

- Use your own images, e.g., facial expression to train a GAN.
- Show the generated images.
- Try latent vectors like (all 1), (all 0.5), (all 0.3), (0, 0, 1, 0,), (one dimension goes from 0 to 1 and other dimensions fixed), ... to see the generated images.



Study images generated with different latent vectors

```
[78]: latent0 = torch.zeros(1, latent_size, 1, 1)
latent1 = torch.ones(1, latent_size, 1, 1)
latentNeg1 = -torch.ones(1, latent_size, 1, 1)

#Let feature_idx dimension = a value and all other dimensions = 0 (or 1, -1)
NewLatent0 = torch.zeros(1, latent_size, 1, 1)
#NewLatent0[:, 3, :, :] = 50
NewLatent0[:, 123, :, :] = -50
```

```
[84]: fake_img = denorm(fake_img)
plt.imshow(fake_img)
```

```
t[84]: <matplotlib.image.AxesImage at 0x7fd9d8d14b50>
```

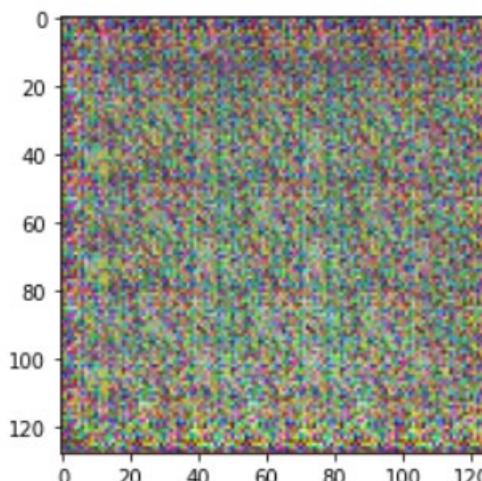


Image size = 128*128*3
Class 1 = 10, class2 = 10
Latent vector size = 128
Batch = 128
Colab run 30K epochs, 2-3hr

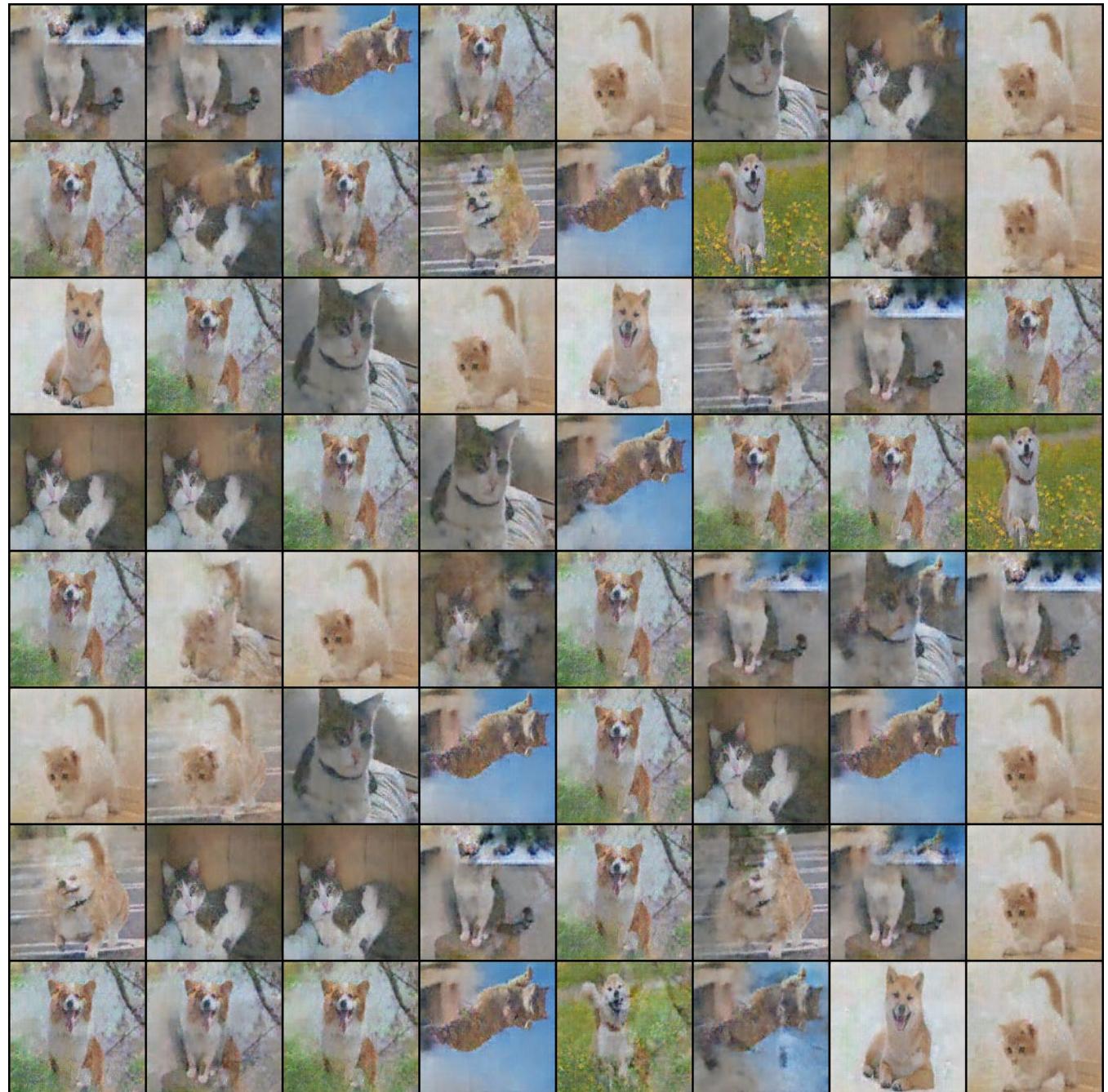


Image size = 128*128*3
Class 1 = 10, class2 = 10
Latent vector size = 128
Batch = 128
Anaconda run 23.4K epochs



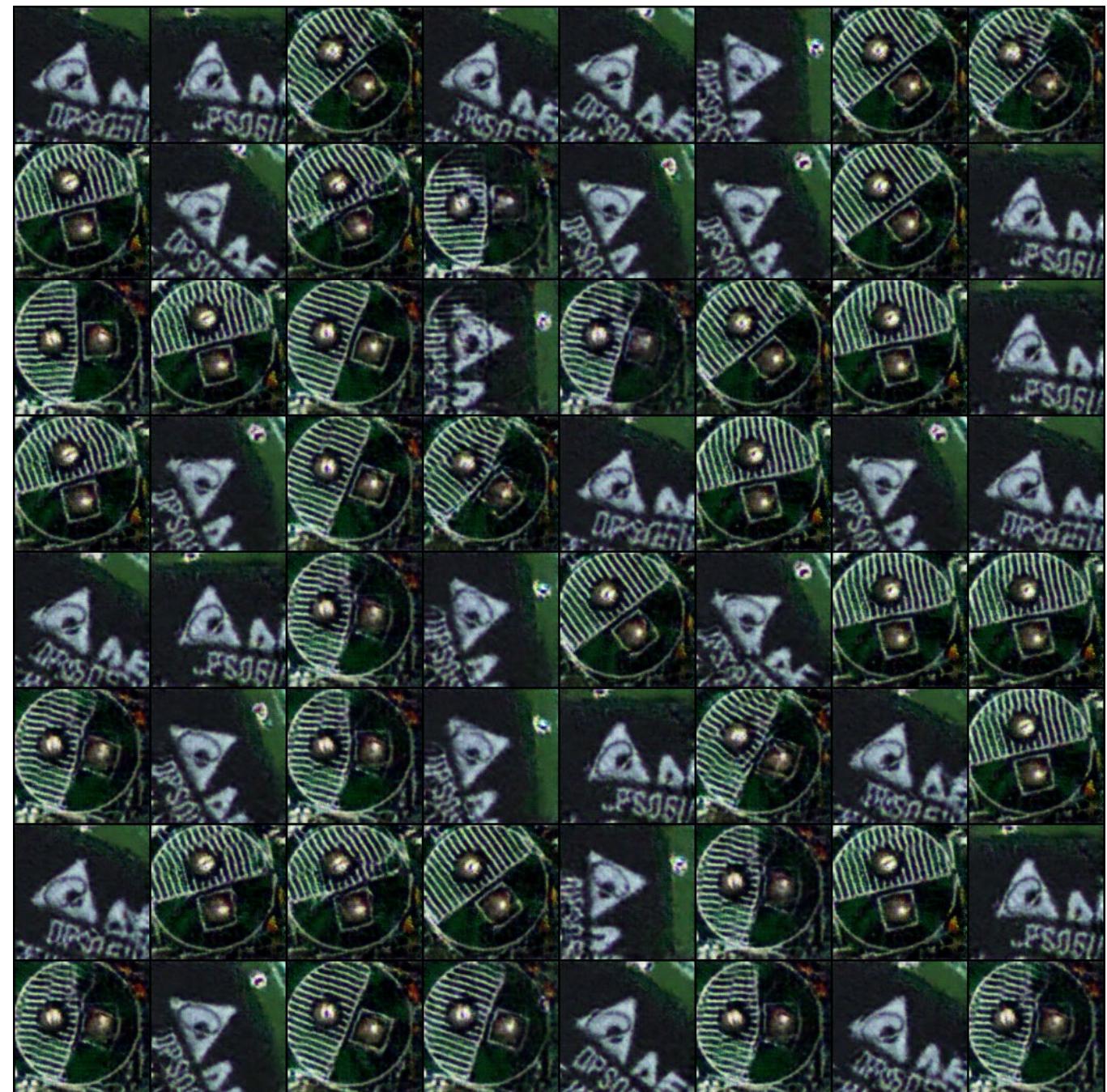
Images generated from G after
5651 epochs

Image size = 180x180x3

Class 1 = 100, Class 2 = 100

Latent vector size = 128
batch=32

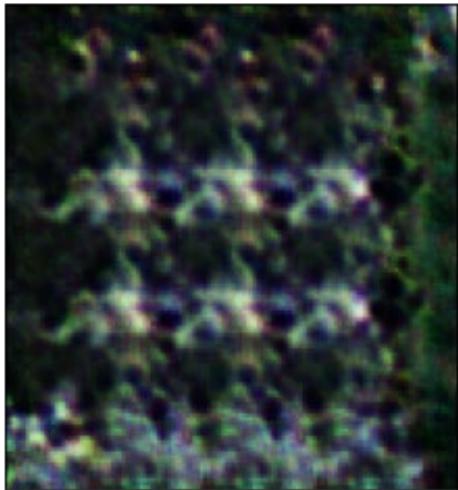
Run on 2080Ti, takes about 15 min.



1085442 Carlos

Images generated from different latent vectors

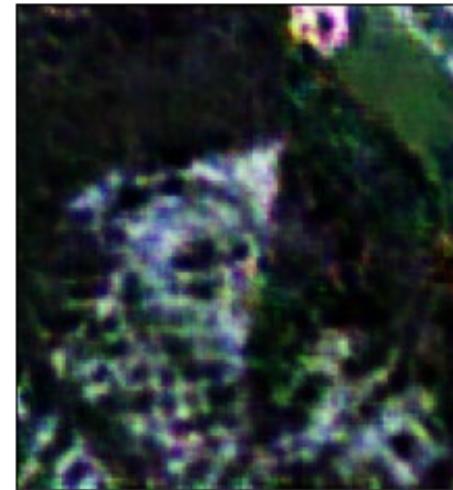
[0, 0, 0...0]



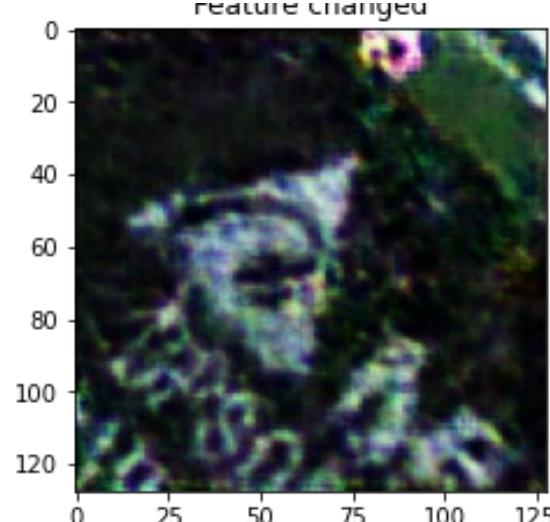
[1, 1, 1...1]



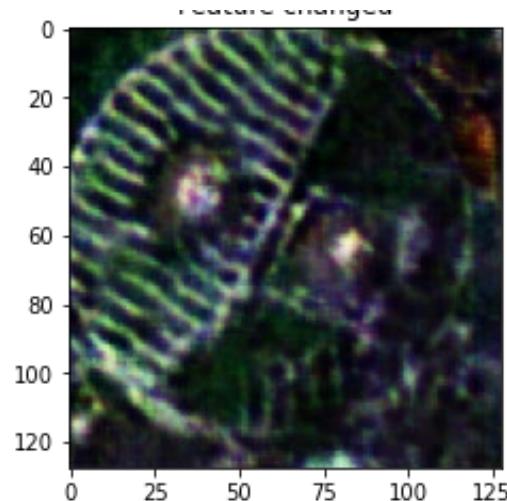
[-1, -1, -1...-1]



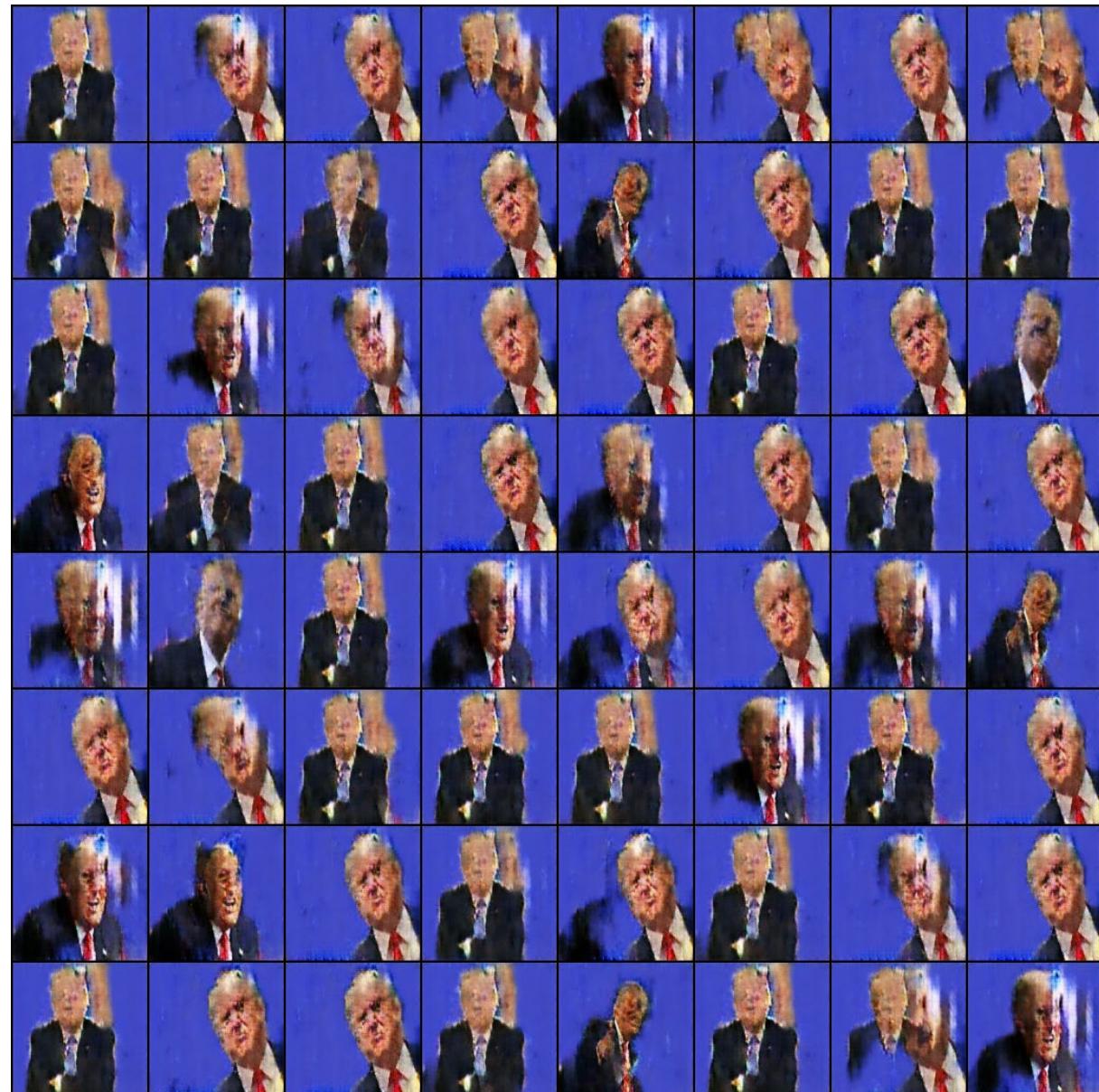
[1, 1, 1...1], feature 3=50



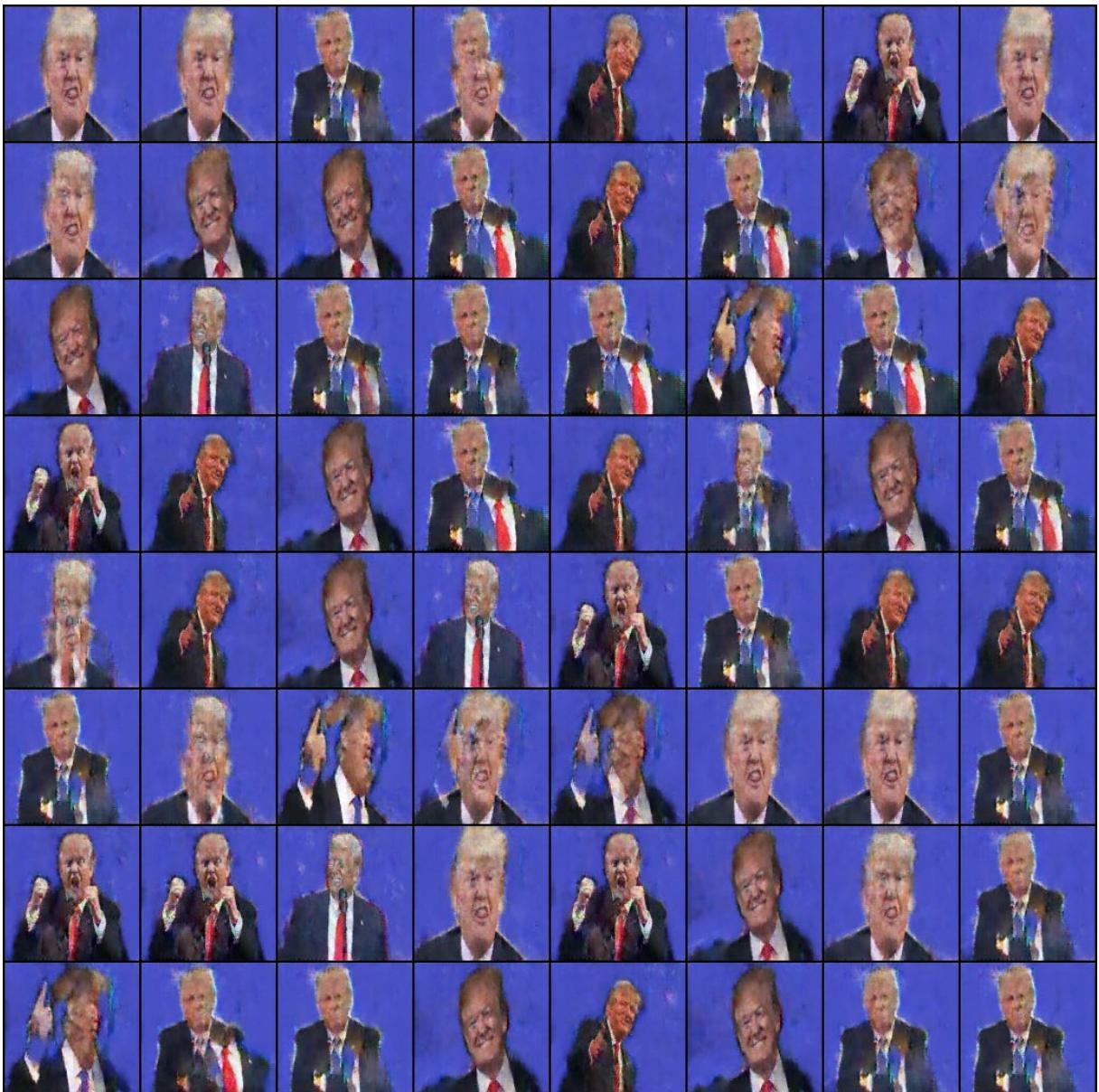
[1, 1, 1...1], feature 123=50



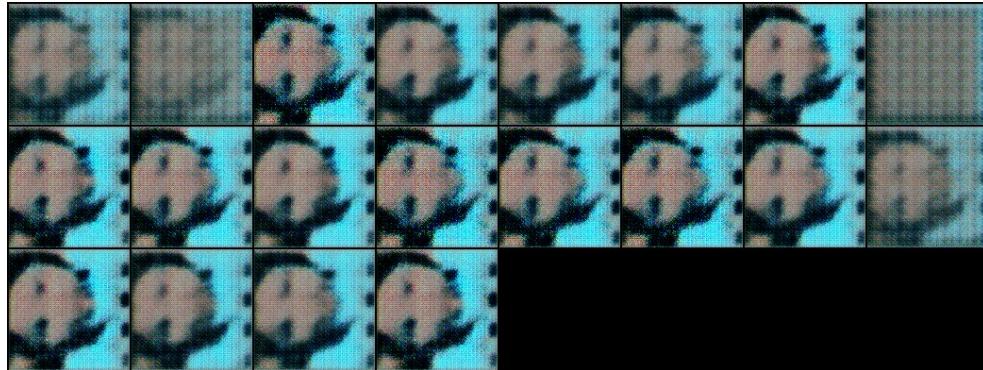
Epoch =8K



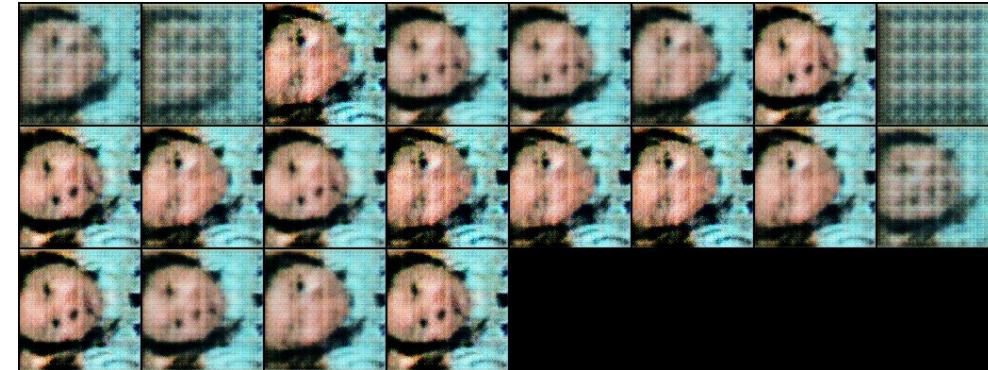
Epoch =20K



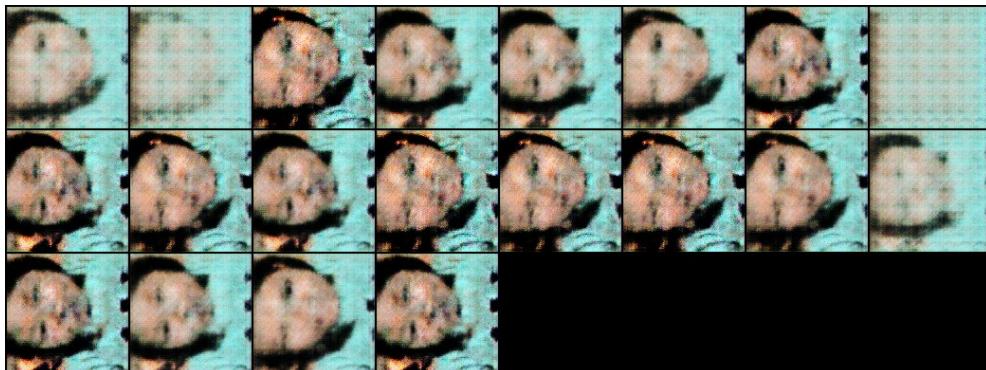
image_size = 128
batch_size = 64
latent_size= 1
epochs = 400



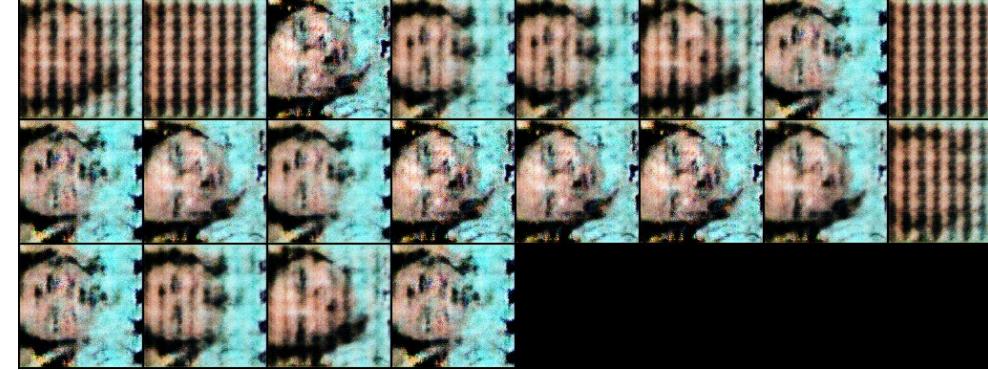
Epoch = 151



251



301

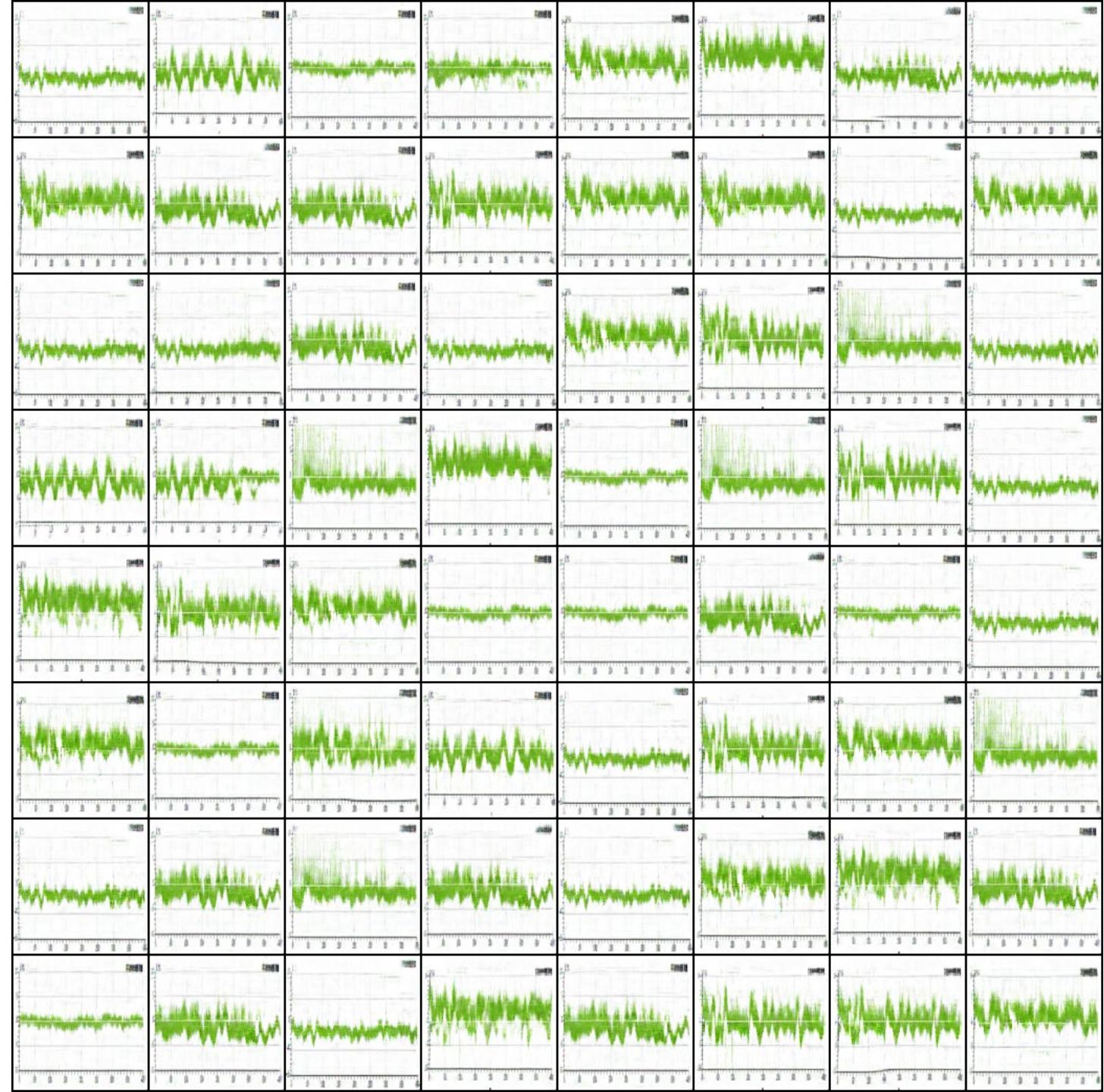


351

1095433 Natty

Images-19951

1071228 陳仁慶



Images generated from G after
2100 epochs

Image size = 128x128x3

Class 1 = 10, Class 2 = 10

Latent vector size = 128
batch=128

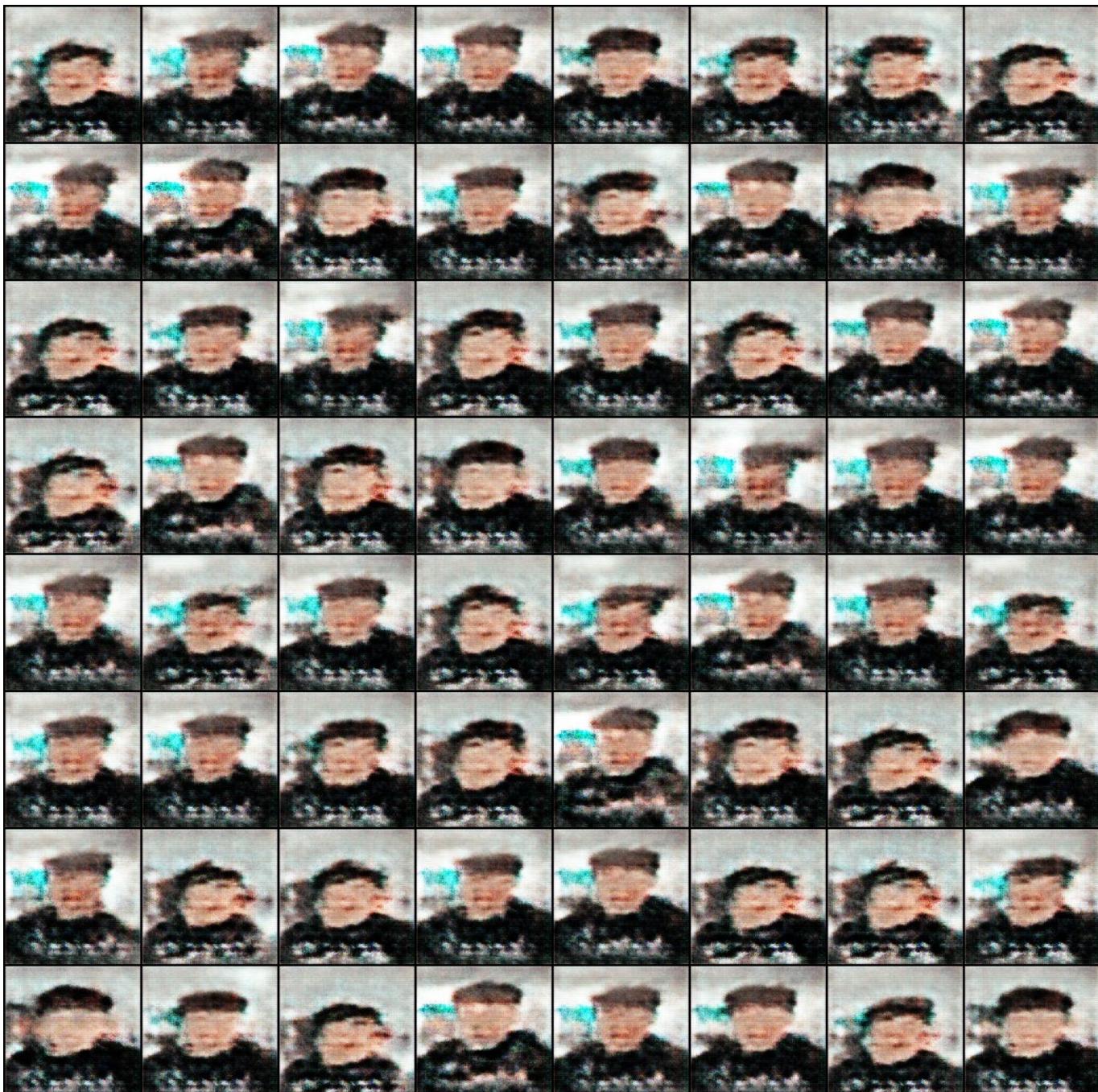
Run on Colab Tesla K80, takes about 120
min.



Images generated from G after
3051 epochs

Image size = 128x128x3
Class 1 = 10, Class 2 = 10
Latent vector size = 128
batch=128

Run on Colab Tesla T4, takes about 60 min.

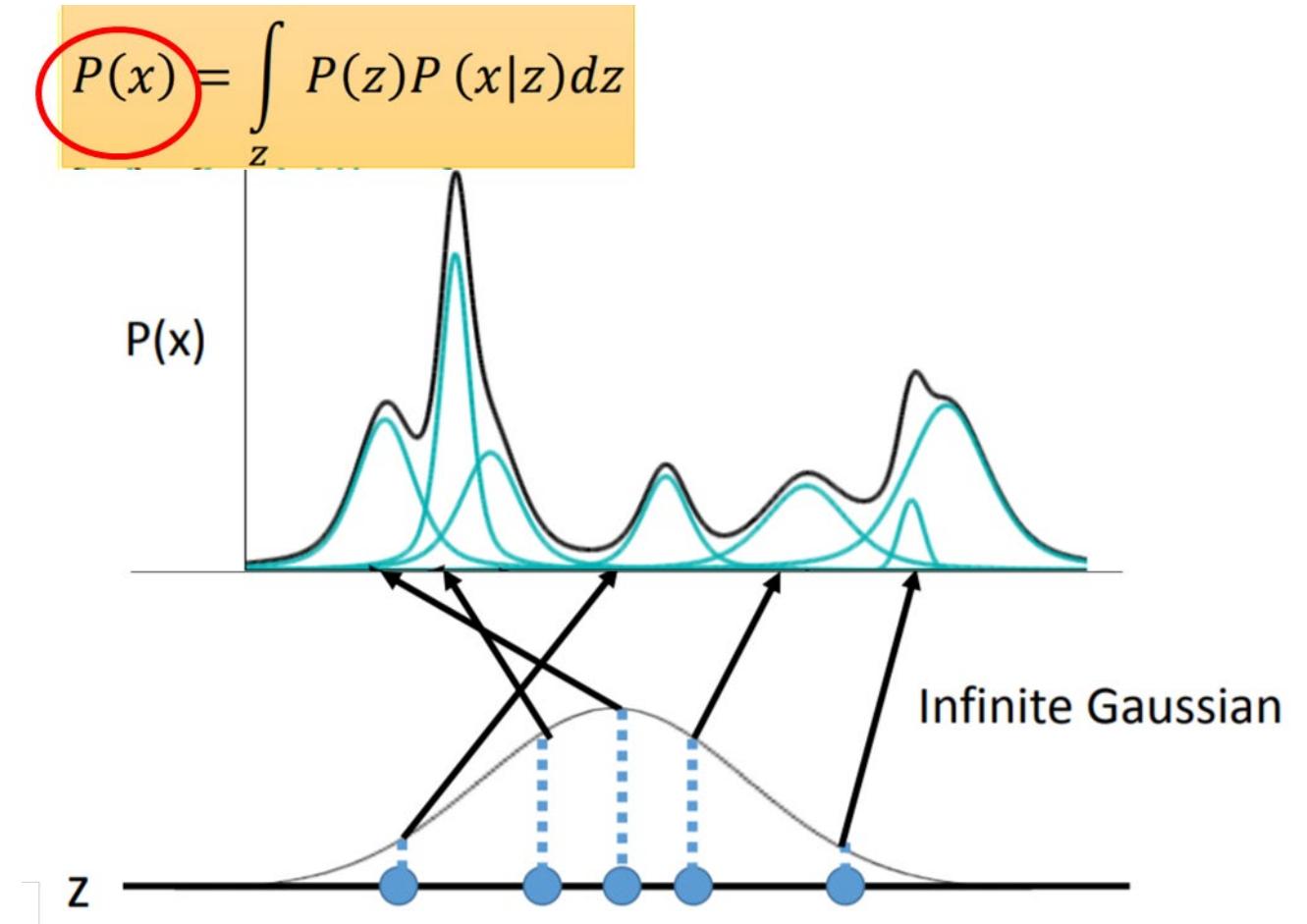
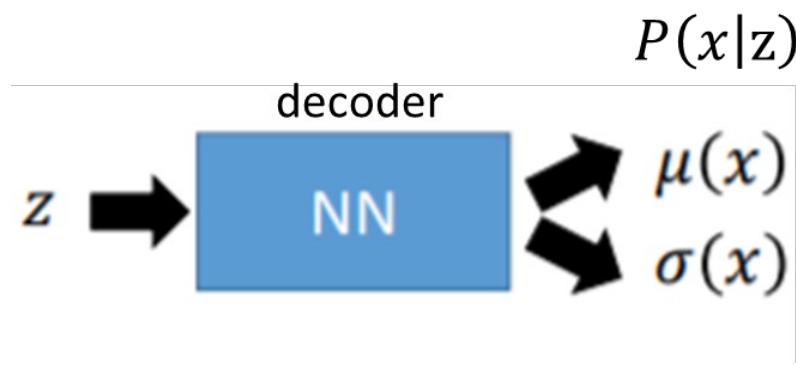


Theory behind GAN, VAE, PPO

The concept of deep generative model

1. Use probability distribution model to model the image generation process or the state-action generation process.
2. With this probability model, we want to maximize the likelihood to observe the training data or observe the trajectories with higher long-term rewards.
3. Maximize the likelihood can be achieved by minimizing the KL divergence.
3. Use DNN to represent a more general probability model.
4. Use DNN to represent a more general divergence measure.

(VAE) 1. Modelling the image generation process as a mixed Gaussian distribution model



(VAE) 2. Given the image generation model, we want to maximize the likelihood to observe the training images

Maximizing Likelihood

$$P(x) = \int_z P(z)P(x|z)dz$$

$$L = \sum_x \log P(x)$$

$P(z)$ is normal distribution

$$x|z \sim N(\mu(z), \sigma(z))$$

$\mu(z), \sigma(z)$ is going to be estimated

Maximizing the likelihood of the observed x

$$L = p(x^1) \times p(x^2) \times p(x^3) \times \cdots p(x^m) = \prod_{i=1, \dots, m} P(x^i)$$

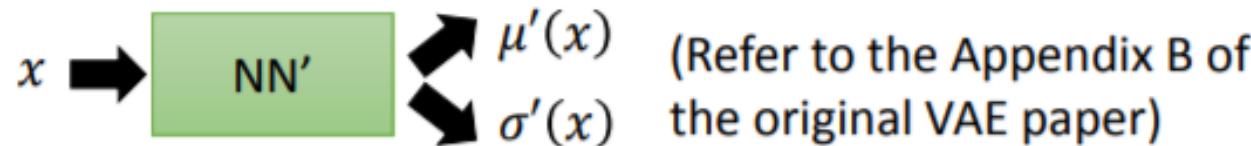
(VAE) 3. Maximize the likelihood can be achieved by minimizing KL divergence

Max. L_b can be done by min. $KL(q(z|x)||P(z))$ and max. $\int_z q(z|x) \log P(x|z) dz$. That is why loss = KLD + MSE (x, \hat{x})

Minimizing $KL(q(z|x)||P(z))$

$$\sum_{i=1}^3 (\exp(\sigma_i) - (1 + \sigma_i) + (m_i)^2)$$

Minimize



Maximizing

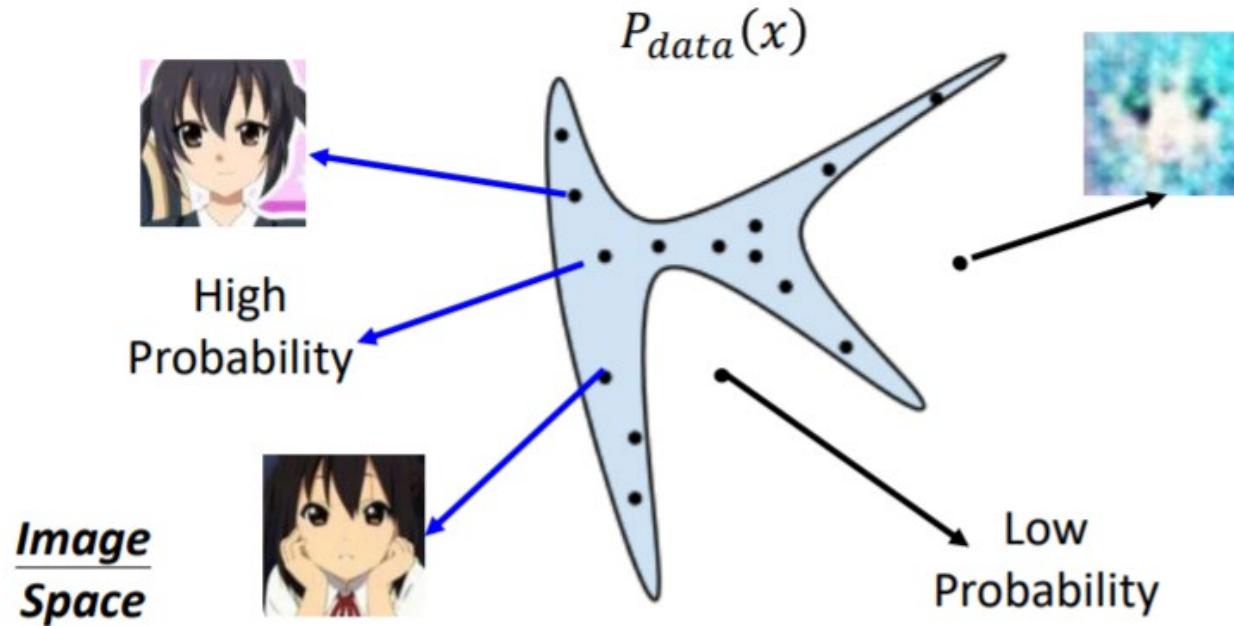
$$\int_z q(z|x) \log P(x|z) dz = E_{q(z|x)}[\log P(x|z)]$$



(GAN) 1. Modelling the generator as a probability distribution model P_G

X: an image (a high-dimensional vector)

- We want to find data distribution $P_{data}(x)$



(GAN) 2. Given the image generation model, we want to maximize the likelihood to observe the training images

- Given a data distribution $P_{data}(x)$ (We can sample from it.)
- We have a distribution $P_G(x; \theta)$ parameterized by θ
 - We want to find θ such that $P_G(x; \theta)$ close to $P_{data}(x)$
 - E.g. $P_G(x; \theta)$ is a Gaussian Mixture Model, θ are means and variances of the Gaussians

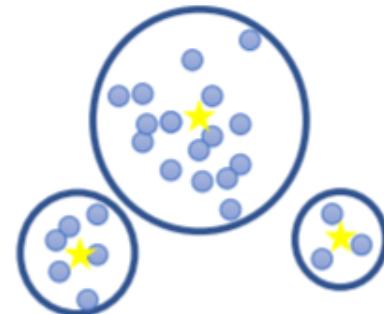
Sample $\{x^1, x^2, \dots, x^m\}$ from $P_{data}(x)$

We can compute $P_G(x^i; \theta)$

Likelihood of generating the samples

$$L = \prod_{i=1}^m P_G(x^i; \theta)$$

Find θ^* maximizing the likelihood



(GAN) 3. Maximize the likelihood can be achieved by minimizing the KL divergence

Maximum likelihood estimation = minimum
KL divergence

$$\theta^* = \arg \max_{\theta} \prod_{i=1}^m P_G(x^i; \theta) = \arg \min_{\theta} KL(P_{data} || P_G)$$

$$\begin{aligned}\theta^* &= \arg \max_{\theta} \prod_{i=1}^m P_G(x^i; \theta) = \arg \max_{\theta} \log \prod_{i=1}^m P_G(x^i; \theta) \\ &= \arg \max_{\theta} \sum_{i=1}^m \log P_G(x^i; \theta) \quad \{x^1, x^2, \dots, x^m\} \text{ from } P_{data}(x) \\ &\approx \arg \max_{\theta} E_{x \sim P_{data}} [\log P_G(x; \theta)] \\ &= \arg \max_{\theta} \int_x P_{data}(x) \log P_G(x; \theta) dx - \int_x P_{data}(x) \log P_{data}(x) dx \\ &= \arg \min_{\theta} KL(P_{data} || P_G) \quad \text{How to define a general } P_G?\end{aligned}$$

$$\int_x P_{data}(x) \log P_{data}(x) dx = \log P_{data}(x)$$

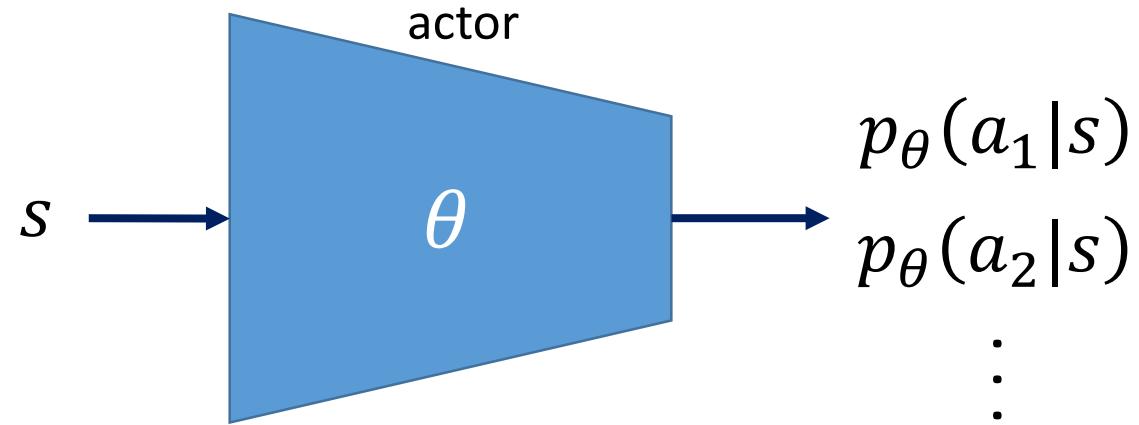
$$D_{KL}(q || p) = \sum_{i=1}^N q(x_i) \log\left(\frac{q(x_i)}{p(x_i)}\right)$$

(Classification) 1. Assuming the training data is generated from $y_1 = P_{w,b}(C_1 | x) = \sigma(w \cdot x + b)$ 2. Maximize the likelihood to observe the training data

Training Data	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">x^1</td><td style="text-align: center;">x^2</td><td style="text-align: center;">x^3</td><td style="text-align: center;">.....</td><td style="text-align: center;">x^N</td></tr> <tr> <td style="text-align: center;">C_1</td><td style="text-align: center;">C_1</td><td style="text-align: center;">C_2</td><td style="text-align: center;">.....</td><td style="text-align: center;">C_1</td></tr> </table>	x^1	x^2	x^3	x^N	C_1	C_1	C_2	C_1
x^1	x^2	x^3	x^N							
C_1	C_1	C_2	C_1							

$$\begin{aligned}
 L(w, b) &= f_{w,b}(x^1)f_{w,b}(x^2)\left(1 - f_{w,b}(x^3)\right)\cdots f_{w,b}(x^N) \\
 -\ln L(w, b) &= \ln f_{w,b}(x^1) + \ln f_{w,b}(x^2) + \ln \left(1 - f_{w,b}(x^3)\right)\cdots \\
 &\quad \hat{y}^n: 1 \text{ for class 1, } 0 \text{ for class 2} \\
 &= \sum_n -\left[\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln \left(1 - f_{w,b}(x^n)\right)\right] \\
 &\quad \text{Cross entropy between two Bernoulli distribution}
 \end{aligned}$$

(PPO) 1. Modelling the state-action mapping network as $p_\theta(a|s)$



(PPO) 2. Given the probability model, we want to maximize the likelihood to observe trajectories with higher long-term rewards

$$\tau = (s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_T, a_T)$$

$$p_\theta(\tau) = p(s_1)p_\theta(a_1|s_1)p(s_2|s_1, a_1)p_\theta(a_2|s_2)p(s_3|s_2, a_2)\dots$$

$$\bar{R}_\theta = \sum R(\tau) p_\theta(\tau) = E_{\tau \sim p_\theta(\tau)}[R(\tau)] \quad R(\tau) = \sum_{t=1}^T r_t$$

$$\max_\theta E[\bar{R}_\theta]$$

$$\begin{aligned} \nabla \bar{R}_\theta &= \sum R(\tau) \nabla p_\theta(\tau) = E_{\tau \sim p_\theta(\tau)}[R(\tau) \nabla \log p_\theta(\tau)] \approx \frac{1}{N} \sum_{n=1}^N R(\tau^n) \nabla \log p_\theta(\tau^n) \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p_\theta(a_t^n | s_t^n) \end{aligned}$$

(PPO) 3. To facilitate off-policy sampling, we minimize the KL divergence

Off-policy

$$\nabla \bar{R}_\theta = E_{(s_t, a_t) \sim \pi_{\theta'}} \left[\frac{p_\theta(a_t | s_t)}{p_{\theta'}(a_t | s_t)} A^{\theta'}(s_t, a_t) \nabla \log p_\theta(a_t^n | s_t^n) \right]$$

Loss function

$$J^{\theta'}(\theta) = E_{(s_t, a_t) \sim \pi_{\theta'}} \left[\frac{p_\theta(a_t | s_t)}{p_{\theta'}(a_t | s_t)} A^{\theta'}(s_t, a_t) \right]$$

Proximal policy optimization (PPO)

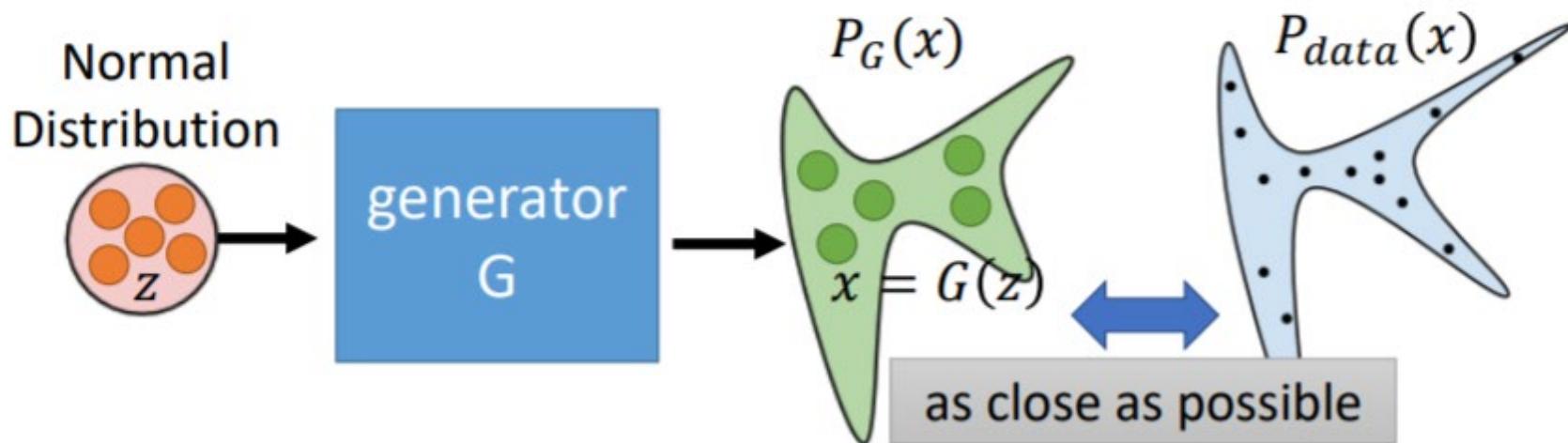
$$J_{PPO}^{\theta'}(\theta) = J^{\theta'}(\theta) - \beta KL(\theta, \theta')$$

$$J_{PPO2}^{\theta'}(\theta) = \sum_{(s_t, a_t)} \min \left(\frac{p_\theta(a_t | s_t)}{p_{\theta'}(a_t | s_t)} A^{\theta'}(s_t, a_t), \text{clip} \left(\frac{p_\theta(a_t | s_t)}{p_{\theta'}(a_t | s_t)}, 1 - \varepsilon, 1 + \varepsilon \right) A^{\theta'}(s_t, a_t) \right)$$

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347.

4. Use DNN to represent a more general probability model

- A generator G is a network. The network defines a probability distribution P_G



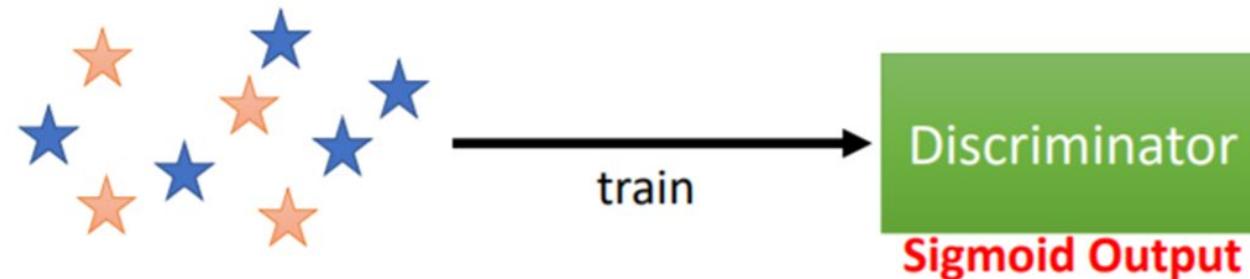
$$G^* = \arg \min_G \underline{Div(P_G, P_{data})}$$

Divergence between distributions P_G and P_{data}
How to compute the divergence?

5. Use DNN to represent a more general divergence measure

How to compute the divergence when a NN is used to represent a more general probability model? – train a discriminator NN to compute the divergence

- ★ : data sampled from P_{data}
- ★ : data sampled from P_G



Example Objective Function for D

$$V(G, D) = E_{x \sim P_{data}}[\log D(x)] + E_{x \sim P_G}[\log(1 - D(x))]$$

(G is fixed)

Training: $D^* = \arg \max_D V(D, G)$

[Goodfellow, et al., NIPS, 2014]

The objective function for D is related to JS divergence

$$V(D, G) = E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))]$$

- Given G , what is the optimal D^* maximizing

$$\begin{aligned} V &= E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))] \\ &= \int_x P_{data}(x) \log D(x) dx + \int_x P_G(x) \log(1 - D(x)) dx \\ &= \int_x [P_{data}(x) \log D(x) + P_G(x) \log(1 - D(x))] dx \end{aligned}$$

Assume that $D(x)$ can be any function

- Given x , the optimal D^* maximizing

$$P_{data}(x) \log D(x) + P_G(x) \log(1 - D(x))$$

The objective function for D is related to JS divergence

$$V(D, G) = E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))]$$

$$\begin{aligned}\max_D V(G, D) &= V(G, D^*) & D^*(x) &= \frac{P_{data}(x)}{P_{data}(x) + P_G(x)} \\&= -2\log 2 + \int_x P_{data}(x) \log \frac{P_{data}(x)}{(P_{data}(x) + P_G(x))/2} dx \\&\quad + \int_x P_G(x) \log \frac{P_G(x)}{(P_{data}(x) + P_G(x))/2} dx \\&= -2\log 2 + \text{KL}\left(P_{data} \parallel \frac{P_{data} + P_G}{2}\right) + \text{KL}\left(P_G \parallel \frac{P_{data} + P_G}{2}\right) \\&= -2\log 2 + 2JSD(P_{data} \parallel P_G) \quad \text{Jensen-Shannon divergence}\end{aligned}$$

The objective function for D is implemented as binary cross entropy

$$V(D, G) = E_{x \sim P_{data}}[\log D(x)] + E_{x \sim P_G}[\log(1 - D(x))]$$

Binary Classifier

D is a binary classifier with sigmoid output (can be deep)

$\{x^1, x^2, \dots, x^m\}$ from $P_{data}(x)$  Positive examples

$\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}$ from $P_G(x)$  Negative examples

Minimize Cross-entropy

Cross entropy:

$$C(f(x^n), \hat{y}^n) = -[\hat{y}^n \ln f(x^n) + (1 - \hat{y}^n) \ln(1 - f(x^n))]$$

(GAN) Two adversarial neural networks G and D

(1) Train generator

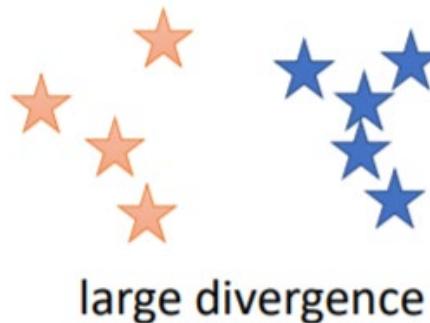
$$G^* = \arg \min_G \text{Div}(P_G, P_{\text{data}})$$



blue star : data sampled from P_{data}
orange star : data sampled from P_G

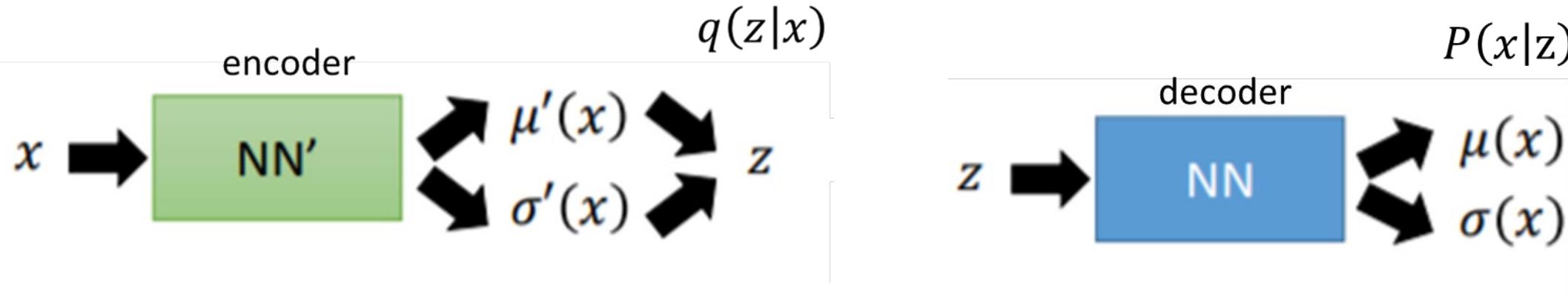
(2) Train discriminator

$$D^* = \arg \max_D V(D, G)$$

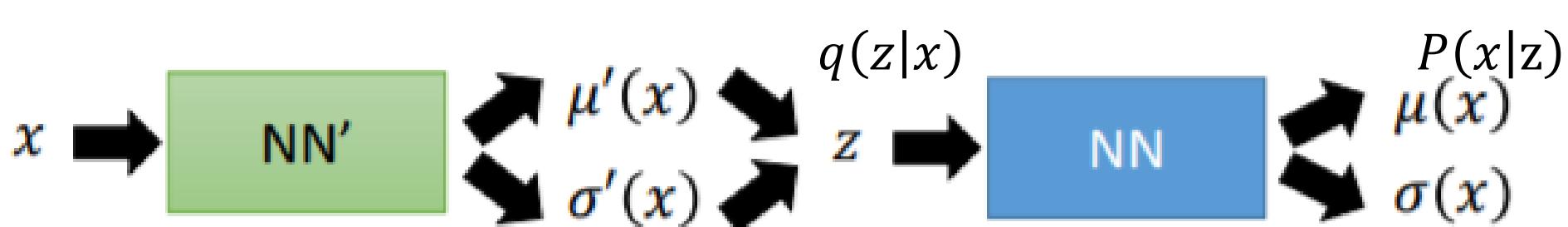


$$V(D, G) = E_{x \sim P_{\text{data}}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))]$$

(VAE) Two sequential neural networks, one encode and one decode

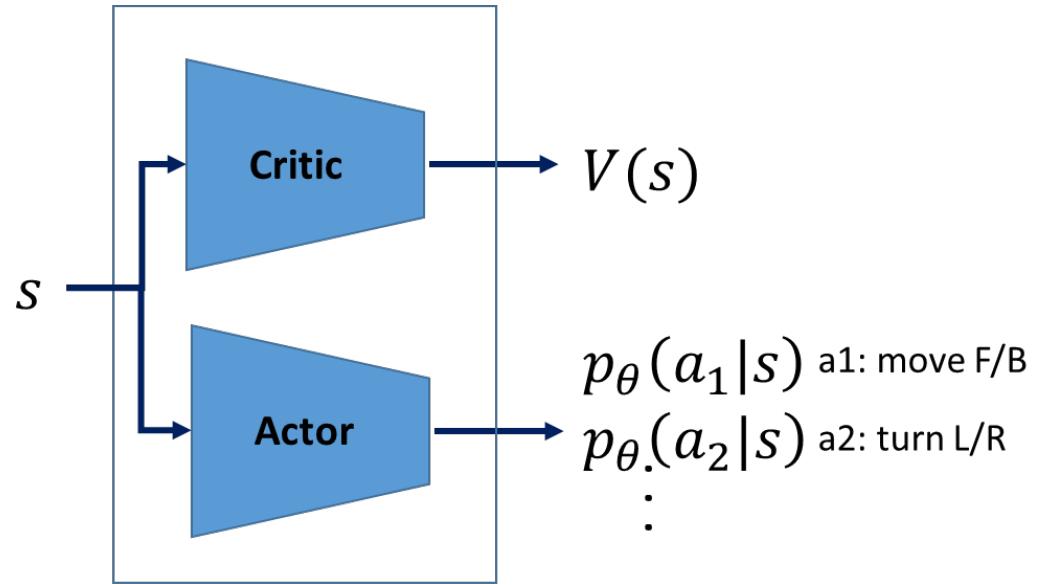


$$L = MSE(x, \hat{x}) + KL(q(z|x) || P(z))$$



(PPO-AC) Two parallel neural networks, one actor and one critic

$$L = c_v L_v + L_\pi - \beta L_{reg}$$



(1) Actor – Learns the best actions (that can have maximum long-term rewards)

$$L_\pi = \sum_{(s_t, a_t)} \min \left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t, a_t), \text{clip} \left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon \right) A^{\theta'}(s_t, a_t) \right)$$

(2) Critic – Learns the expected value of the long-term reward.

$$L_v = \text{MSE of (return} - v)$$

Algorithm

- Given G_0
- Find D_0^* maximizing $V(G_0, D)$

$V(G_0, D_0^*)$ is the JS divergence between $P_{data}(x)$ and $P_{G_0}(x)$

- $\theta_G \leftarrow \theta_G - \eta \partial V(G, D_0^*) / \partial \theta_G \rightarrow$ Obtain G_1
- Find D_1^* maximizing $V(G_1, D)$

Decrease JS divergence(?)

$V(G_1, D_1^*)$ is the JS divergence between $P_{data}(x)$ and $P_{G_1}(x)$

- $\theta_G \leftarrow \theta_G - \eta \partial V(G, D_1^*) / \partial \theta_G \rightarrow$ Obtain G_2
-

Decrease JS divergence(?)

$$G^* = \arg \min_G \max_D V(D, G)$$

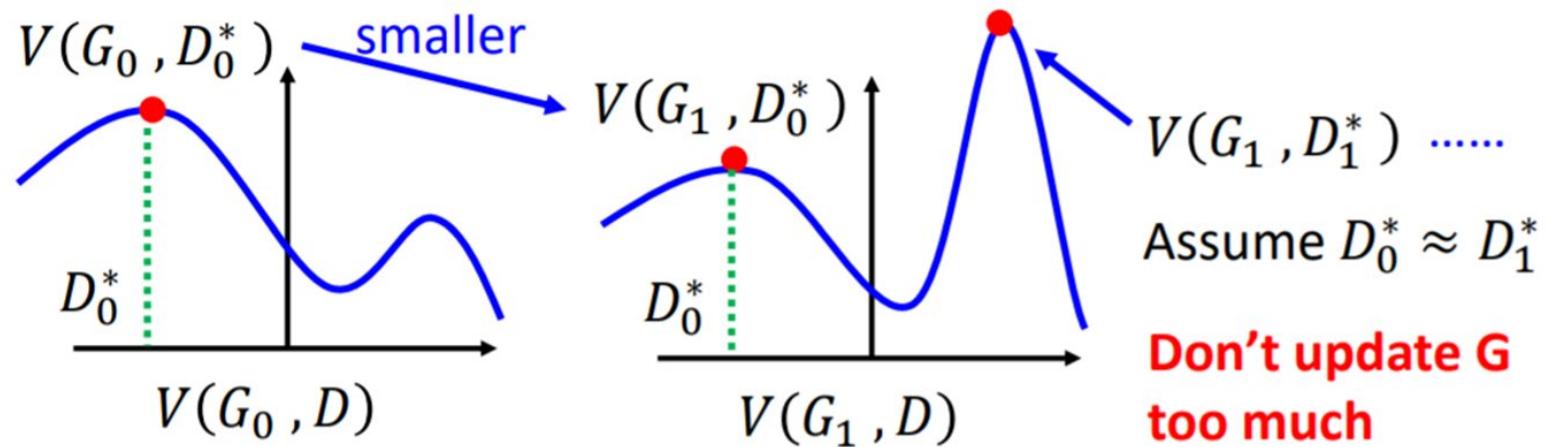
$$D^* = \arg \max_D V(D, G)$$

$$\begin{aligned} & V(D, G) \\ &= E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))] \end{aligned}$$

$$G^* = \arg \min_G \text{Div}(P_G, P_{data})$$

Do not update G too much...

When we update G from G_0 to G_1 , the divergence measurement function $V(G_0, D^*)$ will no longer fully represent the JS divergence between P_G and P_{data}



(PPO): We do not update $p_\theta(a_t|s_t)$ too much so that we can calculate $p_{\theta'}(a_t|s_t)$ off-policy

$$L_\pi = \sum_{(s_t, a_t)} \min \left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t, a_t), \text{clip} \left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1 - \varepsilon, 1 + \varepsilon \right) A^{\theta'}(s_t, a_t) \right)$$

But we should update D as much as possible

- In each training iteration:

- Sample m examples $\{x^1, x^2, \dots, x^m\}$ from data distribution $P_{data}(x)$
- Sample m noise samples $\{z^1, z^2, \dots, z^m\}$ from the prior $P_{prior}(z)$
- Obtaining generated data $\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}, \tilde{x}^i = G(z^i)$
- Update discriminator parameters θ_d to maximize
 - $\tilde{V} = \frac{1}{m} \sum_{i=1}^m \log D(x^i) + \frac{1}{m} \sum_{i=1}^m \log (1 - D(\tilde{x}^i))$
 - $\theta_d \leftarrow \theta_d + \eta \nabla \tilde{V}(\theta_d)$
- Sample another m noise samples $\{z^1, z^2, \dots, z^m\}$ from the prior $P_{prior}(z)$
- Update generator parameters θ_g to minimize
 - $\tilde{V} = \frac{1}{m} \sum_{i=1}^m \log D(x^i) + \frac{1}{m} \sum_{i=1}^m \log (1 - D(G(z^i)))$
 - $\theta_g \leftarrow \theta_g - \eta \nabla \tilde{V}(\theta_g)$

Learning
D

Repeat
k times

Learning
G

Only
Once

In [19]: `def train_discriminator(real_images, opt_d):`

```
# Clear discriminator gradients
opt_d.zero_grad()
```

For loop for k times?

```
# Pass real images through discriminator
real_preds = discriminator(real_images)
real_targets = torch.ones(real_images.size(0), 1, device)
real_loss = F.binary_cross_entropy(real_preds, real_targets)
real_score = torch.mean(real_preds).item()
```

Generate fake images

```
latent = torch.randn(batch_size, latent_size, 1, 1, device)
fake_images = generator(latent.to(device))
```

Pass fake images through discriminator

```
fake_targets = torch.zeros(fake_images.size(0), 1, device)
fake_preds = discriminator(fake_images)
fake_loss = F.binary_cross_entropy(fake_preds, fake_targets)
fake_score = torch.mean(fake_preds).item()
```

Update discriminator weights

```
loss = real_loss + fake_loss
loss.backward()
opt_d.step()
```

```
return loss.item(), real_score, fake_score
```