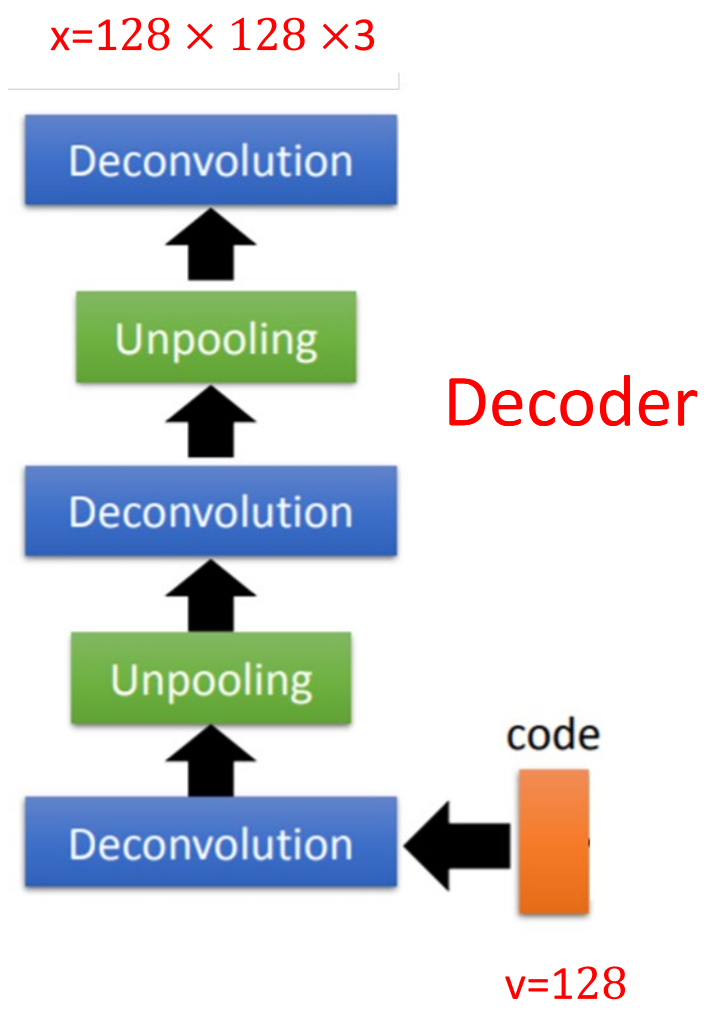
Generative Adversarial Network (GAN)

Practice

- Open "8.1. GAN.ipynb"

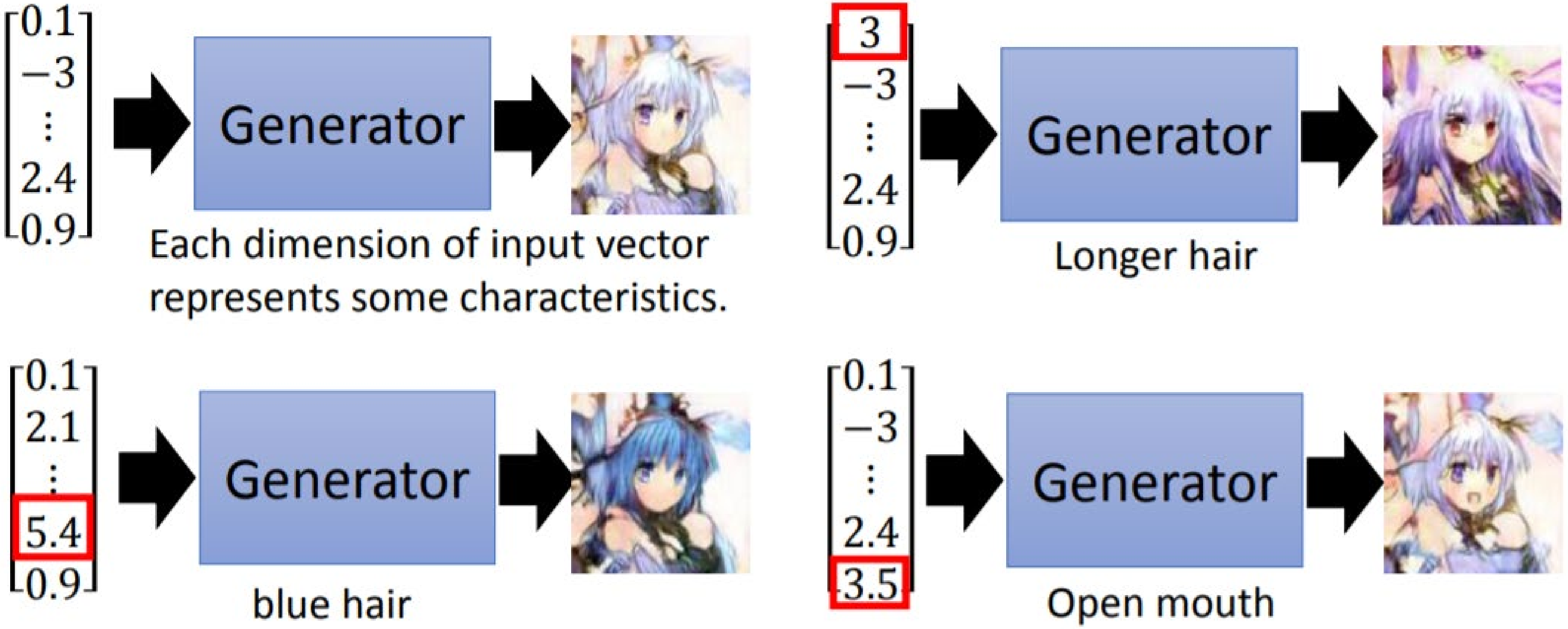
Generator




```
[10]: latent_size= 128
```

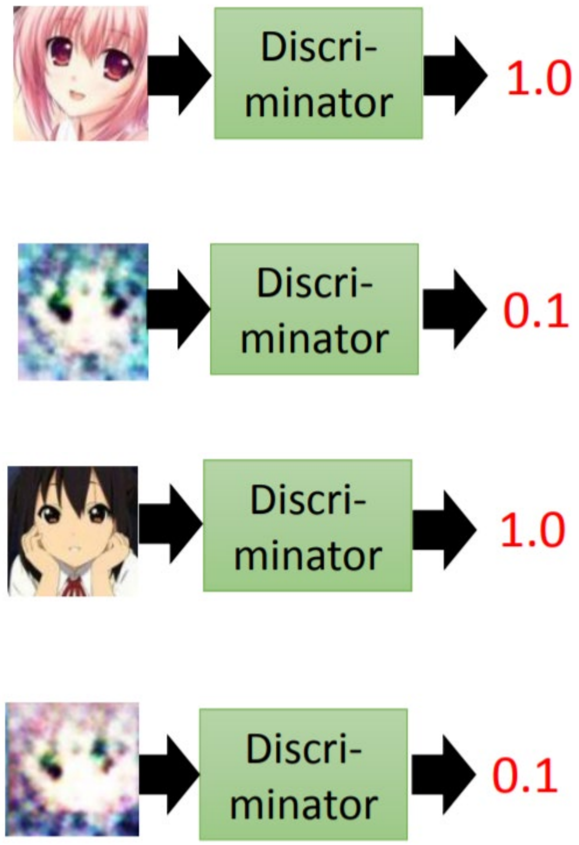
```
[11]: generator = nn.Sequential(
          # in: latent_size x 1 x 1

          nn.ConvTranspose2d(latent_size, 512, kernel_size=4, stride=1,
          nn.BatchNorm2d(512),
          nn.ReLU(True),
          # out: 512 x 4 x 4

          nn.ConvTranspose2d(512, 256, kernel_size=4, stride=2, padding=
          nn.BatchNorm2d(256),
          nn.ReLU(True),
          # out: 256 x 8 x 8

          nn.ConvTranspose2d(256, 128, kernel_size=4, stride=2, padding=
          nn.BatchNorm2d(128),
          nn.ReLU(True),
          # out: 128 x 16 x 16

          nn.ConvTranspose2d(128, 64, kernel_size=4, stride=2, padding=1
          nn.BatchNorm2d(64),
          nn.ReLU(True),
          # out: 64 x 32 x 32

          nn.ConvTranspose2d(64, 32, kernel_size=4, stride=2, padding=1,
          nn.BatchNorm2d(32),
          nn.ReLU(True),
          # out: 32 x 64 x 64

          nn.ConvTranspose2d(32, 3, kernel_size=4, stride=2, padding=1,
          nn.Tanh()
          # out: 3 x 128 x 128
      )
```

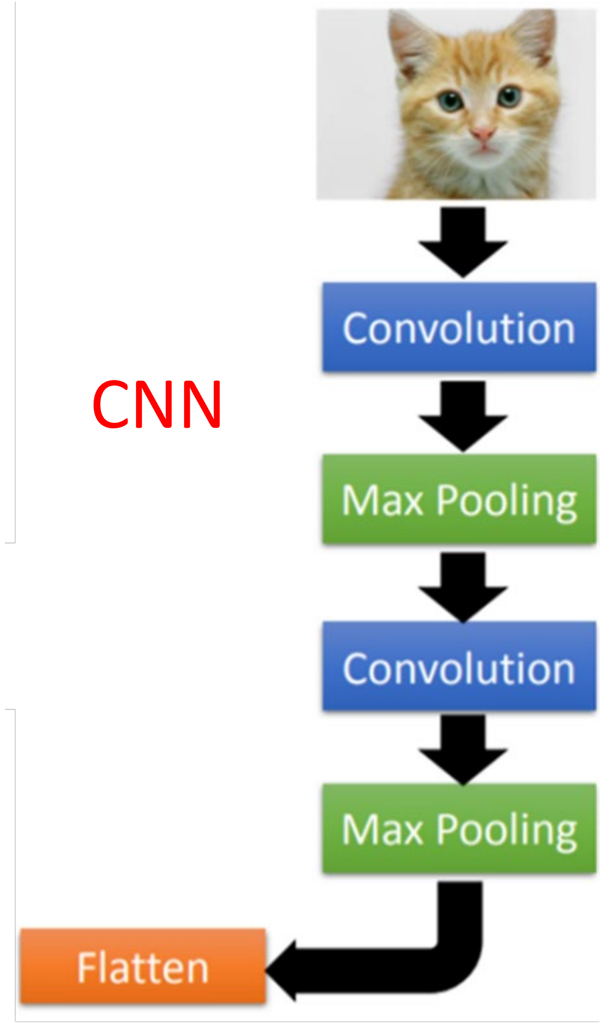
Each dimension in the feature vector represent a property of the output image



Reference: 李弘毅 GAN Lecture 1 (2018): https://youtu.be/DQNNMiAP5lw

Discriminator





[15]:
```
discriminator = nn.Sequential(
    # in: 3 x 128 x 128

    nn.Conv2d(3, 64, kernel_size=4, stride=2, p
    nn.BatchNorm2d(64),
    nn.LeakyReLU(0.2, inplace=True),
    # out: 64 x 64 x 64

    nn.Conv2d(64, 128, kernel_size=4, stride=2,
    nn.BatchNorm2d(128),
    nn.LeakyReLU(0.2, inplace=True),
    # out: 128 x 32 x 32

    nn.Conv2d(128, 256, kernel_size=4, stride=2
    nn.BatchNorm2d(256),
    nn.LeakyReLU(0.2, inplace=True),
    # out: 256 x 16 x 16

    nn.Conv2d(256, 512, kernel_size=4, stride=2
    nn.BatchNorm2d(512),
    nn.LeakyReLU(0.2, inplace=True),
    # out: 512 x 8 x 8

    nn.Conv2d(512, 1024, kernel_size=4, stride=
    nn.BatchNorm2d(1024),
    nn.LeakyReLU(0.2, inplace=True),
    # out: 1024 x 4 x 4

    nn.Conv2d(1024, 1, kernel_size=4, stride=1,
    # out: 1 x 1 x 1

    nn.Flatten(),
    nn.Sigmoid())
```

Reference: 李弘毅 GAN Lecture 1 (2018)

Step1 – Fix G and train D

- Initialize generator and discriminator G D
- In each training iteration:

Step 1: Fix generator G, and update discriminator D

```
[12]: generator.to(device)
```

```
[16]: discriminator.to(device)
```

```
[35]: for epoch in range(epochs):
        if(epoch % 10 ==0):
          print(epoch, end=",")
        for real_images, _ in train_dl:
          # Train discriminator
          loss_d, real_score, fake_score = train_discriminator(real_images.
          # Train generator
          loss_g = train_generator(opt_g)
```

Reference: 李弘毅 GAN Lecture 1 (2018): https://youtu.be/DQNNMiAP5lw

Train discriminator D

- Sample m examples $\{x^1, x^2, \dots, x^m\}$ from database
- Sample m noise samples $\{z^1, z^2, \dots, z^m\}$ from a distribution
- Obtaining generated data $\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}$, $\tilde{x}^i = G(z^i)$

$$D(x^i) \rightarrow 1,\ \log D(x^i) \rightarrow 0$$

$$D(\tilde{x}^i) \rightarrow 0,\ \log(1 - D(\tilde{x}^i)) \rightarrow 0$$

log(0.xxx) <0

Update discriminator parameters θ_d to maximize

- $\tilde{V} = \frac{1}{m}\sum_{i=1}^{m} \underline{logD(x^i)} + \frac{1}{m}\sum_{i=1}^{m} \underline{log\left(1 - D(\tilde{x}^i)\right)}$
- $\theta_d \leftarrow \theta_d \underline{+} \eta \nabla \tilde{V}(\theta_d)$

[19]:
```
def train_discriminator(real_images, opt_d):
    # Clear discriminator gradients
    opt_d.zero_grad()

    # Pass real images through discriminator
    real_preds = discriminator(real_images)
    real_targets = torch.ones(real_images.size(0)
    real_loss = F.binary_cross_entropy(real_preds
    real_score = torch.mean(real_preds).item()

    # Generate fake images
    latent = torch.randn(batch_size, latent_size,
    fake_images = generator(latent.to(device))

    # Pass fake images through discriminator
    fake_targets = torch.zeros(fake_images.size(0
    fake_preds = discriminator(fake_images)
    fake_loss = F.binary_cross_entropy(fake_preds
    fake_score = torch.mean(fake_preds).item()

    # Update discriminator weights
    loss = real_loss + fake_loss
    loss.backward()
    opt_d.step()
    return loss.item(), real_score, fake_score
```

Step2 – Fix D and train G

Step 2: Fix discriminator D, and update generator G

Generator learns to "fool" the discriminator

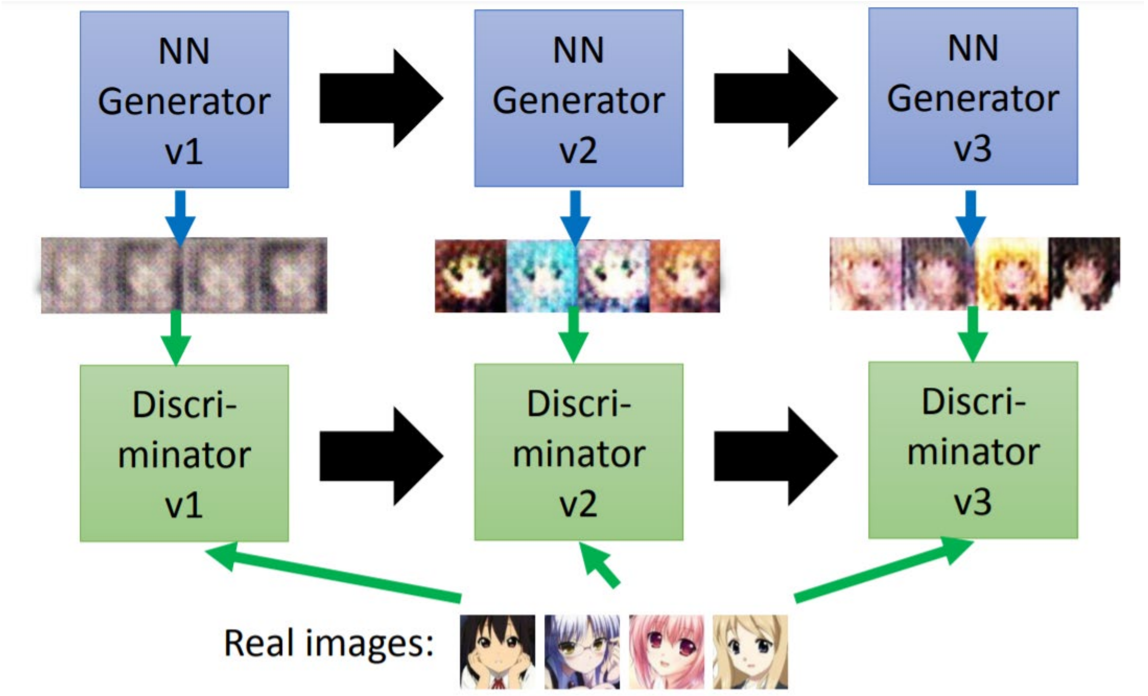
- Sample m noise samples$\{z^1, z^2, \ldots, z^m\}$ from a distribution
- Update generator parameters θ_g to maximize
  - $\tilde{V} = \frac{1}{m}\sum_{i=1}^{m} log\left(D\left(G\left(z^i\right)\right)\right)$
  - $\theta_g \leftarrow \theta_g - \eta \nabla \tilde{V}(\theta_g)$

[35]:
```
for epoch in range(epochs):
  if(epoch % 10 ==0):
    print(epoch, end=",")
  for real_images, _ in train_dl:
    # Train discriminator
    loss_d, real_score, fake_score =
    # Train generator
    loss_g = train_generator(opt_g)
```

[28]:
```
def train_generator(opt_g):
    # Clear generator gradients
    opt_g.zero_grad()

    # Generate fake images
    latent = torch.randn(batch_size, latent_size,
    fake_images = generator(latent)

    # Try to fool the discriminator
    preds = discriminator(fake_images)
    targets = torch.ones(batch_size, 1, device=dev
    loss = F.binary_cross_entropy(preds, targets)

    # Update generator weights
    loss.backward()
    opt_g.step()

    return loss.item()
```

An evolution process

```
[35]: for epoch in range(epochs):
        if(epoch % 10 ==0):
          print(epoch, end=",")
        for real_images, _ in train_dl:
          # Train discriminator
          loss_d, real_score, fake_score = train_discriminator(real_images
          # Train generator
          loss_g = train_generator(opt_g)
```



Reference: 李弘毅 GAN Lecture 1 (2018): https://youtu.be/DQNNMiAP5lw

Visualize fake images generated by *G* during the evolution process

[17]:
```
sample_dir = 'generated'
os.makedirs(sample_dir, exist_ok=True)
```

[34]:
```
fixed_latent = torch.randn(64, latent_size,
# used to generate saved images
```

```
if(epoch % 50 ==0):
  # Log losses & scores (last batch)
  print("Epoch [{}/{}], loss_g: {:.4f}, loss_d:
       epoch+1, epochs, loss_g, loss_d, real_sco
  # Save generated images
  save_samples(epoch+start_idx, fixed_latent, s
```

[18]:
```
def save_samples(index, latent_tensors, show=True):
  fake_images = generator(latent_tensors)
  fake_fname = 'generated-images-{0:0=4d}.png'.format(index)
  save_image(denorm(fake_images), os.path.join(sample_dir, f
  print('Saving', fake_fname)
  if show:
    fig, ax = plt.subplots(figsize=(8, 8))
    ax.set_xticks([]); ax.set_yticks([])
    ax.imshow(make_grid(fake_images.cpu().detach(), nrow=8).
```


8.1. GAN.ipynb
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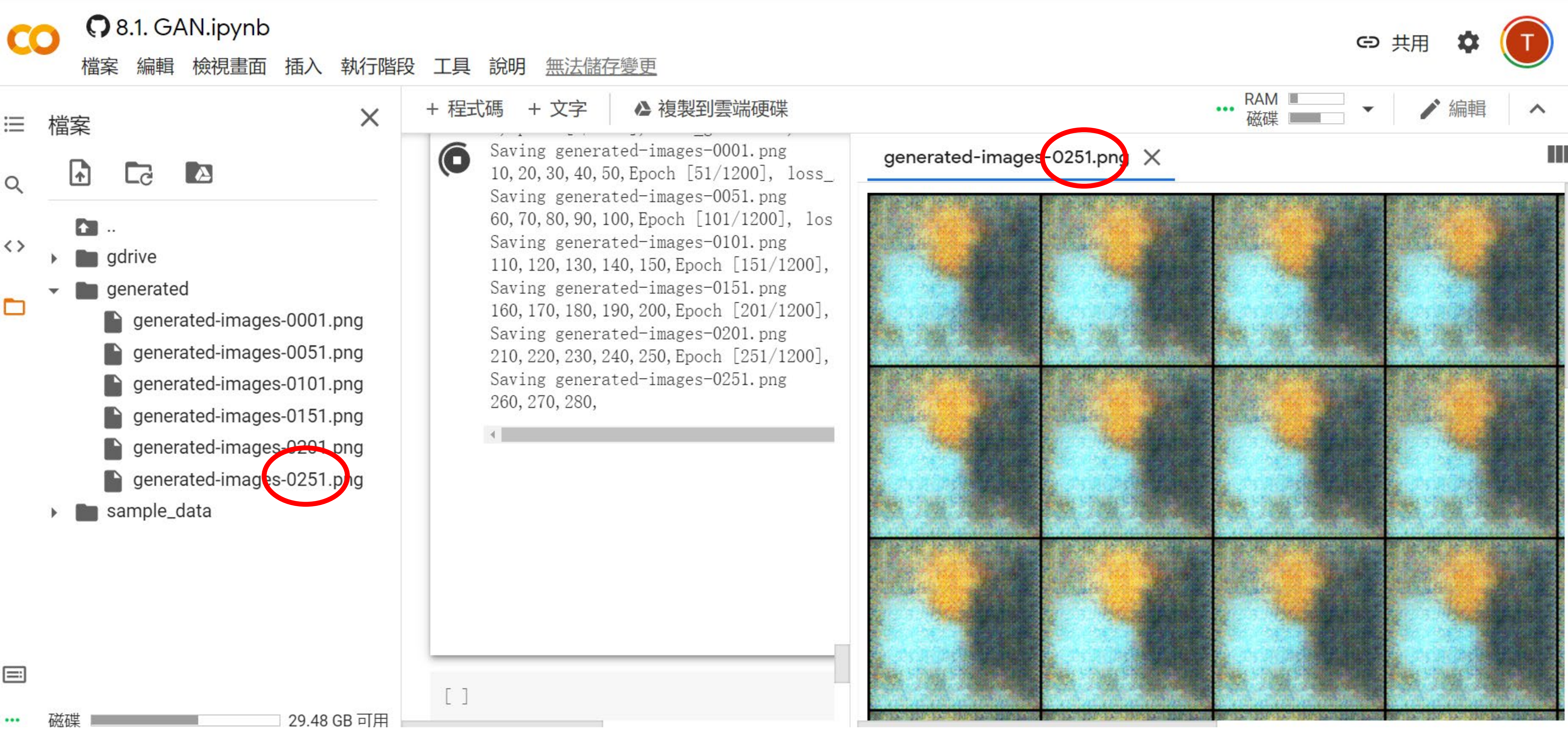
sample_data
Log losses & scores (l
print("Epoch [{}/{}], loss_
epoch+1, epochs, lo
# Save generated images
latent = torch.randn(batch_
save_samples(epoch+start_idx,
#save_samples(epoch+start_idx
0,Epoch [1/1200], loss_g: 5.6803, los
Saving generated-images-0001.png
10,20,30,40,50,Epoch [51/1200], loss_
Saving generated-images-0051.png
60,70,80,90,100,Epoch [101/1200], los
Saving generated-images-0101.png
110,120,130,140,150,Epoch [151/1200],
Saving generated-images-0151.png
160,170,180,190,200,Epoch [201/1200],
Saving generated-images-0201.png
210,220,
generated-images-0201.png
generated-images-0001.png



8.1. GAN.ipynb
檔案 編輯 檢視畫面 插入 執行階段 工具 說明 無法儲存變更
共用
+ 程式碼 + 文字 複製到雲端硬碟
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Saving generated-images-0001.png
10, 20, 30, 40, 50, Epoch [51/1200], loss_
Saving generated-images-0051.png
60, 70, 80, 90, 100, Epoch [101/1200], los
Saving generated-images-0101.png
110, 120, 130, 140, 150, Epoch [151/1200],
Saving generated-images-0151.png
160, 170, 180, 190, 200, Epoch [201/1200],
Saving generated-images-0201.png
210, 220, 230, 240, 250, Epoch [251/1200],
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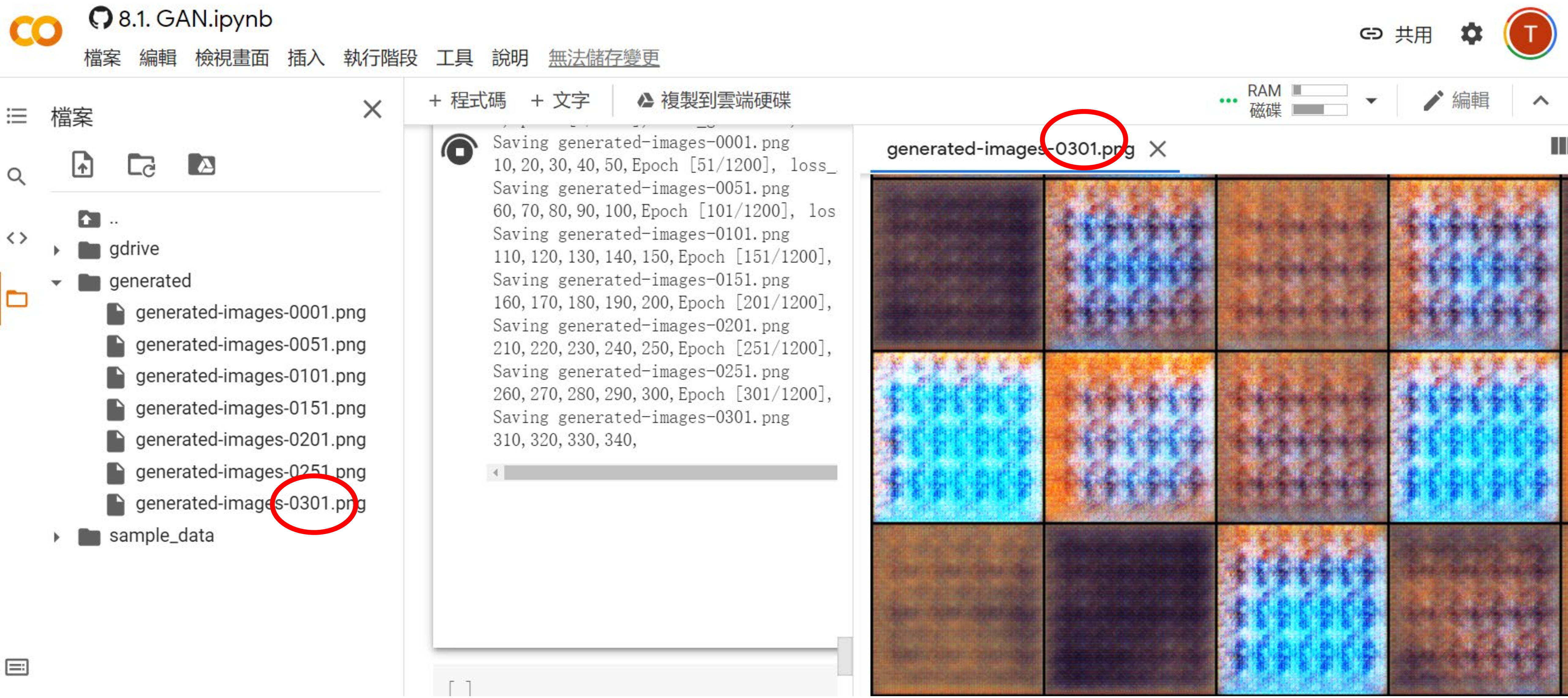
260, 270, 280,
generated-images-0251.png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generated-images-0001.png
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generated-images-0151.png
generated-images-0201.png
generated-images-0251.png
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60, 70, 80, 90, 100, Epoch [101/1200], los
Saving generated-images-0101.png
110, 120, 130, 140, 150, Epoch [151/1200],
Saving generated-images-0151.png
160, 170, 180, 190, 200, Epoch [201/1200],
Saving generated-images-0201.png
210, 220, 230, 240, 250, Epoch [251/1200],
Saving generated-images-0251.png
260, 270, 280, 290, 300, Epoch [301/1200],
Saving generated-images-0301.png
310, 320, 330, 340,
generated-images-0301.png

Epoch =41201

34151

28651

25901

23201

16401

15551

14301

Combine image frames into a video

[37]:
```
img_array = []
for filename in files:
    img = cv2.imread(filename)
    height, width, layers = img.shape
    size = (width,height)
    img_array.append(img)

out = cv2.VideoWriter('GANTrainingVideo.avi',cv2.VideoWriter_fourcc(*'DIVX'), 15, size)

for i in range(len(img_array)):
    out.write(img_array[i])
out.release()
```

Tom & Jerry video https://youtu.be/uDEGITFmhiQ

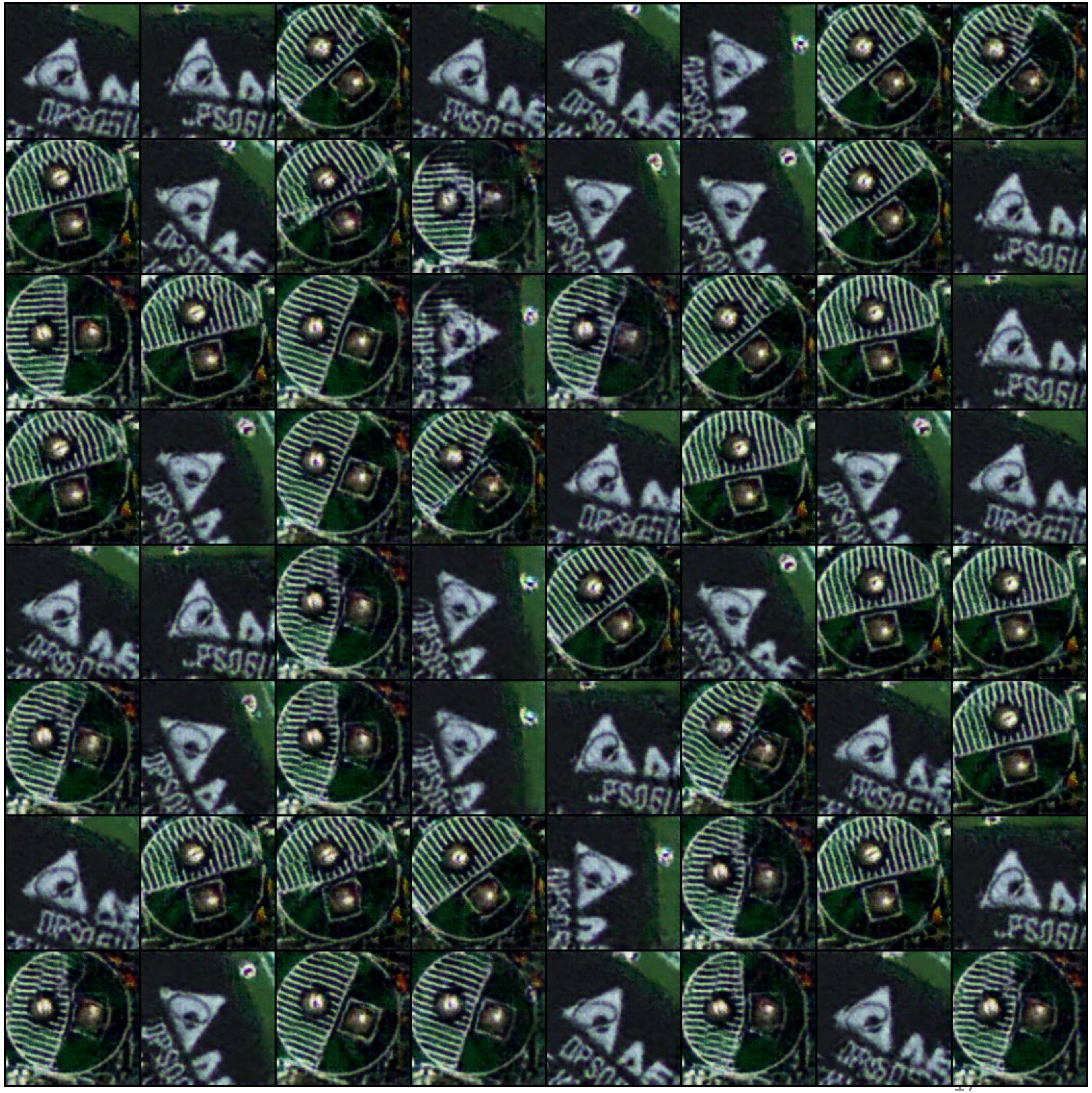
HW7 (1)

- Use your own images, e.g., facial expression to train a GAN.
- Show the generated images.
- Try latent vectors like (all 1), (all 0.5), (all 0.3), (0, 0, 1, 0, ….), (one dimension goes from 0 to 1 and other dimensions fixed), … to see the generated images.

Images generated from *G* after 5651 epochs

Image size = 180x180x3
Class 1 = 100, Class 2 = 100
Latent vector size = 128
batch=32

Run on 2080Ti, takes about 15 min.



Images generated from *G* after 3051 epochs

Image size = 128x128x3
Class 1 = 10, Class 2 = 10
Latent vector size = 128
batch=128

Run on Colab Telsa T4, takes about 60 min.

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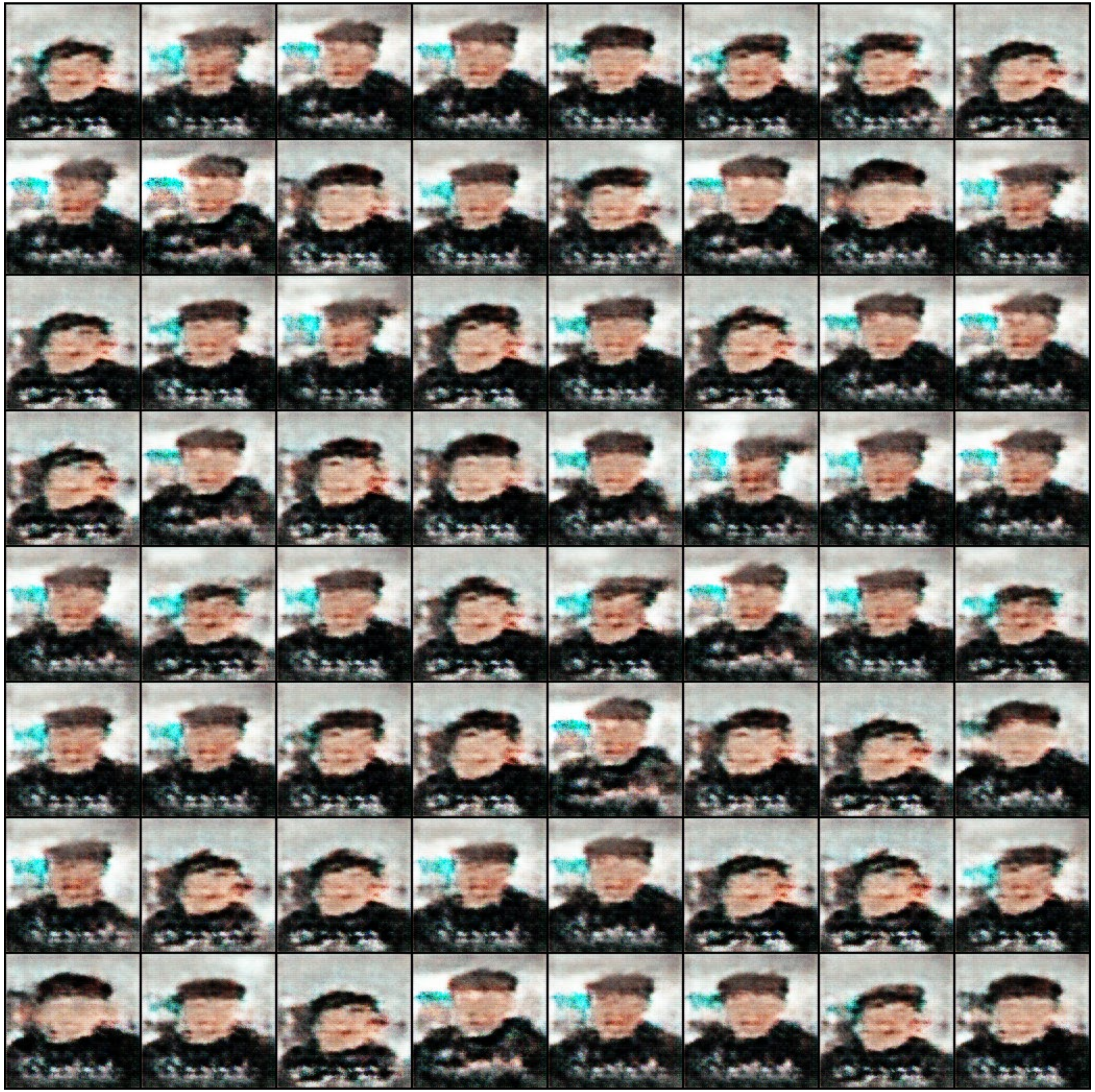
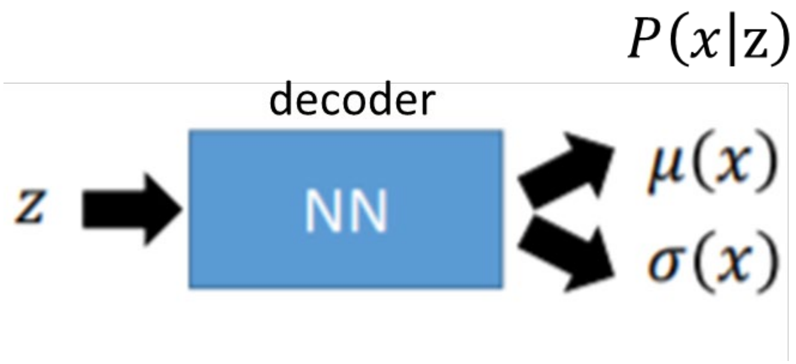


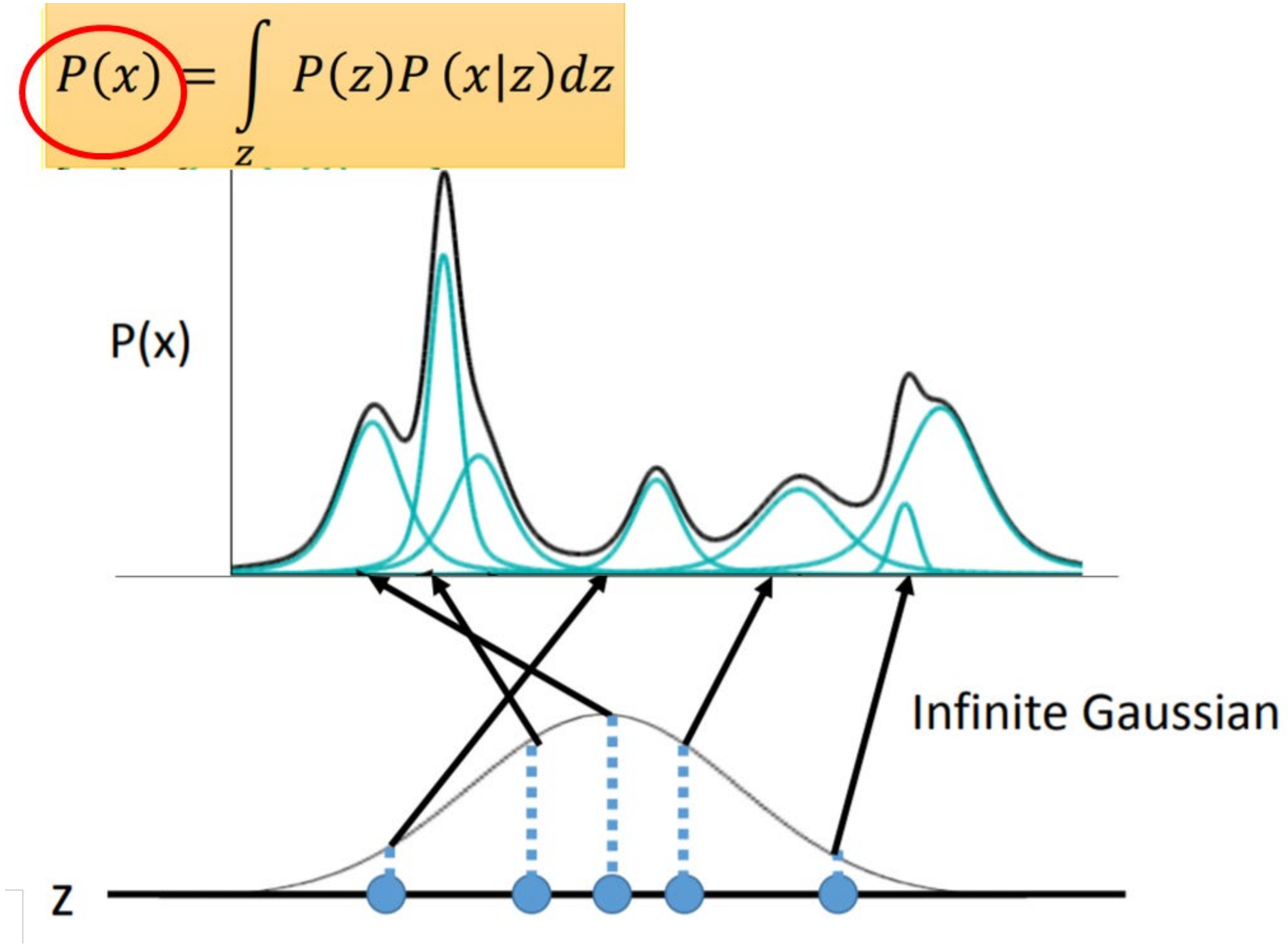
Theory behind GAN

The concept of deep generative model

1. Use probability distribution model to model the image generation process or the state-action generation process.
2. With this probability model, we want to maximize the likelihood to observe the training data or to observe trajectories with higher long-term rewards.
3. Maximum likelihood can be achieved by minimizing KL divergence.
3. Use DNN to represent a more general probability model.
4. Use DNN to represent a more general divergence measure.

(VAE) 1. Modelling the image generation process as a mixed Gaussian distribution model





Reference: 李弘毅 ML Lecture 18 https://youtu.be/8zomhgKrsmQ

(VAE) 2. Given the image generation model, we want to maximize the likelihood to observe the training images

<u>***Maximizing Likelihood***</u>

$$P(x) = \int_z P(z)P(x|z)dz$$

P(z) is normal distribution

$x|z \sim N(\mu(z), \sigma(z))$

$\mu(z), \sigma(z)$ is going to be estimated

$$L = \sum_x logP(x)$$

Maximizing the likelihood of the observed x

$$L = p(x^1) \times p(x^2) \times p(x^3) \times \cdots p(x^m) = \prod_{i=1,\ldots,m} P(x^i)$$

Reference: 李弘毅 ML Lecture 18 https://youtu.be/8zomhgKrsmQ

(VAE) 3. Maximize the likelihood can be achieved by minimizing KL divergence

Max. L_b can be done by min. $KL(q(z|x)||P(z))$ and max. $\int_z q(z|x)logP(x|z)dz$. That is why loss = KLD + MSE (x, $\hat{x}$)

Minimize

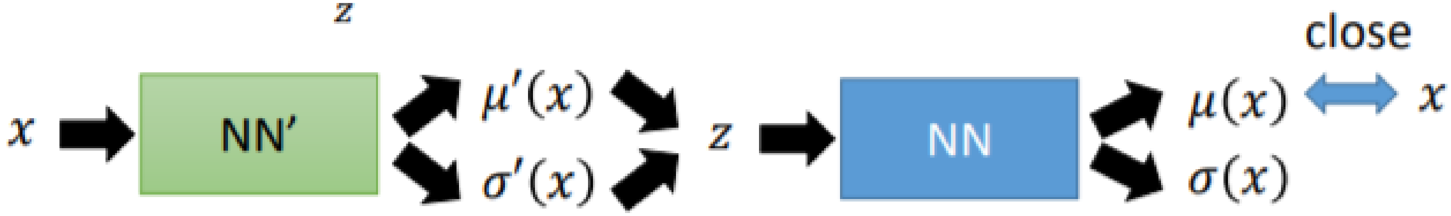
$$\sum_{i=1}^{3}(exp(\sigma_i) - (1 + \sigma_i) + (m_i)^2)$$

Minimizing $KL(q(z|x)||P(z))$

(Refer to the Appendix B of the original VAE paper)

Maximizing

$$\int_z q(z|x)logP(x|z)dz = E_{q(z|x)}[logP(x|z)]$$

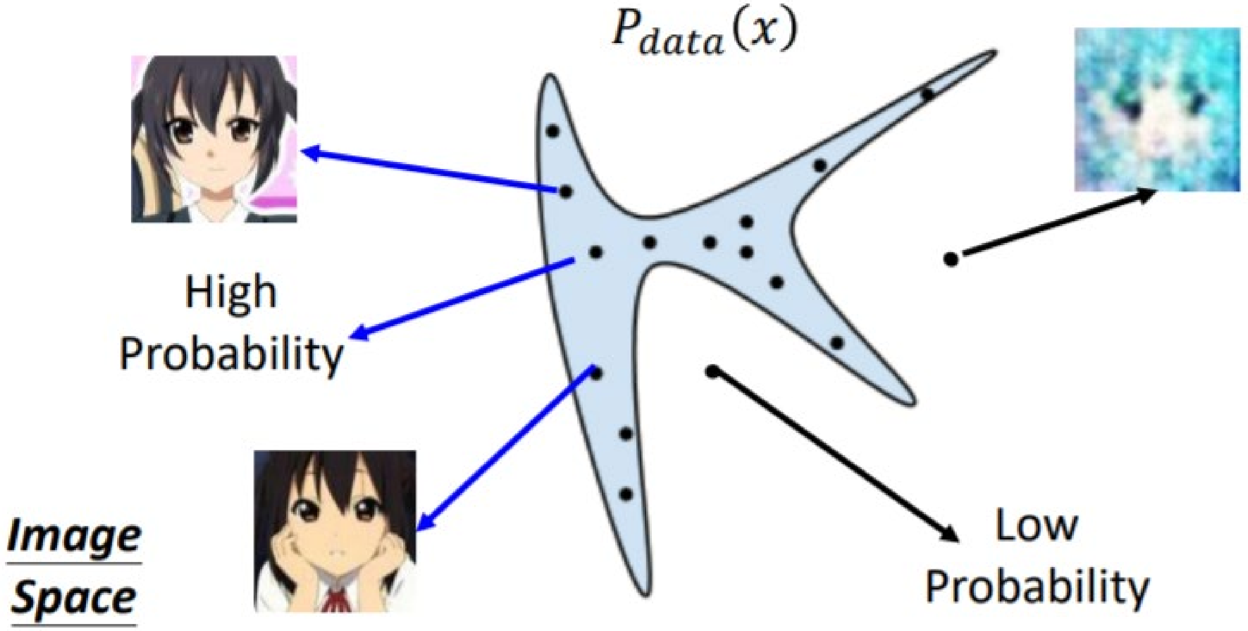


by min. $MSE(x, \hat{x})$

(GAN) 1. Modelling the generator as a probability distribution model P_G

X: an image (a high-dimensional vector)

- We want to find data distribution $P_{data}(x)$



Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

(GAN) 2. Given the image generation model, we want to maximize the likelihood to observe the training images

- Given a data distribution $P_{data}(x)$ (We can sample from it.)
- We have a distribution $P_G(x;\theta)$ parameterized by θ
  - We want to find θ such that $P_G(x;\theta)$ close to $P_{data}(x)$
  - E.g. $P_G(x;\theta)$ is a Gaussian Mixture Model, θ are means and variances of the Gaussians

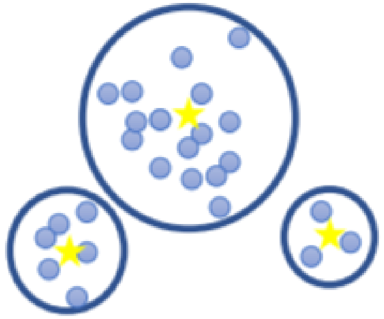
Sample $\{x^1, x^2, \dots, x^m\}$ from $P_{data}(x)$

We can compute $P_G(x^i;\theta)$

Likelihood of generating the samples

$$L = \prod_{i=1}^{m} P_G(x^i;\theta)$$

Find θ^* maximizing the likelihood



Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

(GAN) 3. Maximize the likelihood can be achieved by minimizing the KL divergence

Maximum likelihood estimation = minimum KL divergence

$$\theta^* = arg\max_{\theta}\prod_{i=1}^{m} P_G(x^i;\theta) = arg\min_{\theta} KL(P_{data}||P_G)$$

$$\theta^* = arg\max_{\theta}\prod_{i=1}^{m} P_G(x^i;\theta) = arg\max_{\theta} log\prod_{i=1}^{m} P_G(x^i;\theta)$$

$$= arg\max_{\theta}\sum_{i=1}^{m} logP_G(x^i;\theta)$$

$\{x^1, x^2, \dots, x^m\}$ from $P_{data}(x)$

$$\approx arg\max_{\theta} E_{x\sim P_{data}}[logP_G(x;\theta)]$$

$$= arg\max_{\theta}\int_x P_{data}(x)logP_G(x;\theta)dx - \int_x P_{data}(x)logP_{data}(x)dx$$

$$= arg\min_{\theta} KL(P_{data}||P_G)$$

How to define a general P_G?

$$\int_x P_{data}(x)logP_{data}(x)dx = logP_{data}(x)$$

$$D_{KL}(q||p) = \sum_{i=1}^{N} q(x_i)\log(\frac{q(x_i)}{p(x_i)})$$

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

(Classification) 1. Modelling classification using Bayesian's rule

$$f_{\mu^1,\Sigma^1}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma^1|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu^1)^T(\Sigma^1)^{-1}(x-\mu^1)\right\}$$

$$\boxed{\mu^1 = \begin{bmatrix} 75.0 \\ 71.3 \end{bmatrix} \quad \Sigma^1 = \begin{bmatrix} 874 & 327 \\ 327 & 929 \end{bmatrix}}$$

P(C1)
= 79 / (79 + 61) =0.56

Assuming x^n are sampled from a Gaussian distribution, then we can use maximum likelihood to find the best Gaussian distribution behind them.

$$P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)}$$

$$f_{\mu^2,\Sigma^2}(x) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma^2|^{1/2}} exp\left\{-\frac{1}{2}(x-\mu^2)^T(\Sigma^2)^{-1}(x-\mu^2)\right\}$$

$$\boxed{\mu^2 = \begin{bmatrix} 55.6 \\ 59.8 \end{bmatrix} \quad \Sigma^2 = \begin{bmatrix} 847 & 422 \\ 422 & 685 \end{bmatrix}}$$

P(C2)
= 61 / (79 + 61)
=0.44

If $P(C_1|x) > 0.5$ ➡ x belongs to class 1

Reference: 李弘毅 ML Lecture 4 https://youtu.be/fZAZUYEeIMg

(Classification) 2. Maximize the likelihood to observe the training data

Assuming the training data is generated from $y_1 = P_{w,b}(C_1 | x) = \sigma(w \cdot x + b)$, what is the probability of generating the data?

Training Data	x^1	x^2	x^3	……	x^N
	C_1	C_1	C_2		C_1

$$L(w,b) = f_{w,b}(x^1) f_{w,b}(x^2) \left(1 - f_{w,b}(x^3)\right) \cdots f_{w,b}(x^N)$$

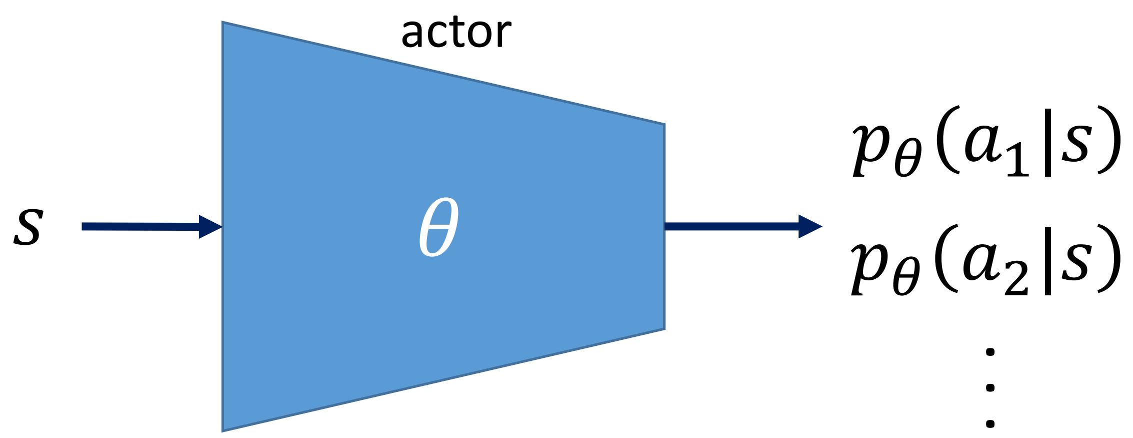
$$-lnL(w,b) = lnf_{w,b}(x^1) + lnf_{w,b}(x^2) + ln\left(1 - f_{w,b}(x^3)\right) \cdots$$

$\hat{y}^n$: 1 for class 1, 0 for class 2

$$= \sum_n -\underbrace{\left[\hat{y}^n lnf_{w,b}(x^n) + (1-\hat{y}^n) ln\left(1 - f_{w,b}(x^n)\right)\right]}_{\text{Cross entropy between two Bernoulli distribution}}$$

Reference: 李弘毅 ML Lecture 5 https://youtu.be/hSXFuypLukA

(PPO) 1. Modelling the state-action mapping network as $p_\theta(a|s)$



(PPO) 2. Given the probability model, we want to maximize the likelihood to observe trajectories with higher long-term rewards

$$\tau = \left(s_1, a_1, r_1, s_2, a_2, r_{2,} \cdots s_{\mathrm{T}}, a_{\mathrm{T}}\right)$$

$$p_\theta(\tau) = p(s_1) p_\theta(a_1|s_1) p(s_2|s_1, a_1) p_\theta(a_2|s_2) p(s_3|s_2, a_2) \cdots$$

$$\bar{R}_\theta = \sum R(\tau)\, p_\theta(\tau) = E_{\tau \sim p_\theta(\tau)}[R(\tau)] \qquad \mathrm{R}(\tau) = \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{r}_{\mathrm{t}}$$

$$\max_\theta E[\bar{R}_\theta]$$

$$\nabla \bar{R}_\theta = \sum R(\tau)\, \nabla p_\theta(\tau) = E_{\tau \sim p_\theta(\tau)}[R(\tau) \nabla \log p_\theta(\tau)] \approx \frac{1}{N} \sum_{n=1}^{N} R(\tau^n) \nabla \log p_\theta(\tau^n)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p_\theta(a_t^n | s_t^n)$$

(PPO) 3. To facilitate off-policy sampling, we minimize the KL divergence

Off-policy

$$\nabla \bar{R}_\theta = E_{(s_t,a_t)\sim\pi_{\theta'}}\left[\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t,a_t) \nabla \log p_\theta(a_t^n|s_t^n)\right]$$

Loss function

$$\mathrm{J}^{\theta'}(\theta) = E_{(s_t,a_t)\sim\pi_{\theta'}}\left[\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t,a_t)\right]$$

Proximal policy optimization (PPO)

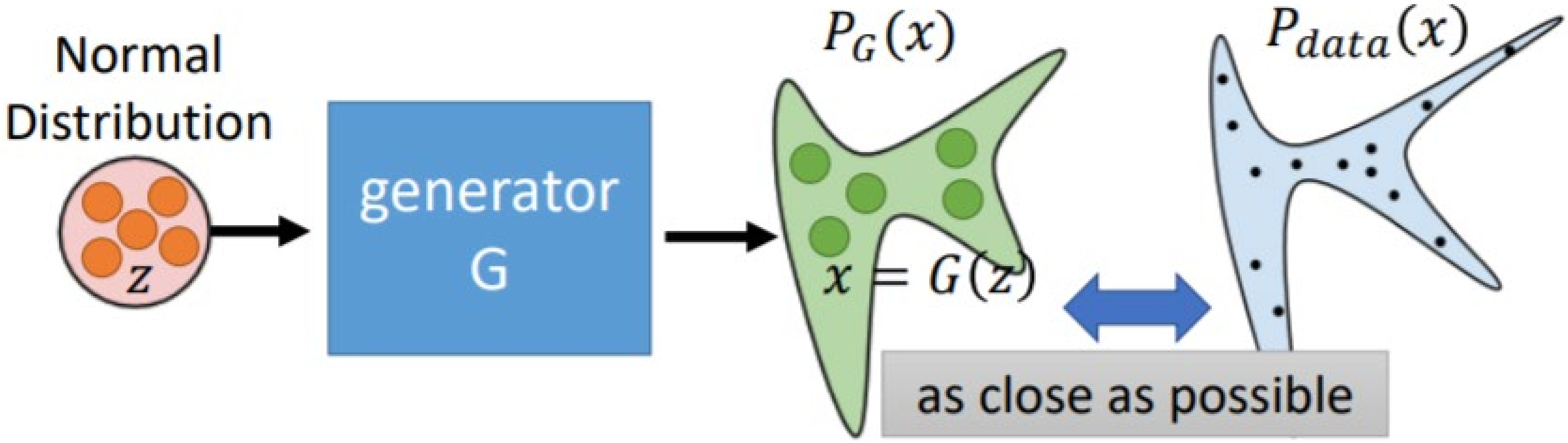
$$J_{PPO}^{\theta'}(\theta) = J^{\theta'}(\theta) - \beta KL(\theta,\theta')$$

$$J_{PPO2}^{\theta'}(\theta) = \sum_{(s_t,a_t)} min\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t,a_t), clip\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon\right) A^{\theta'}(s_t,a_t)\right)$$

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347.

4. Use DNN to represent a more general probability model

- A generator G is a network. The network defines a probability distribution P_G



$$G^* = arg \min_G \underline{Div(P_G, P_{data})}$$

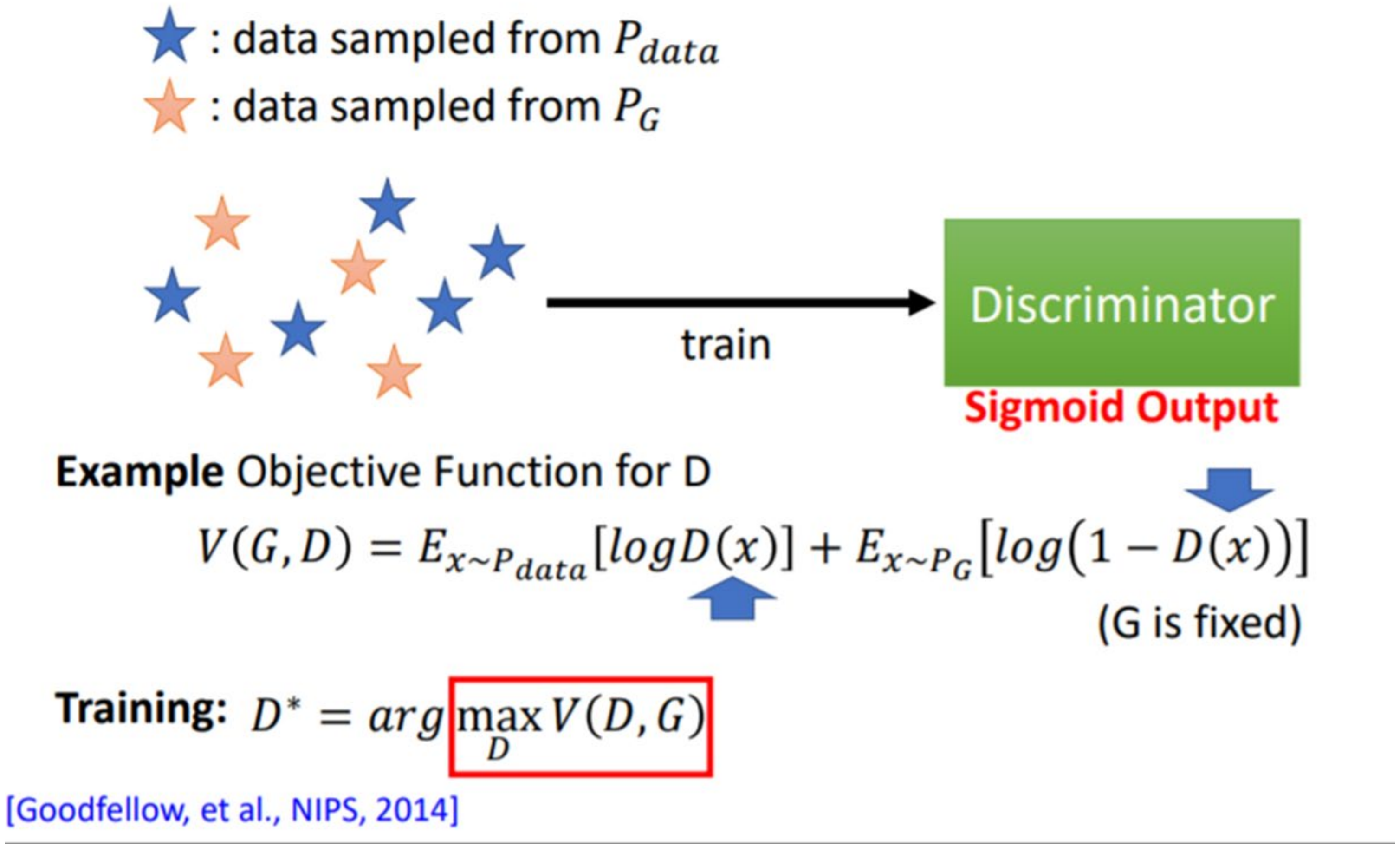
Divergence between distributions P_G and P_{data}

How to compute the divergence?

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

5. Use DNN to represent a more general divergence measure

How to compute the divergence when a NN is used to represent a more general probability model? – train a discriminator *NN* to compute the divergence



Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

The objective function for D is related to JS divergence

$$V(D,G) = E_{x\sim P_{data}}[logD(x)] + E_{x\sim P_G}[\log(1-D(x))]$$

- Given G, what is the optimal D* maximizing

$$V = E_{x\sim P_{data}}[logD(x)] + E_{x\sim P_G}[log(1-D(x))]$$

$$= \int_x P_{data}(x)logD(x)\,dx + \int_x P_G(x)log(1-D(x))\,dx$$

$$= \int_x [P_{data}(x)logD(x) + P_G(x)log(1-D(x))]\,dx$$

Assume that D(x) can be any function

- Given x, the optimal D* maximizing

$$P_{data}(x)logD(x) + P_G(x)log(1-D(x))$$

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

The objective function for *D* is related to JS divergence

$$V(D,G) = E_{x\sim P_{data}}[logD(x)] + E_{x\sim P_G}[\log(1-D(x))]$$

$$\max_D V(G,D) \;= V(G,D^*) \qquad D^*(x) = \frac{P_{data}(x)}{P_{data}(x)+P_G(x)}$$

$$= -2log2 + \int_x P_{data}(x)log\frac{P_{data}(x)}{(P_{data}(x)+P_G(x))/2}dx$$

$$+ \int_x P_G(x)log\frac{P_G(x)}{(P_{data}(x)+P_G(x))/2}dx$$

$$= -2\log 2 + \mathrm{KL}\left(\mathrm{P_{data}}||\frac{\mathrm{P_{data}}+\mathrm{P_G}}{2}\right) + \mathrm{KL}\left(\mathrm{P_G}||\frac{\mathrm{P_{data}}+\mathrm{P_G}}{2}\right)$$

$$= -2log2 + 2JSD(P_{data}||P_G)$$

Jensen-Shannon divergence

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

The objective function for *D* is related to cross entropy

$$V(D,G) = E_{x \sim P_{data}}[logD(x)] + E_{x \sim P_G}[\log(1 - D(x))]$$

Binary Classifier

D is a binary classifier with sigmoid output (can be deep)

$\{x^1, x^2, \dots, x^m\}$ from $P_{data}(x)$ → Positive examples

$\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}$ from $P_G(x)$ → Negative examples

Minimize Cross-entropy

Cross entropy:

$$C(f(x^n), \hat{y}^n) = -\left[\hat{y}^n lnf(x^n) + (1 - \hat{y}^n) ln\left(1 - f(x^n)\right)\right]$$

(GAN) Two adversarial neural networks *G* and *D*

(1) Train generator

$$G^* = arg\min_G Div(P_G, P_{data})$$

small divergence

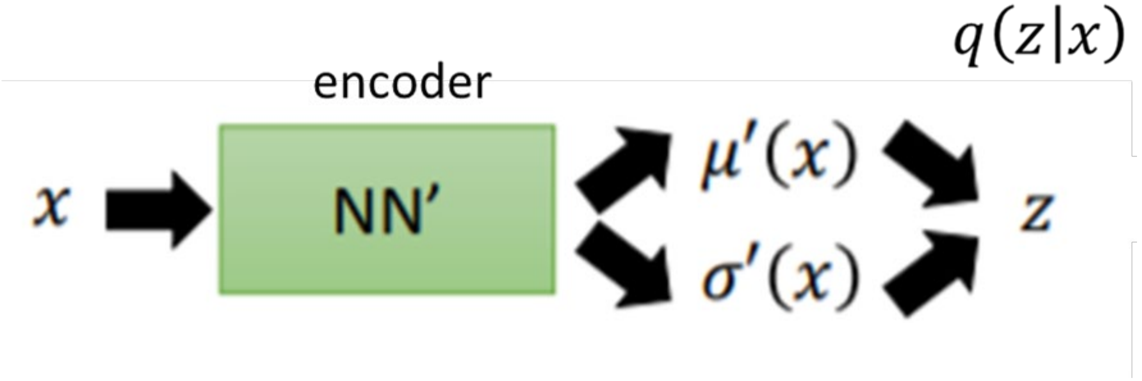
(2) Train discriminator

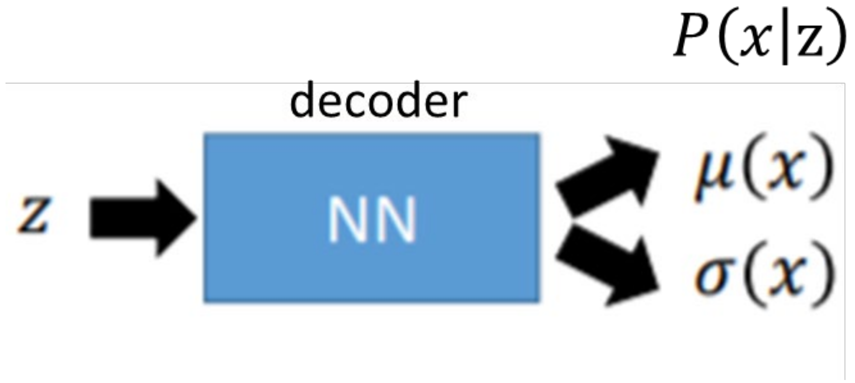
$$D^* = arg\max_D V(D, G)$$

large divergence

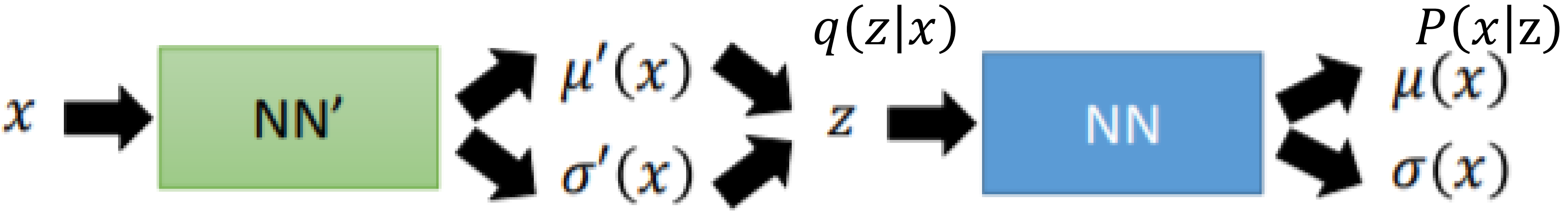
$$V(D, G) = E_{x \sim P_{data}}[log D(x)] + E_{x \sim P_G}[\log(1 - D(x))]$$

(VAE) Two sequential neural networks, one encode and one decode



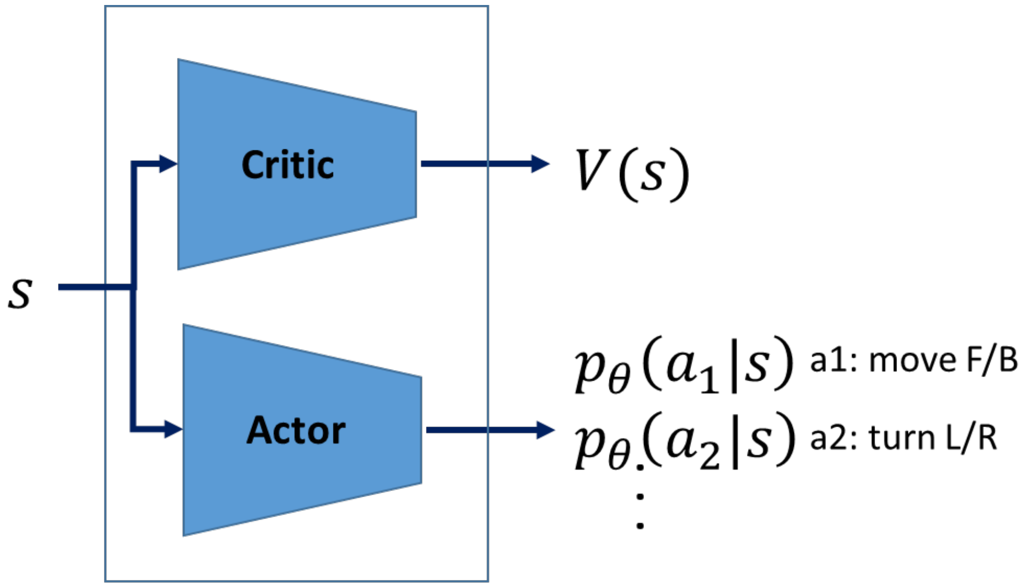


$$L = MSE(x, \hat{x}) + KL(q(z|x)||P(z))$$



(PPO-AC) Two parallel neural networks, one acting and one critic

$$L = c_v L_v + L_\pi - \beta L_{reg}$$



(1) Actor – Learns the best actions (that can have maximum long-term rewards)

$$L_\pi = \sum_{(s_t, a_t)} min\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t, a_t), clip\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon\right) A^{\theta'}(s_t, a_t)\right)$$

(2) Critic – Learns the expected value of the long-term reward.

$$L_v = MSE\ of\ (return - v)$$

Algorithm

$$G^* = arg \min_G \max_D V(D, G)$$

- Given G_0
- Find D_0^* maximizing $V(G_0, D)$

$V(G_0, D_0^*)$ is the JS divergence between $P_{data}(x)$ and $P_{G_0}(x)$

- $\theta_G \leftarrow \theta_G - \eta\, \partial V(G, D_0^*)/\partial \theta_G$ ➡ Obtain G_1
- Find D_1^* maximizing $V(G_1, D)$

Decrease JS divergence(?)

$V(G_1, D_1^*)$ is the JS divergence between $P_{data}(x)$ and $P_{G_1}(x)$

- $\theta_G \leftarrow \theta_G - \eta\, \partial V(G, D_1^*)/\partial \theta_G$ ➡ Obtain G_2
- ……

Decrease JS divergence(?)

$$D^* = arg \max_D V(D, G)$$

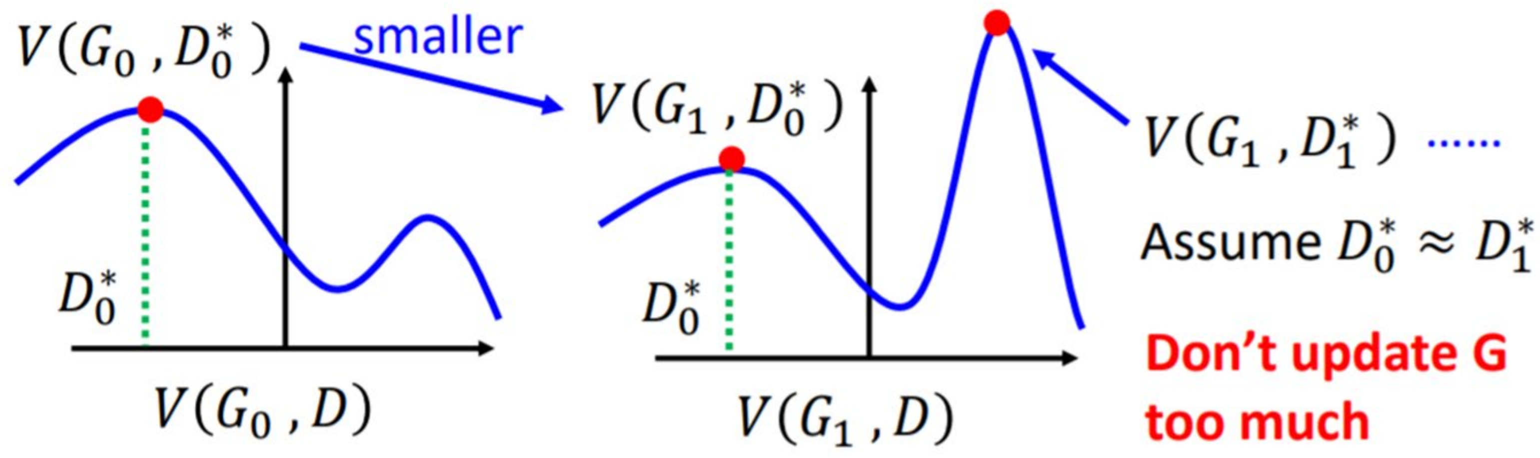
$$V(D, G) = E_{x \sim P_{data}}[log D(x)] + E_{x \sim P_G}[\log(1 - D(x))]$$

$$G^* = arg \min_G Div(P_G, P_{data})$$

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

Do not update G too much…

When you update G from G0 to G1, the V(G0, D) will no longer model the JS divergence between PG and Pdata



Compare with RL PPO: do not update actor too much

$$L_\pi = \sum_{(s_t,a_t)} min\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)} A^{\theta'}(s_t,a_t), clip\left(\frac{p_\theta(a_t|s_t)}{p_{\theta'}(a_t|s_t)}, 1-\varepsilon, 1+\varepsilon\right) A^{\theta'}(s_t,a_t)\right)$$

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

But we should update D as much as possible

- In each training iteration:

Learning D

$\boxed{\text{Repeat k times}}$

- Sample m examples $\{x^1, x^2, \ldots, x^m\}$ from data distribution $P_{data}(x)$
- Sample m noise samples $\{z^1, z^2, \ldots, z^m\}$ from the prior $P_{prior}(z)$
- Obtaining generated data $\{\tilde{x}^1, \tilde{x}^2, \ldots, \tilde{x}^m\}$, $\tilde{x}^i = G(z^i)$
- Update discriminator parameters θ_d to maximize
  - $\tilde{V} = \frac{1}{m}\sum_{i=1}^{m} logD(x^i) + \frac{1}{m}\sum_{i=1}^{m} log\left(1 - D(\tilde{x}^i)\right)$
  - $\theta_d \leftarrow \theta_d + \eta \nabla \tilde{V}(\theta_d)$

Learning G

$\boxed{\text{Only Once}}$

- Sample another m noise samples $\{z^1, z^2, \ldots, z^m\}$ from the prior $P_{prior}(z)$
- Update generator parameters θ_g to minimize
  - $\tilde{V} = \cancel{\frac{1}{m}\sum_{i=1}^{m} logD(x^i)} + \frac{1}{m}\sum_{i=1}^{m} log\left(1 - D\left(G(z^i)\right)\right)$
  - $\theta_g \leftarrow \theta_g - \eta \nabla \tilde{V}(\theta_g)$

Reference: 李弘毅 GAN Lecture 4 (2018) https://youtu.be/DMA4MrNieWo

But we should update D as much as possible

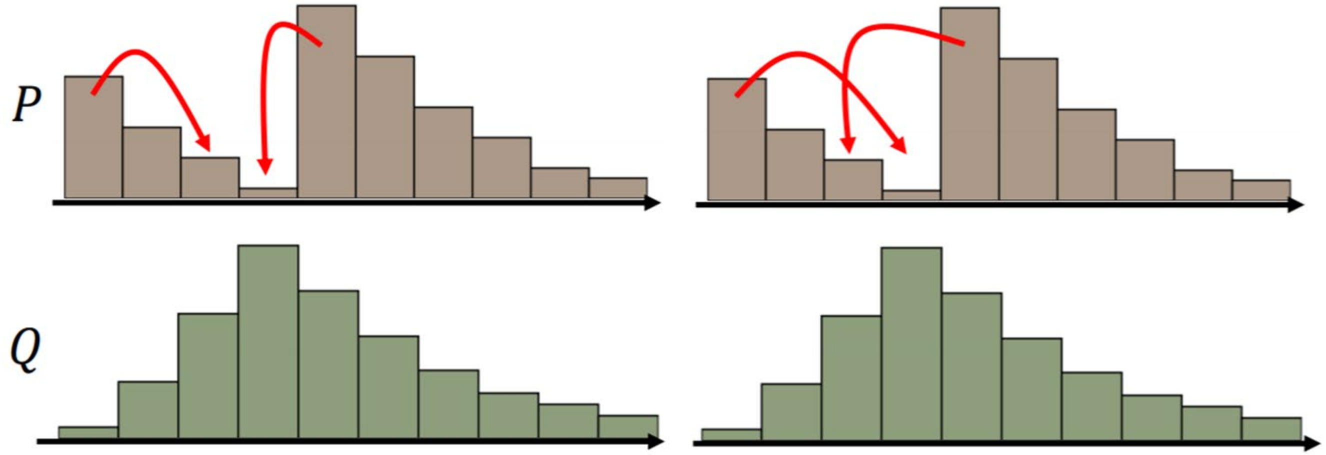
In [19]:

```
def train_discriminator(real_images, opt_d):
    # Clear discriminator gradients
    opt_d.zero_grad()

    # Pass real images through discriminator
    real_preds = discriminator(real_images)
    real_targets = torch.ones(real_images.size(0), 1, dev
    real_loss = F.binary_cross_entropy(real_preds, real_t
    real_score = torch.mean(real_preds).item()

    # Generate fake images
    latent = torch.randn(batch_size, latent_size, 1, 1, c
    fake_images = generator(latent.to(device))

    # Pass fake images through discriminator
    fake_targets = torch.zeros(fake_images.size(0), 1, de
    fake_preds = discriminator(fake_images)
    fake_loss = F.binary_cross_entropy(fake_preds, fake_t
    fake_score = torch.mean(fake_preds).item()

    # Update discriminator weights
    loss = real_loss + fake_loss
    loss.backward()
    opt_d.step()
    return loss.item(), real_score, fake_score
```

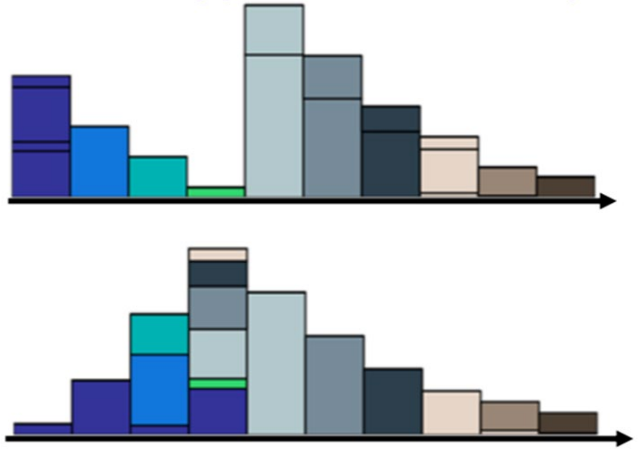
For loop for *k* times?

Wasserstein GAN (WGAN)

W-GAN: Earth move's distance



Best "moving plans" of this example

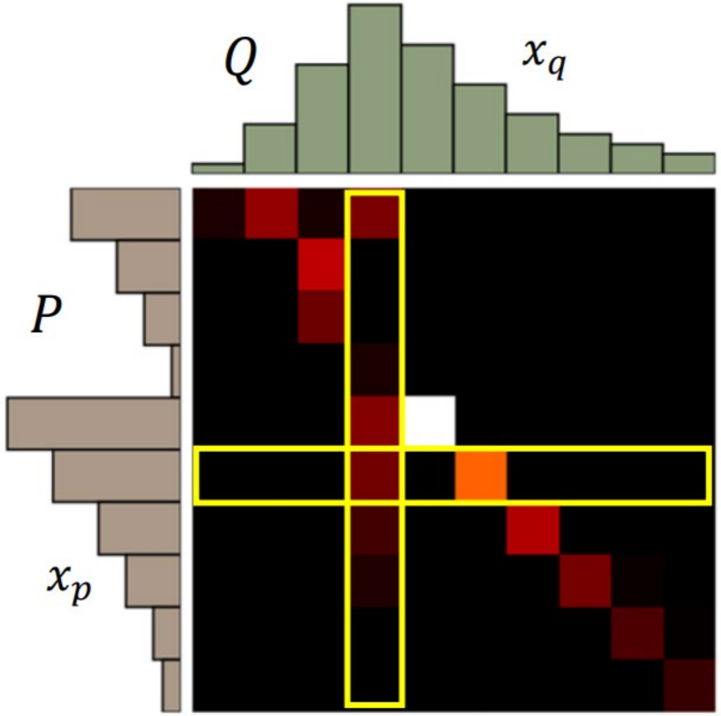


Average distance of a plan γ:

$$B(\gamma) = \sum_{x_p, x_q} \gamma(x_p, x_q) \|x_p - x_q\|$$

Earth Mover's Distance:

$$W(P, Q) = \min_{\gamma \in \Pi} B(\gamma)$$



W-GAN

Evaluate wasserstein distance between P_{data} and P_G

$$V(G,D) = \max_{D \in \underline{1-Lipschitz}} \{E_{x \sim P_{data}}[D(x)] - E_{x \sim P_G}[D(x)]\}$$

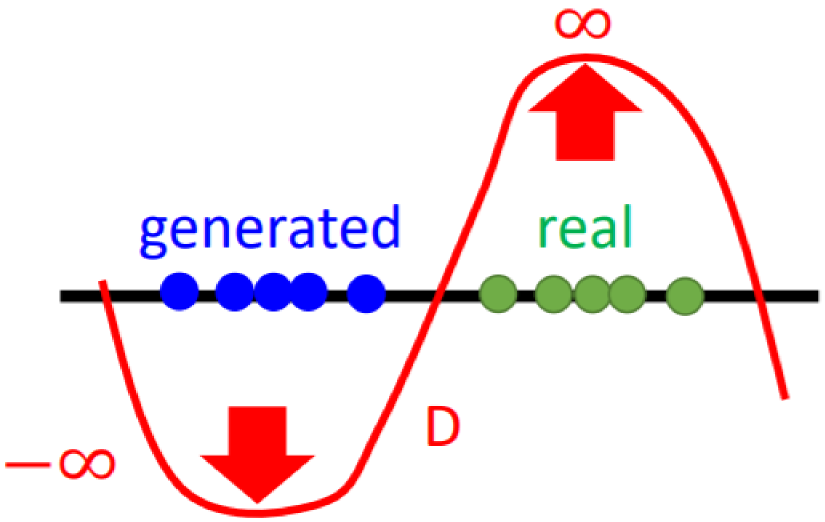
D has to be smooth enough.

Lipschitz Function

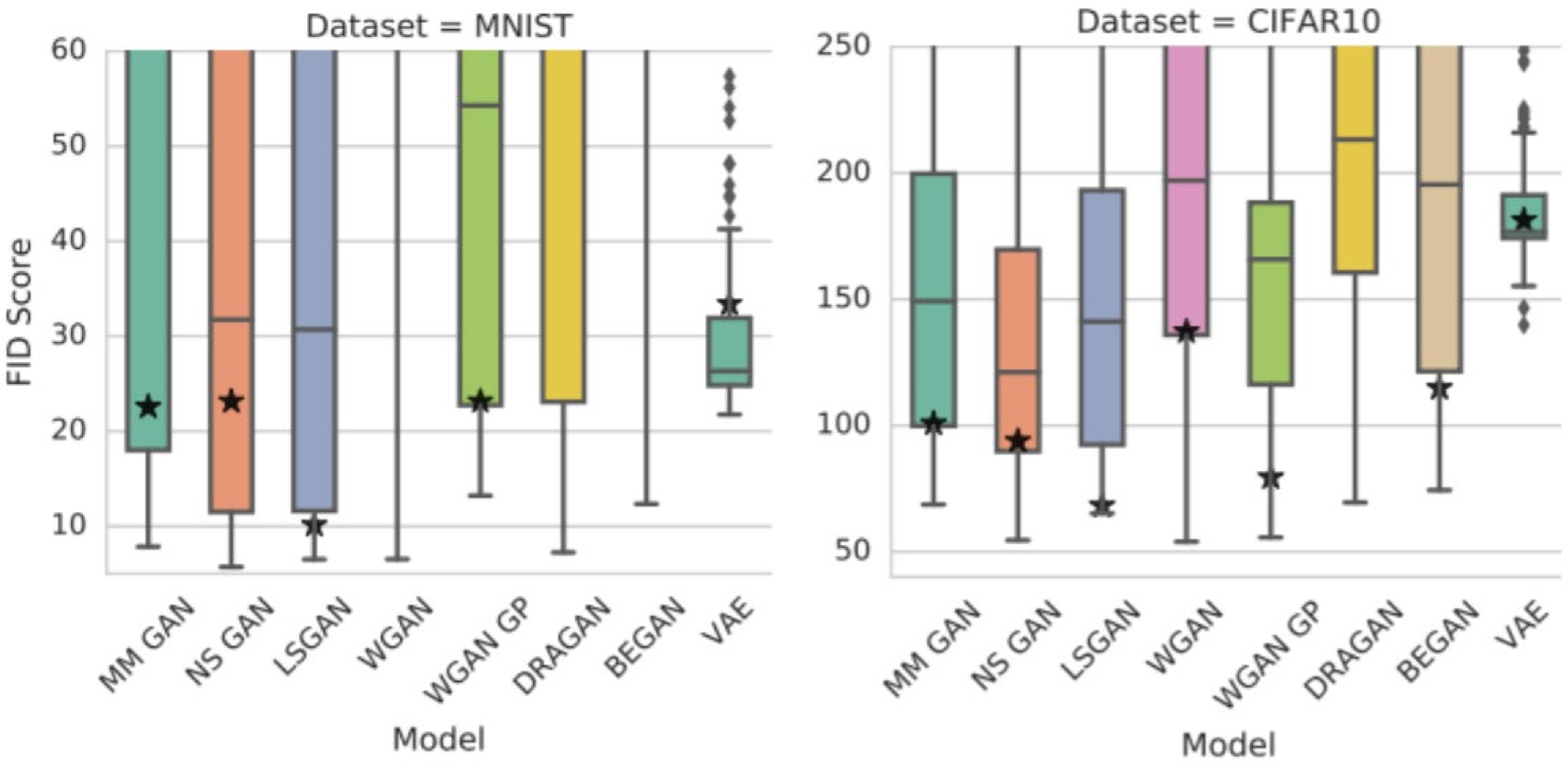
$$\|f(x_1) - f(x_2)\| \le K\|x_1 - x_2\|$$

Output change — Input change

K=1 for "$1 - Lipschitz$"



GAN is sensitive to hyper-parameter tuning and its performance range is large. Different GANs' performances are similar



Reference: 李弘毅 GAN Lecture 1 (2018): https://youtu.be/DQNNMiAP5lw