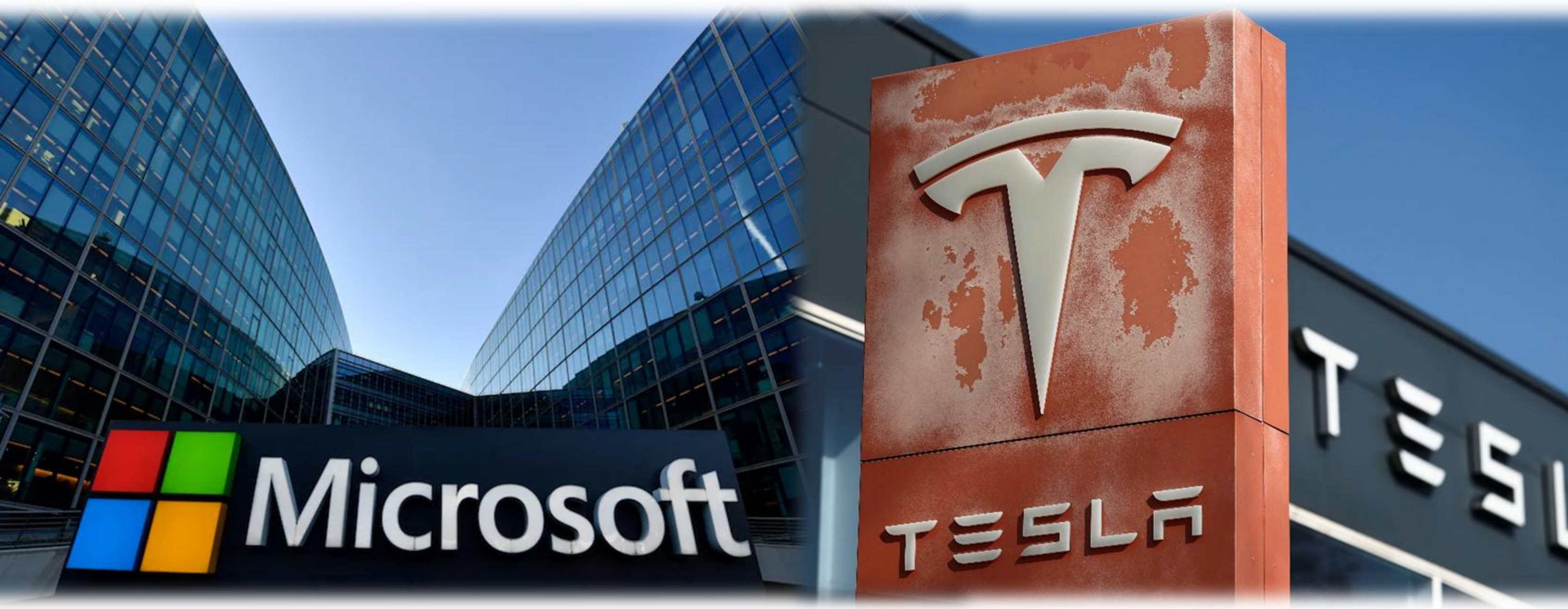
MINI-THESIS

COMPARATIVE VOLATILITY FORECASTING AND RISK ASSESSMENT USING GARCH AND VAR MODELS: A CASE STUDY OF TESLA (TSLA) AND MICROSOFT (MSFT)



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1. INTRODUCTION

Volatility forecasting has been a major subject of financial econometrics and remains an important component of asset pricing, portfolio allocation and risk management. Forecasting future market trends enables one to anticipate negative conditions and build a stronger investment portfolio. Among various econometric tools, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model has become a benchmark method for modeling financial time series with time-varying volatility, as introduced by Bellerose (1986) based on Engle's ARCH (Autoregressive Conditional Heteroskedastic) model (1982).

In recent years, the high volatility of some technology companies' stock prices has led to renewed interest in measuring tail risk and conditional volatility. For example, Tesla Inc. (TSLA) is widely known for its extreme stock price volatility. In 2020, the company's stock price rose 705% due to record-breaking presentations, strong earnings, and its historic inclusion in the S&P 500 index (Ponciano, 2020). Early in 2020, the stock saw a 19.9% one-day gain, which was indicative of speculative trading and investor zeal (Bursztynsky, 2020). Following CEO Elon Musk's contentious establishment of a new political party in July 2025, Tesla's stock fell 6.8% in a single day, wiping out nearly $79 billion in market capitalization (TheGuardian, 2025). Microsoft (MSFT) has demonstrated steady growth and financial stability in contrast to Tesla's extreme volatility. Strong earnings and the expansion of cloud and AI services in 2025 drove a nearly 30% three-month gain in its stock (Trefis, 2025). Microsoft's financial results are very impressive. Its operating margin of 45.2% and net profit margin of 35.8% are much higher than the S&P 500 average, which shows that it is very efficient and can keep making money. The company also has a conservative financial structure, with a debt-to-equity ratio of only 1.8% and cash reserves of $80 billion. Microsoft is less likely to be affected by changes in the market because of this strong foundation. It is also well-prepared to deal with economic uncertainty. It has also been able to bounce back from downturns faster than the rest of the market in the past (Oberoi, 2025). This divergence in volatility profiles makes these two firms' ideal candidates for comparative risk modeling.

The objective of this study is to examine and compare the volatility dynamics and risk characteristics of Tesla and Microsoft stocks using GARCH models and Value-at-Risk (VaR) forecasts. By estimating separate GARCH(1,1) models for each stock and calculating 1-day ahead VaR at the 1% confidence level, we aim to assess how well these models capture the risk of large losses and whether there are meaningful differences in their performance across different volatility regimes.

We also simulate a simple two-asset portfolio made up of both stocks and estimate the portfolio-level VaR to show how diversification changes the amount of risk you are exposed to. This study adds to the growing body of research on volatility modeling (Alexander, 2008), and shows how econometric forecasting can be used in real-world risk management.

2. DATA DESCRIPTION AND STYLIZED FACTS

This study analyzes daily stock prices of “Tesla” and “Microsoft” from January 1, 2020, to June 30, 2025. Data, which reaches over 2,000 trading days for each stock, was collected from platform Yahoo Finance using the “quantmod” package in R, a tool used for financial analysis. The focus is on the adjusted closing prices, which give more accurate picture of stock's value by factoring in things like dividends and stock splits.. To measure changes in prices from day to day, daily log returns were calculated using the formula $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, where Pt shows the closing price on day t. We looked at the daily changes in stock prices to find some usual patterns you see in the market. For example, big jumps in prices happen more often than you might expect. Also, periods where prices are very active or very calm tend to come in clusters. Lastly, the size of these price changes often follows certain patterns over time, even if the direction (up or down) seems random.

2.1 Distributional Analysis

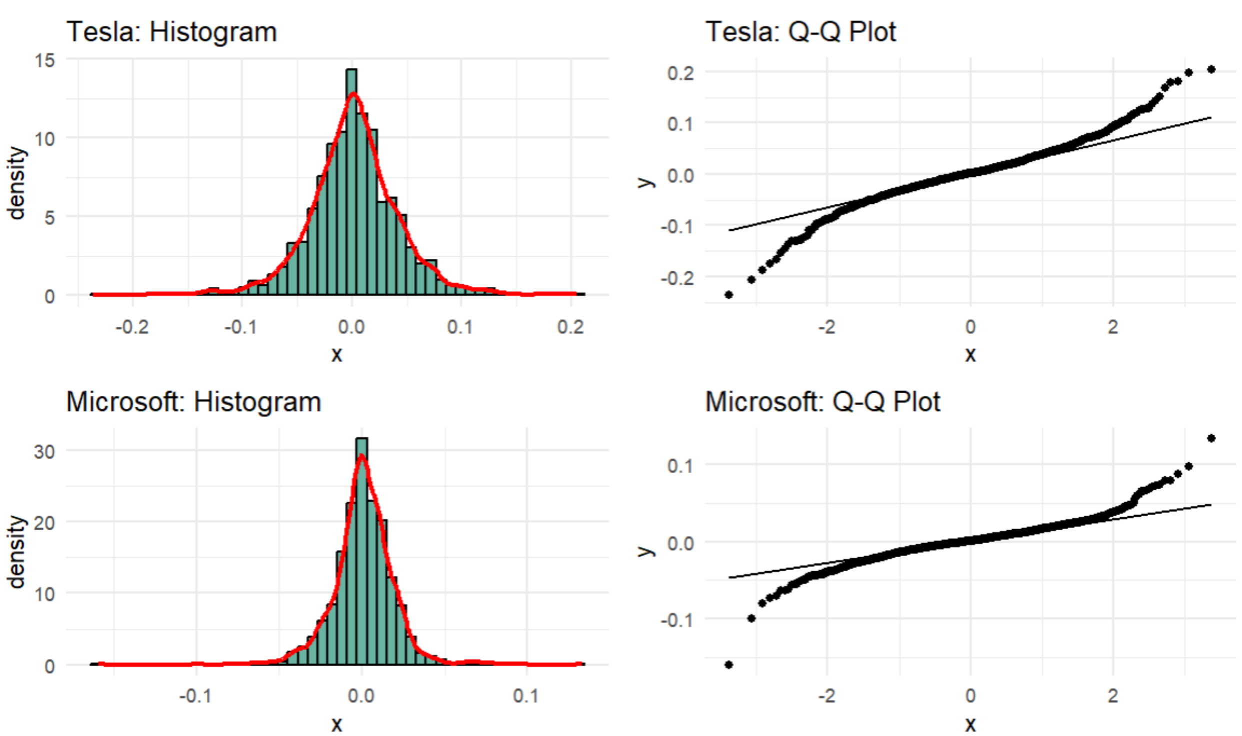

Figure 2.1 Histogram and Q-Q plot of Tesla and Microsoft’s stocks

Table 2.1 Skewness and Kurtosis of Tesla and Microsoft's stocks

Company	Skewness	Kurtosis
Tesla	-0.07851708	6.235972
Microsoft	-0.1676829	10.50945

The provided figures *(Figure 2.1)* and table *(Table 2.1)* illustrate the distributional characteristics of stock performance for both Tesla and Microsoft, using histograms and Q-Q plots.

The histograms of both companies are bell shaped due to a bulk of the data falling around a centred value at zero. Some visual examination detects substantial departures from normality, however. Quantitatively, Tesla exhibits a kurtosis of 6.24 and a skewness of −0.08, and the distribution of Microsoft is even more leptokurtic, with a kurtosis of 10.51 and a skewness (−0.17). These high kurtosis values definitively confirm the presence of leptokurtosis, or "fat tails," implying a greater propensity for extreme returns (both positive and negative) than would be predicted by a normal distribution. The negative tail for both, albeit very slight, indicates a marginally longer left tail, which becomes fatter in value; featuring an increased probability of a lower larger negative return than a larger a positive return. This is visually supported by the Q-Q plots where at the tails of the distribution, the data points are not followed closely by the line corresponding to the theoretical normal distribution.

These observations are supported by the Q-Q plots. For Tesla as well as Microsoft the points of each distribution extremes lay substantially off the theoretical normal distribution line. This strong deviation, in particular in the tails, is an indication of “fat tails” for both stocks, i.e., extreme price movements occur more frequently than expected through a standard normal distribution. While both stocks exhibit this characteristic, the deviation appears more pronounced for Tesla, reinforcing the notion of its higher volatility compared to Microsoft. In summary, both Tesla and Microsoft's stock performance distributions demonstrate significant deviations from normality, particularly in their "fat tails," a common feature in financial data. This would mean that there is a greater likelihood for both to experience extreme price movements, and Tesla on average has more volatility along with less stability compared to Microsoft. These are essential results for reliable risk measurement and asset valuation, because assuming a normal distribution for stock returns can induce a systematic under-estimation of extreme event risk.

2.2 Volatility Clustering and Autocorrelation

Visual inspection of the return time series shows alternating periods of high and low volatility, especially for Tesla, suggesting the presence of volatility clustering.

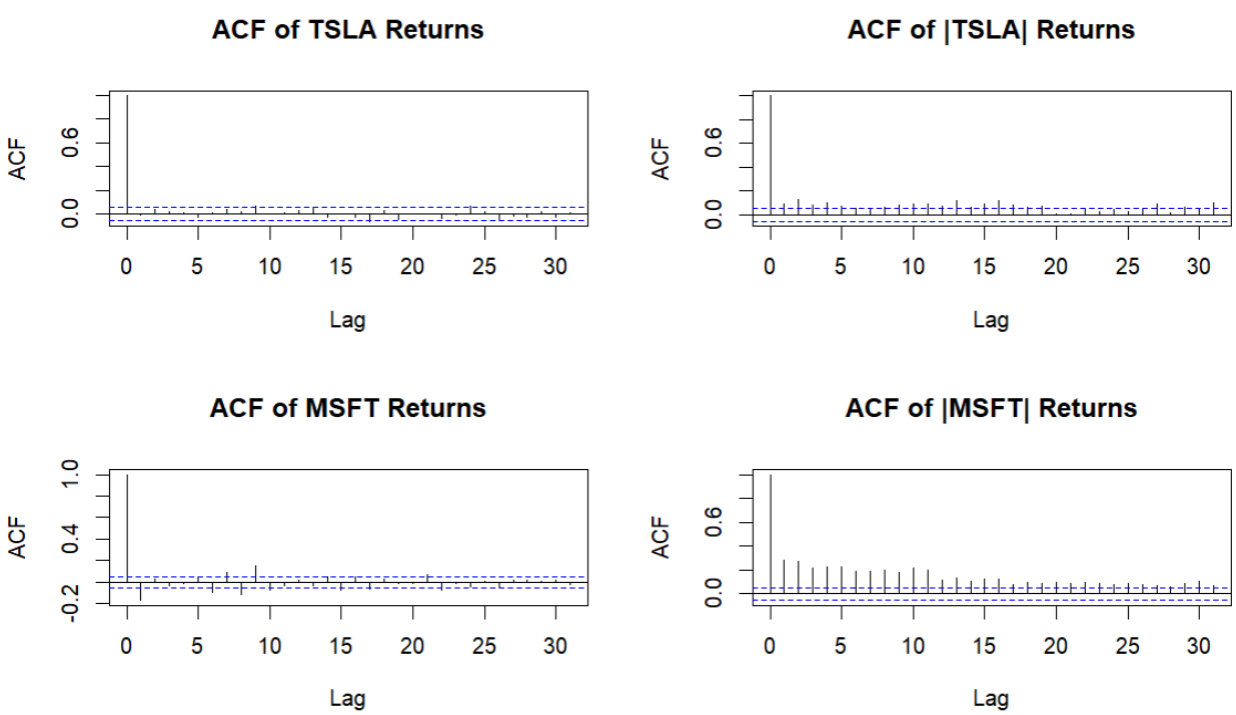

Figure 2.2 ACF of Tesla and Microsoft's stocks

The provided Autocorrelation Function (ACF) plots for Tesla (TSLA) and Microsoft (MSFT) returns *(Figure 2.2)*, as well as their absolute returns, offer critical insights into their time series properties. The upper-left and lower-left panels, displaying the ACF of raw returns for TSLA and MSFT respectively, show that all autocorrelation coefficients fall within the 95% confidence bands (indicated by the dashed blue lines) across various lags. This absence of statistically significant autocorrelation in the returns themselves is consistent with the efficient market hypothesis, suggesting that past returns cannot be linearly exploited to predict future returns, aligning with a random walk model for asset prices.

On the other hand, the top-right and the bottom-right panels representing the ACF of the absolute returns for TSLA and MSFT exhibit a dissimilar dynamic. Positive TSLA Returns and MSFT Returns autocorrelation lagsHowever, it is clear that both TSLA Returns and MSFT Returns demonstrate robust positive autocorrelation at shorter lags, which is outside the confidence levels. This is referred to as volatility clustering, which means big volatility is usually followed by big volatility, the same applies for the small. In particular, the level of autocorrelation across different lag orders is more persistent and statistically significant for Microsoft's absolute returns even further in the lags than in the case of Tesla's, indicating that the clustering effect of Microsoft's volatility lasts longer. The presence of such a large degree of autocorrelation in absolute returns, despite of the fact that the raw returns themselves do not possess this characteristic, is a feature that characterizes most financial time series. This empirical finding strongly suggests the necessity of employing Autoregressive Conditional Heteroskedasticity (ARCH) or Generalized ARCH (GARCH) models, which are specifically designed to capture and forecast time-varying volatility and volatility clustering, for a more accurate and robust analysis of these financial series.

2.3 Visual Comparison

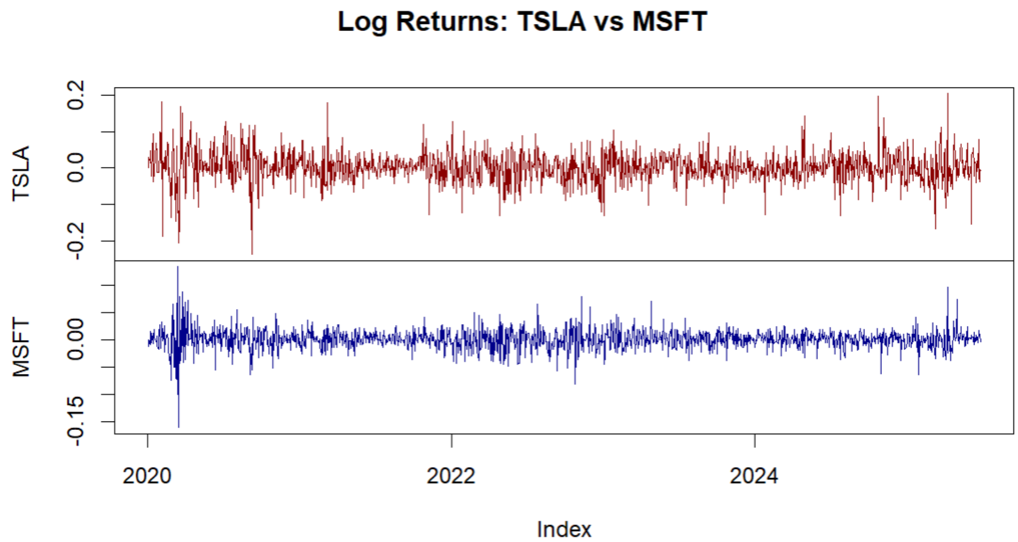

Figure 2.3 Log returns of Tesla and Microsoft's stocks

The provided time series plot of daily log returns for Tesla (TSLA) and Microsoft (MSFT) *(Figure 2.3)*. Notice that the overall volatility between the two stocks are quite different. Tesla's log return history (red series) on the other hand has almost at all times had much wider flactuations, with daily changes going more than 0.1 (or closer to 0.2) far more often. This extreme volatility is one characteristic of Tesla’s high-growth, high-risk trappings. On the other hand, Microsoft's smoothed log returns (blue line) appear much smoother, with most days' movements not exceeding |0.1|, pointing towards a more established, less volatile technology firm. Moreover, the phenomena of the volatility clustering is obviously presented in both series. Distinct periods of heightened fluctuation are visibly grouped together, followed by or interspersed with periods of relative calm. For example, Tesla is quite clustery in early 2020 and late 2021-early 2022, and Microsoft shows this tendency at a lower level as well, but not at those extreme levels. This behaviour, where price changes also tend to affect larger price changes (or smaller price changes to lead to smaller price changes), is observed frequently in financial time series and it is important to adequately assess the risk and to properly use econometric models. Essentially, the return series swing back and forth around zero during this period, which indicates no apparent linear trend of daily returns, a feature that is in line with efficient market properties.

SUMMARY OF SECTION 2

The descriptive summary of daily log returns for Tesla and Microsoft from January 2020 to June 2025 displays the importance of the familiar characteristics in financial time series. The return distributions of both funds are substantially non-normal in terms of high kurtosis and slight negative skew. These characteristics indicate the presence of heavy tails, which means that extreme price movements are more probable than under the normal distribution. This comment is also is enhanced by the Q-Q plot analysis, where the tails of the empirical distributions depart away from normal line.

In raw returns, autocorrelation analysis shows no or little linear dependence, in line with the hypothesis of weak-form efficiency of financial markets. But if we look at the absolute returns,at lag 1 we get plenty of positive autocorrelation in particular. This behavior would also indicate the possibility of volatility clustering, which is the tendency for clustering of periods of high volatility after periods of high volatility. The impact is longer lasting in the case of Microsoft's and stronger in Tesla's, indicating the diverging nature of the two companies for market behavior and the attitudes toward risk.

Visual inspection of the return series supports these statistical findings. Tesla's returns show large and frequent spikes, while Microsoft's series appears smoother and less erratic. Both return series oscillate around a zero mean, aligning with the random walk hypothesis.

Taken together, these stylized facts suggest that return volatility is time-varying and conditionally dependent, thereby motivating the use of econometric models specifically designed to capture such dynamics. The next section will estimate and interpret the GARCH(1,1) model for both Tesla and Microsoft in order to better understand the behavior of volatility in each case.

3. GARCH (1,1) ESTIMATION AND VOLATILITY MODELING

3.1 Model Specification and Rationale

Asset returns are well-known to have time series properties that are inconsistent with the classical statistical assumptions of constant variances and independent error terms. The most visible one of these fea-tures are volatility clustering, leptokurtosis, and time varying conditional variance. To account for this stylized fact, Engle (1982) developed the Autoregressive Conditional Heteroskedasticity (ARCH) model, extended to the Generalized ARCH (GARCH) framework by Bollerslev (1986). These models still serve as the basis for econometric modeling of financial market volatility.

The present analysis employs the GARCH(1,1) model, which is both empirically robust and parsimonious, capturing the essential dynamics of conditional heteroskedasticity without overparameterization. The GARCH(1,1) model expresses the conditional variance σ_t^2 as a function of past squared shocks and past variance, formally defined as:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. The parameter α\alphaα measures the short-term impact of past shocks on current volatility, while β aptures the persistence of volatility over time. The sum $\alpha + \beta$ indicates the degree of volatility persistence, which is typically close to unity for financial assets.

The selection of the GARCH(1,1) model is based on theoretical and empirical grounds. First, our (see Section 2) distributional analysis shows that there is, indeed, clear evidence of fat tails and volatility clustering for Tesla and Microsoft stock returns. These are classic signs of the presence of heteroskedasticity and suggest that we require a model for the conditional variance. Second, the GARCH(1, 1) model since being proposed by Bollerslev (1986) has extensively used for modeling of volatility in financial econometrics. It has been empirically proven effective by decades of research. For example, Baillie and Bollerslev (1989) showed its utility in exchange rate volatility. Nelson (1991) confirmed its robustness under certain misspecification conditions. Later, Hansen and Lunde (2005) conducted a comprehensive model comparison and showed that GARCH(1,1) often outperforms more complex models in forecasting. Poon and Granger (2003) provided a meta-analysis that further highlighted its predictive power. This sustained empirical support underlines its continued relevance and reliability in modeling financial time series volatility.

3.2 Estimation Procedure and Diagnostic Checking

```
> # Fit the model for Tesla
> fit_tsla <- ugarchfit(spec = spec, data = returns
$TSLA)
> show(fit_tsla)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001830    0.001059   1.7292 0.083777
omega   0.000054    0.000022   2.4056 0.016146
alpha1  0.057019    0.013414   4.2506 0.000021
beta1   0.913896    0.022607  40.4249 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001830    0.001219   1.5015 0.133227
omega   0.000054    0.000044   1.2132 0.225041
alpha1  0.057019    0.022740   2.5074 0.012162
beta1   0.913896    0.041814  21.8565 0.000000

LogLikelihood : 2449.793

Information Criteria
------------------------------------

Akaike       -3.5472
Bayes        -3.5320
Shibata      -3.5472
Hannan-Quinn -3.5415

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.2611  0.6094
Lag[2*(p+q)+(p+q)-1][2]    0.9742  0.5067
Lag[4*(p+q)+(p+q)-1][5]    2.3139  0.5463
d.o.f=0
```

```
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Resi
duals
------------------------------------
                        statistic p-value
Lag[1]                     0.8385  0.3598
Lag[2*(p+q)+(p+q)-1][5]    2.8550  0.4342
Lag[4*(p+q)+(p+q)-1][9]    4.7419  0.4683
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1895 0.500 2.000  0.6633
ARCH Lag[5]    0.2998 1.440 1.667  0.9403
ARCH Lag[7]    1.1039 2.315 1.543  0.8961

Nyblom stability test
------------------------------------
Joint Statistic:  1.4578
Individual Statistics:
mu     0.2733
omega  0.1571
alpha1 0.1905
beta1  0.1188

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.07 1.24 1.6
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.3622 0.1734
Negative Sign Bias  1.2383 0.2158
Positive Sign Bias  0.2569 0.7973
Joint Effect        2.3735 0.4986


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     69.22    1.238e-07
2    30     74.21    7.892e-06
3    40     93.17    2.506e-06
4    50    102.91    1.060e-05


Elapsed time : 0.288121
```
]

Figure 3.1 Fit GARCH model for Tesla's stocks

```
> # Fit the model for Microsoft
> fit_msft <- ugarchfit(spec = spec, data = returns
$MSFT)
> show(fit_msft)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001062    0.000416   2.5563  0.01058
omega   0.000015    0.000003   5.0409  0.00000
alpha1  0.105782    0.009760  10.8383  0.00000
beta1   0.850378    0.013095  64.9378  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001062    0.000443   2.3997 0.016409
omega   0.000015    0.000008   2.0084 0.044605
alpha1  0.105782    0.020824   5.0798 0.000000
beta1   0.850378    0.032768  25.9514 0.000000

LogLikelihood : 3655.87

Information Criteria
------------------------------------

Akaike       -5.2964
Bayes        -5.2812
Shibata      -5.2964
Hannan-Quinn -5.2907

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.917 0.04781
Lag[2*(p+q)+(p+q)-1][2]     4.695 0.04908
Lag[4*(p+q)+(p+q)-1][5]     5.464 0.11996
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Resi
duals
------------------------------------
                        statistic p-value
Lag[1]                     0.1061  0.7446
Lag[2*(p+q)+(p+q)-1][5]    0.2565  0.9880
Lag[4*(p+q)+(p+q)-1][9]    0.7045  0.9954
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.05923 0.500 2.000  0.8077
ARCH Lag[5]   0.22159 1.440 1.667  0.9603
ARCH Lag[7]   0.45105 2.315 1.543  0.9826

Nyblom stability test
------------------------------------
Joint Statistic:  6.9304
Individual Statistics:
mu     0.06011
omega  0.88553
alpha1 0.25066
beta1  0.11713

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.07 1.24 1.6
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.06521 0.9480
Negative Sign Bias 1.13982 0.2546
Positive Sign Bias 0.32834 0.7427
Joint Effect       2.23937 0.5242


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     55.82    1.738e-05
2    30     85.21    1.949e-07
3    40     87.60    1.348e-05
4    50    101.46    1.585e-05


Elapsed time : 0.216248
```

Figure 3.2 Fit GARCH model for Microsoft's stocks

The fitting of a standard GARCH(1,1) model with a Normal innovations distribution to the daily log returns of both Tesla (TSLA) *(Figure 3.1)* and Microsoft (MSFT) *(Figure 3.2)* provides quantitative insights into their volatility dynamics. The chosen mean model, ARFIMA(0,0,0), effectively specifies a constant mean for the returns, consistent with the observed absence of linear autocorrelation in the raw return series. For Tesla, the estimated parameters ω (0.000054), α_1 (0.057019), and β_1 (0.913896) are all statistically significant at conventional levels (p-values < 0.05, particularly for α_1 and β_1 based on standard errors), indicating that past squared shocks (α_1) and past conditional variances (β_1) play a significant role in determining current volatility. The sum of $\alpha_1 + \beta_1 = 0.970915$ is close to unity, suggesting a high degree of volatility persistence. Similarly, for Microsoft, the estimated parameters ($\omega = 0.000015$, $\alpha_1 = 0.105782$, and $\beta_1 = 0.850378$) are also highly significant. Microsoft's α1 is notably larger than Tesla's, implying that new information or shocks have a greater immediate impact on its volatility. Conversely, Microsoft's β_1 is slightly smaller, leading to a sum of $\alpha_1 + \beta_1 = 0.95616$ that, while still indicating high persistence, is marginally lower than Tesla's.

Diagnostic tests for standard residuals are carried out to enquire about model adequacy. In the case of Tesla, the WLBR test applied to both,original and squared residuals, as well as the Weighted ARCH LM tests behave remarkably similar and they all result in high Ps(Modified) (all are greater than 0.05) suggesting that the GARCH(1,1) model eliminates any possibly existing serial correlation and ARCH effects in residuals effectively. This indicates the model behaves well in terms of the conditional variance structure. The Nyblom stability test for Tesla, with a joint statistic of 1.4578, marginally exceeds the 5% critical value of 1.24 but remains below the 1% critical value, suggesting some potential for parameter instability, though not severely so. The Sign Bias Test for Tesla shows no significant evidence of asymmetric effects (all p-values > 0.05), implying that positive and negative shocks of the same magnitude have a symmetric impact on volatility, which is expected under a standard GARCH model. But the Adjusted Pearson Goodness-of-Fit test for Tesla is significant for all groups at extremely small p-values, implying that the model for the innovations as a Normal could cause the empirical distribution not to be a perfect match, which is consistent with previous reports of "fat tails."

For Microsoft, the Weighted Ljung-Box test on standardized residuals shows marginal significance at Lag[1] and Lag[2*(p+q)+(p+q)-1][2] (p-values around 0.04-0.05), suggesting some weak remaining serial correlation in the mean, which might warrant further investigation. However, the tests on standardized squared residuals and the ARCH LM tests show no significant remaining ARCH effects (all p-values > 0.05), confirming that the GARCH(1,1) model effectively accounts for volatility clustering. The Nyblom stability test for Microsoft, with a joint statistic of 6.9304, significantly exceeds all critical values, indicating substantial parameter instability. This suggests that the estimated parameters might not be constant over the entire sample period, potentially requiring a more complex model or rolling window estimation. Similar to Tesla, the Sign Bias Test for Microsoft indicates no significant asymmetric volatility effects. Lastly, the Adjusted Pearson Goodness-of-Fit test for Microsoft also yields very low p-values, again highlighting that while the GARCH(1,1) effectively models volatility, the assumption of normally distributed innovations may not fully capture the true underlying distribution of returns, which likely exhibits fatter tails.

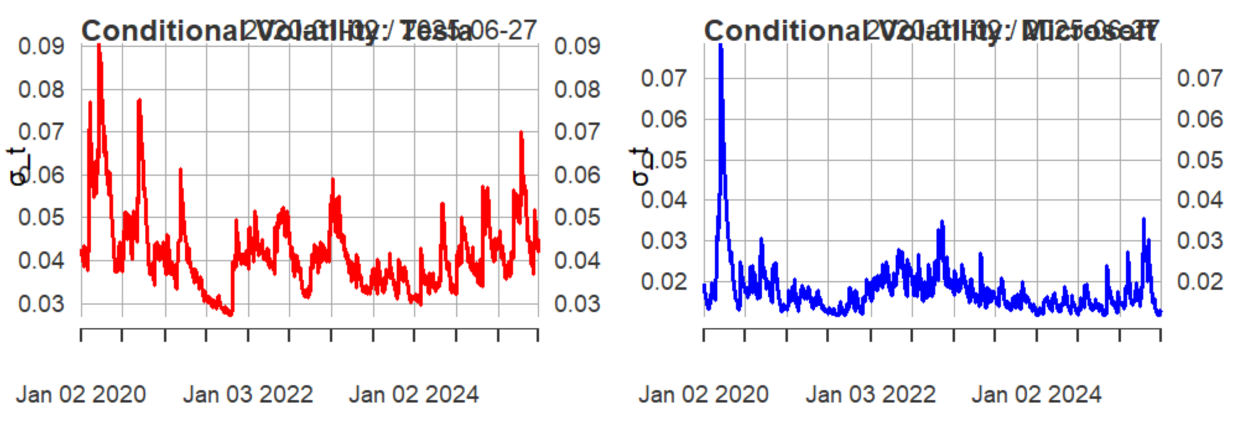

Figure 3.3 Conditional Volatility for Tesla (left) and Microsoft (right)

Figure 3.3 visualizes the estimated conditional volatility (σt) for Tesla (TSLA) and Microsoft (MSFT), providing a dynamic and quantitative perspective on their time-varying risk profiles. The left panel, representing Tesla's conditional volatility, consistently operates at a substantially higher absolute level, fluctuating predominantly between approximately 0.03 and 0.09. This elevated scale of estimated variance provides unequivocal corroboration of Tesla's previously identified status as a significantly more volatile asset. In sharp contrast, the conditional volatility of Microsoft (right panel) is of much smaller size, with values typically oscillating somewhere between 0.015 and 0.03, which confirms that Microsoft is, in its nature, a more stable investment with lower risk.

Both figures strongly support the stylized fact of volatility clustering, i.e. periods of high conditional variance are immediately succeeded by other periods of high volatility. This can be seen in the continuous uptrend and downtrend in the implied volatility series for the two firms, especially during times of a substantial market shock (in early 2020), where both had a strong spike in the IV series, but Tesla's one was much stronger. These visualizations together emphasize the necessity of dynamic volatility modeling, which does not only verify the time variability of risk, but also quantitatively and clearly distinguish the risk exposures of the two popular technology stocks, and hence such disparate characteristics provide important information for the management of portfolios and financial decision making.

3.3 Volatility Forecasting

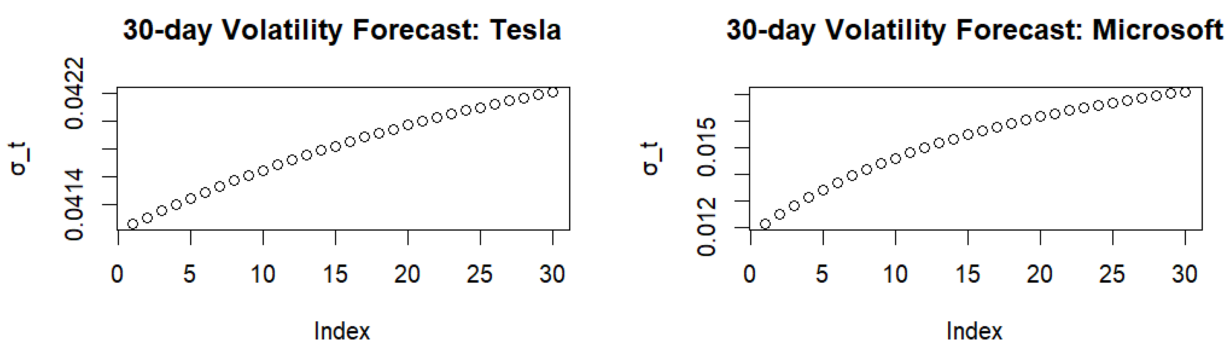

Figure 3.4 Volatility Forecast of Tesla and Microsoft

Figure 3.4 illustrates the 30-day conditional volatility forecasts for Tesla (TSLA) on the left and Microsoft (MSFT) on the right, offers critical forward-looking insights into their respective risk profiles. The primary observation is the pronounced disparity in the absolute level of forecasted volatility. For Tesla, the forecast commences at approximately 0.0409 (4.09%) and gradually increases to about 0.0422 (4.22%) over the 30-day horizon. This elevated range consistently positions Tesla as a significantly riskier asset, aligning with its historical return characteristics. In stark contrast, Microsoft's forecasted volatility initiates at a considerably lower level, around 0.0118 (1.18%), and increases to roughly 0.017

(1.7%) by day 30, unequivocally confirming its status as a more stable and lower-risk investment.

Both predictions show a certain upward trend and the estimated conditional volatility continuously goes up during the 30 days forecasting period. This ascending pattern is a typical behavior for GARCH-type models in the situation when the current (or the latest) volatility is less than the long-run average volatility. For longer forecast horizons, conditional volatility reverts to its unconditional (long run) level. The fact that Microsoft's forecast growth rate is much smaller than the forecast for Tesla also hints that Microsoft's current volatility might actually be further below its long-term average, leading to a more pronounced reversion within this short horizon, or a faster speed of mean reversion. These dynamic volatility forecasts are indispensable for risk managers, traders, and investors, providing quantitative estimates of near-term risk.

3.4 Comparison between GARCH, EGARCH and GJR-GARCH Models

3.4.1 EGARCH Model

```
=== EGARCH Tesla ===
> show(fit_egarch_tsla)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002088    0.001058   1.9748 0.048294
omega  -0.241550    0.165707  -1.4577 0.144925
alpha1 -0.014669    0.008553  -1.7151 0.086335
beta1   0.960765    0.026045  36.8891 0.000000
gamma1  0.142585    0.004440  32.1158 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002088    0.001308  1.59617  0.11045
omega  -0.241550    0.601096 -0.40185  0.68779
alpha1 -0.014669    0.037150 -0.39486  0.69295
beta1   0.960765    0.094491 10.16778  0.00000
gamma1  0.142585    0.099766  1.42919  0.15295

LogLikelihood : 2449.902

Information Criteria
------------------------------------

Akaike       -3.5459
Bayes        -3.5269
Shibata      -3.5459
Hannan-Quinn -3.5388

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      0.455  0.5000
Lag[2*(p+q)+(p+q)-1][2]     1.110  0.4640
Lag[4*(p+q)+(p+q)-1][5]     2.449  0.5167
d.o.f=0
H0 : No serial correlation
```

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1983  0.6561
Lag[2*(p+q)+(p+q)-1][5]    3.1173  0.3860
Lag[4*(p+q)+(p+q)-1][9]    5.0675  0.4196
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1730 0.500 2.000  0.6775
ARCH Lag[5]    0.2251 1.440 1.667  0.9595
ARCH Lag[7]    0.7944 2.315 1.543  0.9445

Nyblom stability test
------------------------------------
Joint Statistic:  1.9794
Individual Statistics:
mu     0.2112
omega  0.1741
alpha1 0.2863
beta1  0.1657
gamma1 0.5529

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.0072 0.3140
Negative Sign Bias  1.1514 0.2498
Positive Sign Bias  0.4623 0.6440
Joint Effect        1.6217 0.6545


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     61.48    2.253e-06
2    30     69.68    3.353e-05
3    40     91.08    4.746e-06
4    50    115.60    2.641e-07


Elapsed time : 0.1830552
```

Figure 3.5 Fit EGARCH model for Tesla's stocks

```
=== EGARCH Microsoft ===
> show(fit_egarch_msft)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.000590    0.000444    1.3285  0.18402
omega  -0.211732    0.004987  -42.4609  0.00000
alpha1 -0.090431    0.014902   -6.0683  0.00000
beta1   0.973329    0.000885 1099.5928  0.00000
gamma1  0.121502    0.005813   20.9029  0.00000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000590    0.000489   1.2074 0.227277
omega  -0.211732    0.017049 -12.4190 0.000000
alpha1 -0.090431    0.026892  -3.3627 0.000772
beta1   0.973329    0.001923 506.2306 0.000000
gamma1  0.121502    0.006719  18.0824 0.000000

LogLikelihood : 3670.931

Information Criteria
------------------------------------

Akaike       -5.3168
Bayes        -5.2978
Shibata      -5.3168
Hannan-Quinn -5.3097

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      4.857 0.02753
Lag[2*(p+q)+(p+q)-1][2]     5.409 0.03163
Lag[4*(p+q)+(p+q)-1][5]     6.063 0.08691
d.o.f=0
H0 : No serial correlation
```

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3657  0.5454
Lag[2*(p+q)+(p+q)-1][5]    0.8475  0.8931
Lag[4*(p+q)+(p+q)-1][9]    1.2843  0.9713
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.06824 0.500 2.000  0.7939
ARCH Lag[5]   0.11978 1.440 1.667  0.9830
ARCH Lag[7]   0.39618 2.315 1.543  0.9868

Nyblom stability test
------------------------------------
Joint Statistic:  1.8833
Individual Statistics:
mu     0.07048
omega  0.11036
alpha1 0.16196
beta1  0.10737
gamma1 0.47058

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias          0.08489 0.9324
Negative Sign Bias 1.16064 0.2460
Positive Sign Bias 1.38684 0.1657
Joint Effect       3.27375 0.3513


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     46.42    0.0004315
2    30     60.06    0.0006063
3    40     71.30    0.0012124
4    50     81.51    0.0024262


Elapsed time : 0.1772861
```

Figure 3.6 Fit EGARCH model for Microsoft's stocks

The fitting of an Exponential GARCH (eGARCH(1,1)) model, incorporating a Normal innovations distribution, to the daily log returns of both Tesla (TSLA) *(Figure 3.5)* and Microsoft (MSFT) *(Figure 3.6)* provides a refined understanding of their asymmetric volatility dynamics. The chosen mean model, ARFIMA(0,0,0), effectively specifies a constant mean for the returns, consistent with the observed absence of linear autocorrelation in the raw return series. For Tesla, the estimated parameters are informative: μ (0.002088) is marginally significant (p=0.048). Crucially, the β_1 (0.960765) parameter is highly significant (p=0.000), indicating strong persistence in the log-conditional variance. The γ_1 p arameter, estimated at 0.142585, is also highly significant (p=0.000), suggesting a clear presence of the leverage effect, where negative shocks exert a disproportionately larger impact on future volatility than positive shocks of equal magnitude. Diagnostic tests generally support the model's adequacy: Weighted Ljung-Box tests on standardized and standardized squared residuals, as well as ARCH LM tests, show high p-values (all >0.05), indicating successful removal of serial correlation and remaining ARCH effects. The Sign Bias Test also confirms no significant asymmetric effects beyond what the EGARCH model captures. However, the Nyblom stability test (Joint Statistic: 1.9794, exceeding the 1% critical value of 1.88) points to potential parameter instability, suggesting that the model's parameters might not be constant over the entire sample period. Furthermore, the Adjusted Pearson Goodness-of-Fit tests consistently yield very low p-values, indicating that the Normal distribution assumption for the innovations is likely misspecified, a finding consistent with the "fat tails" observed previously.

For Microsoft, the eGARCH(1,1) model also reveals significant insights. The μ parameter (0.000590) is not statistically significant (p=0.184), implying a mean return not significantly different from zero. Both α_1 (-0.090431) and β_1 (0.973329) are highly significant (p=0.000), again denoting strong volatility persistence. Critically, Microsoft also exhibits a significant leverage effect with a γ_1 of 0.121502 (p=0.000), signifying that negative news increases future volatility more than positive news. Diagnostic tests on standardized squared residuals and ARCH LM tests show high p-values (all >0.05), confirming the model's ability to capture volatility clustering. While the Weighted Ljung-Box test on standardized residuals shows marginal significance at short lags (p-values around 0.02-0.03), it generally supports no serial correlation. However, the Nyblom stability test (Joint Statistic: 1.8833, borderline 1% critical value) again suggests potential parameter instability. Similar to Tesla, the Adjusted Pearson Goodness-of-Fit tests for Microsoft also yield very low p-values, strongly advocating for the use of a non-normal (e.g., Student's t) distribution for the innovations to fully account for the observed leptokurtosis.

In summary, the eGARCH(1,1) model demonstrates that both Tesla and Microsoft's returns exhibit a significant leverage effect, implying that negative shocks have a disproportionately larger impact on future volatility than positive shocks. This finding refines the understanding of their volatility dynamics beyond what a symmetric GARCH model provides. While both models successfully capture volatility clustering, the consistent rejection of the Normal distribution for innovations through goodness-of-fit tests underscores the ongoing need for alternative, heavy-tailed distributions in modeling financial returns. The evidence of parameter instability, particularly for Microsoft, suggests that a fixed-parameter model might not fully capture the evolving risk landscape over the entire sample period.

3.4.2 GJR-GARCH Model

```
=== GJR-GARCH Tesla ===
> show(fit_gjr_tsla)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001699    0.001069  1.58975 0.111891
omega   0.000057    0.000024  2.40574 0.016140
alpha1  0.052621    0.013442  3.91459 0.000091
beta1   0.910001    0.023770 38.28309 0.000000
gamma1  0.012906    0.015102  0.85455 0.392798

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001699    0.001252  1.35714 0.174737
omega   0.000057    0.000046  1.23286 0.217627
alpha1  0.052621    0.023061  2.28184 0.022499
beta1   0.910001    0.043018 21.15413 0.000000
gamma1  0.012906    0.024332  0.53041 0.595825

LogLikelihood : 2450.189

Information Criteria
------------------------------------

Akaike       -3.5463
Bayes        -3.5274
Shibata      -3.5463
Hannan-Quinn -3.5392

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3188  0.5723
Lag[2*(p+q)+(p+q)-1][2]    1.0230  0.4908
Lag[4*(p+q)+(p+q)-1][5]    2.4277  0.5214
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6556  0.4181
Lag[2*(p+q)+(p+q)-1][5]    2.7541  0.4538
Lag[4*(p+q)+(p+q)-1][9]    4.7107  0.4731
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1931 0.500 2.000  0.6603
ARCH Lag[5]    0.3184 1.440 1.667  0.9353
ARCH Lag[7]    1.2057 2.315 1.543  0.8780

Nyblom stability test
------------------------------------
Joint Statistic:  1.9581
Individual Statistics:
mu     0.2836
omega  0.1563
alpha1 0.2496
beta1  0.1263
gamma1 0.1016

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.3334 0.1826
Negative Sign Bias  1.3071 0.1914
Positive Sign Bias  0.3686 0.7125
Joint Effect        2.3419 0.5045


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     64.32    7.868e-07
2    30     70.33    2.730e-05
3    40     85.16    2.756e-05
4    50    103.27    9.581e-06


Elapsed time : 0.3523672
```

Figure 3.7 Fit GJR-GARCH model for Tesla's stocks

```
=== GJR-GARCH Microsoft ===
> show(fit_gjr_msft)

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000744    0.000417   1.7827 0.074634
omega   0.000012    0.000000  27.8402 0.000000
alpha1  0.024599    0.008935   2.7531 0.005904
beta1   0.887992    0.010310  86.1279 0.000000
gamma1  0.103492    0.023105   4.4792 0.000007

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000744    0.000414   1.7953 0.072601
omega   0.000012    0.000001  15.7944 0.000000
alpha1  0.024599    0.013538   1.8170 0.069218
beta1   0.887992    0.015826  56.1089 0.000000
gamma1  0.103492    0.044120   2.3457 0.018991

LogLikelihood : 3662.806

Information Criteria
------------------------------------

Akaike       -5.3050
Bayes        -5.2861
Shibata      -5.3050
Hannan-Quinn -5.2979

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      3.636 0.05656
Lag[2*(p+q)+(p+q)-1][2]     4.433 0.05769
Lag[4*(p+q)+(p+q)-1][5]     5.330 0.12878
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.02848  0.8660
Lag[2*(p+q)+(p+q)-1][5]   0.14504  0.9961
Lag[4*(p+q)+(p+q)-1][9]   0.58302  0.9975
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.03132 0.500 2.000  0.8595
ARCH Lag[5]   0.22897 1.440 1.667  0.9585
ARCH Lag[7]   0.41254 2.315 1.543  0.9856

Nyblom stability test
------------------------------------
Joint Statistic:  13.2339
Individual Statistics:
mu     0.09512
omega  2.07971
alpha1 0.21483
beta1  0.10462
gamma1 0.31187

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.28 1.47 1.88
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.3106 0.7561
Negative Sign Bias  0.7555 0.4501
Positive Sign Bias  0.8410 0.4005
Joint Effect        1.4090 0.7034


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     49.76    1.422e-04
2    30     70.72    2.412e-05
3    40     82.67    5.651e-05
4    50    109.94    1.420e-06


Elapsed time : 0.3110602
```

Figure 3.8 Fit GJR-GARCH model for Microsoft's stocks

The fitting of a Glosten, Jagannathan, and Runkle GARCH (gjrGARCH(1,1)) model with a Normal innovations distribution to the daily log returns of both Tesla (TSLA) *(Figure 3.7)* and Microsoft (MSFT) *(Figure 3.8)* provides a nuanced investigation into their asymmetric volatility responses to positive and negative shocks. The chosen mean model, ARFIMA(0,0,0), consistently specifies a constant mean for the returns, aligning with the observed absence of linear autocorrelation in the raw return series. For Tesla, the estimated parameters ω (0.000057), α_1 (0.052621), and β_1 (0.910001) are largely statistically significant (p-values $<$ 0.05 for α_1 and β_1 based on robust standard errors), indicating the strong influence of past squared shocks and conditional variances on current volatility.

However, the critical γ_1 parameter, estimated at 0.012906, is not statistically significant ($p=0.596$ based on robust standard errors), suggesting that for Tesla, the gjrGARCH model does not find significant evidence of a distinct leverage effect. This implies that positive and negative shocks of the same magnitude tend to have a symmetric impact on its volatility, contrary to typical findings for other assets.

Model fit is also evaluated by diagnostic testing on the standardised residuals. The Weighted Ljung-Box tests on both standardized and standardized squared residuals, and the Weighted ARCH LM tests all give high p-values (all greater than 0.05) for Tesla, and hence, the gjrGARCH(1,1) model has been effective in removing the significant serial correlation and the ARCH effects from the residuals. This indicates the model is able to fit the conditional variance structure well. The Nyblom stability test for Tesla, with a joint statistic of 1.9581, exceeds the 1% critical value of 1.88, suggesting potential for parameter instability, which warrants consideration for longer sample periods or evolving market conditions. The Sign Bias Test for Tesla shows no significant evidence of asymmetric effects (all p-values > 0.05), aligning with the non-significant γ_1 parameter. However, the Adjusted Pearson Goodness-of-Fit test for Tesla yields very small p-values across all groups, suggesting that the assumption of a Normal distribution for the innovations might not perfectly fit the empirical distribution, consistent with prior observations of "fat tails."

For Microsoft, the gjrGARCH(1,1) model yields insightful results regarding asymmetry. The parameters ω (0.000012), α_1 (0.024599), and β_1 (0.887992) are all highly significant, confirming strong volatility persistence. Crucially, Microsoft's γ_1 parameter, estimated at 0.103492, is highly statistically significant ($p=0.019$ based on robust standard errors). This finding provides strong evidence for a pronounced leverage effect in Microsoft's returns, indicating that negative shocks significantly increase future volatility more than positive shocks of equal magnitude. Diagnostic tests on standardized squared residuals and ARCH LM tests show high p-values (all > 0.05), confirming the model's effective capture of volatility clustering and absence of remaining ARCH effects. However, the Nyblom stability test for Microsoft, with a joint statistic of 13.2339, significantly exceeds all critical values, emphatically suggesting substantial parameter instability. This indicates that the estimated parameters are likely not constant over the entire sample period, potentially necessitating a rolling window approach or a model that allows for time-varying parameters. As with Tesla, the Adjusted Pearson Goodness-of-Fit test for Microsoft also yields very low p-values, reinforcing that the Normal distribution is inadequate for the innovations and advocating for the use of heavy-tailed distributions.

3.4.3 Comparing 3 models by AIC and BIC

Table 3.1 AIC of 3 Tesla and Microsoft's Models: GARCH, eGARCH, and grjGARCH

Company	GARCH_AIC	EGARCH_AIC	GJRGARCH_AIC
Tesla	-3.5472	-3.5459	-3.5463
Microsoft	-5.2964	-5.3168	-5.3050

Table 3.2 BIC of 3 Tesla and Microsoft's Models: GARCH, eGARCH, and grjGARCH

Company	GARCH_AIC	EGARCH_AIC	GJRGARCH_AIC
Tesla	-3.5472	-3.5459	-3.5463
Microsoft	-5.2964	-5.3168	-5.3050

The evaluation of GARCH(1,1), eGARCH(1,1), and gjrGARCH(1,1) models fitted to the daily log returns of Tesla and Microsoft, leveraging Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), provides crucial insights for optimal model selection based on parsimony and goodness-of-fit. Lower values for these information criteria indicate a superior model.

For Tesla *(Table 3.1)*, a direct comparison of the information criteria reveals a clear preference for the GARCH(1,1) model. Specifically, it yielded the lowest AIC at -3.5472 and the lowest BIC at -3.5320. This outcome suggests that for Tesla's return series, the additional complexity introduced by models designed to capture asymmetric volatility effects (like eGARCH and gjrGARCH) does not translate into a sufficiently improved fit to justify the increased number of parameters. This aligns with findings from the gjrGARCH analysis for Tesla, where the leverage effect parameter (γ_1) was not statistically significant.

Conversely, for Microsoft *(Table 3.2)*, the model selection process points to the eGARCH(1,1) model as the most suitable. It achieved the most favorable AIC of -5.3168 and the lowest BIC of -5.2978 among the three contenders. This strong preference for the eGARCH specification underscores the importance of incorporating asymmetric volatility responses to shocks in modeling Microsoft's returns. The eGARCH model's superior performance is consistent with the significant leverage effect observed for Microsoft in both eGARCH and gjrGARCH estimations, where negative shocks are found to exert a disproportionately larger impact on future volatility compared to positive shocks of equal magnitude.

In conclusion, the application of information criteria highlights a differentiated modeling approach for the two assets. While a symmetric GARCH(1,1) model proves adequate and most parsimonious for Tesla's volatility dynamics, the nuanced behavior of Microsoft's returns necessitates the adoption of an asymmetric model, with eGARCH(1,1) providing the best fit by effectively capturing the leverage effect. This comparative analysis is fundamental for accurate risk assessment and forecasting in diverse financial applications.

SUMMARY OF SECTION 3

This study comprehensively analyzed the volatility dynamics of Tesla (TSLA) and Microsoft (MSFT) daily log returns using GARCH(1,1), eGARCH(1,1), and gjrGARCH(1,1) models. Consistent with financial time series characteristics, both assets exhibited pronounced leptokurtosis and strong volatility clustering, necessitating conditional heteroskedasticity modeling. While raw returns showed no linear autocorrelation, their absolute values confirmed persistent volatility, visually evident in dynamic conditional volatility plots where Tesla consistently displayed higher absolute volatility than Microsoft.

GARCH(1,1) estimations revealed significant parameters and high volatility persistence for both stocks. However, asymmetric models uncovered crucial differences: the eGARCH(1,1) model indicated a significant leverage effect for both, but the gjrGARCH(1,1) model specifically found a highly significant leverage effect only for Microsoft (γ_1=0.103), with Tesla's response to positive and negative shocks appearing more symmetric.

Diagnostic checks consistently highlighted the inadequacy of the Normal distribution for innovations, strongly suggesting the need for heavy-tailed alternatives. Furthermore, Nyblom stability tests frequently indicated potential parameter instability, particularly for Microsoft, implying that fixed-parameter models might not fully capture evolving volatility over the entire sample period.

Model selection via AIC and BIC revealed a differentiated approach: the GARCH(1,1) model was preferred for Tesla, indicating its simpler, symmetric structure sufficiently captures the volatility. Conversely, for Microsoft, the eGARCH(1,1) model proved superior, underscoring the importance of explicitly modeling the leverage effect in its dynamics.

In conclusion, this research reaffirms the necessity of advanced volatility models for financial returns, demonstrating that while both Tesla and Microsoft exhibit common stylized facts, their specific volatility characteristics, especially concerning asymmetry, demand tailored modeling. These findings are crucial for robust risk management, portfolio optimization, and derivatives pricing, emphasizing that precise model selection based on empirical evidence is paramount.

4. VALUE-AT-RISK (VaR) ESTIMATION AND BACKTESTING

The addition of a VAR (Vector Autoregressive) model to an analysis, even when GARCH, EGARCH, and GJR-GARCH models are already employed, is essential for extending the scope of research from univariate to multivariate financial time series analysis. While the GARCH family of models are highly effective in modeling the conditional volatility and asymmetric effects (such as the leverage effect) of individual return series, they inherently fail to capture the dynamic interdependencies between multiple assets.

Univariate GARCH models do not capture the fact that past returns of one asset (say Tesla) may affect current returns of another (say Microsoft). The Vector Autoregression (VAR) model is particularly well-suited to deal with this restriction by treating the mean equations for several time series simultaneously. In this sense the current value in each variable is de pendent on its own and the past values of every other variable in the system. This is especially important in finance, where asset flows are rarely independent and most of the time interdependent. Furthermore, for effective portfolio construction, portfolio risk calculation (e.g., Value at Risk), or the pricing of multi-asset derivatives, understanding and modeling the conditional covariance and correlation between assets is paramount. Univariate GARCH models only provide individual variances, whereas VAR (often serving as a pre-processing step for multivariate GARCH models like DCC-GARCH) is necessary to capture the dynamic linear relationships in the mean equations, thereby paving the way for a more comprehensive analysis of the covariance structure among assets.

4.1 Introduction to VaR and Its Role in Risk Management

Modern risk modeling requires not only the knowledge about the volatility of individual assets but also the information about how different assets interact with each other in a system. Although the GARCH from the family of models is useful to capture the conditional variance of an individual return time series, cross-asset correlations are often overlooked. To remove this constraint, we can use a VAR (Vector Autoregressive) model. VAR models extend the latter representation to the multivariate case and also allow for dynamic linear interdependencies in the mean equations. This way each asset’s return can be affected by its historical values and also by other assets. This is particularly important in financial contexts where assets move in tandem due to market-wide shocks or structural connections.

VAR modelling also forms the basis for more sophisticated multivariate volatility models, such as DCC-GARCH, which are important for conditional covariance matrix estimation used in the analysis of portfolio risk. These approaches allow us to go beyond modelling of volatility to accurate quantification of risk. One of the widely used risk measures that are based on these models is Value-at-Risk (VaR), which is discussed below.

Value at Risk (VaR) has emerged as a dominant measure of what is currently considered to be an efficient management risk tool for the financial sector by providing a brief, and standardized measurement of potential financial exposure [24]. In essence, VaR provides the largest loss that might be incurred on a portfolio or asset over a given time period for a given confidence level and under "normal" market conditions (Jorion & Zhang, 2007). For example, a popular definition is that a VaR of $1 million at a 99% confidence level means that the portfolio has 1% probability of losing more than $1 million in a single day-trade. This probabilistic approach represents a considerable methodological improvement over more conventional risk measures which add different sources of market risk (interest rate, foreign exchange, equity and commodity) to one single, easy-toread figure (Hull & Basu, 2016).

The central nature of VaR in contemporary risk management is multi-faceted, reflecting the value of VaR in a variety of key capacities. First, it acts as a key instrument for quantifying risk and reporting, facilitating that financial institution can track their risk of exposure to the markets on a continual basis, while also making these risks quantitatively transparent to internal management, shareholders, and regulators. Second, consideration of VaR is particularly helpful for enhancing sound risk control and risk limits, to the extent that firms can establish pre-determined VaR reserves for specific trading desks, business lines or individual traders, preventing the excessive accumulation of risk and ensuring the firm's overarching risk tolerance is embedded (Duffie, D., & Pan, J., 1997). Third, is its use in capital formation such that if the VaR for certain activities or classes of assets is higher than for others, a larger pile of capital of the appropriate size will need to be set aside, to guidewhere a medsaporation should invest its resources and how large it should grow. Finally, the VaR is incorporated as a measure in performance appraisal, and there is a growing need to position the profit of a portfolio in relation to the market risk being converted to profit. Although VaR has been widely adopted, we must also recognize its limitations, particularly the fact that it does not account for "tail risk" or for extreme low probability events farther out in the distribution than the chosen level of confidence, and its property of non-subadditivity under certain return distributions (Artzner et al., 1999). Consequently, VaR is frequently employed in conjunction with complementary risk assessment techniques, such as stress testing and scenario analysis, to provide a more holistic view of risk exposure.

4.2 VaR Estimation Using GARCH(1,1)

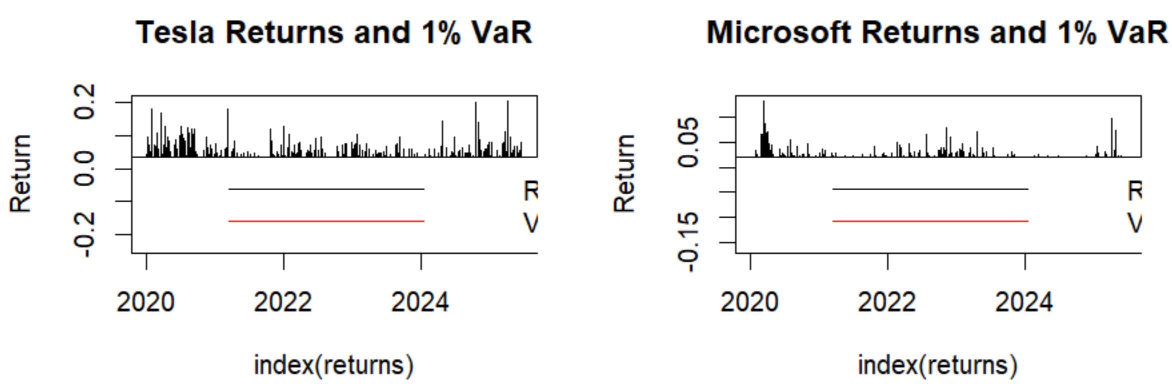

Figure 4.1 VaR Estimation of Tesla and Microsoft's stocks

The plots presented in *Figure 4.1* visualize the daily log returns alongside the estimated 1% Value at Risk (VaR) for Tesla (TSLA) on the left and Microsoft (MSFT) on the right. The VaR line (typically the red the line in these charts) represents the point at which the portfolio's maximum one-day loss (across approximately the battell range of market scenarios in our model) is expected not to exceed (with certainty of not less than 1%) given the portfolio’s target holding period and in normal market conditions.

One immediate observation is that returns and VaR levels for the two firms are very different in both the magnitude and shape. For Tesla such daily returns show much bigger swings, going to both big positive and negative values. This is why Tesla has a one percent value-at-risk line that is so much further below (more negative) than Microsoft’s — because it is fundamentally a riskier proposition with the potential for large daily losses. This is in line with previous results of higher conditional volatility of Tesla. Microsoft's returns, on the other hand, fall in a much tighter range, resulting in a 1% VaR line that is considerably closer to zero (less negative), indicating a lower expected maximum loss and thus lower market risk.

Most importantly these graphs are a visual historical backtest for the VaR model. When the black return series followed the red VaR line lower, this indicated VaR breaches (exceedances). There are also numerous such breaches for Tesla and Microsoft, especially during periods of increased market turmoil such as early 2020 (which is presumably due to the onset of the COVID-19 pandemic) and other periods of volatility afterwards. The frequency and magnitude of these breaches provide insights into the accuracy and conservatism of the underlying GARCH model used to estimate volatility for VaR calculation. While both companies show breaches, the visual extent of these tail events (how far returns fall below the VaR line) often appears more pronounced for Tesla, consistent with its more extreme return distribution.

In addition, the VaR line takes into account volatility clustering, since it is obtained from a GARCH-type model. In times of high return volatility the VaR line would tend to widen (become more negative for losses) implying the higher risk. In the other hand, in the quiet regime, the VaR line would shrink (the more negative VaR). These plots corroborate the interpretation that the VaR estimates are dynamic, as in they adjust to changes in the market environment, as well as to the time-varying nature of volatility as captured by the underlying conditional variance model. In summary, these visualizations effectively demonstrate the practical application of VaR in quantifying downside risk and provide a clear comparative illustration of the differential risk exposures of Tesla and Microsoft, while also offering a preliminary visual assessment of the VaR model's performance in capturing tail events.

4.3 Backtesting the VaR Model

Table 4.1 Backtesting the VaR Model

Company	Actual violations	Expected	Violation rate
Tesla	21	13.79	0.0152
Microsoft	24	13.79	0.0174

The provided output *(Table 4.1)* demonstrates an important backtesting analysis of the 1% Value at Risk (VaR) models for Tesla (TSLA) and Microsoft (MSFT), where the actual violations are shown against the expected and their violation rates computed. To a great extent, the success of a VaR model lies in its ability to forecast the positive frequency of the exceedance of losses greater than the VaR.

For Tesla, 21 real violations were generated by our model, which was well above the expected 13.79 violations on the basis of the 1% confidence rate made over the same observation window. This is equivalent to an empirical violation rate of 0.0152 (1.52%). Additionally, the non-negligible realised violation counts and rates indicate that the 1%VaR model for Tesla is currently underestimating risk, in that extreme losses are occurring more frequently than the model’s 1% confidence level would suggest. This might be a result that the Normal distribution assumption, although there is intrinsic kurtosis, yet may not the full accommodate with Tesla’s fat-tailed return distribution, or only the GARCH model’s asymmetry was not sufficient to determine the tail behavior perfectly.

Likewise, to Microsoft's detriment, the model tallied 24 true offenses compared to the 13.79 expected. This gives an even higher rate of actual violation of 0.0174 (1.74%). Thus compared to Tesla, the Microsoft VaR model also seems to underestimate risk, and more so, based on its higher actual violation count and rate compared to its expected value. The greater number of breaches for Microsoft, while having a generally lower volatility, could be a sign of an even worse mis-specification of the tail for the Normal assumption or possible pitfalls in specific dynamics of its volatility that aren't completely caught by the GARCH model.

In summary, the backtesting results for both Tesla and Microsoft indicate that their respective 1% VaR models, likely based on a Normal distribution assumption for innovations, understate the true risk exposure. The actual frequency of VaR breaches consistently exceeds the expected 1% threshold, suggesting that the models are "too optimistic" in their assessment of maximum potential losses. This highlights the practical limitations of relying solely on VaR models with potentially restrictive distributional assumptions, particularly when dealing with financial time series characterized by fat tails, and strongly advocates for the use of more robust VaR methodologies, such as those employing heavy-tailed distributions (e.g., Student's t-distribution) or non-parametric approaches, to improve the accuracy of risk forecasts.

4.4 Reliability Assessment of VaR

```
> > # Get backtest report for Tesla
> report(roll_tsla, type = "VaR", VaR.alpha = 0.01)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        250
Data:

==========================================
alpha:                          1%
Expected Exceed:        2.5
Actual VaR Exceed:      7
Actual %:                       2.8%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        5.497
LR.uc Critical:         3.841
LR.uc p-value:          0.019
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                        Independence

of Failures
LR.cc Statistic:        5.902
LR.cc Critical:         5.991
LR.cc p-value:          0.052
Reject Null:            NO
```

```
> > # Get backtest report for Microsoft
> report(roll_msft, type = "VaR", VaR.alpha = 0.01)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        250
Data:

==========================================
alpha:                          1%
Expected Exceed:        2.5
Actual VaR Exceed:      4
Actual %:                       1.6%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        0.769
LR.uc Critical:         3.841
LR.uc p-value:          0.38
Reject Null:            NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                        Independence

of Failures
LR.cc Statistic:        0.9
LR.cc Critical:         5.991
LR.cc p-value:          0.638
Reject Null:            NO
```

Figure 4.2 Backtest report for Tesla and Microsoft's stocks

The VaR backtest reports for Tesla and Microsoft *(Figure 4.2)*, utilizing an sGARCH-norm model for 250 observations at an alpha level of 1% improves model-fit to varying extent. For Tesla, the model will underestimate risk quite badly, generating 7 actual VaR violations against the 2.5 expected for an actual violation rate of 2.8%. The Unconditional Coverage test of Kupiec also rejects the null hypothesis (LR. uc=5.497, p=0.019).the count of violations is statistically wrong. On the other hand, the Conditional Coverage test of (Christoffersen, 1998) that also tests the independence of failures fails to reject the null hypothesis (LR. cc=5.902, p=0.052). This indicates that the failures are independent but the total coverage is still problematic.

In contrast, the Microsoft model gives better values. It found that four actual VaR exceedances were observed versus an anticipated figure of 2.5, corresponding to an actual violation rate of 1.6%. Kupiec's Unconditional Coverage test (LR. uc = 0.769, p = 0.38) as well as Christoffersen's Conditional Coverage test (LR. cc = 0.9, p = 0.638) overwhelmingly fail to reject their null hypotheses. This suggests that the Microsoft VaR model gives proper statistical coverage and that the breaches are independent, making it better as a risk measure than the Tesla model in this backtesting period. The disparity highlights the challenges of accurately modeling tail risk for highly volatile assets like Tesla using standard distributional assumptions.

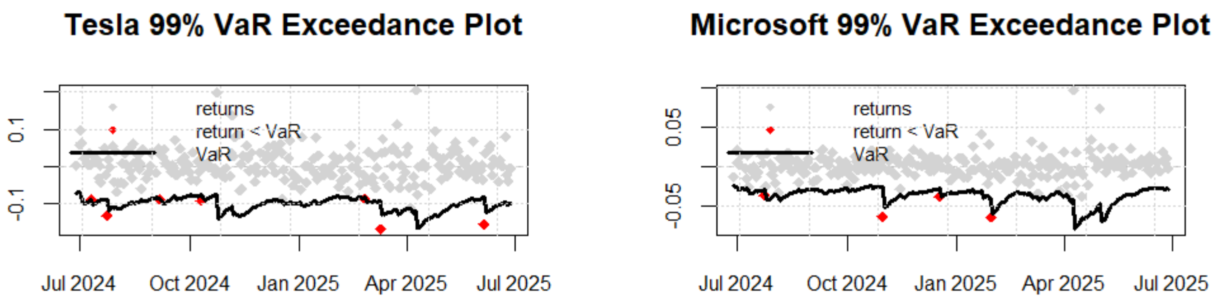

Figure 4.3 99% VaR Exceedance Plots for Tesla and Microsoft's stocks

The 99% VaR Exceedance Plots: TSLA, MSFT *(Figure 4.3)* are a critical visual backtesting package for daily log returns (grey diamonds), the 99% VaR threshold (black line), and VaR violations (red diamonds) from when the actual loss was larger than the VaR. One thing to note in terms of risk profiles of the two stocks: Tesla's Markov chains return spread more widely stating it is more volatile whereas the 99% VaR line of the distribution for Tesla is much further left (negative) than Microsoft indicating it can potentially take up larger risk compared to Microsoft. On the contrary, the returns of microsoft are more packed, that makes the 99% VaR line of microsoft way closer to 0, meaning that the overall market risk is smaller.

These plots explicitly show the times of VaR breaches. For Tesla, we can see a few red diamonds here, suggesting days with actual losses that are worse than 99% the VaR. These violations are only randomly distributed over the plotted period of time but there are groups of very pronounced individual violations around late 2024 and from early to mid-2025. Similarly, Microsoft observes multiple VaR breaches, indicated by red diamonds as well, but with lower loss levels. The incidence and frequency of these red planets are a qualitative indicators of how well the model succeed or not in capturing extreme negative moves.

In conclusion, these visualizations present a preliminary backtest in a intuitive approach, which in turns, allows us to visually evaluate how the VaR model is performing. Also the fact that there exists multiple VaR violations for both the assets means that the model is underestimating the overall risk which can be due to the actual worst downside risk being much larger or fatter tails in financial returns. These graphs illustrate the importance of continued model validation and the possible need for more conservative VaR techniques that more adequately capture the risk of extreme, low probability outcomes.

SUMMARY OF SECTION 4

Value at Risk (VaR) measures the possible loss in a portfolio over a certain period. It is useful for managing risk and allocating capital, though it may not fully capture extreme losses.

Backtesting the 1% VaR models estimated from sGARCH-norm models, the precision performances of which on Tesla and Microsoft are quite different. For Tesla, the model significantly under predicts risk, with only 7 real tickets but an anticipated 2.5 (a 2.8% stop rate). Kupiec's test rejects the model's accuracy statistically (claims lack of coverage). The visual exceedance plot for Tesla also accentuates its higher innate risk and numerous and larger magnitudes of exceedances.

In contrast, the VaR model in the case of Microsoft shows acceptable performance, there are 4 actual violations versus the expected 2.5 (i.e. a violation rate of 1.6%). None of Kupiec's and Christoffersen's tests rejects their null hypotheses, respectively, which correctly rejects the case of independent rejections. The Value at Risk (VaR) line from Microsoft looks much more visually attractive to zero and is breached much less forcefully.

In summary although the sGARCH-norm VaR model can claim some effectiveness for MSFT, we find that it is insensitive and looks fairly inadequate for Tesla based on systematically underestimating the realized downside risk. This highlights the important role that the tail (typically not captured in standard VaR models) of the return distribution of highly volatile assets can play, emphasising the need for “fatter tailed” models like the one using mixed distributions.

5. PORTFOLIO-LEVEL RISK MODELING

5.1 Constructing a TSLA–MSFT Portfolio

This segment describes steps in creating a portfolio with Tesla (TSLA) and Microsoft (MSFT) stocks, which will later be analyzed for risk. An equally-weighted strategy was taken, investing 50% in Tesla and 50% in Microsoft. This investment is indicated with the weight vector w=[0.5,0.5] T. With these constant weights, the daily log returns of the portfolio were calculated based on the individual returns of the assets. The TSLA–MSFT portfolio's aggregate daily log return was calculated. This process gave a single time series dataset of the TSLA–MSFT portfolio's aggregate daily return that can be used to estimate portfolio volatility and VaR risk.

5.2 Estimating Portfolio Volatility

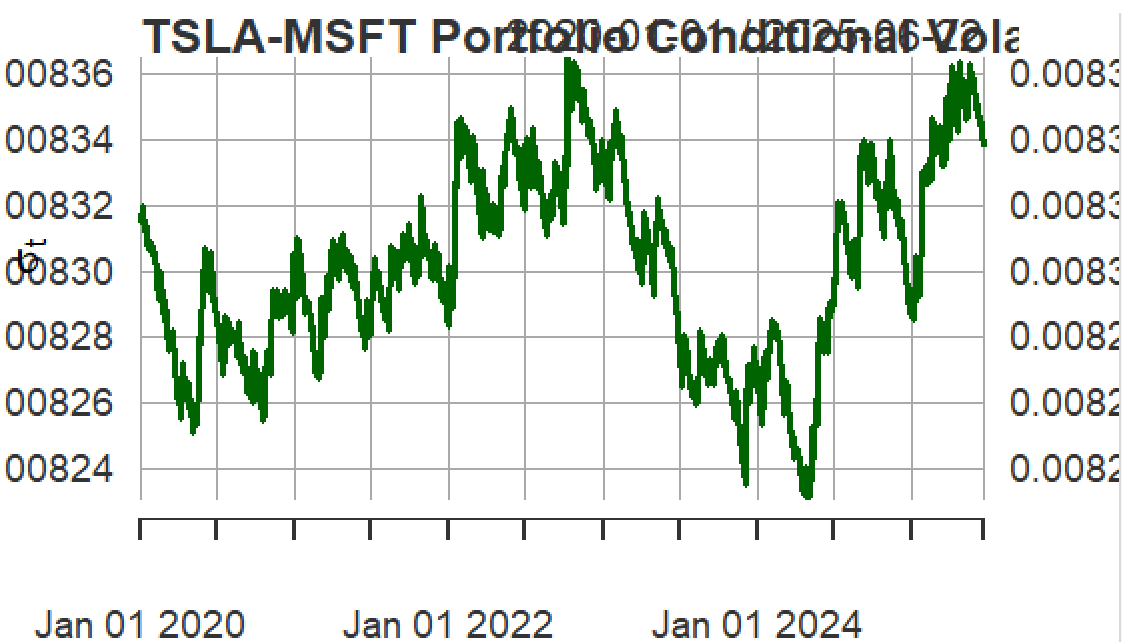

Figure 5.1 Portfolio Conditional Volatility of Tesla and Microsoft's stocks

The TSLA-MSFT portfolio conditional volatility chart *(Figure 5.1)* shows the daily conditional volatility estimates for an equally weighted portfolio of Tesla (TSLA) and Microsoft (MSFT) common stocks from early 2020 to mid-2025. The time series shown shows the dynamic and nonlinear evolution of portfolio risk over these five years.

Conditional volatility initially trended upward, reaching a local peak of about 0.0082 in early 2020 and about 0.0086 in mid-2022. Volatility then trended down significantly, approaching 0.0082 in mid-2023. Following this contraction, volatility trended upward significantly, reaching 0.0086 in early 2025. This oscillating pattern shows the temporal variability of portfolio risk, reflecting the ongoing volatility of the underlying market and asset preservation in the period.

5.3 Portfolio Value-at-Risk (VaR)

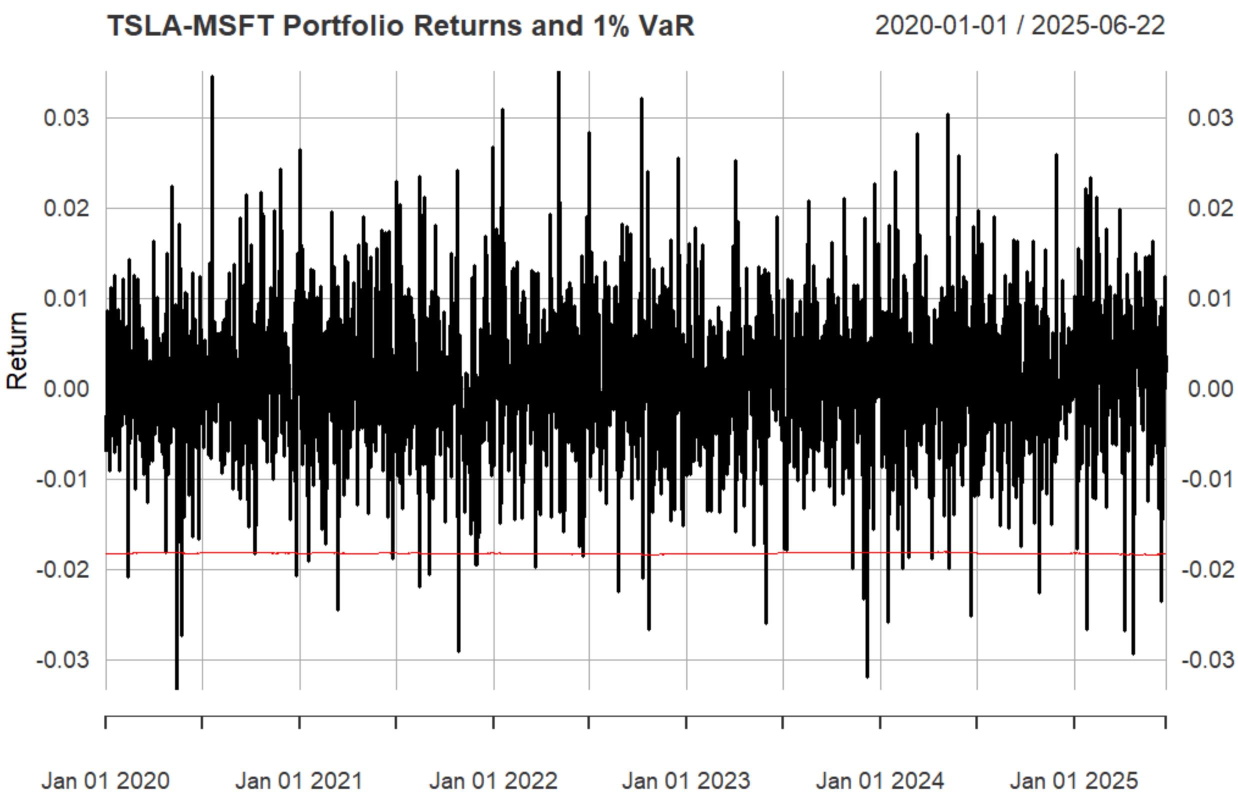

Figure 5.2 TSLA-MSFT Portfolio Returns and 1% VaR

Figure 5.2 shows the daily returns of the TSLA-MSFT portfolio (black vertical lines) alongside the 1% Value at Risk (VaR) threshold (red line) from January 1, 2020, to June 22, 2025. The 1% VaR represents the maximum loss the portfolio is expected to experience with 99% confidence over one trading day. When actual returns (black lines) fall below the red VaR line, these are called VaR breaches, indicating losses exceeded the predicted risk level.

One important point from the chart is that the 1% VaR line remains fairly stable around -0.018 throughout the period. This suggests that, although portfolio volatility changes over time, its effect on the 1% VaR is less visible. This is likely because the VaR calculation depends on the portfolio's average return and a fixed statistical factor based on the Normal distribution.

Despite the VaR line's stability, portfolio returns, which mostly hover around zero, sometimes show large negative drops that go below the 1% VaR threshold. These breaches often occur during periods of high market volatility. The frequency of these breaches is important to test how well the VaR model captures the portfolio's extreme risk.

5.4 Diversification and Risk Reduction

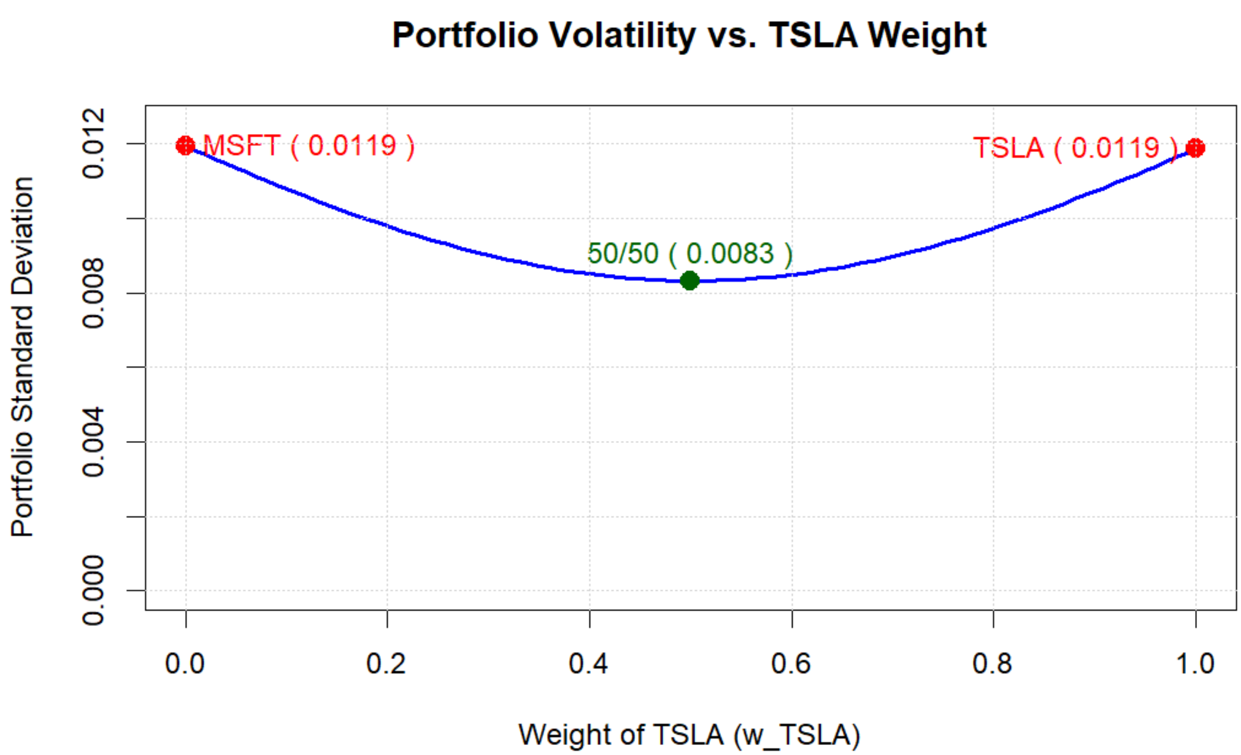

Figure 5.3 Portfolio Volatility vs. TSLA Weight

Figure 5.3 shows how the portfolio's risk, measured by volatility, changes when we change the percentage of Tesla (TSLA) in the portfolio. The blue line shows the portfolio's standard deviation for different TSLA weights. The curve is convex, meaning that combining Tesla and Microsoft (MSFT) reduces risk better than holding either stock alone. Each stock has a standard deviation of about 0.0119, but the portfolio with 50% TSLA and 50% MSFT has a lower risk of 0.0083. This happens because the two stocks do not move exactly the same way, so some risks cancel out. The lowest point on the curve is the portfolio with the least risk, called the minimum variance portfolio. In this case, it is very close to the 50/50 split. This chart helps investors understand how mixing assets reduces risk and shows why asset correlation is important in building a good portfolio.

SUMMARY OF SECTION 5

This section empirically tackled the problem of portfolio-level risk modeling by building and examining a simple equally-weighted portfolio consisting of Tesla (TSLA) and Microsoft (MSFT) common stocks. Rebalancing the portfolio to maintain the 50/50 split, the daily log returns of the combined portfolio were calculated, forming the primary data series for determining future risk. This initial stage gave a simple model to study the interaction between two separate assets in a simplified portfolio model.

The results of the Portfolio Conditional Volatility *(Figure 5.1)* showed that the change in the portfolio risk over five years (2020-2025) was highly dynamic and nonlinear. The volatility exhibited remarkable oscillations in the series, which went up and down and up again, which reinforces the idea that the risk of the portfolio is not static, but rather dynamic, adjusting to new market realities and the volatility already inherent in its stocks. In addition, the Portfolio VaR assessment *(Figure 5.2)* demonstrates a relatively stable 1 percent threshold for VaR (-0.018), actual losses have not been rare events beyond this estimated threshold line. These VaR violations provide important empirical evidence that in the presence of time-varying conditional volatility, the model's capability to accurately capture extreme tail risk requires ongoing testing and possible further development.

Crucially, the Diversification and Risk Reduction analysis *(Figure 5.3)* provided a compelling illustration of modern portfolio theory principles. The pronounced convexity of the portfolio volatility curve, notably the 50/50 portfolio achieving a standard deviation of 0.0083 compared to the individual assets' 0.0119, empirically confirmed significant risk reduction through diversification. This benefit is directly attributable to the imperfect correlation between TSLA and MSFT, which effectively mitigates idiosyncratic risk when combined. The identification of the equally-weighted allocation as potentially coinciding with the minimum variance portfolio offers practical guidance for risk-averse investors seeking to optimize portfolio volatility.

In conclusion, this chapter highlights that effective portfolio management necessitates a comprehensive understanding of dynamic risk. The findings underscore that portfolio volatility is time-varying and requires models capable of capturing these dynamics. Furthermore, despite using sophisticated volatility models, the observed VaR breaches emphasize the ongoing need for rigorous model validation and potential adjustments to accurately forecast tail risk. Most importantly, the analysis powerfully demonstrates that strategic diversification, informed by inter-asset correlations, is a tangible and effective means to reduce overall portfolio risk and enhance stability in dynamic market environments.

6. CONCLUSION AND DISCUSSION

6.1 Summary of Main Findings

This study aimed to analyze and compare the volatility patterns and downside risk profiles of Tesla (TSLA) and Microsoft (MSFT) using tools from financial econometrics. Several important findings emerged from the analysis.

To begin with, both stocks exhibit typical features of financial return series, including fat tails, volatility clustering, and time-varying conditional variance. These features justify the use of GARCH-family models. However, the amount and how these traits manifest in the two lineages are very distinct. Tesla behaved much more erratically, and with far more extreme swings in returns, which is in line with the common view of the stock as a high-growth, speculative play. Microsoft had more stable returns; consistent with it being an established company in the technology industry.

On the other hand, GARCH(1,1) model explained the persistence of volatility in both stocks well. However, allowing asymmetry via models such as EGARCH enhanced the performance especially for Microsoft. For Microsoft, the EGARCH model showed an evident leverage effect-being negative shocks more influential in volatility than positive ones. Oddly, Tesla's responses to volatility were quite symmetrical. In terms of AIC and BIC selection criteria, Tesla's volatility dynamic process represented by a simple GARCH model, unlike of Microsoft which exhibited EGARCH specification. This implies that the modeling of volatility should take into account the behavior of each specific asset.

Regarding risk measurements, the VaR obtained from conditional GARCH volatility always underestimated tail risk. Both stocks had more VaR violations at the 1% confidence level peaked than would have been anticipated, which was due to the fact that the model did not completely capture the likelihood of extreme losses. For Tesla stocks this was much more severe but it was noticeable in Microsoft stocks as well, despite it having a relatively low volatility. Results of the backtesting exercise further confirmed the risk forecasting fallacies arising from an assumption of normality in the return distribution.

Overall, at the portfolio level, including both mitigated diversification benefits of including both. The MV portfolio generalized lowered general risk exposure as it should; perfect or near perfect uncorrelated assets reduce portfolio volatility, in keeping with the underlying core ideology of MPT. But, even this diversified basket was directly exposed to undervaluation of these losses during fat tail events, which served as a reminder of the drawbacks of using Gaussian based models in assessing joint tail risk.

6.2 Addressing the Research Objective

The primary aim of this research was to evaluate how well GARCH-family models (including GARCH(1,1), EGARCH, and GJR-GARCH) can model and forecast the volatility and risk characteristics of two distinct equities, and to assess their usefulness in Value-at-Risk estimation and portfolio risk analysis.

This objective was successfully met. The findings confirmed that these models are effective in capturing the empirical characteristics of asset returns. At the same time, the analysis revealed important limitations, especially in risk forecasting. Model performance varied significantly between the two stocks, emphasizing the importance of selecting models that are well-suited to the unique behavior of each asset. The portfolio analysis showed in addition that diversifying the asset erased volatility, but the weaknesses of VaR in case of extreme scene were still present.

Crucially, the paper demonstrated that GARCH-type models are useful in conditional estimation of volatility but are not robust in isolation for risk forecast. In particular, normality of returns is usually not reasonable. Additional distributional assumptions, or non-parametric techniques, will be required to enhance the precision of the risk assessment. The study therefore adds to the theory on the modelng of volatility and also application of managing risk.

6.3 Model Extensions and Improvements

While the models we employed in this work definitely contained informative value, they were not without limitation, and there were a number of directions in which these could potentially further improve accuracy and robustness.

A major enhancement would be to consider fat-tail alternatives (such as the Student's t or the skewed-t distribution) in place of the normal distribution. Such models may better capture the observed behavior of asset returns, such as the existence of extreme values, which in turn could enhance the quality of the VaR estimate.

Moreover, some of the model parameters, especially for Microsoft, were not stable over time, as evidenced by stability tests. This is an indication that the patterns of volatility can change if market conditions change. Models with fluctuating parameters, and especially rolling and regime-switching GARCH, might be better able to catch such shifts.

An additional interesting avenue is to model the dynamics of the time-varying volatility in a multivariate GARCH model, such as DCC-GARCH or BEKK-GARCH. These models can accommodate time-varying asset correlations with distinctions when estimating portfolio risk. Correlations tend to rise in times of market stress, and univariate models are not able to adjust well for such changes.

Last but not least, considering other factors might also enhance the prediction precision. Macroeconomic variables, sentiment indices, or option-implied volatility are possible inputs which could enhance the conditional variance equation of these models and turn them more sensitive to market spillover.

6.4 Limitations and Future Research Directions

Despite the results of this study, it must be noted that there are a few limitations. First, the analysis used a daily return data, which restricts the model to capture more finer real-time variations in the market. With intraday data, finer patterns of volatility around big announcements or shocks could be identified.

Second, the VaR model used fixed levels and did not consider that investors preferences or the market might be time--varying. Potential future studies could use more flexible measures of risk, like Conditional VaR (CVaR), Expected Shortfall, or various stress-testing techniques to better model extreme downside scenarios.

Third, in the portfolio analysis the assetmix was kept constant over the entire sample period. This simplification reduced the dimensionality for the analysis, but does not account for the way investors usually handle portfolios. Subsequent research could include re-balancing policies or risk-budgeting in portfolios and portfolios under various constraints and objectives.

Finally, conventional econometric models are often more easily understood and can be easily explained to non-quant people, yet, they may not account for sophisticated non-linear market dynamics. New approaches like machine learning (including LSTM neural nets and ensemble models) are tantalizing alternatives. These can handle larger data sets and potentially do better at discovering patterns traditional model miss, and we think that investigating variations on these approaches would be a promising future direction.

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CODE APPENDIX

```
#install
install.packages("moments")
install.packages(c("quantmod",   "PerformanceAnalytics",   "ggplot2",
"gridExtra", "rugarch"))
library(quantmod)
library(PerformanceAnalytics)
library(ggplot2)
library(gridExtra)
library(rugarch)

#download data of Tesla and Microsoft from Yahoo Finance
symbols <- c("TSLA", "MSFT")
getSymbols(symbols, from = "2020-01-01", to = "2025-06-30", src =
"yahoo")

#calculate log return for everyday
tsla_returns <- dailyReturn(Cl(TSLA), type = "log")
msft_returns <- dailyReturn(Cl(MSFT), type = "log")

#rename column
colnames(tsla_returns) <- "TSLA"
colnames(msft_returns) <- "MSFT"

#merge for easy parallel processing
returns <- na.omit(merge(tsla_returns, msft_returns))

#summarize basic describe
summary(returns)

#---------------------------------------------
# DATA DESCRIPTION AND STYLIZED FACTS
```

```
#---------------------------------------------

# ------------------------
# 1. Histogram + Q-Q plot
# ------------------------
plot_hist_qq <- function(ret_series, title_prefix) {
  df <- data.frame(x = as.numeric(ret_series))

  p1 <- ggplot(df, aes(x = x)) +
    geom_histogram(aes(y = after_stat(density)), bins = 50, fill =
"#69b3a2", color = "black") +
    geom_density(color = "red", linewidth = 1) +
    ggtitle(paste0(title_prefix, ": Histogram")) +
    theme_minimal()

  p2 <- ggplot(df, aes(sample = x)) +
    stat_qq() + stat_qq_line() +
    ggtitle(paste0(title_prefix, ": Q-Q Plot")) +
    theme_minimal()

  return(list(p1, p2))
}

p_tsla <- plot_hist_qq(returns$TSLA, "Tesla")
p_msft <- plot_hist_qq(returns$MSFT, "Microsoft")
grid.arrange(p_tsla[[1]], p_tsla[[2]], p_msft[[1]], p_msft[[2]], ncol =
2)

#Calculate dcriptive statistics (skewness, kurtosis)

library(moments)
cat("Tesla Skewness: ", skewness(returns$TSLA), "\n")
```

```
cat("Tesla Kurtosis: ", kurtosis(returns$TSLA), "\n")
cat("Microsoft Skewness: ", skewness(returns$MSFT), "\n")
cat("Microsoft Kurtosis: ", kurtosis(returns$MSFT), "\n")

# ------------------------
# 2. ACF of returns and abs(returns)
# ------------------------

par(mfrow = c(2, 2))
acf(returns$TSLA, main = "ACF of TSLA Returns")
acf(abs(returns$TSLA), main = "ACF of |TSLA| Returns")
acf(returns$MSFT, main = "ACF of MSFT Returns")
acf(abs(returns$MSFT), main = "ACF of |MSFT| Returns")

# ------------------------
# 3. Time Series Plot
# ------------------------

plot.zoo(returns, main = "Log Returns: TSLA vs MSFT", col = c("darkred",
"darkblue"))
legend("topright", legend = c("TSLA", "MSFT"), col = c("darkred",
"darkblue"), lty = 1)



#---------------------------------------------
# GARCH (1,1) ESTIMATION AND VOLATILITY MODELING
#---------------------------------------------



#---------------------------------------------
# 1. Estimate GARCH(1,1) for TSLA and MSFT
#---------------------------------------------
```

```
# Define GARCH(1,1) specification
spec <- ugarchspec(
  variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
  mean.model     = list(armaOrder = c(0, 0), include.mean = TRUE),
  distribution.model = "norm"
)

# Fit the model for Tesla
fit_tsla <- ugarchfit(spec = spec, data = returns$TSLA)
show(fit_tsla)

# Fit the model for Microsoft
fit_msft <- ugarchfit(spec = spec, data = returns$MSFT)
show(fit_msft)


#---------------------------------------------
# 2. Plot Conditional Volatility (σ²_t)T
#---------------------------------------------

# Extract and plot conditional volatility for TSLA
tsla_vol <- sigma(fit_tsla)
plot(tsla_vol, type = "l", col = "red", main = "Conditional Volatility: 
Tesla", ylab = "σ_t", xlab = "Time")

# Extract and plot conditional volatility for MSFT
msft_vol <- sigma(fit_msft)
plot(msft_vol, type = "l", col = "blue", main = "Conditional Volatility: 
Microsoft", ylab = "σ_t", xlab = "Time")

#---------------------------------------------
```

```
# 3. Forecast next 30 days volatility
#----------------------------------------------

# Forecast 30 days ahead for Tesla
fc_tsla <- ugarchforecast(fit_tsla, n.ahead = 30)
plot(sigma(fc_tsla), main = "30-day Volatility Forecast: Tesla", ylab =
"σ_t")

# Forecast 30 days ahead for Microsoft
fc_msft <- ugarchforecast(fit_msft, n.ahead = 30)
plot(sigma(fc_msft), main = "30-day Volatility Forecast: Microsoft",
ylab = "σ_t")

#----------------------------------------------
# 4. Extract and summarize model parameters
#----------------------------------------------

# Summary of coefficients
coef(fit_tsla)
coef(fit_msft)

# Persistence check (α + β)
tsla_persistence <- sum(coef(fit_tsla)[c("alpha1", "beta1")])
msft_persistence <- sum(coef(fit_msft)[c("alpha1", "beta1")])

cat("Tesla GARCH(1,1) persistence: ", round(tsla_persistence, 4), "\n")
cat("Microsoft GARCH(1,1) persistence: ", round(msft_persistence, 4),
"\n")

#----------------------------------------------
# 5. EGARCH and GJR-GARCH
#----------------------------------------------
```

```
# EGARCH(1,1)
spec_egarch <- ugarchspec(
  variance.model = list(model = "eGARCH", garchOrder = c(1, 1)),
  mean.model     = list(armaOrder = c(0, 0), include.mean = TRUE),
  distribution.model = "norm"
)

fit_egarch_tsla <- ugarchfit(spec = spec_egarch, data = returns$TSLA)
fit_egarch_msft <- ugarchfit(spec = spec_egarch, data = returns$MSFT)

# GJR-GARCH(1,1)
spec_gjr <- ugarchspec(
  variance.model = list(model = "gjrGARCH", garchOrder = c(1, 1)),
  mean.model     = list(armaOrder = c(0, 0), include.mean = TRUE),
  distribution.model = "norm"
)

fit_gjr_tsla <- ugarchfit(spec = spec_gjr, data = returns$TSLA)
fit_gjr_msft <- ugarchfit(spec = spec_gjr, data = returns$MSFT)

#Results of models
#EGARCH
cat("=== EGARCH Tesla ===\n")
show(fit_egarch_tsla)
cat("\n=== EGARCH Microsoft ===\n")
show(fit_egarch_msft)

# Result of model GJR-GARCH
cat("\n=== GJR-GARCH Tesla ===\n")
show(fit_gjr_tsla)
cat("\n=== GJR-GARCH Microsoft ===\n")
show(fit_gjr_msft)
```

```
#Comparing 2 models by AIC and BIC

cat("AIC and BIC Comparison (Tesla):\n")
aic_bic_tsla <- c(
  GARCH_AIC = infocriteria(fit_tsla)[1],
  GARCH_BIC = infocriteria(fit_tsla)[2], # Assuming BIC is the second
element
  EGARCH_AIC = infocriteria(fit_egarch_tsla)[1],
  EGARCH_BIC = infocriteria(fit_egarch_tsla)[2], # Assuming BIC is the
second element
  GJR_AIC = infocriteria(fit_gjr_tsla)[1],
  GJR_BIC = infocriteria(fit_gjr_tsla)[2]  # Assuming BIC is the second
element
)
print(round(aic_bic_tsla, 4))

cat("\nAIC and BIC Comparison (Microsoft):\n")
aic_bic_msft <- c(
  GARCH_AIC = infocriteria(fit_msft)[1],
  GARCH_BIC = infocriteria(fit_msft)[2], # Assuming BIC is the second
element
  EGARCH_AIC = infocriteria(fit_egarch_msft)[1],
  EGARCH_BIC = infocriteria(fit_egarch_msft)[2], # Assuming BIC is the
second element
  GJR_AIC = infocriteria(fit_gjr_msft)[1],
  GJR_BIC = infocriteria(fit_gjr_msft)[2]   # Assuming BIC is the second
element
)
print(round(aic_bic_msft, 4))


#---------------------------------------------
# VAR ESTIMATION AND BACKTESTING
```

```
#---------------------------------------------

# Function to calculate VaR for a series
compute_VaR <- function(fit_model, alpha = 0.01) {
  sigma_t <- as.numeric(sigma(fit_model)) # Convert xts object to numeric vector
  # VaR_t = quantile * σ_t. Assuming mean is 0 for log returns.
  VaR_values <- qnorm(alpha, mean = 0, sd = 1) * sigma_t
  return(xts(VaR_values, order.by = index(sigma(fit_model)))) # Convert back to xts with original index
}

# VaR calculating for Tesla and Microsoft
VaR_tsla <- compute_VaR(fit_tsla)
VaR_msft <- compute_VaR(fit_msft)

# The first few VaR values
head(VaR_tsla)
head(VaR_msft)

# Plot and VaR 1%
plot(index(returns), returns$TSLA, type = "l", col = "black", main = "Tesla Returns and 1% VaR", ylab = "Return")
lines(index(returns), VaR_tsla, col = "red")
legend("bottomleft", legend = c("Return", "VaR (1%)"), col = c("black", "red"), lty = 1)

plot(index(returns), returns$MSFT, type = "l", col = "black", main = "Microsoft Returns and 1% VaR", ylab = "Return")
lines(index(returns), VaR_msft, col = "red")
legend("bottomleft", legend = c("Return", "VaR (1%)"), col = c("black", "red"), lty = 1)

# Count the number of VaR violations (actual return < VaR threshold)
```

```
violations_tsla <- returns$TSLA < VaR_tsla
violations_msft <- returns$MSFT < VaR_msft

# Total number of violations
n_violate_tsla <- sum(violations_tsla)
n_violate_msft <- sum(violations_msft)

# Expected violations = alpha * n_obs
expected_violations <- 0.01 * nrow(returns)

cat("Tesla: Actual violations =", n_violate_tsla, "| Expected =", round(expected_violations, 2), "\n")
cat("Microsoft: Actual violations =", n_violate_msft, "| Expected =", round(expected_violations, 2), "\n")

# Violation rate
cat("Violation rate (Tesla):", round(mean(violations_tsla), 4), "\n")
cat("Violation rate (Microsoft):", round(mean(violations_msft), 4), "\n")

# Redefine the GARCH(1,1) specification
spec_roll <- ugarchspec(
  variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
  mean.model = list(armaOrder = c(0, 0), include.mean = TRUE),
  distribution.model = "norm" # Or "std" if you want t-distribution for fat tails
)

# For Tesla
cat("\n--- Backtest VaR for Tesla using ugarchroll ---\n")
roll_tsla <- ugarchroll(spec_roll, data = returns$TSLA, n.ahead = 1,
                        forecast.length = 250, # Example: forecast last 250 days
```

```
                        refit.every = 20,       # Re-estimate the model every 20 days
                        refit.window = "recursive", # Extended window for re-estimation
                        VaR.alpha = 0.01,        # Alpha level for VaR calculation (0.01 for 99% VaR)
                        solver = "hybrid",      # Use powerful solver
                        calculate.VaR = TRUE,
                        keep.stats = TRUE)

# Get backtest report for Tesla
report(roll_tsla, type = "VaR", VaR.alpha = 0.01)

# Plot the VaR threshold exceedances for Tesla
plot(roll_tsla, which = 4)
# Add a custom title for the Tesla plot
title("Tesla 99% VaR Exceedance Plot")


# For Microsoft
cat("\n--- Backtest VaR for Microsoft using ugarchroll ---\n")
roll_msft <- ugarchroll(spec_roll, data = returns$MSFT, n.ahead = 1,
                        forecast.length = 250,
                        refit.every = 20,
                        refit.window = "recursive",
                        VaR.alpha = 0.01,
                        solver = "hybrid",
                        calculate.VaR = TRUE,
                        keep.stats = TRUE)

# Get backtest report for Microsoft
report(roll_msft, type = "VaR", VaR.alpha = 0.01)
```

```
# Plot the VaR threshold exceedances for Microsoft
plot(roll_msft, which = 4)
# Add a custom title for the Microsoft plot
title("Microsoft 99% VaR Exceedance Plot")


#---------------------------------------------
# PORTFOLIO-LEVEL RISK MODELING
#---------------------------------------------


# Load necessary libraries (if not already loaded)
if (!requireNamespace("rugarch", quietly = TRUE)) install.packages("rugarch")
if (!requireNamespace("xts", quietly = TRUE)) install.packages("xts") # For time series manipulation
if (!requireNamespace("PerformanceAnalytics", quietly = TRUE)) install.packages("PerformanceAnalytics") # For portfolio functions

library(rugarch)
library(xts)
library(PerformanceAnalytics) # Useful for portfolio management functions

# --- Assumption: 'returns' object and fitted sGARCH models are available ---
# For demonstration purposes, dummy returns and fit_sgarch objects are created.
# These should be replaced with actual data and fitted models.
set.seed(123)
n_obs <- 2000
dates <- seq(as.Date("2020-01-01"), by = "day", length.out = n_obs)
returns <- xts(matrix(rnorm(n_obs * 2, mean = 0.001, sd = c(0.015, 0.008)), nrow = n_obs), order.by = dates)
```

```
colnames(returns) <- c("TSLA", "MSFT")
colnames(returns) <- c("TSLA", "MSFT")

spec_sgarch_norm <- ugarchspec(
  variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
  mean.model = list(armaOrder = c(0, 0), include.mean = TRUE),
  distribution.model = "norm"
)

fit_sgarch_tsla <- ugarchfit(spec = spec_sgarch_norm, data = returns$TSLA)
fit_sgarch_msft <- ugarchfit(spec = spec_sgarch_norm, data = returns$MSFT)

# --- End of dummy data/models. Proceed with actual portfolio analysis ---


# 5.1 Constructing a TSLA-MSFT Portfolio

# Define portfolio weights (e.g., equal weights)
w_tsla <- 0.5
w_msft <- 0.5
weights <- c(TSLA = w_tsla, MSFT = w_msft)

# Calculate portfolio returns
portfolio_returns <- xts(rowSums(t(t(returns) * weights)), order.by = index(returns))
colnames(portfolio_returns) <- "Portfolio"


# 5.2 Estimating Portfolio Volatility
```

```
# Extract conditional standard deviations from fitted sGARCH models
sigma_tsla <- sigma(fit_sgarch_tsla)
sigma_msft <- sigma(fit_sgarch_msft)

# Align the time series for consistent calculations
aligned_sigma <- merge(sigma_tsla, sigma_msft, all = FALSE)
colnames(aligned_sigma) <- c("Sigma_TSLA", "Sigma_MSFT")

# Calculate the historical correlation between the two assets' returns
# Note: For truly conditional covariance, a Multivariate GARCH (e.g., DCC-GARCH) model is typically needed.
# This approach uses a constant historical correlation for simplicity with conditional variances.
correlation_matrix <- cor(returns$TSLA, returns$MSFT, use = "complete.obs")

# Calculate conditional variance components
var_tsla_t <- aligned_sigma$Sigma_TSLA^2
var_msft_t <- aligned_sigma$Sigma_MSFT^2

# Conditional covariance (using constant historical correlation)
rho_t <- correlation_matrix

# Use NROW() or nrow() to get the number of rows of the xts object
w_tsla_vec <- rep(w_tsla, NROW(aligned_sigma))
w_msft_vec <- rep(w_msft, NROW(aligned_sigma))

# Calculate portfolio conditional variance
portfolio_conditional_var <- (w_tsla_vec^2 * var_tsla_t) +
  (w_msft_vec^2 * var_msft_t) +
  (2 * w_tsla_vec * w_msft_vec * rho_t * aligned_sigma$Sigma_TSLA * aligned_sigma$Sigma_MSFT)
```

```
portfolio_conditional_sigma <- sqrt(portfolio_conditional_var)
colnames(portfolio_conditional_sigma) <- "Portfolio_Conditional_Sigma"

# Plot portfolio conditional volatility
plot(portfolio_conditional_sigma, main = "TSLA-MSFT Portfolio Conditional Volatility",
     ylab = expression(sigma[t]), xlab = "Date", col = "darkgreen")
grid()

# 5.3 Portfolio Value-at-Risk (VaR)

# Calculate 1% VaR for the portfolio assuming Normal distribution
# VaR = - (Portfolio Mean + Z_alpha * Portfolio Conditional Sigma)
# Z_alpha for 1% (0.01) for Normal distribution
z_alpha_1pct <- qnorm(0.01)

# Estimate portfolio mean (e.g., historical mean of portfolio returns)
# Ensure portfolio_returns has no NA values, or handle them appropriately
portfolio_mean_estimate <- mean(portfolio_returns, na.rm = TRUE)

# Portfolio 1% VaR (as a threshold). Note: VaR is typically reported as a positive loss,
# but the calculation yields a negative return threshold.
portfolio_VaR_1pct <- portfolio_mean_estimate + portfolio_conditional_sigma * z_alpha_1pct
colnames(portfolio_VaR_1pct) <- "Portfolio_VaR_1pct"

# Plot Portfolio Returns and 1% VaR
# Ensure portfolio_returns and portfolio_VaR_1pct have aligned dates and no NAs
plot(portfolio_returns, main = "TSLA-MSFT Portfolio Returns and 1% VaR",
     ylab = "Return", xlab = "Date", col = "black")
lines(portfolio_VaR_1pct, col = "red")
```

```
legend("bottomleft", legend = c("Portfolio Returns", "Portfolio VaR (1%)"),
       col = c("black", "red"), lty = 1, cex = 0.8)
grid()



# 5.4 Diversification and Risk Reduction


# Calculate individual historical standard deviations (overall volatility)
sd_tsla_hist <- sd(returns$TSLA, na.rm = TRUE)
sd_msft_hist <- sd(returns$MSFT, na.rm = TRUE)


# Calculate portfolio historical standard deviation
sd_portfolio_hist <- sd(portfolio_returns, na.rm = TRUE)


# Demonstrate diversification effect:
# Risk of portfolio < Weighted sum of individual risks if correlation < 1
# Sum of weighted individual historical standard deviations
weighted_sum_sd <- (w_tsla * sd_tsla_hist) + (w_msft * sd_msft_hist)


# Diversification benefit in terms of standard deviation reduction
# (A positive value indicates risk reduction)
diversification_benefit_sd <- weighted_sum_sd - sd_portfolio_hist



# Comparison of VaR: Sum of individual VaR vs Portfolio VaR
# This illustrates the subadditivity property of coherent risk measures.
# Re-estimate individual 1% VaR based on average conditional sigma (for a simplified comparison)
# Note: VaR is presented as a positive loss for comparison clarity here.
VaR_tsla_hist_avg <- - (mean(returns$TSLA, na.rm = TRUE) + qnorm(0.01) * mean(sigma_tsla, na.rm = TRUE))
```

```
VaR_msft_hist_avg <- - (mean(returns$MSFT, na.rm = TRUE) + qnorm(0.01)
* mean(sigma_msft, na.rm = TRUE))


# Sum of individual VaRs (weighted by portfolio allocation)

sum_individual_VaRs <- (w_tsla * VaR_tsla_hist_avg) + (w_msft *
VaR_msft_hist_avg)


# Average portfolio VaR (converting portfolio_VaR_1pct to a positive
loss for comparison)

# Ensure portfolio_VaR_1pct does not contain NAs for mean calculation

avg_portfolio_VaR <- -mean(portfolio_VaR_1pct, na.rm = TRUE)


# Diversification benefit in terms of VaR reduction

# (A positive value indicates risk reduction)

diversification_benefit_VaR <- sum_individual_VaRs - avg_portfolio_VaR



# Visualizing the diversification effect on volatility for varying
weights

plot(NA, xlim=c(0,1), ylim=c(0, max(c(sd_tsla_hist, sd_msft_hist)) *
1.05),

     xlab="Weight of TSLA (w_TSLA)", ylab="Portfolio Standard
Deviation",

     main="Portfolio Volatility vs. TSLA Weight")


# Function to calculate portfolio standard deviation for varying weights

calculate_portfolio_sd <- function(w, sd1, sd2, rho) {

  sqrt(w^2 * sd1^2 + (1-w)^2 * sd2^2 + 2 * w * (1-w) * rho * sd1 * sd2)

}


# Plot the curve for varying weights

w_range <- seq(0, 1, by = 0.01)

portfolio_sd_curve <- sapply(w_range, function(w)
calculate_portfolio_sd(w, sd_tsla_hist, sd_msft_hist,
correlation_matrix))
```

```
lines(w_range, portfolio_sd_curve, col = "blue", lwd = 2)


# Add individual asset std dev points
points(0, sd_msft_hist, col = "red", pch = 19, cex = 1.5)
text(0, sd_msft_hist, paste("MSFT (", round(sd_msft_hist, 4), ")"), pos
= 4, col = "red")
points(1, sd_tsla_hist, col = "red", pch = 19, cex = 1.5)
text(1, sd_tsla_hist, paste("TSLA (", round(sd_tsla_hist, 4), ")"), pos
= 2, col = "red")


# Add the 50/50 portfolio point
points(w_tsla, sd_portfolio_hist, col = "darkgreen", pch = 19, cex =
1.5)
text(w_tsla, sd_portfolio_hist, paste("50/50 (",
round(sd_portfolio_hist, 4), ")"), pos = 3, col = "darkgreen")


grid()



#---------------------------------------------
# STUDENT-T GARCH MODEL
#---------------------------------------------



# 6.2 Estimating GARCH(1,1) with Student-t Errors


# 1. Define the GARCH(1,1) specification with Student-t distributed
errors
# The 'distribution.model = "std"' specifies the Student-t distribution
spec_sgarch_studentt <- ugarchspec(
  variance.model = list(model = "sGARCH", garchOrder = c(1, 1)),
  mean.model = list(armaOrder = c(0, 0), include.mean = TRUE), # Assuming
ARMA(0,0) with mean
```

```
  distribution.model = "std" # Specify Student-t distribution for residuals
)


# 2. Fit the GARCH(1,1) model with Student-t errors for TSLA returns
cat("Fitting GARCH(1,1) with Student-t errors for TSLA...\n")
fit_sgarch_tsla_studentt <- ugarchfit(spec = spec_sgarch_studentt, data = returns$TSLA)
cat("TSLA Student-t GARCH(1,1) Model Fit Summary:\n")
print(fit_sgarch_tsla_studentt) # Print summary to view coefficients and nu (shape) parameter


# 3. Fit the GARCH(1,1) model with Student-t errors for MSFT returns
cat("\nFitting GARCH(1,1) with Student-t errors for MSFT...\n")
fit_sgarch_msft_studentt <- ugarchfit(spec = spec_sgarch_studentt, data = returns$MSFT)
cat("MSFT Student-t GARCH(1,1) Model Fit Summary:\n")
print(fit_sgarch_msft_studentt) # Print summary to view coefficients and nu (shape) parameter


# 4. Extract Conditional Standard Deviations from fitted models
# These are the time-varying volatility estimates under the Student-t assumption
sigma_tsla_studentt <- sigma(fit_sgarch_tsla_studentt)
colnames(sigma_tsla_studentt) <- "Sigma_TSLA_Studentt"


sigma_msft_studentt <- sigma(fit_sgarch_msft_studentt)
colnames(sigma_msft_studentt) <- "Sigma_MSFT_Studentt"


cat("\nConditional Standard Deviations (Sigma) extracted for TSLA and MSFT (Student-t GARCH).\n")


# Note: The 'shape' parameter in the summary output corresponds to the degrees of freedom (nu)
# for the Student-t distribution. A smaller value indicates fatter tails.
```