EE 6417

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ME17B179

Assignment - 1

1.3) Dynamical flow networks

flow rate matrix : F

Outflow rate Vector : f_0

Linear Model : $\dot{q} = Cq + u$

$C = -L^T - \text{diag}(f_0)$

$L = \text{diag}(F \mathbb{1}_n) - F$

(i) No inflows, $u_i = 0$ for all i

Linear model : $\dot{q} = Cq$

$= (-L^T - \text{diag}(f_0))\, q$

Multiply both sides with $\mathbb{1}_n^T$

$\mathbb{1}_n^T \dot{q} = 0 - \mathbb{1}_n^T f_0 q$

$\therefore$ The Total rate of change is -ve

$\therefore$ The total mass in the system does not increase with time.

(ii) $\dot{q}_i(t) = \sum_{j=1, j\neq i}^{n} \left(f_{ij} q_j(t) - f_{ij} q_i(t) \right) - f_{0i} q_i(t) + u_i$

$$F = [f_{ij}]_{ij}$$

$$L = \text{diag}(F \mathbb{1}_n) - F$$

$$= \begin{bmatrix} \Sigma f_{1j} & & 0 \\ & \ddots & \\ 0 & & \Sigma f_{nj} \end{bmatrix} - [f_{ij}]_{ij}$$

$$\Rightarrow \begin{bmatrix} \Sigma f_{1j} - f_{11} & & -f_{ij} \\ & \ddots & \\ -f_{ij} & & \Sigma f_{nj} - f_{nn} \end{bmatrix}.$$

we know $C = -L^T - \text{diag}\, f_0$

$$C = \begin{bmatrix} -(\Sigma f_{1j} - f_{11} - f_{01}) & & f_{ji} \\ & \ddots & \\ f_{ji} & & -(\Sigma f_{nj} - f_{nn} - f_{0n}) \end{bmatrix}$$

(iii) Show that column sums of C are non-positive.

$$\sum_{i=1}^{N} C_{ij} = f_{ii} - f_{0i} - \sum_{j=1}^{n} f_{ij} + \sum_{i\neq j, i=1}^{n} f_{ji}$$

$$= \cancel{\sum_{i=1}^{n} f_{ji}} - \cancel{\sum_{j=1}^{n} f_{ij}} - f_{0i} = -f_{0i} \leq 0 //$$

2.6) (i) Given : A is non-negative

Condition for reducible matrix :

$$\sum_{i=1}^{n} A^{i} \leq 0$$

Let A be $\begin{bmatrix} 1/2 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$

here $A + A^2 > 0$

which says A is irreducible

$\therefore$ The given statement is false

(ii) Condition for primitive matrix :

$A^k > 0$ for some $k \in N$

we know $A_{ij} > 0$, $A_{ik} > 0$ for all $i, j \in \{1, \dots, n\}$

by this we know all entries of A is +ve

$\therefore$ whenever a positive matrix like A is multiplied with a positive matrix $\longrightarrow$ it produces positive entries too

$\therefore$ A is primitive

2.20)

(i) Perron eigenvector is the dominant eigen vector that satisfies $Ax = \mu x$, where μ here is dominant eigen value.

dominant eigen value is the largest real valued eigen value among all the possible eigen values.

∴ We can theoretically say that perron eigen value & eigen vector is the solution of the above constraint equation.

(ii) Given: perron eigenvector x is +ve & normalized

$$Ax = \mu x$$

we know perron eigen value correspond to the perron eigen vector, i.e, μ is the perron eigen value and the dominant one which satisfies $(Ax_i)/x_i$

∴ This satisfies the given condition

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2.17)

(i) Given: Converges at x^* for $\omega \neq 0$

$$\cancel{x^*} = \cancel{(1-\omega)}\,\cancel{x^*} + \frac{\omega}{a_{ii}}\left[b_i - \sum_{j=1, j\neq i}^{n} a_{ij}\, x_j^*\right]$$

$$\cancel{\omega}\,\underline{a_{ii}\, x_i^*} = \cancel{\omega}\left[b_i - \underline{\sum_{j=1, j\neq i}^{n} a_{ij}\, x_j^*}\right]$$

$$\sum_{j=1}^{n} a_{ij}\, x^* = b_i$$

$$\Rightarrow A x^* = b$$

(ii) $$e_i(k+1) - e_i(k) = x_i(k+1) - x_i(k)$$

$$= \frac{\omega}{a_{ii}}\left[b_i - \sum_{j=1, j\neq i}^{n} a_{ij}\, x_j(k)\right] - \omega x_i(k)$$

$$= \frac{\omega}{a_{ii}}\left[\underbrace{b_i - \sum_{j=1, j\neq i}^{n} a_{ij}\, x_j^*}_{\cancel{a_{ii}\, x_i^*(k)}} - \sum_{j=1, j\neq i}^{n} a_{ij} e_j\right] - \omega e_i(k) - \cancel{\omega x_i^*(k)}$$

$$e_i(k+1) = (1-\omega)\, e_i(k) - \frac{\omega}{a_{ii}} \sum_{j=1, j\neq i}^{n} a_{ij}\, e_j(k)$$

Coding Problems

EE6417
Assignment 1
Chella Thiyagarajan N
ME17B179

Question 1:

1. **Complete Graph:**

```
# weights for complete graph
complete_w = np.array([[1/5, 1/5, 1/5, 1/5, 1/5],
                       [1/5, 1/5, 1/5, 1/5, 1/5],
                       [1/5, 1/5, 1/5, 1/5, 1/5],
                       [1/5, 1/5, 1/5, 1/5, 1/5],
                       [1/5, 1/5, 1/5, 1/5, 1/5]])
```

(a). What value (state) do the nodes converge to ?
`Ans: [0. 0. 0. 0. 0.]`

(b). Is it equal to the average of the initial values?
Ans: Yes

2. **Cycle Graph:**

```
# weights for cycle graph
cycle_w = np.array([[1/3, 1/3, 0, 0, 1/3],
                    [1/3, 1/3, 1/3, 0, 0],
                    [0, 1/3, 1/3, 1/3, 0],
                    [0, 0, 1/3, 1/3, 1/3],
                    [1/3, 0, 0, 1/3, 1/3]])
```

(a). What value (state) do the nodes converge to ?
`Ans: [-7.222e-19 -7.222e-19 -7.222e-19 -7.222e-19 -7.222e-19]`

(b). Is it equal to the average of the initial values?
Ans: Almost Yes

3. **Star Graph:**

```
# weights for star graph
star_w = np.array([[1/5, 1/5, 1/5, 1/5, 1/5],
                   [1/2, 1/2, 0, 0, 0],
                   [1/2, 0, 1/2, 0, 0],
                   [1/2, 0, 0, 1/2, 0],
                   [1/2, 0, 0, 0, 1/2]])
```

(a). What value (state) do the nodes converge to ?

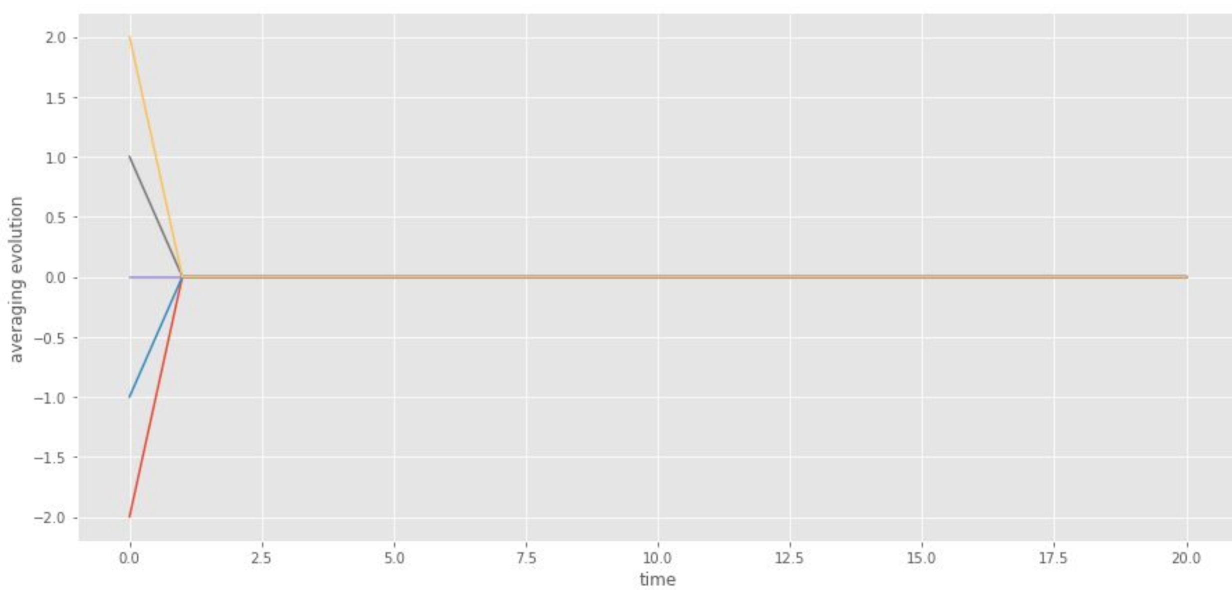
`Ans: [-0.462 -0.462 -0.462 -0.462 -0.462]`

(b). Is it equal to the average of the initial values?

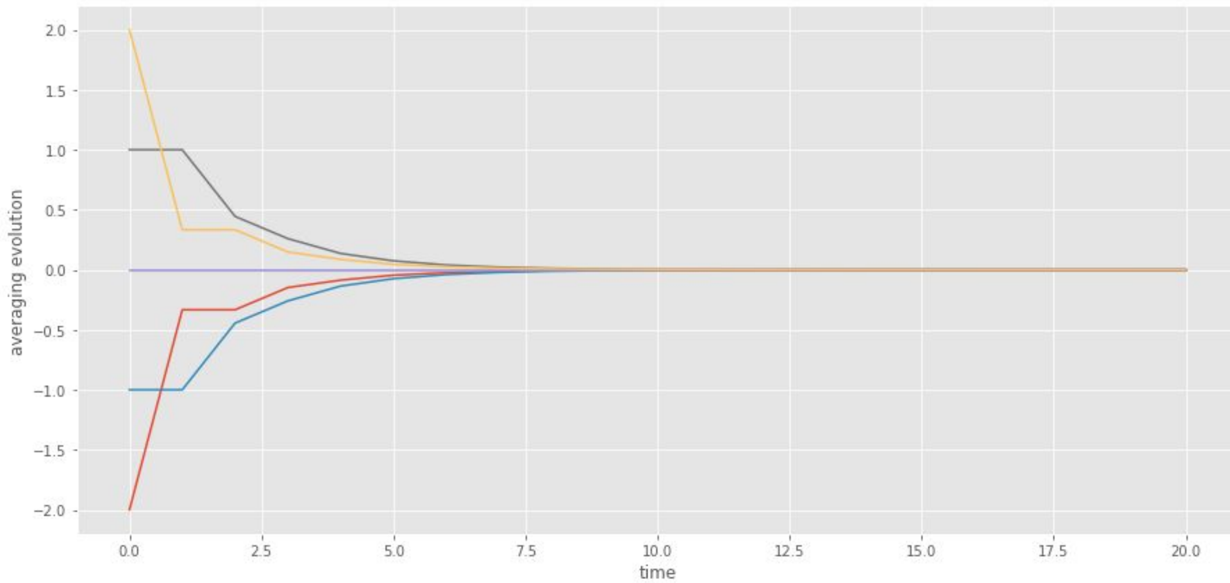
Ans: No

Question 2:

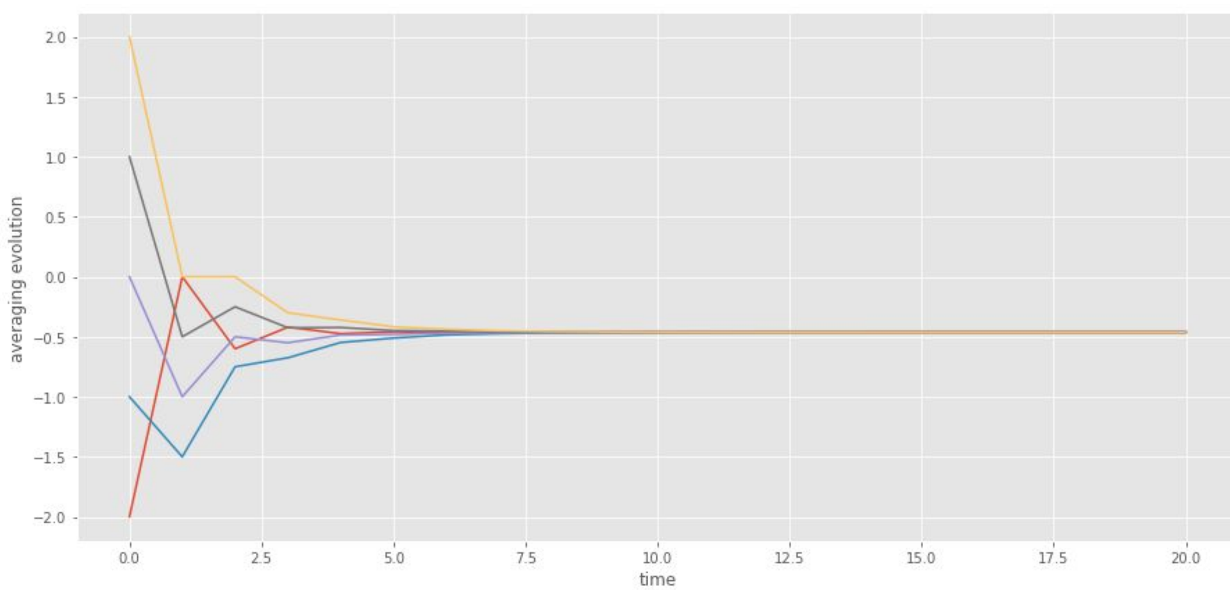
Complete Graph:



Cycle Graph:



Star Graph:



Question 3:

Part (a):

Through empirical experiments, I can conclude that graphs with a more dense adjacency matrix (more edges) the faster the state seems to converge. Disconnected graphs show different behaviour than dense and normally connected graphs, final states converge but not to the same value but to different values and these end states may not strictly be averages of initial states.

Part (b):

The initial state only affects the final state or values it converges to. It does not affect the aspect of convergence.

Part (c):

Experiment details: 5 times the network was created with different or random attributes each time. We will look at the results below:

Erdos Renyi Graphs:
All the graphs converged, but they converged to a final state which is a little bit offset to the average of the initial state. Eg. offsets like -0.187, 0.151, etc.,

Small World Graphs:
All the graphs converged, and all nodes in all the graphs converged to the average of the initial states.

Scale-Free Graphs:
All the graphs converged, but they converged to a final state which is a little bit offset to the average of the initial state. Eg. offsets like 0.352, 0.637, etc.,

Question 4 and Question 5:

Answers to Question 4 and 5 could be found in the python jupyter notebook cell outputs that I have attached with this assignment zip file.

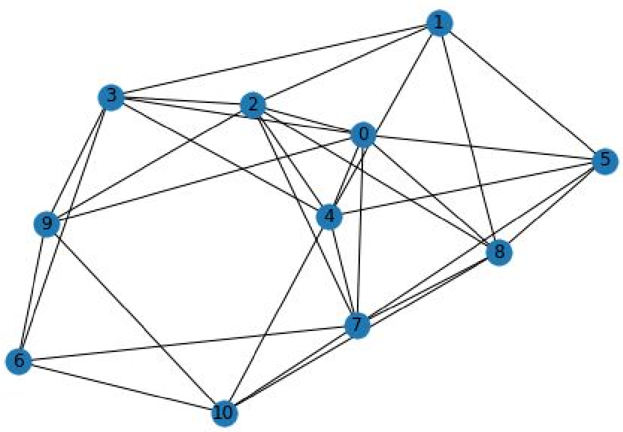
Sample Results to Question 3,4 and 5:

Erdos Renyi graph - trial 1 - initial state (a):

Initial state = [-5 -4 -3 -2 -1 0 1 2 3 4 5]

Answers for Question 3:

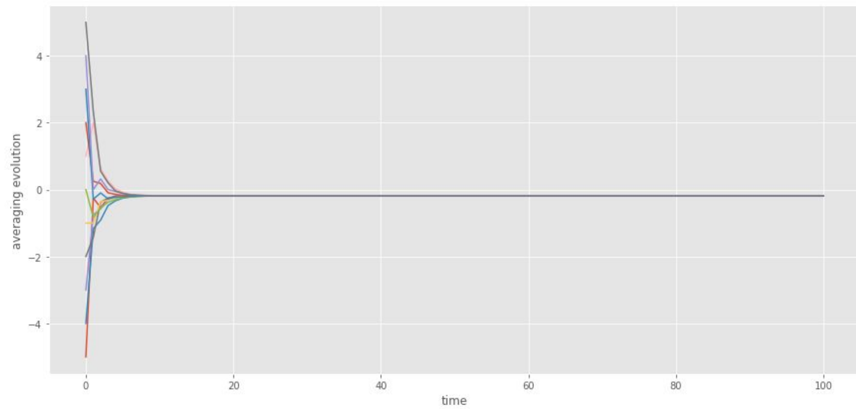
Erdos Renyi graph with nodes = 11 probab = 0.5



Row-stochastic matrix A

```
[[0.125 0.    0.125 0.125 0.125 0.125 0.    0.125 0.125 0.125 0.   ]
 [0.    0.167 0.167 0.167 0.167 0.167 0.    0.    0.167 0.    0.   ]
 [0.125 0.125 0.125 0.125 0.125 0.    0.    0.125 0.125 0.125 0.   ]
 [0.143 0.143 0.143 0.143 0.143 0.    0.143 0.    0.    0.143 0.   ]
 [0.125 0.125 0.125 0.125 0.125 0.125 0.    0.125 0.    0.    0.125]
 [0.167 0.167 0.    0.    0.167 0.167 0.    0.167 0.167 0.    0.   ]
 [0.    0.    0.    0.2   0.    0.    0.2   0.2   0.    0.2   0.2  ]
 [0.125 0.    0.125 0.    0.125 0.125 0.125 0.125 0.125 0.    0.125]
 [0.143 0.143 0.143 0.    0.    0.143 0.    0.143 0.143 0.    0.143]
 [0.167 0.    0.167 0.167 0.    0.    0.167 0.    0.    0.167 0.167]
 [0.    0.    0.    0.    0.167 0.    0.167 0.167 0.167 0.167 0.167]]
```

State Trajectory



Final State = [-0.187 -0.187 -0.187 -0.187 -0.187 -0.187 -0.187 -0.187 -0.187 -0.187
 -0.187]
Average of Initial State = 0.0

Answers for Question 4:
JORDAN FORM
[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
 [0,0.506145806946601,0,0,0,0,0,0,0,0,0],
 [0,0,0.357178555290559,0,0,0,0,0,0,0,0],
 [0,0,0,0.194144765982543,0,0,0,0,0,0,0],
 [0,0,0,0,0.153501519545285,0,0,0,0,0,0],
 [0,0,0,0,0,0.105259177814723,0,0,0,0,0],
 [0,0,0,0,0,0,0.0610383862441997,0,0,0,0],
 [0,0,0,0,0,0,0,-0.0433936662417764,0,0,0],
 [0,0,0,0,0,0,0,0,-0.194572618801303,0,0],
 [0,0,0,0,0,0,0,0,0,-0.206124340653444,0],
 [0,0,0,0,0,0,0,0,0,0,-0.280796633746434]]

Dominant Eigen Value = 0.9999999999999988
Dominant Eigen Vector = [0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3]

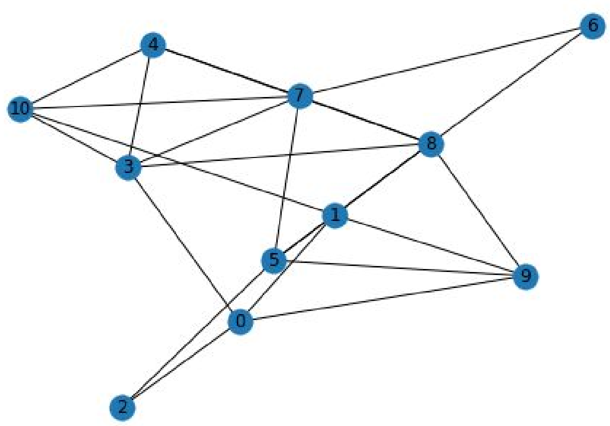
Answers for Question 5:
Lets state the Definitions and coditions for perron frobenius theorem:
1. Existence of a real eigen value that is dominant than other eigen values
Dominant Eigen Value = 0.9999999999999988
Non-Negative Test: Matrix is Non Negative
2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real
All Eigen Values = [1. , 0.51, 0.36,-0.28,-0.19,-0.21,-0.04, 0.19, 0.15, 0.06, 0.11]
Irreducible Test: Matrix is irreducible
3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values
verification = All Eigen Values = [1. , 0.51, 0.36,-0.28,-0.19,-0.21,-0.04, 0.19, 0.15, 0.06, 0.11]
Primitive Test: Matrix is primitive

Erdos Renyi graph - trial 1 - initial state (b)

Initial state = [-3 -4 -4 2 -2 3 -1 -1 -1 4 -1]

Answers for Question 3:

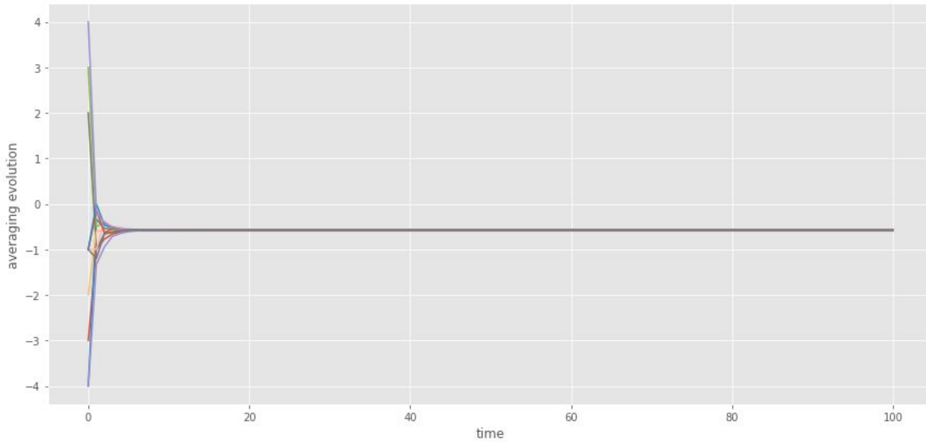
Erdos Renyi graph with nodes = 11 probab = 0.5



Row-stochastic matrix A

```
[[0.2   0.2   0.2   0.2   0.    0.    0.    0.    0.    0.2   0.   ]
 [0.167 0.167 0.    0.    0.    0.167 0.    0.    0.167 0.167 0.167]
 [0.333 0.    0.333 0.    0.    0.333 0.    0.    0.    0.    0.   ]
 [0.167 0.    0.    0.167 0.167 0.    0.    0.167 0.167 0.    0.167]
 [0.    0.    0.    0.2   0.2   0.    0.    0.2   0.2   0.    0.2  ]
 [0.    0.167 0.167 0.    0.    0.167 0.    0.167 0.167 0.167 0.   ]
 [0.    0.    0.    0.    0.    0.    0.333 0.333 0.333 0.    0.   ]
 [0.    0.    0.    0.143 0.143 0.143 0.143 0.143 0.143 0.    0.143]
 [0.    0.125 0.    0.125 0.125 0.125 0.125 0.125 0.125 0.125 0.   ]
 [0.2   0.2   0.    0.    0.    0.2   0.    0.    0.2   0.2   0.   ]
 [0.    0.2   0.    0.2   0.2   0.    0.    0.2   0.    0.    0.2  ]]
```

State Trajectory



Final State = [-0.576 -0.576 -0.576 -0.576 -0.576 -0.576 -0.576 -0.576 -0.576 -0.576
 -0.576]
Average of Initial State = -0.7272727272727273

Answers for Question 4:
JORDAN FORM
[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
 [0,0.674925122919032,0,0,0,0,0,0,0,0,0],
 [0,0,0.467425635204565,0,0,0,0,0,0,0,0],
 [0,0,0,0.381034850329062,0,0,0,0,0,0,0],
 [0,0,0,0,0.247402404300770,0,0,0,0,0,0],
 [0,0,0,0,0,0.151787574686275,0,0,0,0,0],
 [0,0,0,0,0,0,0.0128532201400223,0,0,0,0],
 [0,0,0,0,0,0,0,-0.0286789020306052,0,0,0],
 [0,0,0,0,0,0,0,0,-0.121389742071480,0,0],
 [0,0,0,0,0,0,0,0,0,-0.219804374433157,0],
 [0,0,0,0,0,0,0,0,0,0,-0.331031979520674]]

Dominant Eigen Value = 1.0000000000000004
Dominant Eigen Vector = [-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3]

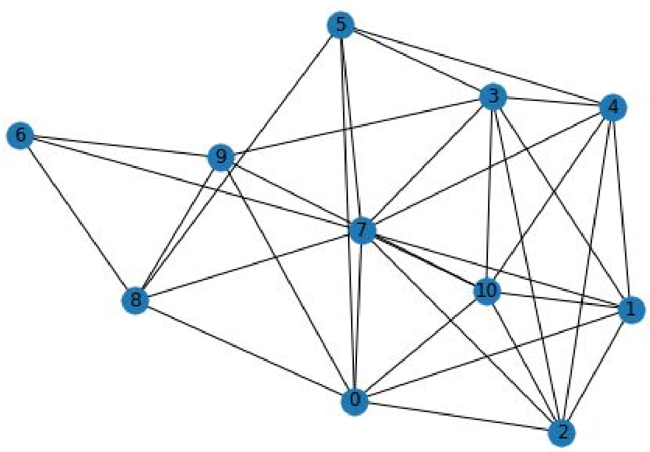
Answers for Question 5:
Lets state the Definitions and coditions for perron frobenius theorem:
1. Existence of a real eigen value that is dominant than other eigen values
Dominant Eigen Value = 1.0000000000000004
Non-Negative Test: Matrix is Non Negative
2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real
All Eigen Values = [1. , 0.67,-0.33, 0.47, 0.38, 0.25,-0.22,-0.12,-0.03, 0.01, 0.15]
Irreducible Test: Matrix is irreducible
3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values
verification = All Eigen Values = [1. , 0.67,-0.33, 0.47, 0.38, 0.25,-0.22,-0.12,-0.03, 0.01, 0.15]
Primitive Test: Matrix is primitive

small world graph - trial 1 - initial state (a)

Initial state = [-5 -4 -3 -2 -1 0 1 2 3 4 5]

Answers for Question 3:

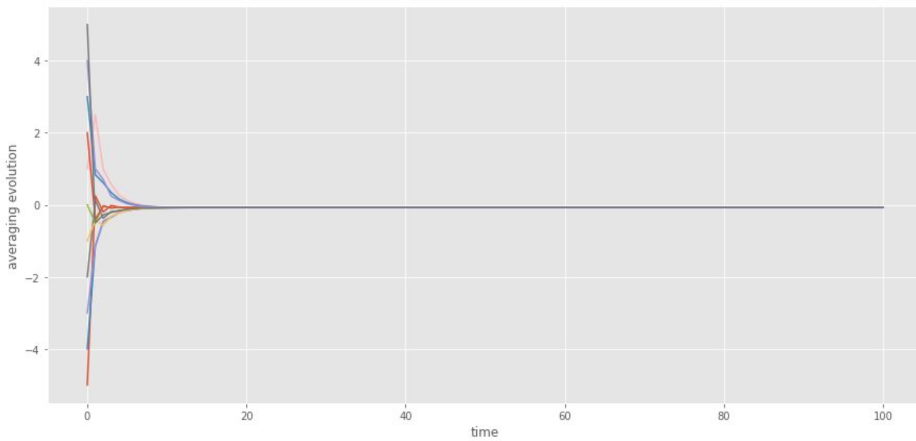
Erdos Renyi graph with nodes = 11 m = 7 probab = 0.5



Row-stochastic matrix A

```
[[0.125 0.125 0.125 0.    0.    0.125 0.    0.125 0.125 0.125 0.125]
 [0.143 0.143 0.143 0.143 0.143 0.    0.    0.143 0.    0.    0.143]
 [0.143 0.143 0.143 0.143 0.143 0.    0.    0.143 0.    0.    0.143]
 [0.    0.125 0.125 0.125 0.125 0.125 0.    0.125 0.    0.125 0.125]
 [0.    0.143 0.143 0.143 0.143 0.143 0.    0.143 0.    0.    0.143]
 [0.167 0.    0.    0.167 0.167 0.167 0.    0.167 0.167 0.    0.   ]
 [0.    0.    0.    0.    0.    0.    0.25  0.25  0.25  0.25  0.   ]
 [0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.    0.1  ]
 [0.167 0.    0.    0.    0.    0.167 0.167 0.167 0.167 0.167 0.   ]
 [0.167 0.    0.    0.167 0.    0.    0.167 0.    0.167 0.167 0.167]
 [0.125 0.125 0.125 0.125 0.125 0.    0.    0.125 0.    0.125 0.125]]
```

State Trajectory



Final State = [-0.078 -0.078 -0.078 -0.078 -0.078 -0.078 -0.078 -0.078 -0.078 -0.078
-0.078]
Average of Initial State = 0.0

Answers for Question 4:
JORDAN FORM
[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
[0,0.566160597883840,0,0,0,0,0,0,0,0,0],
[0,0,0.295365159103198,0,0,0,0,0,0,0,0],
[0,0,0,0.201373406896400,0,0,0,0,0,0,0],
[0,0,0,0,0.166227560482306,0,0,0,0,0,0],
[0,0,0,0,0,3.50697995545135e-65,0,0,0,0,0],
[0,0,0,0,0,0,2.77688303003900e-66,0,0,0,0],
[0,0,0,0,0,0,0,-0.0382167632192046,0,0,0],
[0,0,0,0,0,0,0,0,-0.100531489374073,0,0],
[0,0,0,0,0,0,0,0,0,-0.171258438642665,0],
[0,0,0,0,0,0,0,0,0,0,-0.265548604558374]]

Dominant Eigen Value = 1.0000000000000004
Dominant Eigen Vector = [-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3]

Answers for Question 5:
Lets state the Definitions and coditions for perron frobenius theorem:
1. Existence of a real eigen value that is dominant than other eigen values
Dominant Eigen Value = 1.0000000000000004
Non-Negative Test: Matrix is Non Negative
2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real
All Eigen Values = [1.00e+00, 5.66e-01,-2.66e-01, 2.95e-01, 2.01e-01, 1.66e-01,-1.71e-01,
-1.01e-01,-3.82e-02,-3.83e-18, 3.42e-19]
Irreducible Test: Matrix is irreducible
3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values

verification = All Eigen Values = [1.00e+00, 5.66e-01,-2.66e-01, 2.95e-01, 2.01e-01, 1.66e-01,-1.71e-01, -1.01e-01,-3.82e-02,-3.83e-18, 3.42e-19]

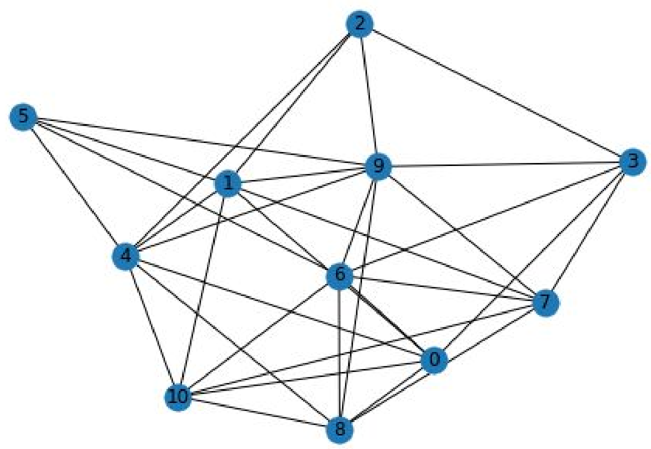
Primitive Test: Matrix is primitive

small world graph - trial 1 - initial state (b)

Initial state = [0 2 1 -3 3 2 -3 1 2 0 3]

Answers for Question 3:

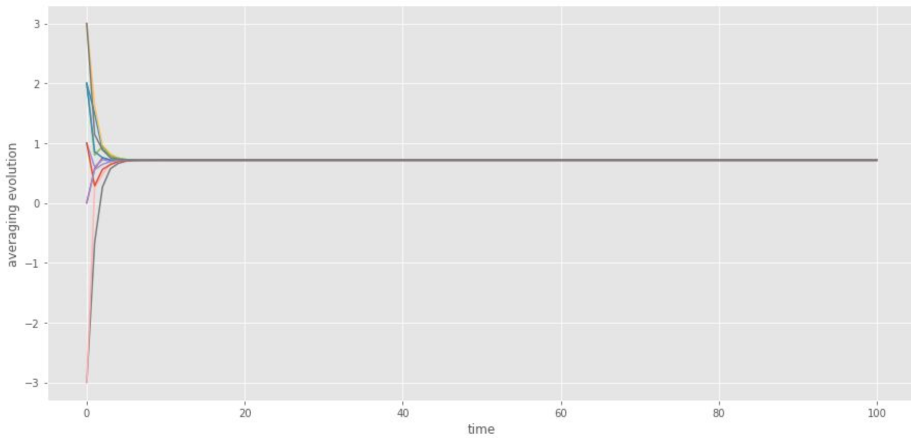
Erdos Renyi graph with nodes = 11 m = 7 probab = 0.5



Row-stochastic matrix A

```
[[0.143 0.143 0.    0.143 0.143 0.    0.143 0.    0.143 0.    0.143]
 [0.125 0.125 0.125 0.    0.125 0.125 0.    0.125 0.    0.125 0.125]
 [0.    0.2   0.2   0.2   0.2   0.    0.    0.    0.    0.2   0.   ]
 [0.167 0.    0.167 0.167 0.    0.    0.167 0.167 0.    0.167 0.   ]
 [0.125 0.125 0.125 0.    0.125 0.125 0.    0.    0.125 0.125 0.125]
 [0.    0.2   0.    0.    0.2   0.2   0.2   0.    0.    0.2   0.   ]
 [0.125 0.    0.    0.125 0.    0.125 0.125 0.125 0.125 0.125 0.125]
 [0.    0.143 0.    0.143 0.    0.    0.143 0.143 0.143 0.143 0.143]
 [0.143 0.    0.    0.    0.143 0.    0.143 0.143 0.143 0.143 0.143]
 [0.    0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.   ]
 [0.143 0.143 0.    0.    0.143 0.    0.143 0.143 0.143 0.    0.143]]
```

State Trajectory



Final State = [0.714 0.714 0.714 0.714 0.714 0.714 0.714 0.714 0.714 0.714 0.714]
Average of Initial State = 0.7272727272727273

Answers for Question 4:
JORDAN FORM
[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
 [0,0.419481728878838,0,0,0,0,0,0,0,0,0],
 [0,0,0.350032156363008,0,0,0,0,0,0,0,0],
 [0,0,0,0.271709706189524,0,0,0,0,0,0,0],
 [0,0,0,0,0.172178575281837,0,0,0,0,0,0],
 [0,0,0,0,0,0.130300365999914,0,0,0,0,0],
 [0,0,0,0,0,0,-0.0163627452401537,0,0,0,0],
 [0,0,0,0,0,0,0,-0.0794819568270286,0,0,0],
 [0,0,0,0,0,0,0,0,-0.134536786038138,0,0],
 [0,0,0,0,0,0,0,0,0,-0.198790286968345,0],
 [0,0,0,0,0,0,0,0,0,0,-0.290324408433106]]

Dominant Eigen Value = 0.9999999999999993
Dominant Eigen Vector = [-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3]

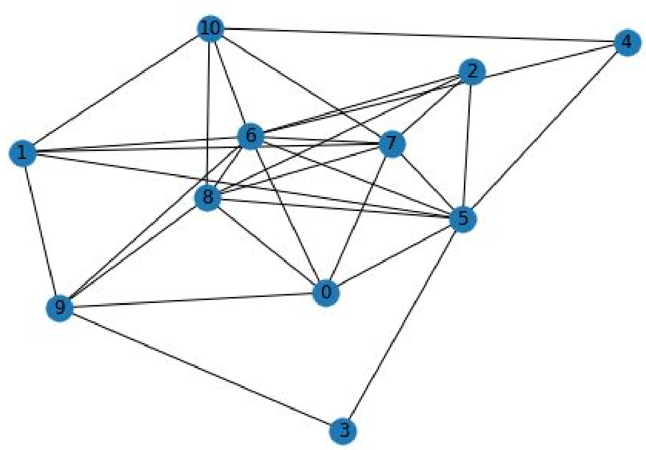
Answers for Question 5:
Lets state the Definitions and coditions for perron frobenius theorem:
1. Existence of a real eigen value that is dominant than other eigen values
Dominant Eigen Value = 0.9999999999999993
Non-Negative Test: Matrix is Non Negative
2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real
All Eigen Values = [1. , 0.42,-0.29, 0.35, 0.27, 0.17, 0.13,-0.2 ,-0.02,-0.08,-0.13]
Irreducible Test: Matrix is irreducible
3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values
verification = All Eigen Values = [1. , 0.42,-0.29, 0.35, 0.27, 0.17, 0.13,-0.2 ,-0.02,-0.08,-0.13]
Primitive Test: Matrix is primitive

scale free graph - trial 1 - initial state (a)

Initial state = [-5 -4 -3 -2 -1 0 1 2 3 4 5]

Answers for Question 3:

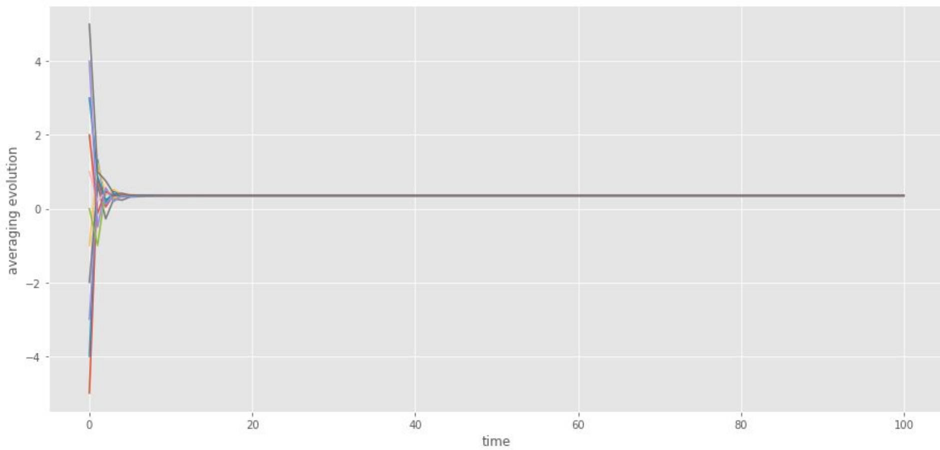
Erdos Renyi graph with nodes = 11 m = 5



Row-stochastic matrix A

```
[[0.167 0.    0.    0.    0.    0.167 0.167 0.167 0.167 0.167 0.   ]
 [0.    0.167 0.    0.    0.    0.167 0.167 0.167 0.    0.167 0.167]
 [0.    0.    0.2   0.    0.    0.2   0.2   0.2   0.2   0.    0.   ]
 [0.    0.    0.    0.333 0.    0.333 0.    0.    0.    0.333 0.   ]
 [0.    0.    0.    0.    0.25  0.25  0.25  0.    0.    0.    0.25 ]
 [0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.    0.   ]
 [0.1   0.1   0.1   0.    0.1   0.1   0.1   0.1   0.1   0.1   0.1  ]
 [0.125 0.125 0.125 0.    0.    0.125 0.125 0.125 0.125 0.    0.125]
 [0.125 0.    0.125 0.    0.    0.125 0.125 0.125 0.125 0.125 0.125]
 [0.167 0.167 0.    0.167 0.    0.    0.167 0.    0.167 0.167 0.   ]
 [0.    0.167 0.    0.    0.167 0.    0.167 0.167 0.167 0.    0.167]]
```

State Trajectory



Final State = [0.352 0.352 0.352 0.352 0.352 0.352 0.352 0.352 0.352 0.352 0.352]

Average of Initial State = 0.0

Answers for Question 4:

JORDAN FORM

[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
 [0,0.508982961131745,0,0,0,0,0,0,0,0,0],
 [0,0,0.385083772700799,0,0,0,0,0,0,0,0],
 [0,0,0,0.295068320758633,0,0,0,0,0,0,0],
 [0,0,0,0,0.210567398978488,0,0,0,0,0,0],
 [0,0,0,0,0,0.117155563843477,0,0,0,0,0],
 [0,0,0,0,0,0,0.0795207920165786,0,0,0,0],
 [0,0,0,0,0,0,0,-0.0428107472978880,0,0,0],
 [0,0,0,0,0,0,0,0,-0.130065662915250,0,0],
 [0,0,0,0,0,0,0,0,0,-0.181664019430279,0],
 [0,0,0,0,0,0,0,0,0,0,-0.330727268675193]]

Dominant Eigen Value = 1.0000000000000009

Dominant Eigen Vector = [0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3,0.3]

Answers for Question 5:

Lets state the Definitions and coditions for perron frobenius theorem:

1. Existence of a real eigen value that is dominant than other eigen values

Dominant Eigen Value = 1.0000000000000009

Non-Negative Test: Matrix is Non Negative

2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real

All Eigen Values = [1. ,-0.33, 0.51, 0.39,-0.18,-0.13,-0.04, 0.3 , 0.21, 0.08, 0.12]

Irreducible Test: Matrix is irreducible

3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values

verification = All Eigen Values = [1. ,-0.33, 0.51, 0.39,-0.18,-0.13,-0.04, 0.3 , 0.21, 0.08, 0.12]

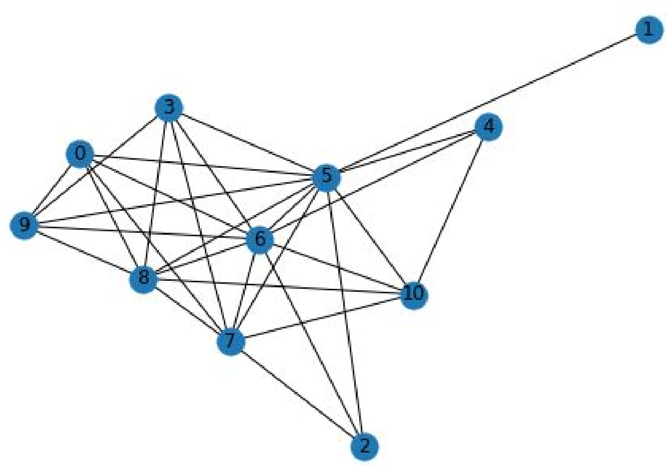
Primitive Test: Matrix is primitive

scale free graph - trial 1 - initial state (b)

Initial state = [-5 1 2 -1 4 -5 -3 1 2 0 0]

Answers for Question 3:

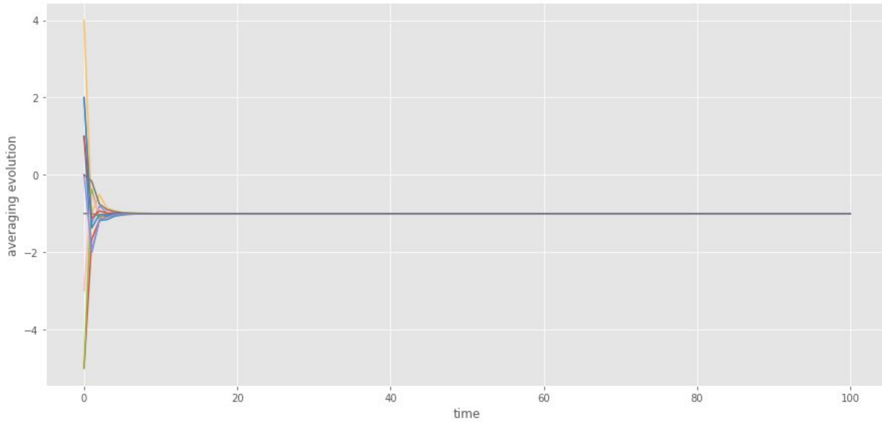
Erdos Renyi graph with nodes = 11 m = 5



Row-stochastic matrix A

```
[[0.167 0.    0.    0.    0.    0.167 0.167 0.167 0.167 0.167 0.   ]
 [0.    0.5   0.    0.    0.    0.5   0.    0.    0.    0.    0.   ]
 [0.    0.    0.25  0.    0.    0.25  0.25  0.25  0.    0.    0.   ]
 [0.    0.    0.    0.167 0.    0.167 0.167 0.167 0.167 0.167 0.   ]
 [0.    0.    0.    0.    0.25  0.25  0.25  0.    0.    0.    0.25 ]
 [0.091 0.091 0.091 0.091 0.091 0.091 0.091 0.091 0.091 0.091 0.091]
 [0.1   0.    0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1   0.1  ]
 [0.125 0.    0.125 0.125 0.    0.125 0.125 0.125 0.125 0.    0.125]
 [0.125 0.    0.    0.125 0.    0.125 0.125 0.125 0.125 0.125 0.125]
 [0.167 0.    0.    0.167 0.    0.167 0.167 0.    0.167 0.167 0.   ]
 [0.    0.    0.    0.    0.167 0.167 0.167 0.167 0.167 0.    0.167]]
```

State Trajectory



Final State = [-1. -1. -1. -1. -1. -1. -1. -1. -1. -1. -1.]
Average of Initial State = -0.36363636363636365

Answers for Question 4:
JORDAN FORM
[[1.00000000000000,0,0,0,0,0,0,0,0,0,0],
 [0,0.554329914032282,0,0,0,0,0,0,0,0,0],
 [0,0,0.431267482715854,0,0,0,0,0,0,0,0],
 [0,0,0,0.339531194075623,0,0,0,0,0,0,0],
 [0,0,0,0,0.166666666666667,0,0,0,0,0,0],
 [0,0,0,0,0,0.165117987274161,0,0,0,0,0],
 [0,0,0,0,0,0,3.99933727274842e-65,0,0,0,0],
 [0,0,0,0,0,0,0,-0.0383834345328899,0,0,0],
 [0,0,0,0,0,0,0,0,-0.0912235842079833,0,0],
 [0,0,0,0,0,0,0,0,0,-0.165097125545452,0],
 [0,0,0,0,0,0,0,0,0,0,-0.254633342902504]]

Dominant Eigen Value = 1.0
Dominant Eigen Vector = [-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3,-0.3]

Answers for Question 5:
Lets state the Definitions and coditions for perron frobenius theorem:
1. Existence of a real eigen value that is dominant than other eigen values
Dominant Eigen Value = 1.0
Non-Negative Test: Matrix is Non Negative
2. Condition for Irreducible matrix = dominant eigen value must be strictly positive and real
All Eigen Values = [1.00e+00, 5.54e-01, 4.31e-01,-2.55e-01, 3.40e-01,-1.65e-01,-9.12e-02,
 -2.29e-18,-3.84e-02, 1.67e-01, 1.65e-01]
Irreducible Test: Matrix is irreducible
3. Condition for Primitive matrix = magnitude of dominant eigen values must be greater than the magnitude of other eigen values
verification = All Eigen Values = [1.00e+00, 5.54e-01, 4.31e-01,-2.55e-01, 3.40e-01,-1.65e-01,-9.12e-02,

-2.29e-18,-3.84e-02, 1.67e-01, 1.65e-01]
Primitive Test: Matrix is primitive