ID6040
Introduction to Robotics

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Robot Motion Planning

- Motion Planning
- Trajectory Planning
  - Joint Space Planning
    - Cubic polynomial functions
    - Higher order polynomials

Refer: John J Craig, Intr. to Robotics

Motion Planning

The goal of motion planning is to generate a function according to which a robot will move.
Function generation depends on task.

A motion *Path* is defined as a sequence of robot configurations in particular order without regard for timing of these configurations.

Motion *trajectory* is a time specified path data.

Trajectory refers to a time history of position, velocity, and acceleration for each degree of freedom

Path planning develops feasible path data for a specific task

Trajectory planning develops corresponding trajectory data to control the robot to follow the path.

Path shapes are described in terms of functions of joint angles.

• Each path point is specified in terms of a desired position and orientation wrt base frame.

• These points are converted to set of joint angles through inverse kinematics

• A smooth function is found for each of the n joints that pass through the via points and reach end point.

• The time required for each segment is the same for each joint so that all joints will reach the via point at the same time.

$$\theta(0) = \theta_0 \qquad \dot{\theta}(0) = \dot{\theta}_0$$

$$\theta(t_f) = \theta_f \qquad \dot{\theta}(t_f) = \dot{\theta}_f$$

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

$$\theta(0) = \theta_0 \qquad \theta(t_f) = \theta_f \Longrightarrow \theta_0 = a_0 \qquad \theta_f = a_0 + a_1 t_f + a_2 t_f^{\ 2} + a_3 t_f^{\ 3}$$

For the case of initial and final velocity zero

$$\dot{\theta}(0) = \dot{\theta}_0 = 0 \qquad \dot{\theta}(t_f) = \dot{\theta}_f = 0 \Longrightarrow 0 = a_1 \qquad 0 = a_1 + 2a_2 t_f + 3a_3 t_f^{\ 2}$$

Solving these, we get

$$a_0 = \theta_0$$

$$a_1 = 0$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0)$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0)$$

Example:

A single link robot with a rotary joint is motionless at $\theta=15$ degrees. It is desired to move the joint in a smooth manner to $\theta=75$ degrees in 3 secs. Find the coefficients of a cubic that accomplishes this motion and brings the manipulator to rest at the goal. Plot the position, velocity, and acceleration of the joint as a function of time.

$$a_0 = \theta_0 = 15$$

$$\theta(t) = 15 + 20t^2 - 4.4t^3$$

$$a_1 = 0$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) = 20$$

position

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) = -4.44$$

velocity

acceleration

Path with Via points

$$\dot{\theta}(0) = \dot{\theta}_0$$

$$\dot{\theta}(t_f) = \dot{\theta}_f$$

$$\theta_0 = a_0$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^{\ 2} + a_3 t_f^{\ 3}$$

$$\dot{\theta}(0) = a_1$$

$$\dot{\theta}(t_f) = a_1 + 2a_2 t_f \ + 3a_3 t_f^{\ 2} = \dot{\theta}_f$$

$$a_0 = \theta_0$$

$$a_1 = \dot{\theta}_f$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f - \dot{\theta}_0)$$

Exercise

Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous acceleration at the intermediate via point. The initial angle is θ_0, the via point is θ_v, and the goal point is θg

$$\theta(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

$$\theta(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

$$t = t_{fi}; i = 1,2$$

If we want to specify position, velocity and acceleration at the beginning and end of a path segment, a quintic polynomial is required, namely

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$\theta_0 = a_0$$

$$\theta_f = a_0 + a_1 t_f + a_2 t_f^{\ 2} + a_3 t_f^{\ 3} + a_4 t_f^{\ 4} + a_5 t_f^{\ 5}$$

$$\dot{\theta}(0) = a_1$$

$$\dot{\theta}(t_f) = a_1 + 2a_2 t_f \ + 3a_3 t_f^{\ 2} + 4a_4 t_f^{\ 3} + 5a_5 t_f^{\ 4}$$

$$\ddot{\theta}(0) = 2a_2$$

$$\ddot{\theta}(t_f) = 2a_2 \ + 6a_3 t_f \ + 12a_4 t_f^{\ 2} + 20a_5 t_f^{\ 3}$$

$$a_0 = \theta_0$$

$$a_1 = \dot{\theta}_0$$

$$a_2 = \ddot{\theta}_0 / 2$$

$$a_3 = \frac{20\theta_f - 20\theta_0 - (8\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3}$$

$$a_4 = \frac{30\theta_0 - 30\theta_f + (14\dot{\theta}_f + 16\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2}{2t_f^4}$$

$$a_5 = \frac{12\theta_f - 12\theta_0 - (6\dot{\theta}_f + 6\dot{\theta}_0)t_f - (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^5}$$

Linear function with parabolic blend

$$t_b \cdot \ddot{\theta}_b = \frac{\theta_h - \theta_b}{t_h - t_b} \quad (1)$$

$$\theta_b = \theta_0 + \frac{1}{2}\ddot{\theta}_b t_b^2 \quad (2)$$

$$(2) \Rightarrow \ddot{\theta}_b t_b^2 = 2(\theta_b - \theta_0) \quad (3)$$

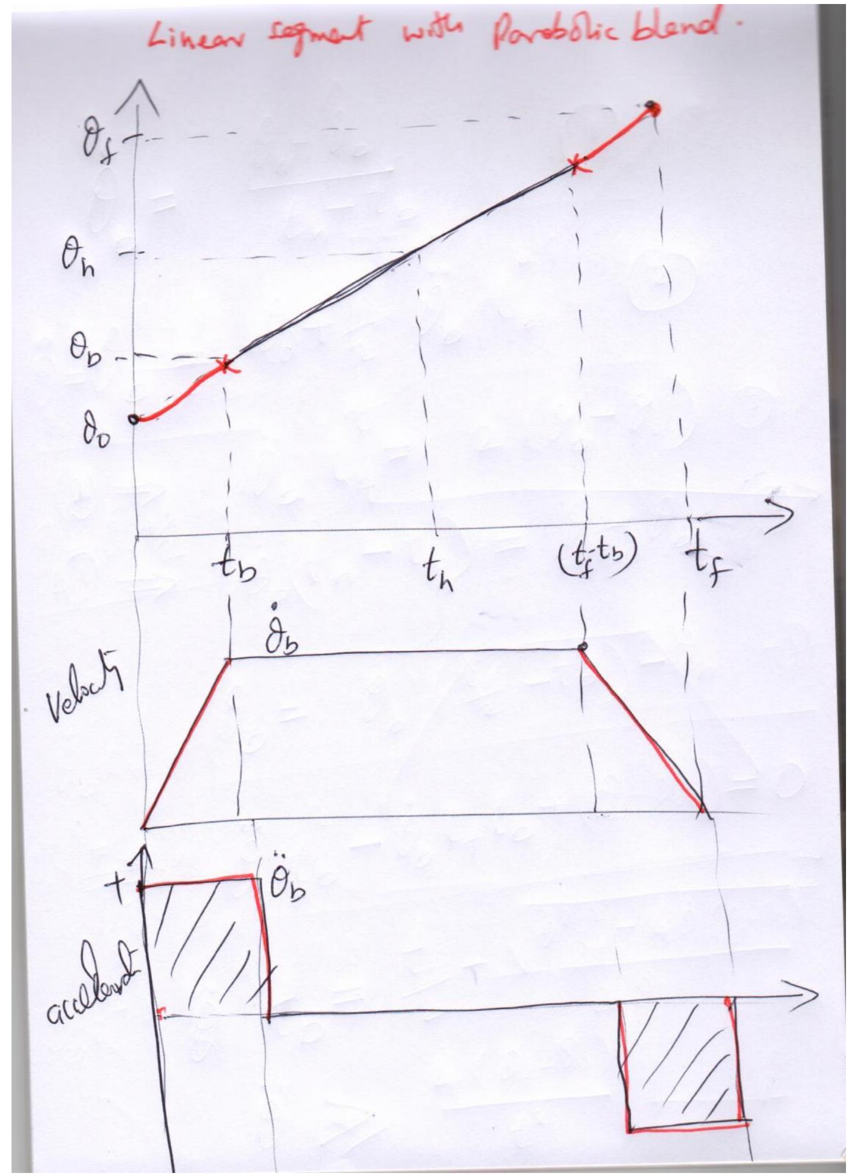
$$(1) \Rightarrow \dot{\theta}_b (t_h - t_b) = \theta_h - \theta_b \quad -(4)$$

$$\theta_b = \ddot{\theta}_b t_b \left(\frac{t_f}{2} - t_b\right) - \theta_h \quad -(5)$$

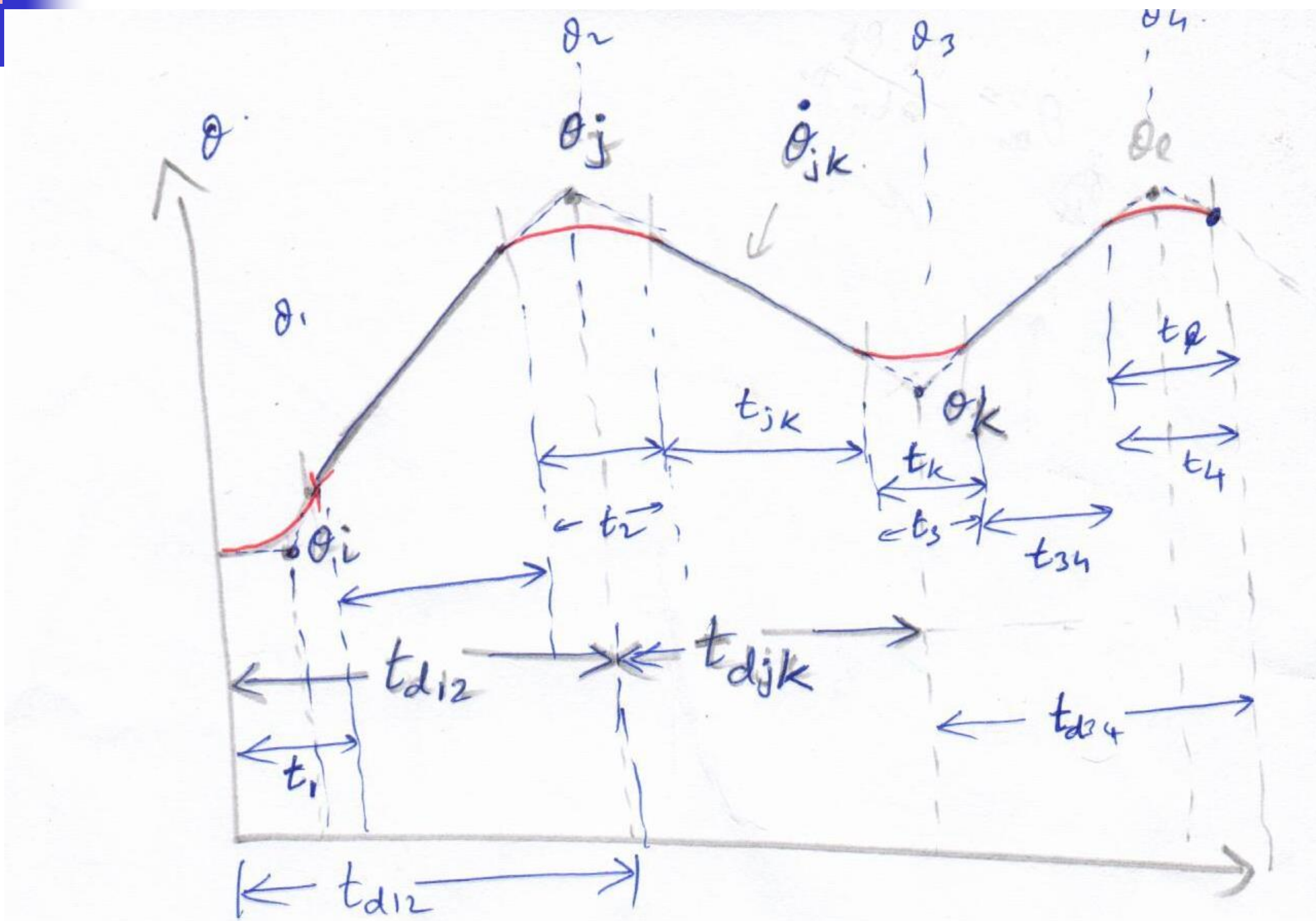
$$\Rightarrow \boxed{\ddot{\theta}_b t_b^2 - \ddot{\theta}_b t_f t_b + (\theta_f - \theta_0) = 0}$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^2 - 4\ddot{\theta}_b(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

$$\ddot{\theta} \geq \frac{4(\theta_f - \theta_0)}{t_f^2}$$



Linear function with parabolic blends for path with via points



Interior Path Points

$j = 2, 3$
$k = 3, 4$

$$\dot{\theta}_{jk} = \frac{\theta_k - \theta_j}{t_{djk}} \simeq \dot{\theta}_{23} = \frac{\theta_3 - \theta_2}{t_{d23}}$$

$$\dot{\theta}_{34} = \frac{\theta_4 - \theta_3}{t_{d34}}$$

$$\ddot{\theta}_j = \mathrm{SGN}\left(\dot{\theta}_{jk} - \dot{\theta}_{j-1,j}\right)\left|\ddot{\theta}_j\right|$$

$$= \simeq \ddot{\theta}_2 = \mathrm{SGN}\left(\dot{\theta}_{23} - \dot{\theta}_{12}\right)\left|\ddot{\theta}_2\right|$$

$$\ddot{\theta}_3 = \mathrm{SGN}\left(\dot{\theta}_{34} - \dot{\theta}_{23}\right)\left|\ddot{\theta}_3\right|$$

$$t_j = \frac{\left(\dot{\theta}_{jk} - \dot{\theta}_{j-1,k}\right)}{\ddot{\theta}_j} \simeq t_2 = \frac{\left(\dot{\theta}_{23} - \dot{\theta}_{12}\right)}{\ddot{\theta}_2}$$

$$t_3 = \frac{\left(\dot{\theta}_{34} - \dot{\theta}_{23}\right)}{\ddot{\theta}_3}$$

$$t_{jk} = t_{djk} - \frac{t_j}{2} - \frac{t_k}{2} \simeq t_{23} = t_{d23} - t_2/2 - t_3/2$$

First Segment:

$$\ddot{\theta}_1 t_1 = \frac{\theta_2 - \theta_1}{t_{d12} - t_1/2} \quad \text{--- (1)}$$

$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \left(\frac{2(\theta_2 - \theta_1)}{\ddot{\theta}_1}\right)} \quad \text{--- (2)}$$

$$\dot{\theta}_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - t_1/2} \quad \text{--- (3)}$$

$$t_{12} = t_{d12} - t_1 - \frac{t_2}{2}$$

$$\boxed{\begin{array}{l} k = 3 \\ l = 4 \end{array}}$$

Last Segment. (connecting θ_3, θ_4).

$$\ddot{\theta}_4 = \mathrm{SGN}(\theta_3 - \theta_4)\,|\ddot{\theta}_4|$$

$$t_4 = t_{d34} - \sqrt{(t_{d34})^2 - \left(\frac{2(\theta_4 - \theta_3)}{\ddot{\theta}_4}\right)}$$

$$\dot{\theta}_{34} = \frac{\theta_4 - \theta_3}{t_{d34} - t_4/2}$$

$$t_{34} = t_{d34} - t_4 - t_3/2$$

Home work

1. Sketch graphs of position, velocity, and acceleration for a two segment continuous acceleration spline. Sketch them for a joint for which θ_0 =5 deg., θ_v =15 deg., θ_g=-10 deg. And each segment lasts 2 secs.

2. Calculate $\dot{\theta}_{12}, \dot{\theta}_{23}, t_{1}, t_2, t_3$ for a two-segment linear spline with parabolic blends. For this joint θ_1 =5 deg., θ_2 =15 deg., θ_3=-40 deg. Assume t_{d12}= t_{d23}=2 secs, default acceleration at blends = 60 degrees/sec^2. Sketch plots of position, velocity and acceleration of θ
3. A single link robot with a rotary joint is motionless at $\theta = -5$ deg. It is desired to move the joint in a smooth manner to θ=80 degrees in 4 secs. and stop smoothly. Compute the corresponding parameters of a linear trajectory with parabolic blends. Plot the position, velocity, and acceleration of the joint as a function of time.

THANK YOU