# End Semester Assignment

17/06/2021


Chella Thiyagarajan N
*ME17B179*
*me17b179@smail.iitm.ac.in*


## I. Question 1

### A. Part (a)

Mass-Damper system having atleast 5 nodes:

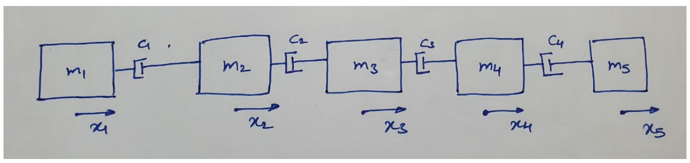


Fig. 1. Mass Damper system with 5 nodes

Assumed damping coefficients are =

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \end{bmatrix}$$

The System equations corresponding to the above system:

1) $m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) = 0$
2) $m_2\ddot{x}_2 + c_1(\dot{x}_2 - \dot{x}_1) + c_2(\dot{x}_2 - \dot{x}_3) = 0$
3) $m_3\ddot{x}_3 + c_2(\dot{x}_3 - \dot{x}_2) + c_3(\dot{x}_3 - \dot{x}_4) = 0$
4) $m_4\ddot{x}_4 + c_3(\dot{x}_4 - \dot{x}_3) + c_4(\dot{x}_4 - \dot{x}_5) = 0$
5) $m_5\ddot{x}_5 + c_4(\dot{x}_5 - \dot{x}_4) = 0$

System Equations Matrix Form:

$$\begin{bmatrix} m_1\ddot{x}_1 \\ m_2\ddot{x}_2 \\ m_3\ddot{x}_3 \\ m_4\ddot{x}_4 \\ m_5\ddot{x}_5 \end{bmatrix}$$

$$= \begin{bmatrix} -c_1 & c_1 & 0 & 0 & 0 \\ c_1 & -(c_1+c_2) & c_2 & 0 & 0 \\ 0 & c_2 & -(c_2+c_3) & c_3 & 0 \\ 0 & 0 & c_3 & -(c_3+c_4) & c_4 \\ 0 & 0 & 0 & c_4 & -c_4 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix}$$

### B. Part (b)

The matrix form from the previous subsection resembles the equation given in example 3.1

$$\dot{p} = -DRD^T M^{-1} p$$

where

$$\dot{p} = \begin{bmatrix} m_1\ddot{x}_1 \\ m_2\ddot{x}_2 \\ m_3\ddot{x}_3 \\ m_4\ddot{x}_4 \\ m_5\ddot{x}_5 \end{bmatrix}$$

$$M^{-1}p = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix}$$

$$-DRD^T = \begin{bmatrix} -c_1 & c_1 & 0 & 0 & 0 \\ c_1 & -(c_1+c_2) & c_2 & 0 & 0 \\ 0 & c_2 & -(c_2+c_3) & c_3 & 0 \\ 0 & 0 & c_3 & -(c_3+c_4) & c_4 \\ 0 & 0 & 0 & c_4 & -c_4 \end{bmatrix}$$

with p the vector of momenta of the masses associated to the vertices, M the diagonal mass matrix, R the diagonal matrix of damping coefficients of the dampers attached to the edges, and $H(p) = (p^T M^{-1} p)/2$ the total kinetic energy of the masses. The vector of velocities $v = M^{-1}p$ converges to a vector in the kernel of L = $DRD^T$. By substituting damper coefficient values in $DRD^T$ matrix we get:

$$L = \begin{bmatrix} 10 & -10 & 0 & 0 & 0 \\ -10 & 30 & -20 & 0 & 0 \\ 0 & -20 & 50 & -30 & 0 \\ 0 & 0 & -30 & 70 & -40 \\ 0 & 0 & 0 & -40 & 40 \end{bmatrix}$$

### C. Part (c)

Proposition 4.2: The flow-Laplacian matrix L satisfies $L + L^T >= 0$ if and only if it is balanced; that is, not only $\mathbf{1}^T L = 0$ (column sums zero) but also $L\mathbf{1} = 0$ (row sums zero).
I have implemented $L + L^T >= 0$, $1^T L = 0$ and $L1 = 0$ in python code and all the conditions are satisfied. Therefore L is balanced.

Note: Please refer to Question_1.ipynb jupyter notebook for code and further details

## II. Question 2

I have used isl_wise_train_detail_03082015_v1.csv file outsourced by Indian Railways. I have constructed a directed graph with station nodes which are immediately connected by a train without any intermediate stations, while the weights on edges represent the number of trains directly linking two stations. My constructed graph has 4344 station nodes and 16992 directed edges.

### A. Part (a)

Average path length is a concept in network topology that is defined as the average number of steps along the shortest paths for all possible pairs of network nodes. It is a measure of the efficiency of information or mass transport on a network.Average path length is one of the three most robust measures of network topology, along with its clustering coefficient and its degree distribution. The average path length distinguishes an easily negotiable network from one, which is complicated and inefficient, with a shorter average path length being more desirable. However, the average path length is simply what the path length will most likely be. The network itself might have some very remotely connected nodes and many nodes, which are neighbors of each other. Average path length is calculated for weekly connected components. As a fact our network has 3 weekly connected components. Results =

1) Weakly Connected Component 1
   a) Number of Nodes = 4328
   b) Number of Directed Edges = 16967
   c) Average Path Length = 7.85583
2) Weakly Connected Component 2
   a) Number of Nodes = 8
   b) Number of Directed Edges = 11
   c) Average Path Length = 1.57142
3) Weakly Connected Component 3
   a) Number of Nodes = 8
   b) Number of Directed Edges = 14
   c) Average Path Length = 3.0

The max value of average path length is around 8 which gives us the insight that in India any station can be reached from any station in 8 steps or trains. Which really shows that how interconnected our whole railway network is inorder to connect major cities and rurals from Jammu to Kanyakumari.

### B. Part (b)

It is the shortest distance between the two most distant nodes in the network. In other words, once the shortest path length from every node to all other nodes is calculated, the diameter is the longest of all the calculated path lengths. The diameter is representative of the linear size of a network. Network Diameter is calculated for Strongly connected components. As a fact our network has 13 weekly connected components. Results =

1) Strongly Connected Component 1
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
2) Strongly Connected Component 2
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
3) Strongly Connected Component 3
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
4) Strongly Connected Component 4
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
5) Strongly Connected Component 5
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
6) Strongly Connected Component 6
   a) Number of Nodes = 4321
   b) Number of Directed Edges = 16960
   c) Network Diameter = 35
7) Strongly Connected Component 7
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
8) Strongly Connected Component 8
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
9) Strongly Connected Component 9
   a) Number of Nodes = 1
   b) Number of Directed Edges = 0
   c) Network Diameter = 0
10) Strongly Connected Component 10
    a) Number of Nodes = 1
    b) Number of Directed Edges = 0
    c) Network Diameter = 0
11) Strongly Connected Component 11
    a) Number of Nodes = 1
    b) Number of Directed Edges = 0
    c) Network Diameter = 0
12) Strongly Connected Component 12
    a) Number of Nodes = 5
    b) Number of Directed Edges = 8
    c) Network Diameter = 4
13) Strongly Connected Component 1
    a) Number of Nodes = 8
    b) Number of Directed Edges = 14
    c) Network Diameter = 7

Maximum diameter in our Indian Railway network is 35 which is a very big number of steps to reach most apart stations in our network, this tells about the corner cases that our Railway network lacks and sheds some light on future developments.

### C. Part (c)

I have found 3 weakly connected components and 13 strongly connected components from our Indian Railways network. More than one weakly connected components tells us that there are stations which are impossible to reach or travel from, but from our previous analysis majority of the station are under one big network, only 2 disjoint 8 station networks are the other two, so this doesn't cause big disadvantages to the inter connectivity all over India. But the problem is that we have more than 3 strongly connected components which needs to be looked into.

### D. Part (d)

I have considered four centrality measures among all the centrality measures available. These centrality's were chosen with our coursework and previous assignments in mind. Chosen Centrality's are

1) Degree Centrality
2) Eigen Vector Centrality
3) Katz Centrality
4) Pagerank Centrality

katz centrality uses $\alpha$ = 1/(2*$\rho$) where $\rho$ is the maximum value of the spectrum of adjacency matrix. Pagerank centrality uses $\alpha$ = 0.5. I have calculated all the above mentioned 4 centralities for every node available in the Indian Railways network and stored them as excel sheets as follows

1) q2_degree_centrality.xlsx
2) q2_eigen_vector_centrality.xlsx
3) q2_katz_centrality.xlsx
4) q2_pagerank_centrality.xlsx

Note: Please refer to Question_2.ipynb jupyter notebook for code and further details

## III. Question 3

I have considered 57buses dataset for this problem. Important part of this question is to calculate average controllability centrality using the given paper. Given a complex network with n nodes and an associated stable linear dynamics matrix A, the Average Energy Controllability Centrality measure for node i is given by

$$C_{CE}(i) = tr(W_i)$$

where

$$W_i$$

is the infinite-horizon controllability Gramian that satisfies

$$AW_i + W_iA^T + e_i(e_i)^T = 0$$

where $e_i$ has a 1 in the $i^{th}$ entry and zeros elsewhere. The tricky part here is to calculate the stable system matrix A. I have explored other related papers that could shed more light on calculating system matrix A. Grid stabilization through VSC-HVDC using wide area measurements by Alexander Fuchs provides more knowledge about the relation ship between Y and A matrix and the method to calculate Y matrix. However TA has facilitated us by providing the Y matrix through prebuilt matlab functions. I have taken the given Y matrix and calculated laplacian matrix of Y to achieve the system matrix A. Later this matrix is used to calculate $W_i$ for each i or bus in the European Grid network. trace of each $W_i$ provides us the Average Energy Controllability Centrality measure for each bus. Finally I have chosen 10 best locations that maximize average controllability, as discussed in the paper. Those 10 best locations are

1) 31
2) 33
3) 32
4) 30
5) 19
6) 20
7) 57
8) 42
9) 25
10) 56

Note: Please refer to Question_3.ipynb jupyter notebook for code and further details

## IV. Question 4

Directed graph has been constructed for the given edge connections and given weights. The graphical visualization of the given DiGraph:

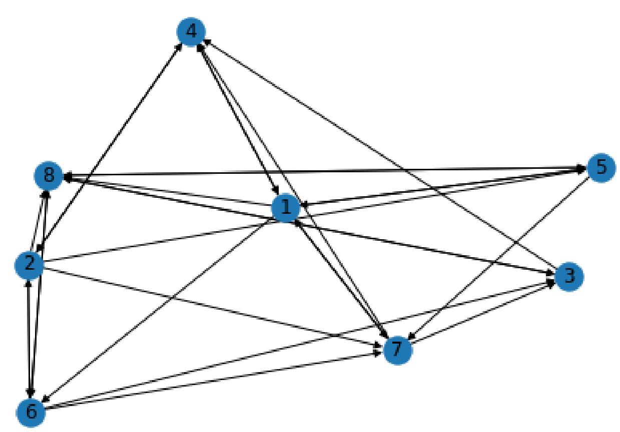


Fig. 2. Given DiGraph

It's respective adjacency matrix:

$$A = 0.015(\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix})$$

The above adjacency matrix is written with the node order [1, 4, 5, 6, 7, 8, 2, 3].

### A. Part (a)

Cause Centrality Ranking in ascending order:

| Node | Value |
|---|---|
| 3 | 1.030573 |
| 4 | 1.031142 |
| 8 | 1.046029 |
| 7 | 1.046030 |
| 5 | 1.046256 |
| 6 | 1.061484 |
| 2 | 1.076717 |
| 1 | 1.076717 |

Effect Centrality Ranking in ascending order:

| Node | Value |
|---|---|
| 3 | 1.079332 |
| 5 | 1.095450 |
| 4 | 1.096038 |
| 2 | 1.108955 |
| 6 | 1.110678 |
| 7 | 1.111277 |
| 1 | 1.124904 |
| 8 | 1.127097 |

### B. Part (b)

Here we need to achieve the first three ranks given by C as 5,4 and 8. In order to achieve it I have used the approach which searches through all the combinations of 5 edges and arrive at the right one. The algorithm returned more than one possible right combinations. I will present the first returned five edge combination:

$$((3,1),(3,2),(3,5),(3,6),(4,5))$$

It's corresponding Digraph visualization is Fig. 3.

### C. Part (c)

The same algorithmic search approach is also adopted to solve this part and the smallest number of edges required to achive that the node 5 becomes the first ranked node in C and node 7 becomes the first ranked node in E is 4. Result:

$$((2,3),(3,1),(3,5),(4,6))$$

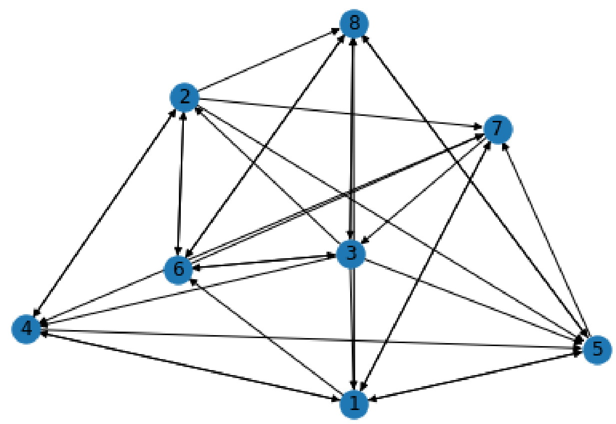


Fig. 3. ((3, 1), (3, 2), (3, 5), (3, 6), (4, 5)) included DiGraph

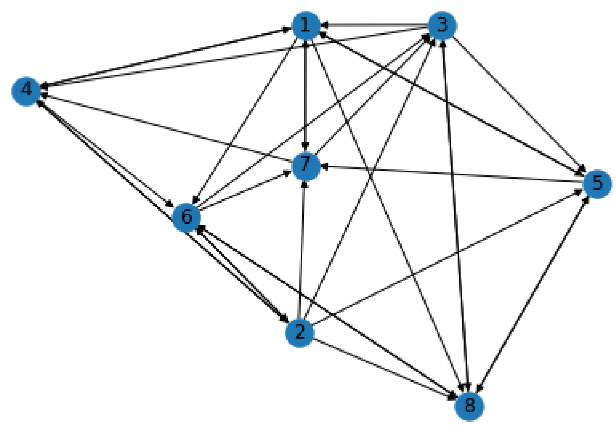


Fig. 4. ((2, 3), (3, 1), (3, 5), (4, 6)) included DiGraph

It's corresponding Digraph visualization is Fig. 4.

Note: Please refer to Question_4.ipynb jupyter notebook for code and further details

## References


[1] Modeling of physical network systems, Arjan van der Schaft, Johann Bernoulli Institute for Mathematics and Computer Science, Jan C. Willems Center for Systems and Control, University of Groningen.

[2] Optimal Sensor and Actuator Placement in Complex Dynamical Networks, Tyler H. Summers, John Lygeros, ETH Zurich, Zurich, Switzerland.

[3] Grid stabilization through VSC-HVDC using wide area measurements, Alexander Fuchs (Member, IEEE) , Sébastien Mariéthoz (Member, IEEE), Mats Larsson, and Manfred Morari (Fellow, IEEE).