Manipulator Kinematics

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Contents

- Kinematics
  - Object Location and Motion
  - Transformation Matrices
  - Homogeneous Transformations
- Forward Kinematics
- Inverse Kinematics
- Differential Relationships

Industrial Robots (Manipulators)

Industrial robots

Pick and Place
Assembly
Welding
Painting
Machining

Kinematics- Fundamentals

Object Location and Motion

Contents:

Object location

- Position of a point in space
- Location of a rigid body in space
- Homogeneous transformation matrix

Object Motion

Translation (Transformation Matrix) and Inverse
Basic Rotation (Transformation Matrices) and Inverse.
General Rotation

Examples.
Properties of homogeneous transformation matrix.
Examples.

Given an object in a physical world,

- how to describe its position and orientation, and
- how to describe its change of position and orientation due to motion

are two basic issues we need to address before talking about having a robot moving physical objects around.

The term location refers to the position and orientation of an object.

Co-ordinate frames

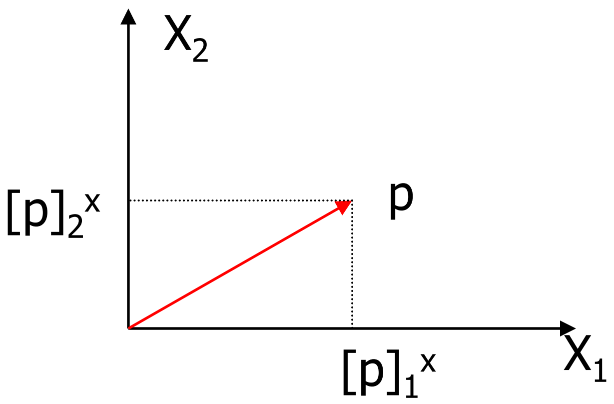
If 'p' is a vector in R^n, and $X=\{x^1, x^2, x^3 \ldots x^n\}$ be a complete orthonormal set of R^n, then the co-ordinates of p with respect to X are denoted as $[p]^x$ and are defined as

$$p = \sum_{k=1}^{n} [p]_k^X x^k$$

The complete orthonormal set X is sometimes called an orthonormal co-ordinate frame.

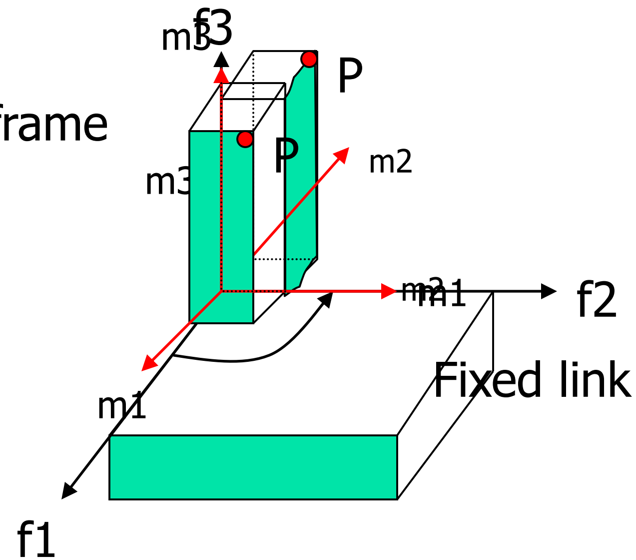
The 'k'th co-ordinate of p wrt X is

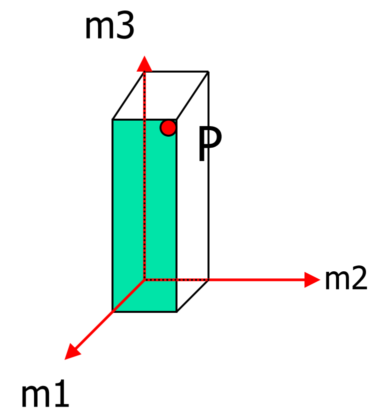
$$[p]_k^x = p.x^k$$

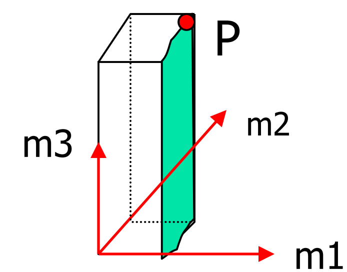


Co-ordinate Transformation

Represent position of p wrt fixed frame
f={f1,f2,f3}







The two sets of co-ordinates of P are given by

$[p]^M = [p.m1, p.m2, p.m3]$

$[p]^F = [p.f1, p.f2, p.f3]$

The co-ordinate transformation problem is to find the co-ordinates of p wrt F, given the co-ordinates of p wrt M.

Co-ordinate Transformation Matrix

Let F={f^1, f^2, f^3, .. f^n} and M={m^1, m^2, m^3, .. m^n } be co-ordinate frames of R^n with F being an orthonormal frame. Then for each point p in R^n,

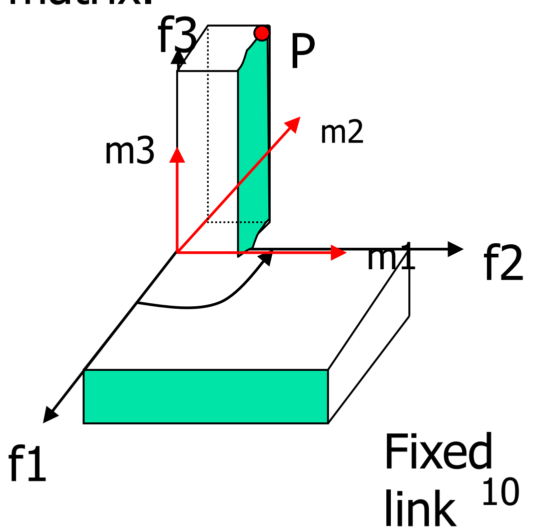
$$[p]^F = A\,[p]^M$$

$$\textit{where}\ \text{A is an nxn matrix defined by}\ A_{kj} = f^k.m^j \quad \text{for}\ 1 \le \text{k, j} \le \text{n}$$

The matrix A is known as Co-ordinate transformation matrix.

$$A = \begin{bmatrix} f^1.m^1 & f^1.m^2 & f^1.m^3 \\ f^2.m^1 & f^2.m^2 & f^2.m^3 \\ f^3.m^1 & f^3.m^2 & f^3.m^3 \end{bmatrix}$$

Example:



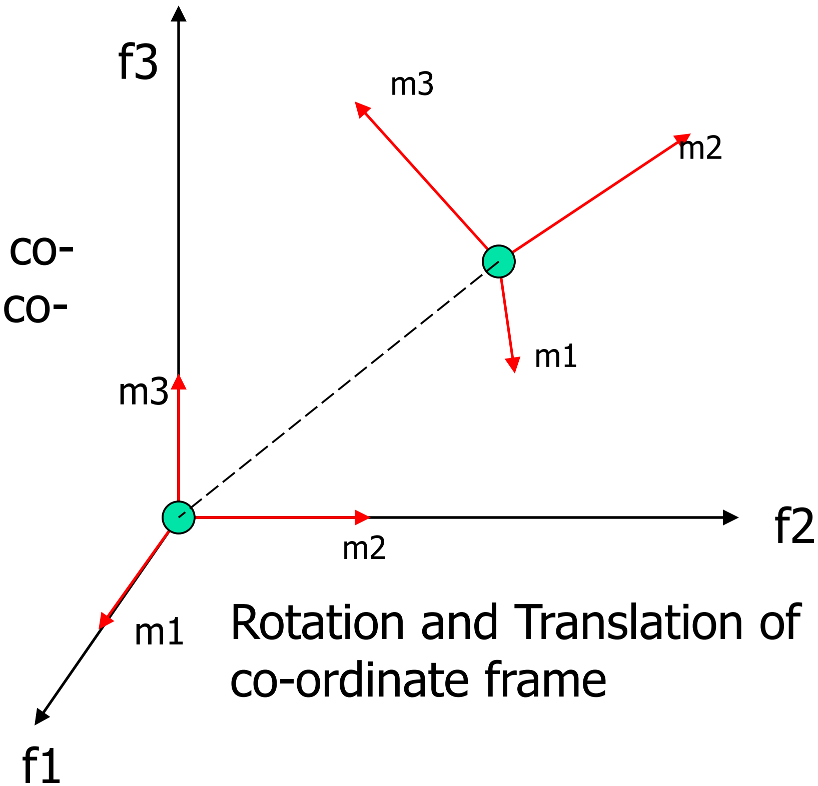
Inverse Co-ordinate Transformation

Let F and M be two orthonormal co-ordinate frames in R^n, having the same origin, and let A be the co-ordinate Transformation matrix that maps M co-ordinates to F co-ordinates, then the transformation matrix which maps F coordinates into M coordinates is given by A^{-1} , where

$$A^{-1}=A^T$$

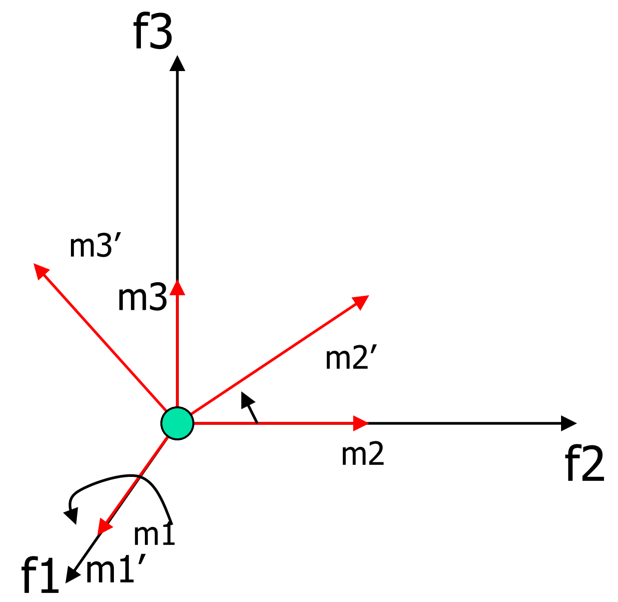
Rotations

In order to specify the position and orientation of the mobile tool in terms of a co-ordinate frame attached to the fixed base, co-ordinate transformations involving both rotations and translations are required.



Rotation and Translation of co-ordinate frame

Fundamental rotations



If the mobile coordinate frame is obtained from the fixed coordinate frame F by rotating M about one of the unit vectors of F, then the resulting coordinate transformation matrix is called a *fundamental rotation matrix*.

In the space R3, there are 3 possibilities.

$$R_1(\phi) = \begin{bmatrix} f^1.m^{1'} & f^1.m^{2'} & f^1.m^{3'} \\ f^2.m^{1'} & f^2.m^{2'} & f^2.m^{3'} \\ f^3.m^{1'} & f^3.m^{2'} & f^3.m^{3'} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & f^2.m^{2'} & f^2.m^{3'} \\ 0 & f^3.m^{2'} & f^3.m^{3'} \end{bmatrix}$$

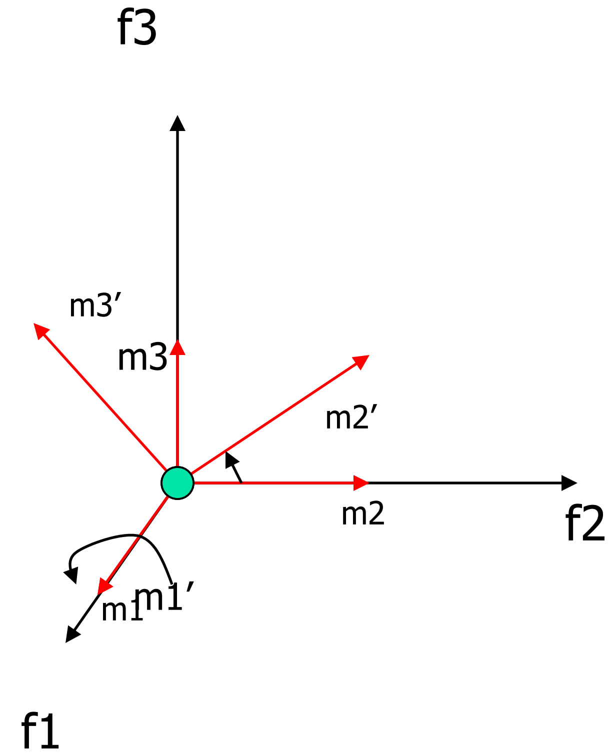
$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

Fundamental rotations

$$R_1(\phi) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$R_2(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix}$$

$$R_3(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Pattern

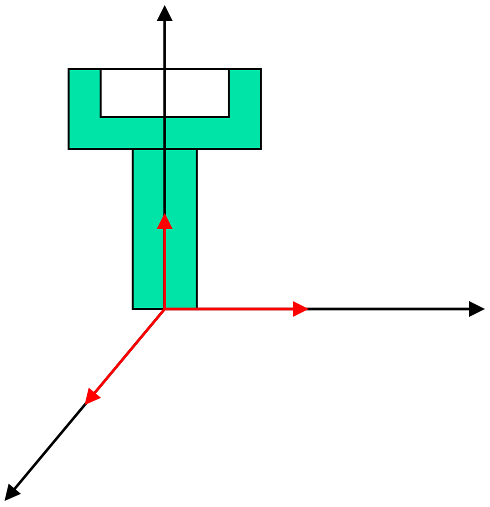
The k^{th} row and the k^{th} column of $R_k(\phi)$ are identical to the k th row and the k th column of identity matrix. In the remaining 2x2 matrix, the diagonal terms are $\cos(\phi)$ while the off diagonal terms are $\pm\sin(\phi)$. The sign of the off diagonal term above the diagonal is $(-1)^k$.

Example

Composite Rotations

A Sequence of fundamental rotations about the unit vectors cause composite rotations.

Algorithm for composite rotation

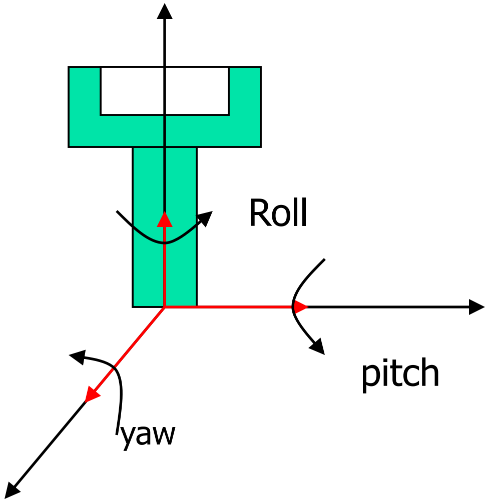


1. Initialise rotation matrix to R=I, which corresponds to F and M being coincident
2. If the mobile frame M is rotated by an amount ϕ about the k^{th} unit vector of F, then pre-multiply R by $R_k(\phi)$. -> $[R_k(\phi).R]$
3. If the mobile frame M is rotated by an amount , ϕ about it's own k^{th} vector, then post-multiply R by $R_k(\phi)$. -> $[R . R_k(\phi)]$
4. If there are more rotations go back to 2. The resulting matrix maps M to F

Yaw-Pitch-Roll Transformation matrix

$$R(\theta) = R_3(\theta_3) R_2(\theta_2) R_1(\theta_1) I$$

$$YPR = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & C_1S_2C_3 + S_1S_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & C_1S_2S_3 - S_1C_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$



Class Work

Suppose we rotate tool about the fixed axes, starting with yaw of $\pi/2$, followed by pitch of $-\pi/2$ and finally, a roll of $\pi/2$, what is the resulting composite rotation matrix?

Homework1

Suppose a point P at the tool tip has mobile co-ordinates $[p]^M = [0,0,.6]^T$, Find $[p]^F$ following YPR transformation of 45, 60 and 90 degrees respectively.

Homogeneous co-ordinates

- We need pure rotations and translations to characterize the position and orientation of a point relative to the co-ordinate frame attached to the base.
- While a rotation can be represented by a 3x3 matrix, it is not possible to represent translation by the same.
- We need to move to a higher dimensional space, the four dimensional space of homogeneous co-ordinates.

Definition: Let q be a point in R^3, and let F be an orthonormal coordinate frame of R^3 . If σ is any non zero scale factor, then the homogeneous coordinates of q with respect to F are denoted as $[q]^F$ and defined:

$$[q]^F = [\sigma q_1, \sigma q_2, \sigma q_3, \sigma]$$

In robotics we use σ=1 for convenience

Homogeneous Transformation matrix

If a physical point in three-dimensional space is expressed in terms of its homogeneous co-ordinates and we want to change from one coordinate frame to another, we use a 4x4 homogeneous transformation matrix.

In general T is

$$T = \left[\begin{array}{c:c} R & p \\ \hdashline \eta^T & \sigma \end{array}\right]$$

The 3x3 matrix R is a rotation matrix

P is a 3x1 translation vector

η is a perspective vector, set to zero

In terms of a robotic arm, P represents the position of the tool tip, R its orientation.

- The fundamental operations of rotations and translations can each be regarded as special cases of the general 4x4 homogeneous transformation matrix.

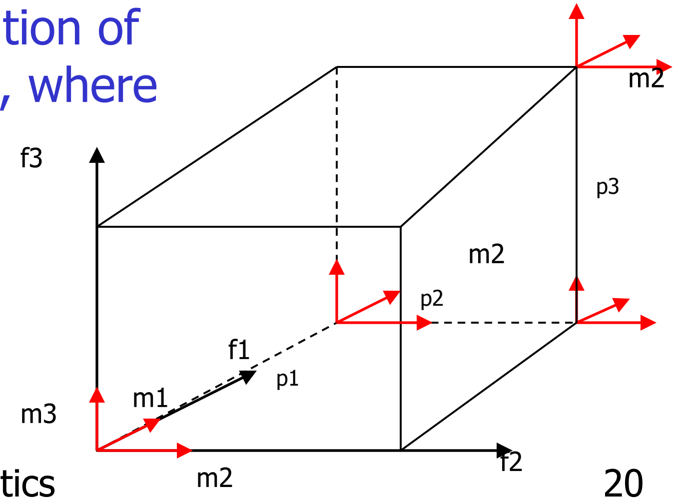
$$Rot(\phi, k) = \left[\begin{array}{ccc|c} & & & 0 \\ & R_k(\phi) & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad 1 \le k \le 3$$

$\mathrm{Rot}(\phi, \mathrm{k})$ is the k^{th} fundamental homogeneous rotation matrix

Using homogeneous coordinates, translations also can be represented by 4x4 matrices.

In terms of homogeneous coordinate frames, the translation of M can be represented by a 4x4 matrix, denoted Tran (p), where

$$Tran(p) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & p_1 \\ 0 & 1 & 0 & p_2 \\ 0 & 0 & 1 & p_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$



Tran (p) is known as the fundamental homogeneous Translation matrix

Inverse Homogeneous Transformation

If T be a homogeneous transformation matrix with rotation R and translation p between two orthonormal coordinate frames and if $\eta=0,\sigma=1$, then the inverse transformation is:

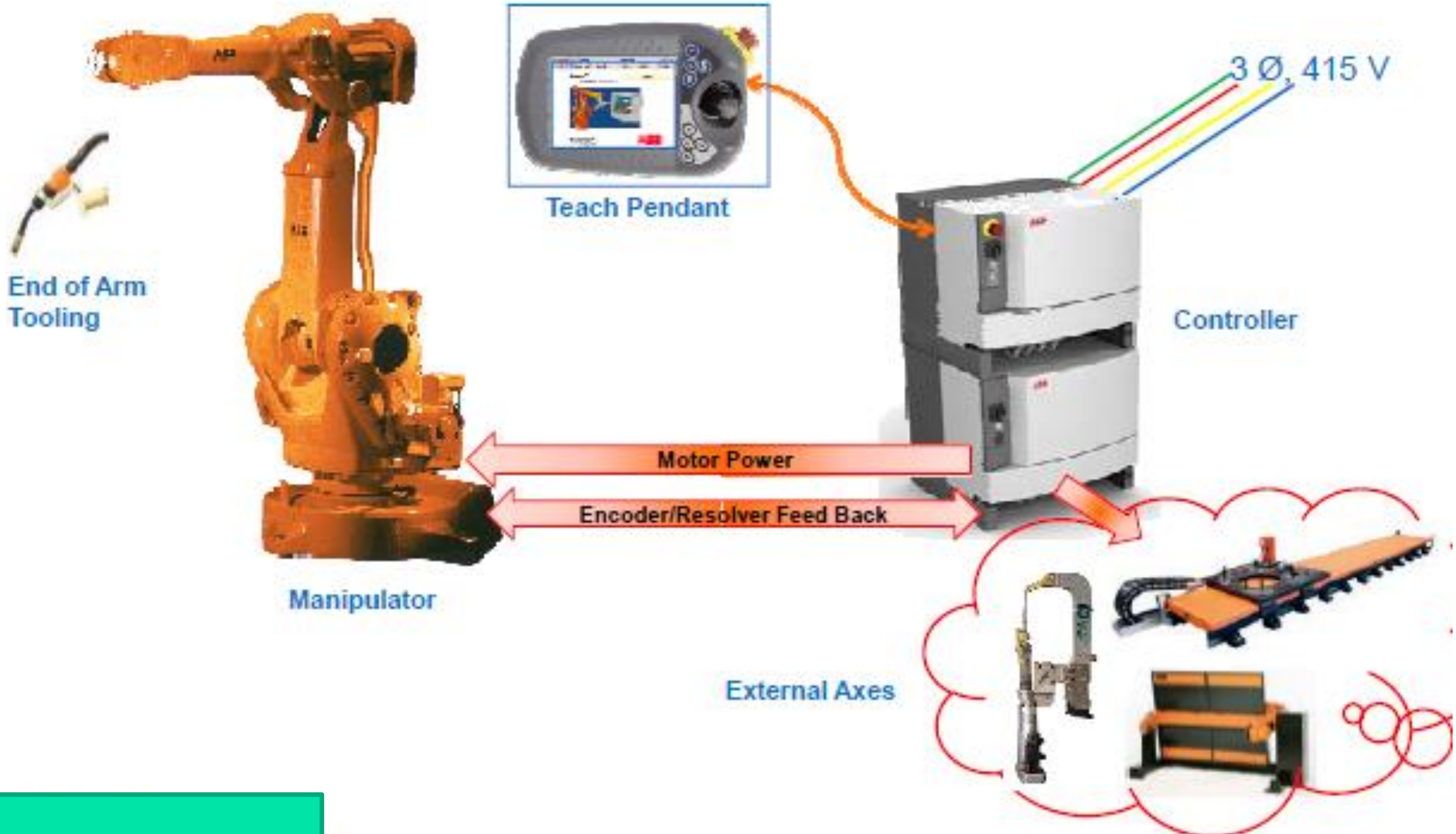
$$T^{-1} = \left[\begin{array}{ccc:c} & R^T & & -R^T p \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

Composite Homogeneous Transformations

Example:

Self Study: Screw Transformation, screw pitch

Components of an Industrial Robot



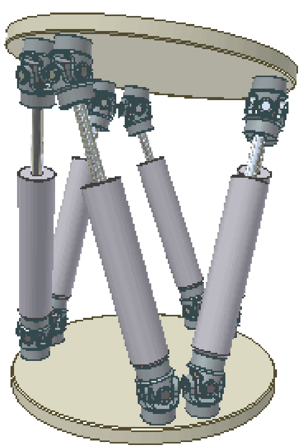
Robot Morphology

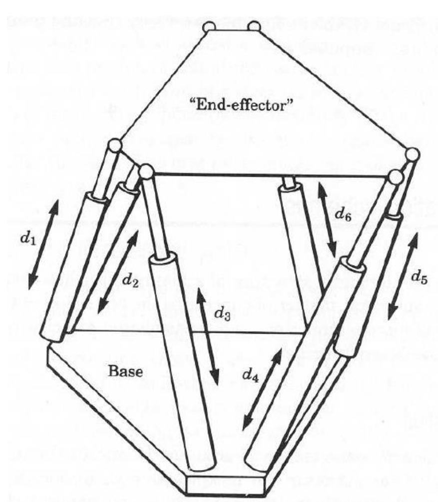
Basic characteristics:

- Kinematics chain
- Degree of freedom
- Architecture
- Work space
- Payload
- Precision

Kinematic Chain: Manipulators

Serial Manipulators





Parallel Manipulators

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Degrees of Freedom (dof)

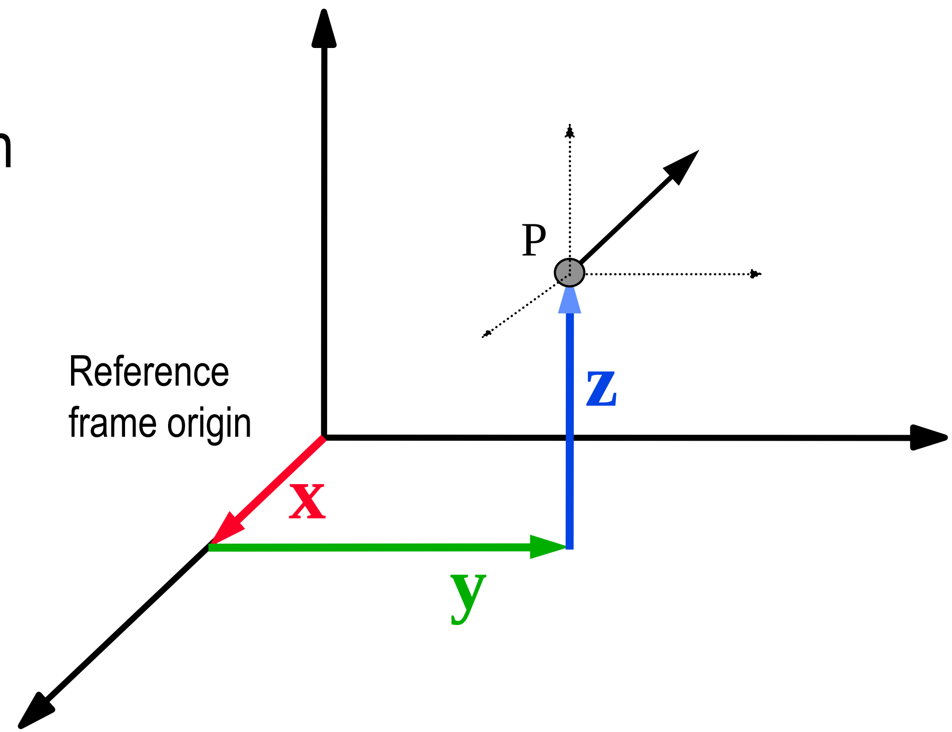
In general, **degrees of freedom** (DOF) are the set of independent displacements that specify completely the displaced or deformed position of the body or system.

- In robotics, *degrees of freedom* is often used to describe the number of directions that a robot can move a joint.
- A human arm is considered to have 7 DOF. A shoulder gives pitch, yaw and roll, an elbow allows for pitch, and a wrist allows for pitch, yaw and roll. Only 3 of those movements would be necessary to move the hand to any point in space, but people would lack the ability to grasp things from different angles or directions.
- A robot (or object) that has mechanisms to control all 6 physical DOF is said to be holonomic. An object with fewer controllable DOF than total DOF is said to be non-holonomic, and an object with more controllable DOF than total DOF (such as the human arm) is said to be redundant.

Degrees of freedom

Positioning

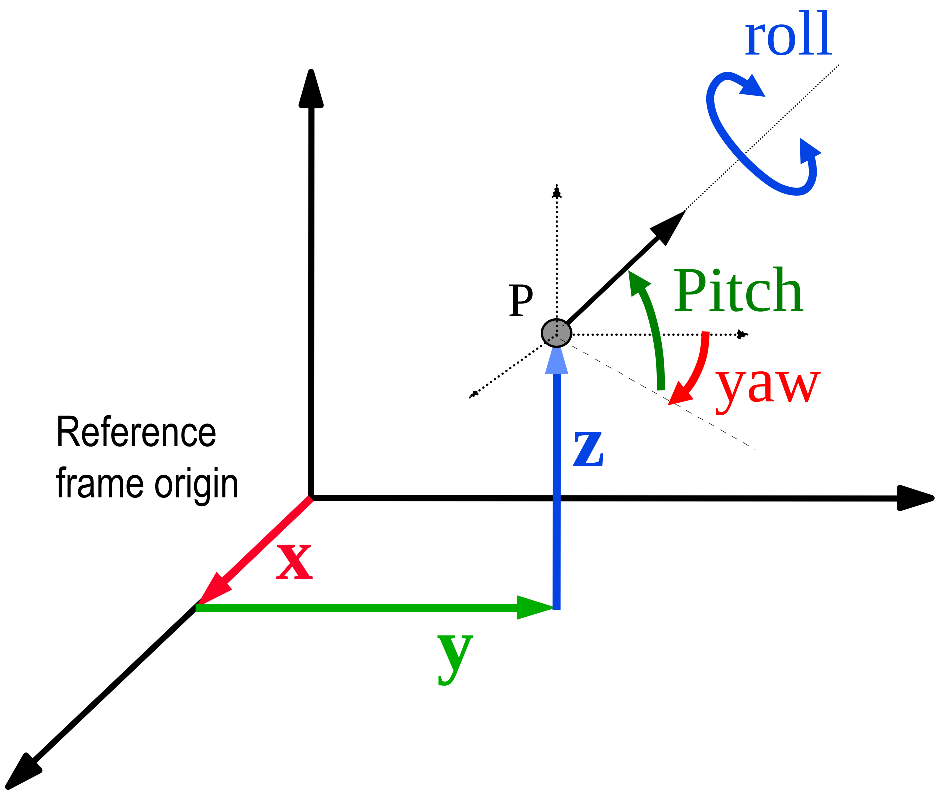
Positioning the end effector in the 3D space, requires three DoF, either obtained from rotations or displacements.



Degrees of freedom

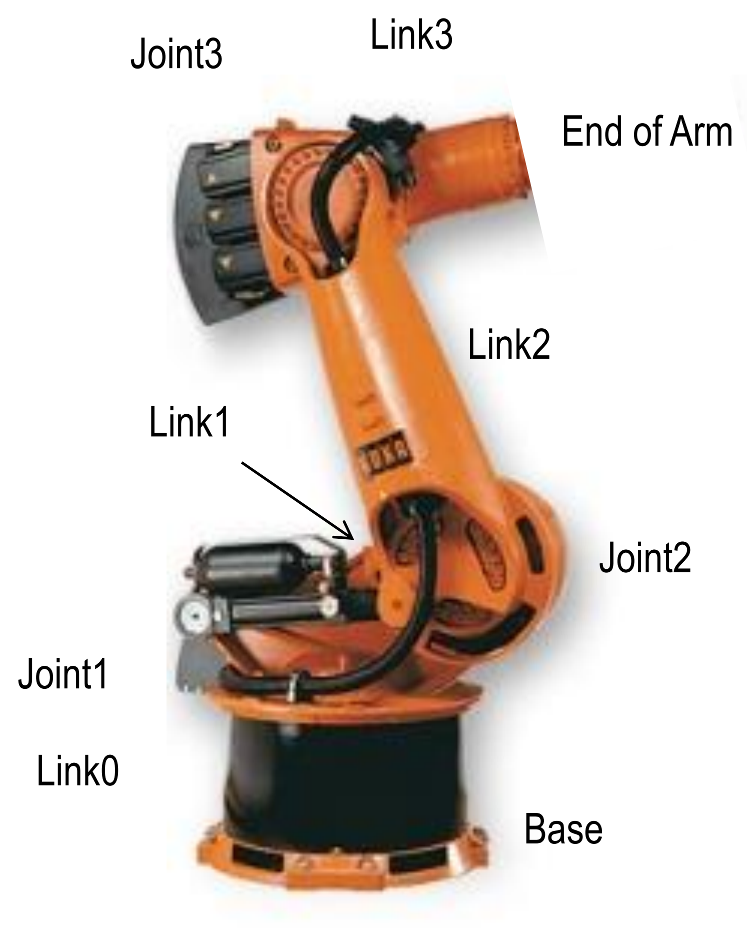
Orientation

Orienting the end effector in the 3D space, requires three additional DoF to produce the three rotations.

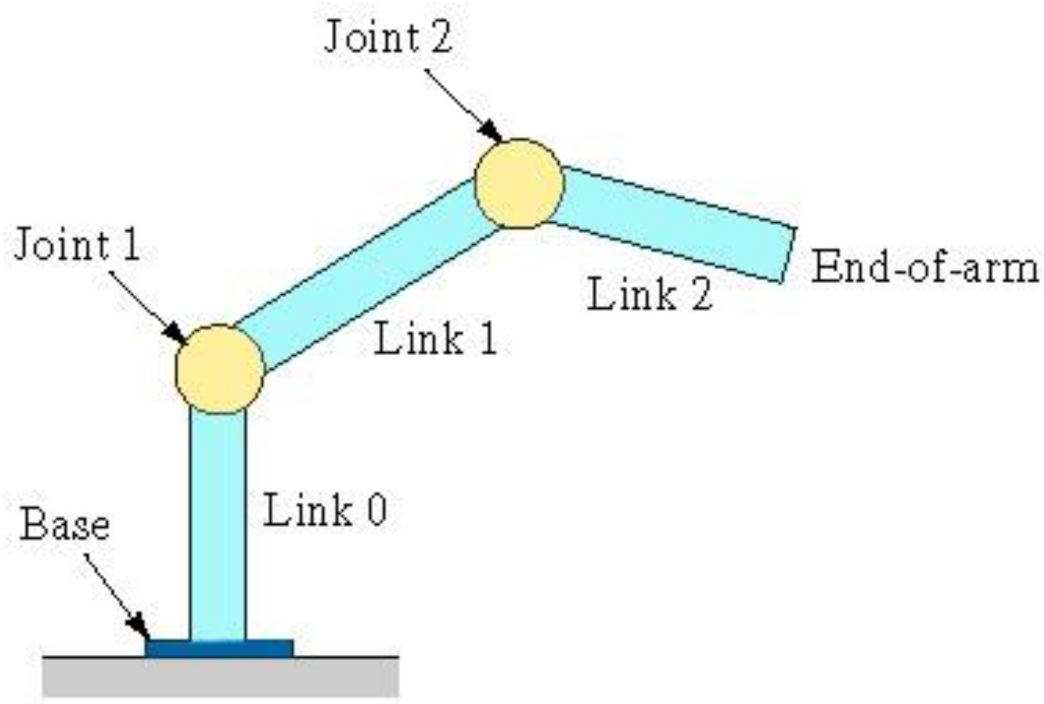


Robot Anatomy

- Manipulator consists of joints and links
  - Joints provide relative motion
  - Links are rigid members between joints
  - Various joint types: linear and rotary
  - Each joint provides a "degree-of-freedom"
  - Most robots possess five or six degrees-of-freedom
- Robot manipulator consists of two sections:
  - Body-and-arm – for positioning of objects in the robot's work volume
  - Wrist assembly – for orientation of objects

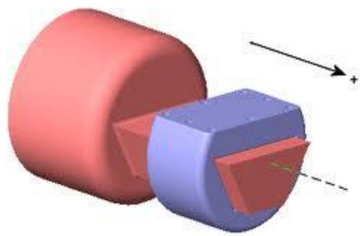


Robot Anatomy

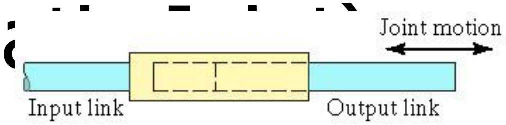


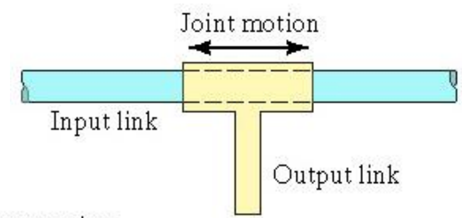
Robot manipulator - a series of joint-link combinations

Manipulator Joints

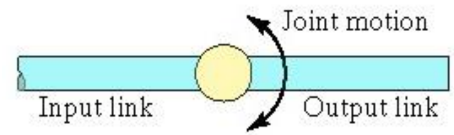


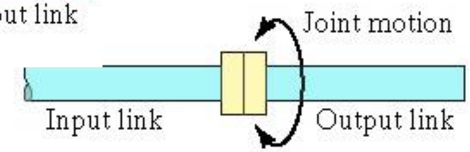
- Translational motion (prisma
  - Linear joint (type P) (or L,O)





- Rotary motion
  - Rotational joint (type R) (R,





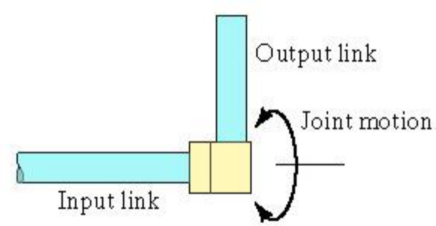
<u>Other Types of Joints:</u>

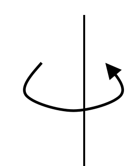
-Cylindrical (Sliding and Turning)

-Screw (Helical Motion)

-Spherical (or ball)

-Planar (sliding on a plane)





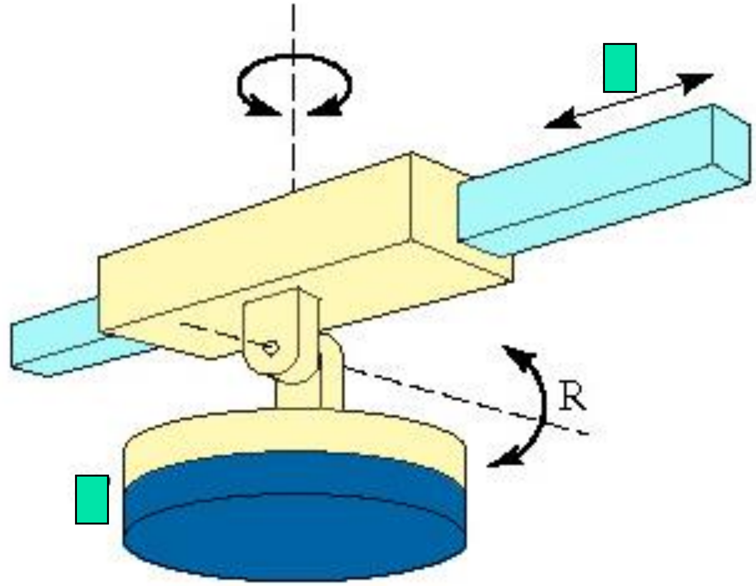
Symbol

Architecture: Robot Body-and-Arm Configurations

- **Robot architecture** is the combination and disposition of the different kind of joints that configure the robot kinematical chain.
- Five common body-and-arm configurations for industrial robots:
  1. Polar coordinate body-and-arm assembly
  2. Cylindrical body-and-arm assembly
  3. Cartesian coordinate body-and-arm assembly
  4. Jointed-arm body-and-arm assembly
  5. Selective Compliance Assembly Robot Arm (SCARA)
- Function of body-and-arm assembly is to position an end effector (e.g., gripper, tool) in space

Polar Coordinate Body-and-Arm Assembly

- Notation RP

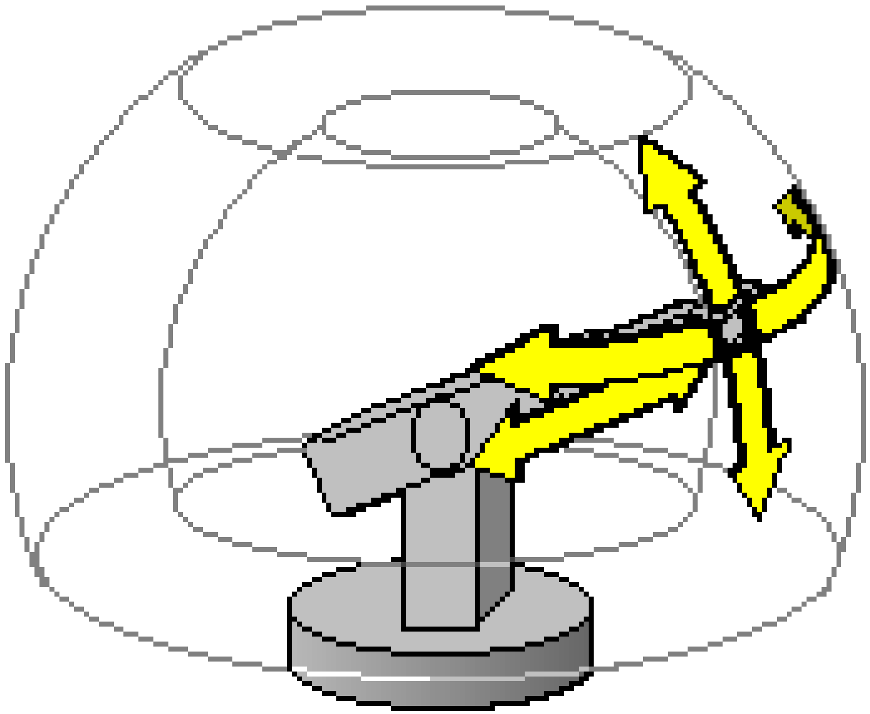


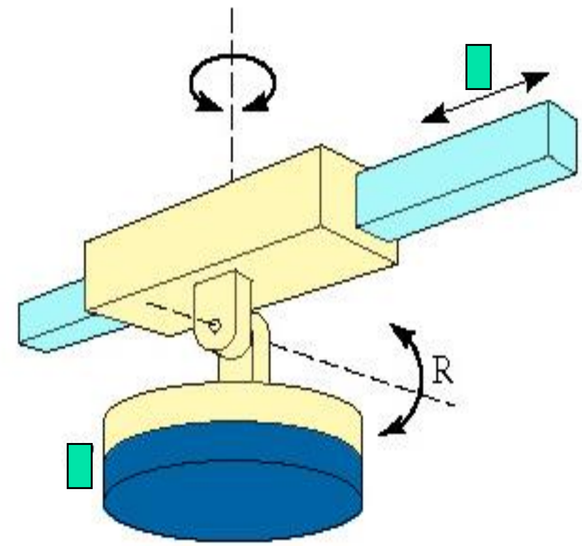
- Consists of a sliding arm (P joint)

Work Envelope

- The gross work envelope of a robot is defined as the locus of points in three dimensional space that can be reached by the wrist.
- Positioning axes (Major axes)

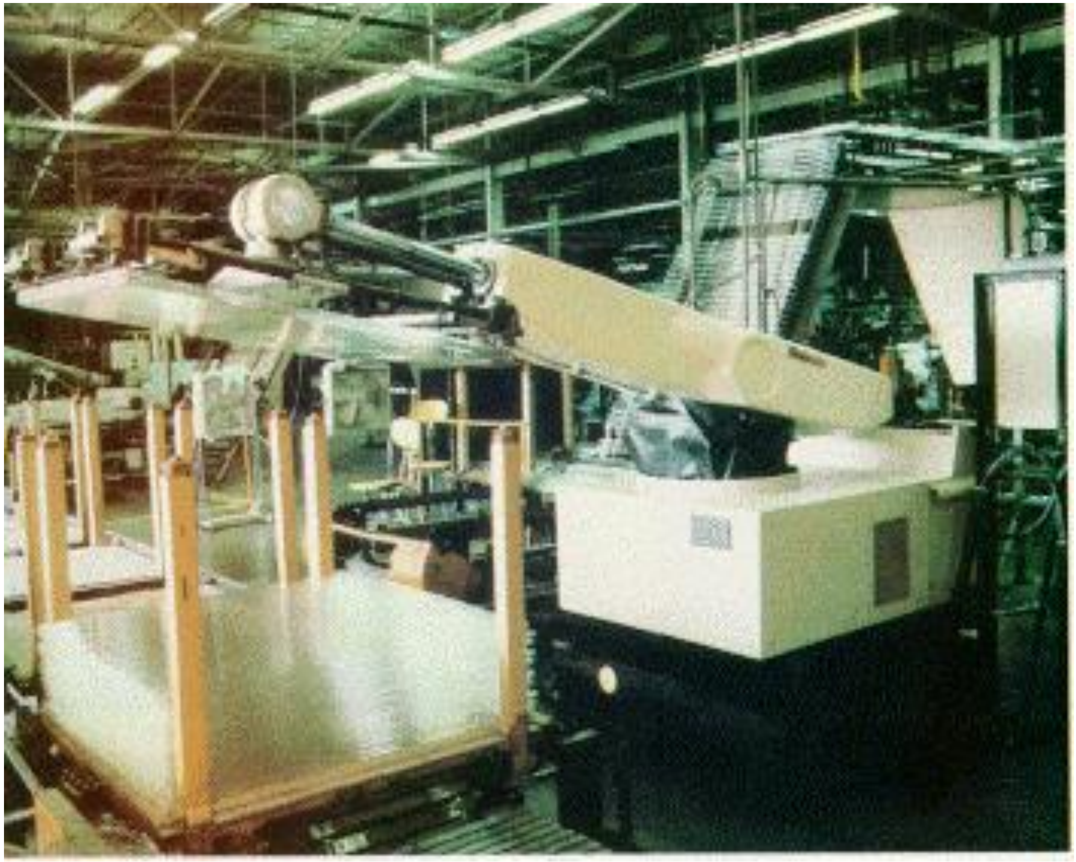
Classical Architectures





Polar Work Space

Classical Architectures



Example of a Polar Work Space Robot

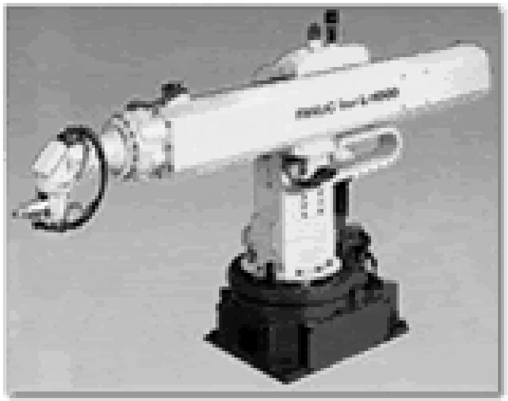
Polar Robot Characteristics

Robot	Joints	Observations
Polar	1a. Rotation: θ 2a. Rotation: φ 3a. Linear: ρ	**Advantages:** • large reach from a central support • It can bend to reach objects on the floor • motors 1 and 2 close to the base **Drawbacks:** • complex kinematics model • difficult to visualize

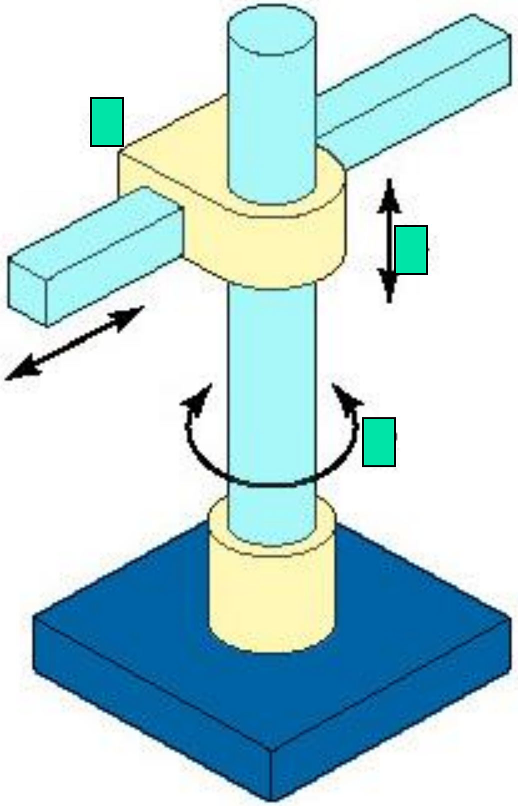
Cylindrical Body-and-Arm Assembly

- Notation RPP:

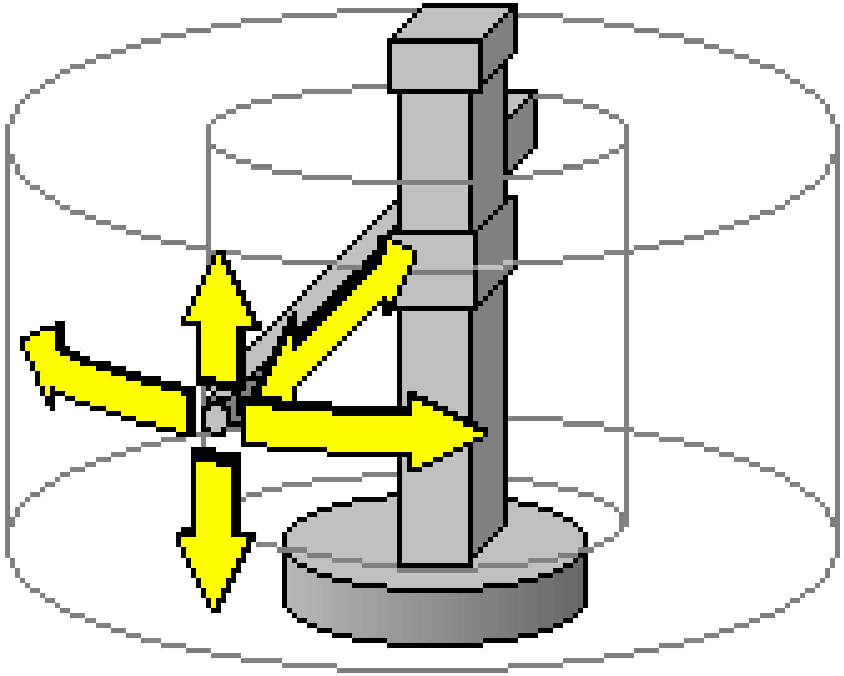
- Consists of a vertical column, relative to which an arm assembly is moved up or down

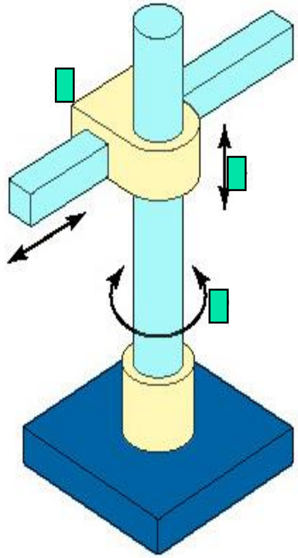


can be moved relative to the



Classical Architectures





Cylindrical Work Space

Classical Architectures

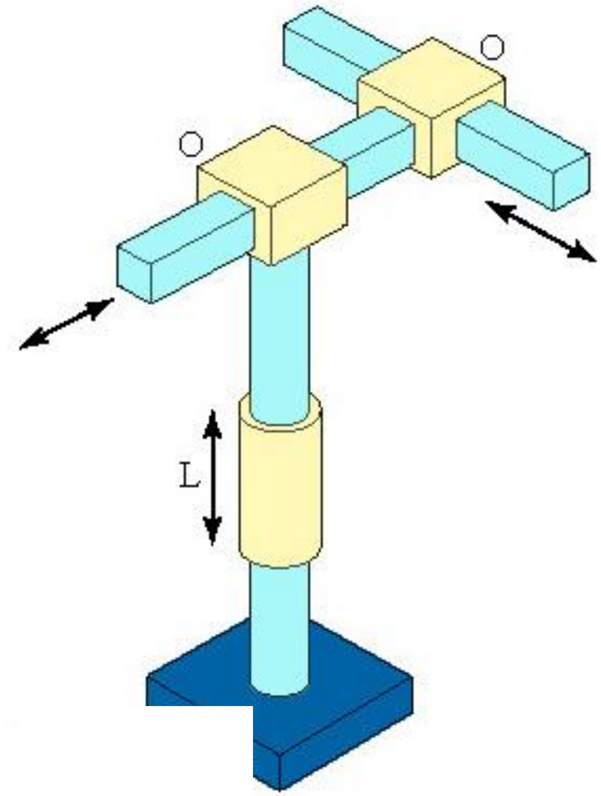
Example of a Cylindrical Work Space Robot

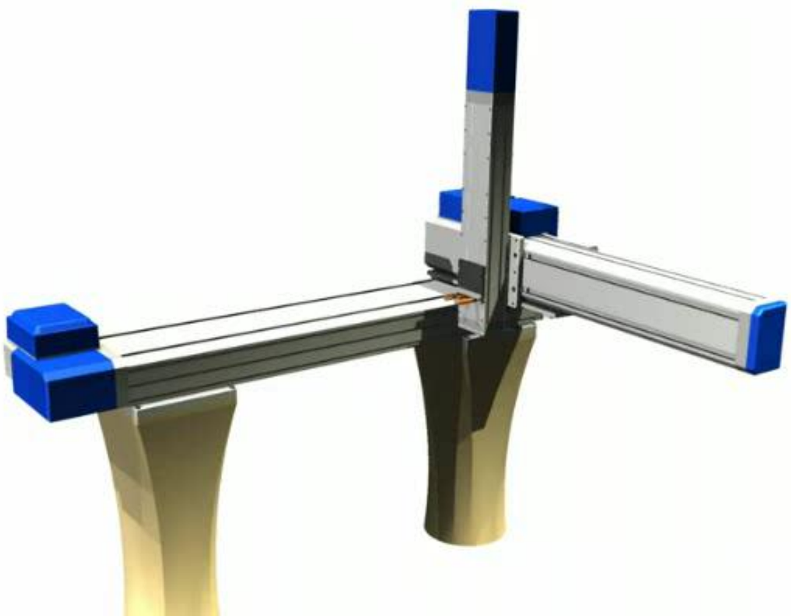
Cylindrical Robot Characteristics

Robot	Joints	Observations
Cylindrical	1a. Rotation: θ 2a. Linear: Z 3a. Linear: ρ	**Advantages:** • simple kinematical model • easy to display • good accessibility to cavities and open machines • large forces when using hydraulic actuators **Drawbacks:** • restricted working volume • requires guides protection (linear)

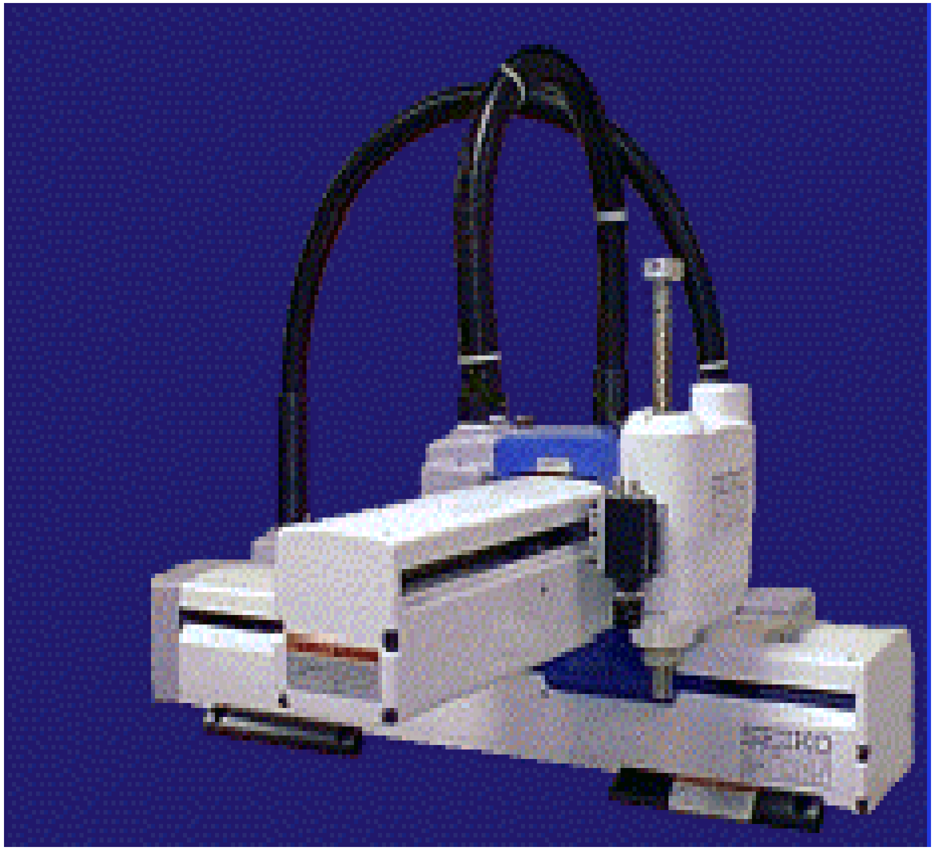
Cartesian Coordinate Body-and-Arm As

- Notation PPP:
- Consists of three sliding joints





Classical Architectures



Example of a Cartesian Work Space Robot

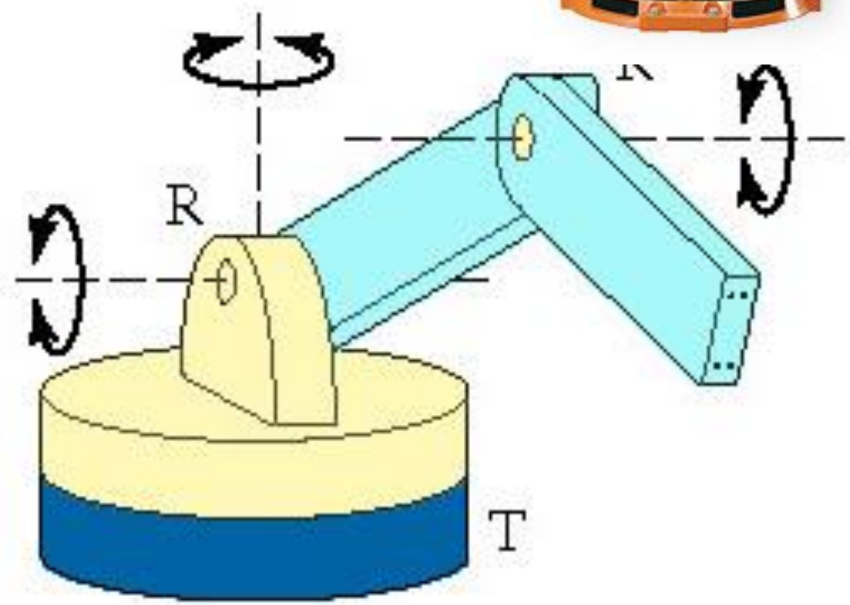
Cartesian Robot Characteristics

Robot	Joints	Observations
Cartesian	1a. Linear: X 2a. Linear: Y 3a. Linear: Z	**Advantages:** • linear movement in three dimensions • simple kinematical model • rigid structure • easy to display • possibility of using pneumatic actuators, which are cheap, in *pick&place* operations • constant resolution **Drawbacks:** • requires a large working volume • the working volume is smaller than the robot volume (crane structure) • requires free area between the robot and the object to manipulate • guides protection

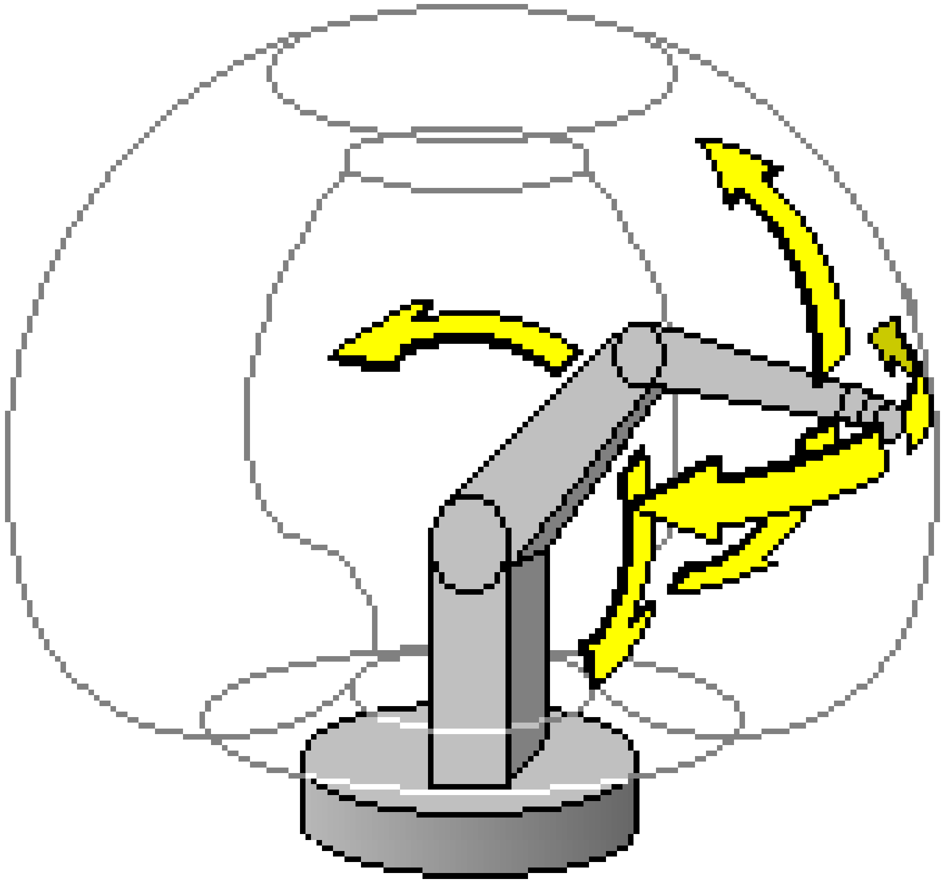
Jointed-Arm Robot

- Notation RRR:

- General configuration of a human arm



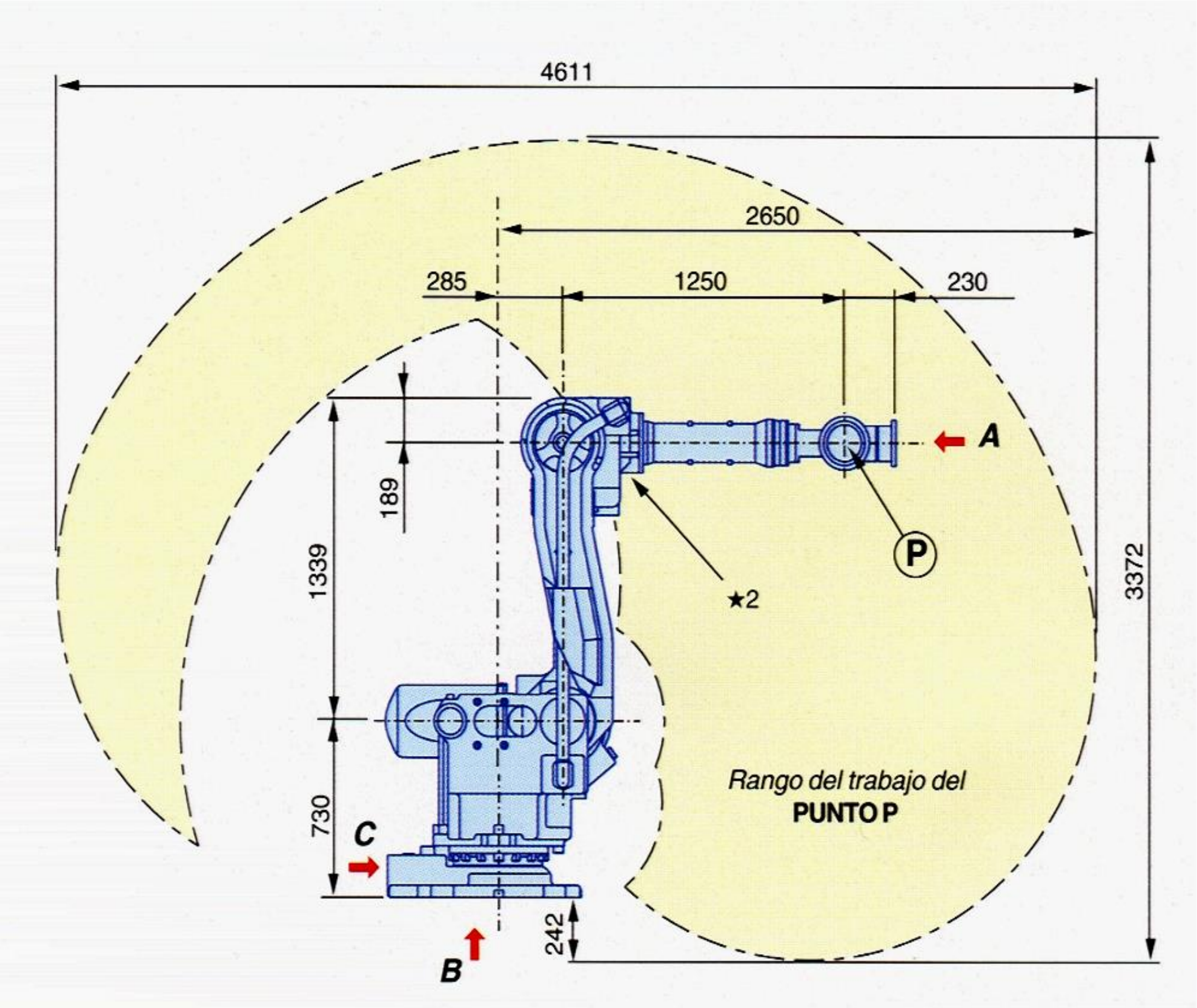
Classical Architectures



Angular Work Space

Classical Architectures

Angular Work Space (R+R+R)

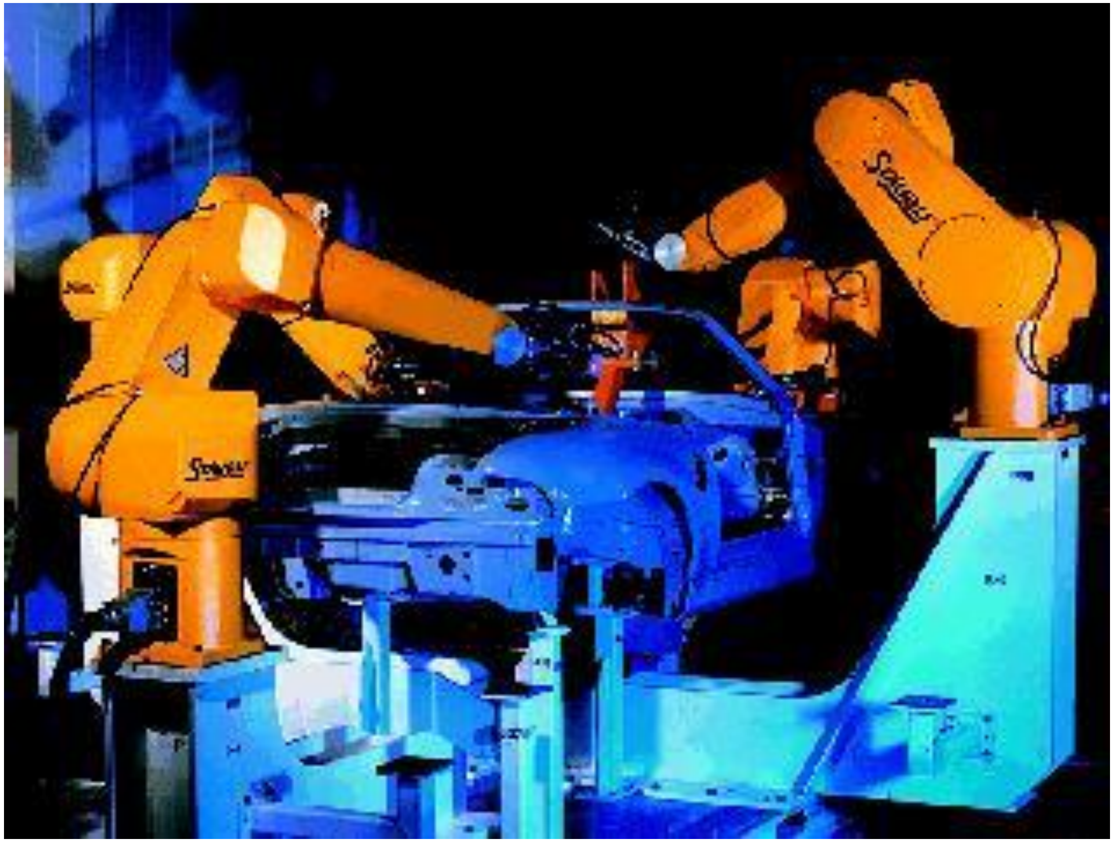


Working space of a robot with angular joints

Angular Robot Characteristics

Robot	Joints	Observations
Angular	1a. rotation θ_1 2a. rotation θ_2 3a. rotation θ_3	**Advantages:** • maximum flexibility • large working volume with respect to the robot size joints easy to protect (angular) • can reach the upper and lower side of an object **Drawbacks:** • complex kinematical model • linear movements are difficult • no rigid structure when stretched

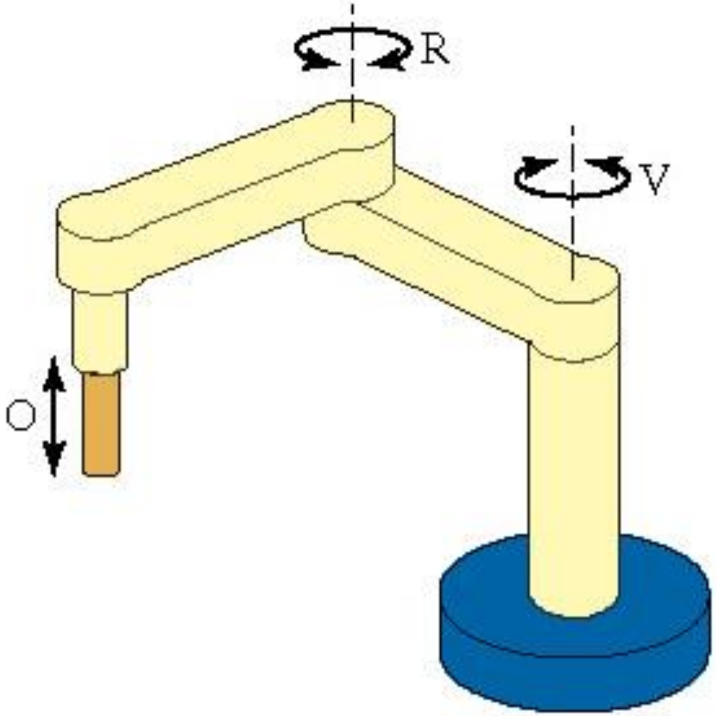
Classical Architectures

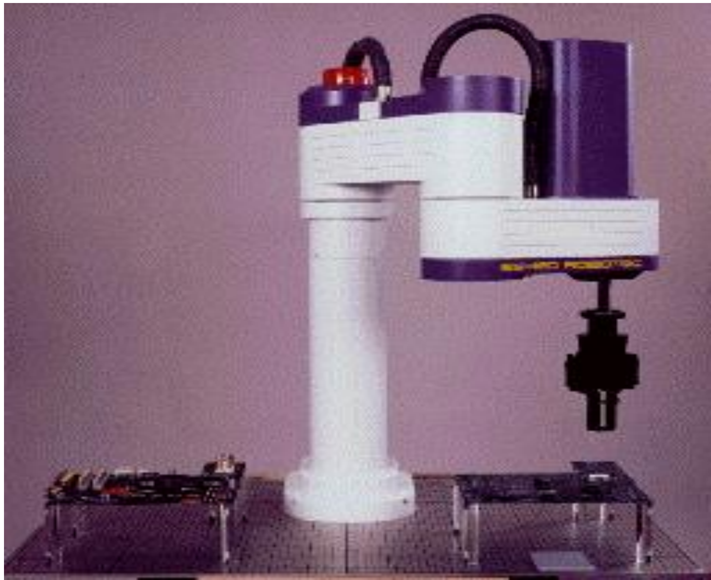


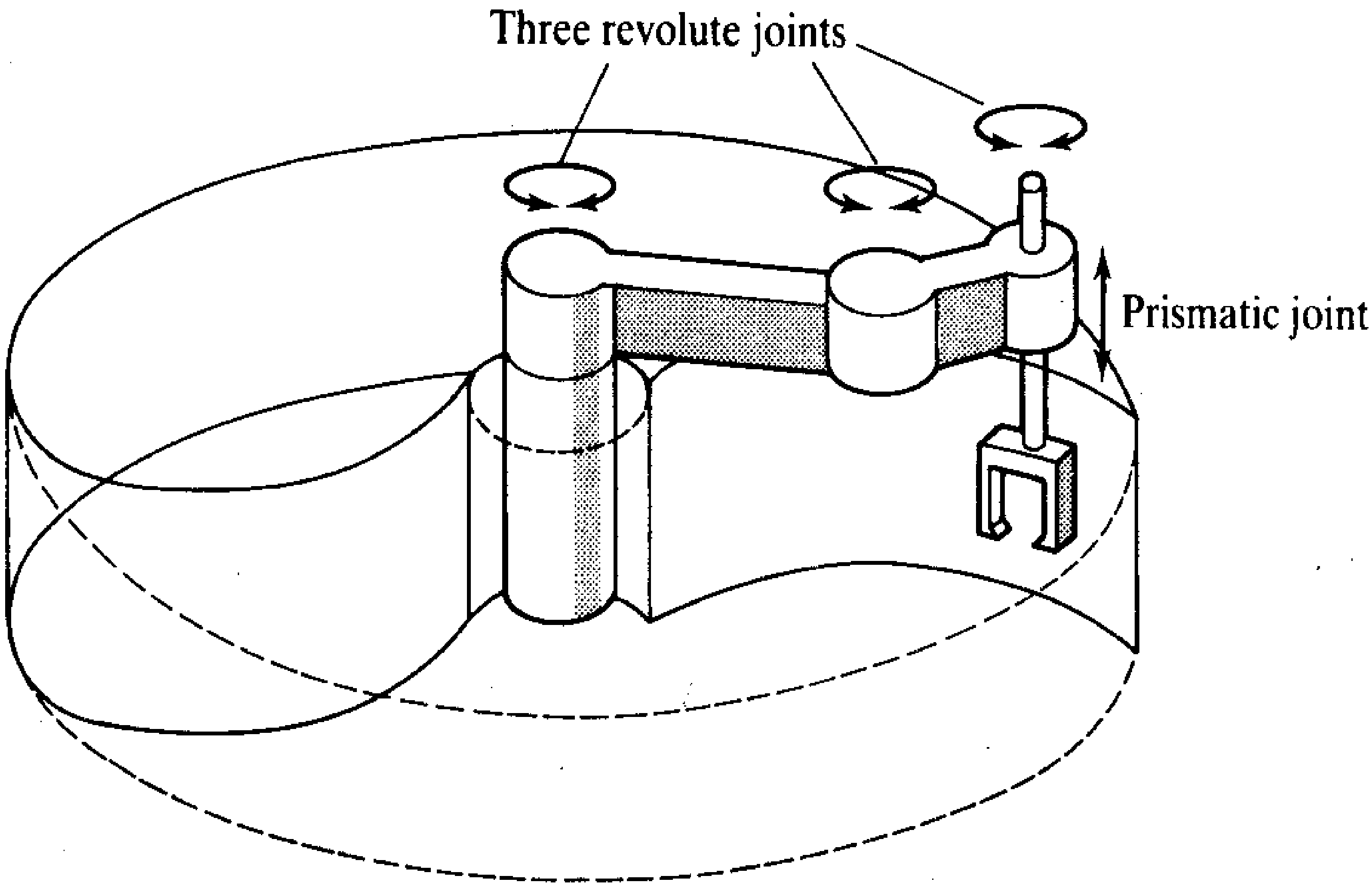
Example of Angular Work Space Robots (R+R+R)

SCARA Robot

- Notation RRP
- SCARA stands for Selective Compliance Assembly Robot Arm
- Similar to jointed-arm robot except that vertical axes are used for shoulder and elbow joints to be compliant in horizontal



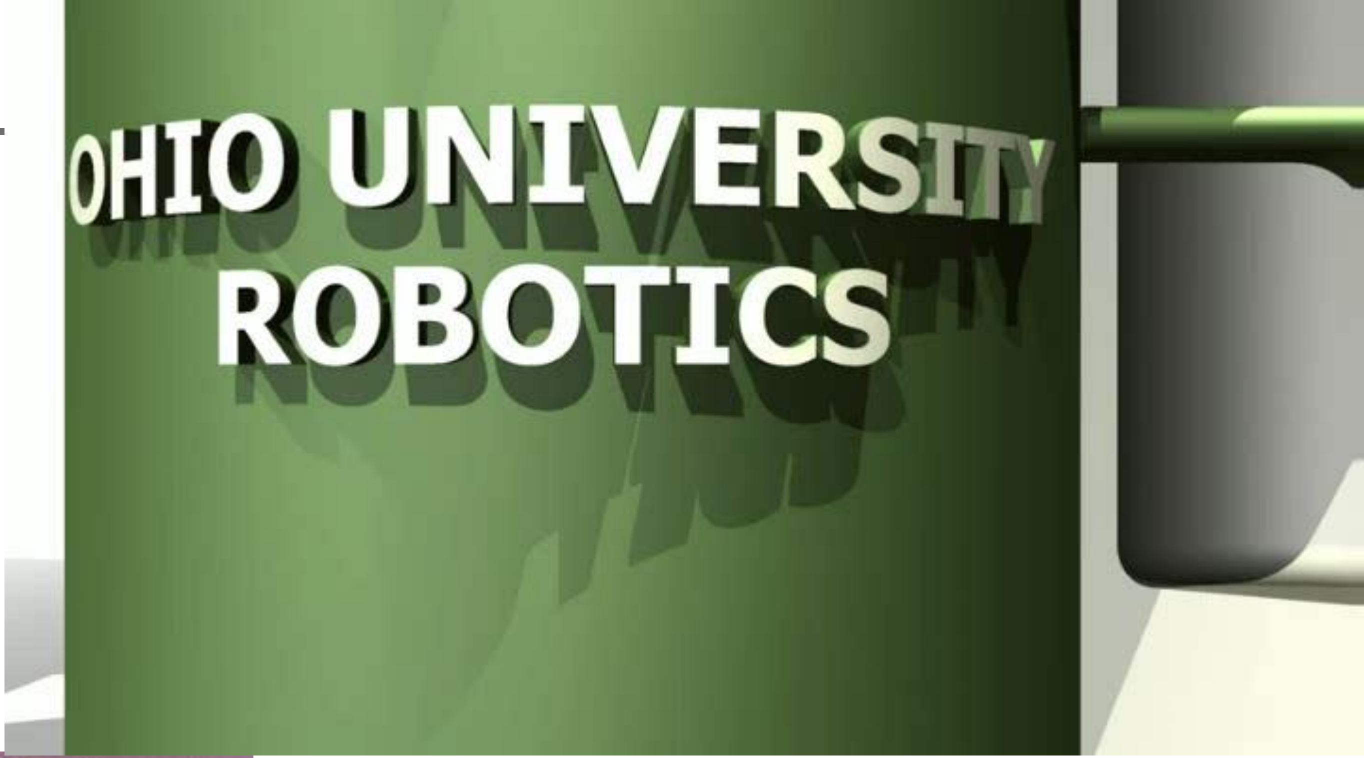


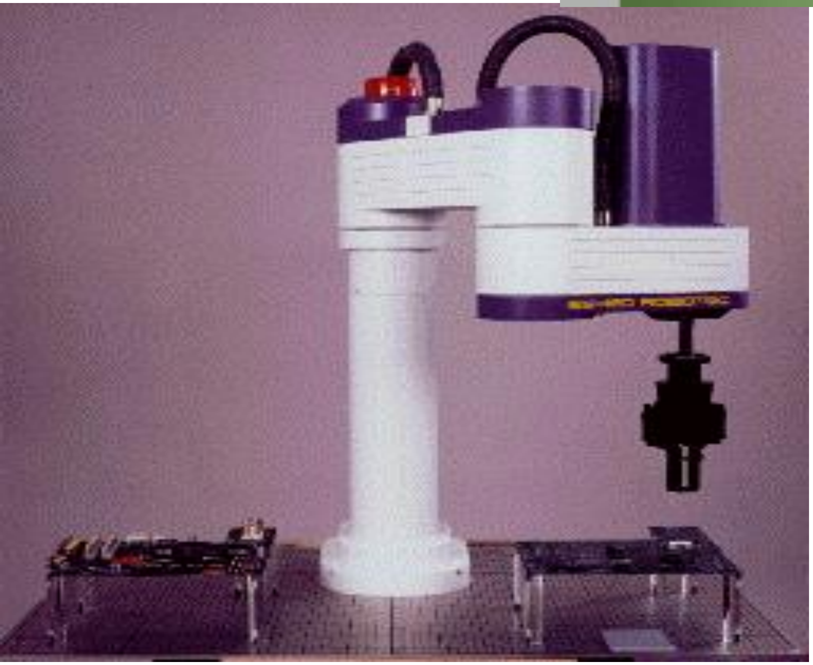


Architecture "SCARA"

Architecture R-R-P with cylindrical coordinates

(SCARA: Selective Compliance Assembly Robotic Arm)





SCARA Robot :

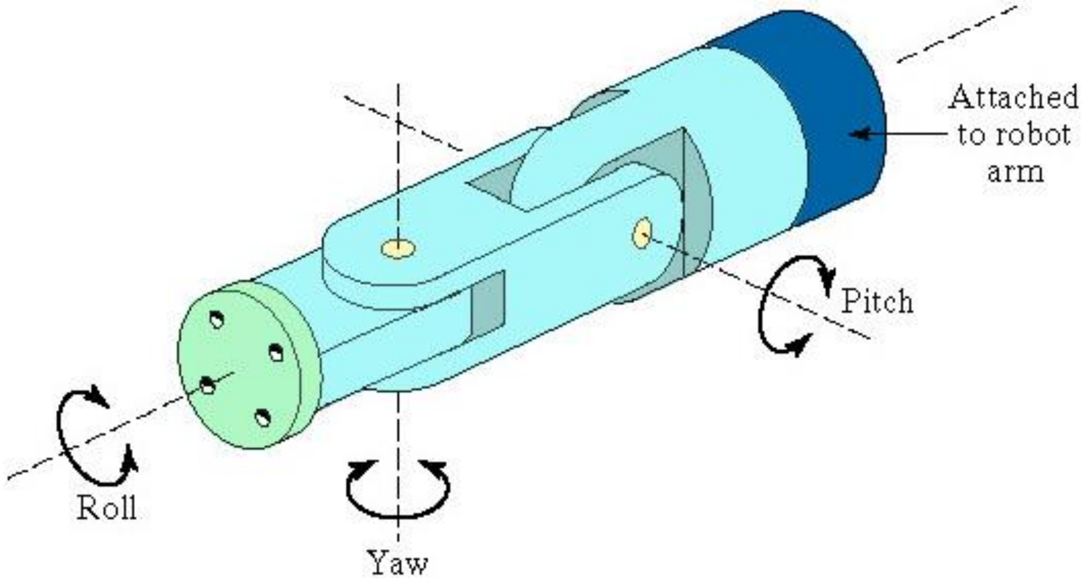
SCARA Robot Characteristics

Robot	Joints			Observations
SCARA	1a.	rotation	θ_1	**Advantages:**
	2a.	rotation	θ_2	• high speed and precision
	3a.	rotation	θ_3	
				Drawbacks:
				• only vertical axes

Wrist Configurations

- Wrist assembly is attached to end-of-arm
- End effector is attached to wrist assembly
- Function of wrist assembly is to orient end effector
  - Body-and-arm determines global position of end effector
- Two or three degrees of freedom:

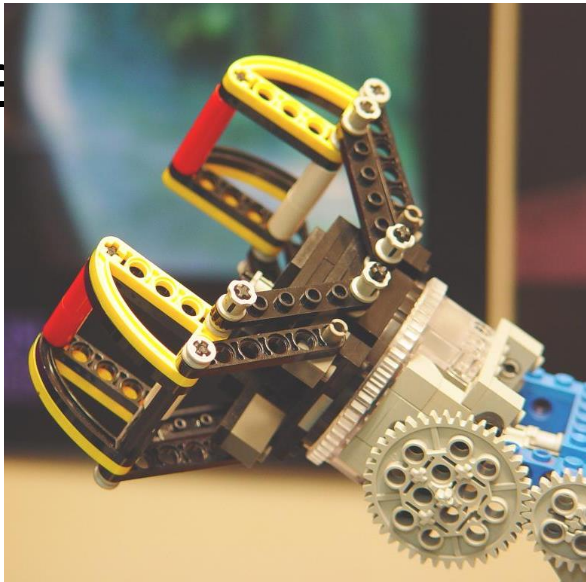
Wrist Configuration



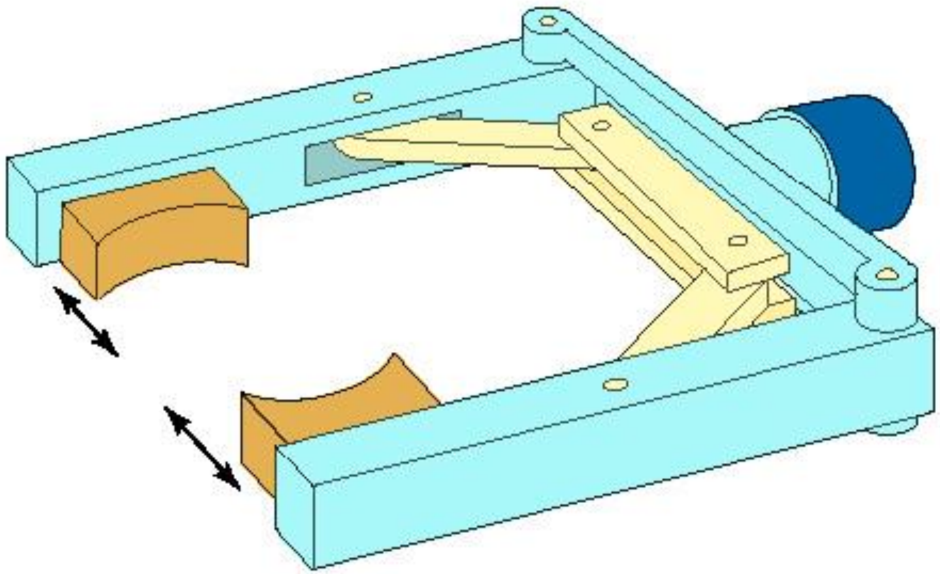
- Typical wrist assembly has two or three

End Effectors

- The special tooling for a robot that enables it to perform a specific task
- Two types:
  - Grippers – to grasp and manipulate objects (e.g., parts) during work cycle
  - Tools – to perform a process, e
    welding, spray painting

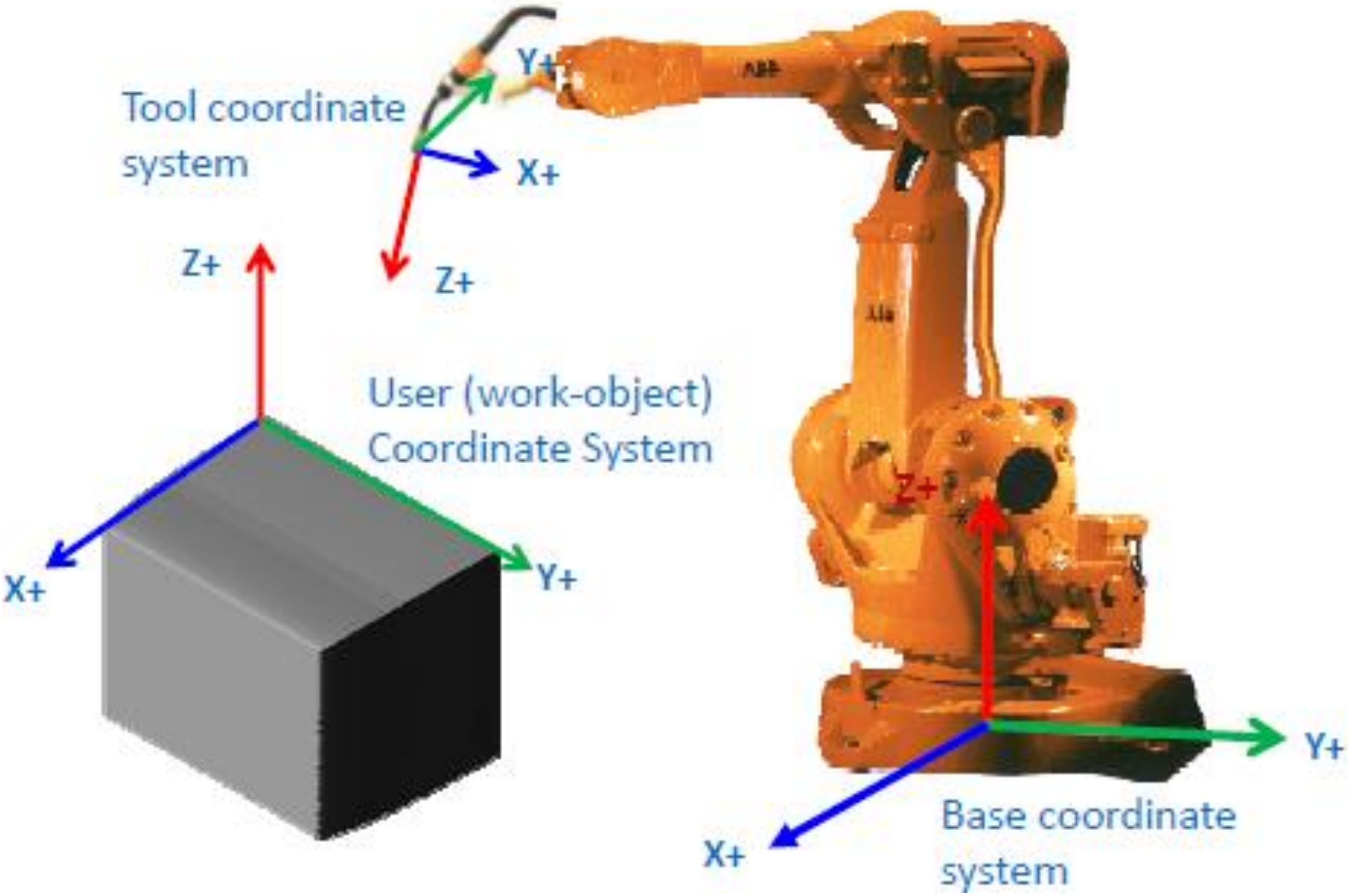


Robot Mechanical Gripper

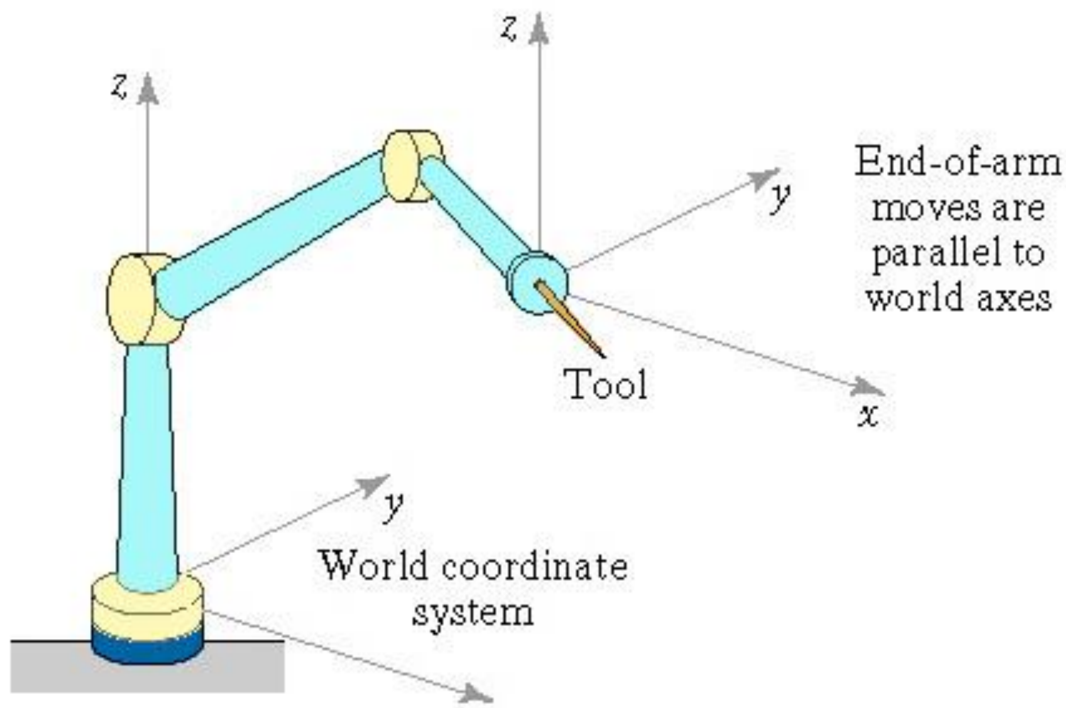


- A two-finger mechanical gripper for

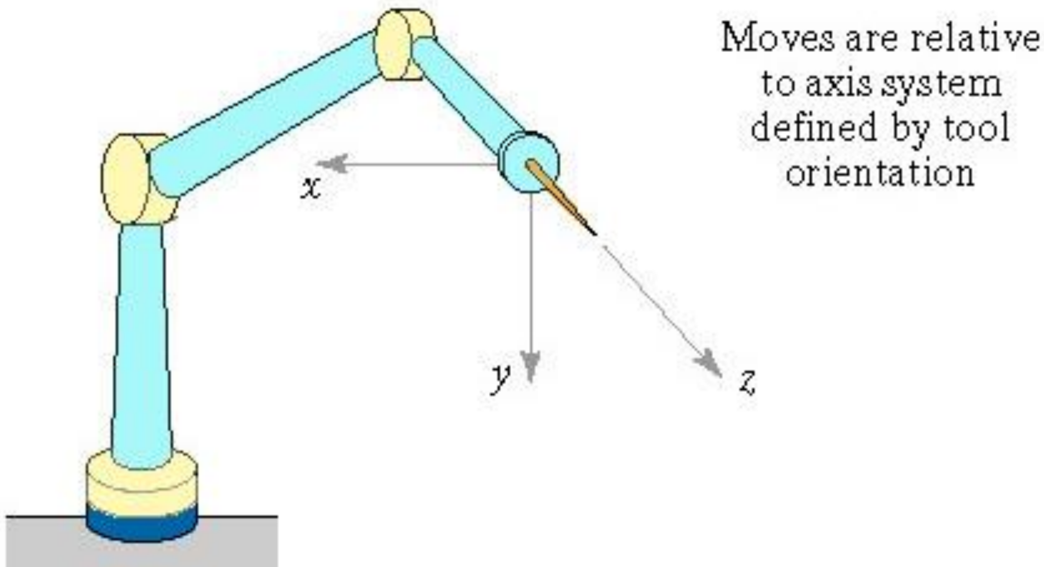
Coordinate Systems of a Robot



World Coordinate System



Tool Coordinate System



- Alignment of the axis system is defined relative to the orientation of the wrist faceplate (to which the end effector is attached)

Links

- A link is a solid mechanical structure connecting two joints
- Link Parameters

  Length, twist angle, joint angle, distance
- Joint Variable

  Link parameter that is variable


End Effectors
• Machine tools
– Grinding, sanding
– Inserting screws
– Drilling
– Hammering
• Paint sprayer
• Gripper, clamp
• Multi-digit hand

Forward Kinematics -Arm Equation

Contents:

Kinematic Parameters
- Joint Parameters
- Link Parameters

Assignment of coordinate frames

Normal, Sliding and Approach Vectors

Denavit-Hartenberg (DH) representation

The arm matrix
Arm equation

Examples.

Direct Kinematics

- In order to manipulate objects in space, it is required to control both the position and orientation of the tool/end effector in three- dimensional space.
- A relationship between the joint variables and the position and orientation of the tool is to be formulated.

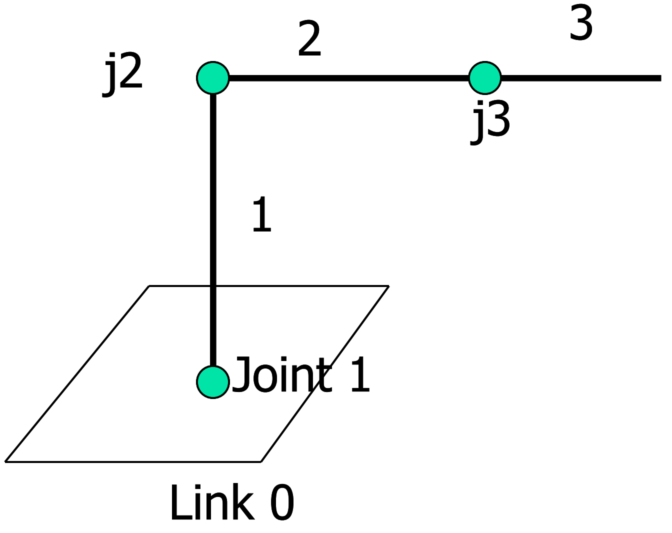
Direct Kinematics Problem:

Given the vector of joint variables of a robotic manipulator, determine the position and orientation of the tool with respect to a co-ordinate frame attached to the robot base.

It is necessary to have a concise formulation of a general solution to the direct kinematics problem.

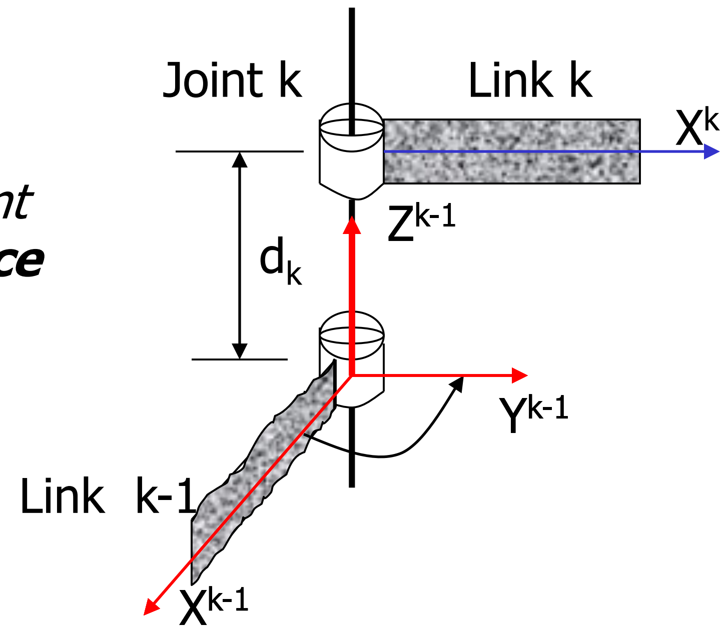
Links and Joints- Numbering

- By convention, the joints and links of a robotic arm are numbered outward starting with the fixed base, which is 0. The last link is the tool or end-effector.
- For an n-axis robot, there are n+1 links interconnected by n joints
- Joint k connects link k-1 to link k



Joint Parameters

The relative position and orientation of two successive links can be specified by two *joint parameters*, **joint angle** *and* ***joint distance***



Joint k connects link k-1 to link k. The parameters associated with joint k are defined w.r.t z^{k-1}, which is aligned with the axis of joint k.

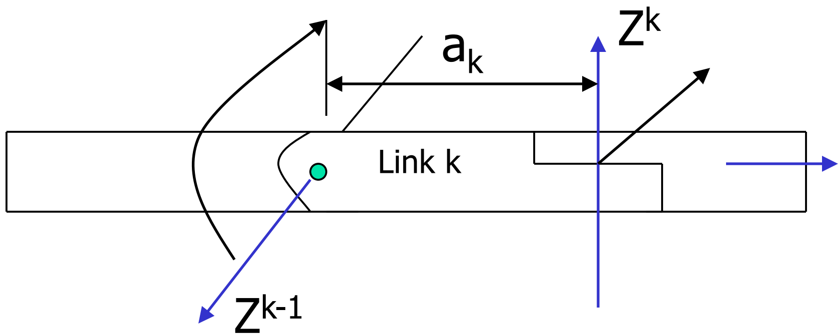
The joint angle, θ_k , is the rotation about z^{k-1} needed to make axis x^{k-1} parallel with axis x^k.

Joint distance d_k, is the translation along z^{k-1} needed to make axis x^{k-1} intersect with axis x^k.

Thus joint angle is a rotation about axis of joint k, while joint distance is a translation along joint axis

For each joint, it will always be the case that one of these parameters is fixed.

Link Parameters



The relative position and orientation of the axes of two successive joints can be specified by two link *parameters,* ***link length*** *and* ***link twist angle***

Link k connects joint k to joint k+1. The parameters associated with link k are defined w.r.t x^k, which is a common normal between the axes of joint k and k+1.

Link length, a_k, is the translation along x^k needed to make axis z^{k-1} intersect z^k

Twist angle, α_k, is the rotation about x^k needed to make axis z^{k-1} parallel with axis z^k

The two link parameters are always constant and are specified as part of the mechanical design.

Kinematic Parameters……

Arm Parameter	Symbol	Revolute Joint (R)	Prismatic Joint (P)
Joint Angle	θ	Variable	Fixed
Joint Distance	d	Fixed	Variable
Link Length	a	Fixed	Fixed
Link Twist angle	α	Fixed	Fixed

For an n-axis robot manipulator, the 4n kinematic parameters constitute the *minimal set needed to specify the kinematic configuration of the robot.*

Normal, Sliding and Approach Vectors

- The orientation of a tool can be represented in *Joint coordinates* by YPR convention.
- In rectangular or Cartesian coordinates the same can be represented by a rotation matrix R= [$\mathbf{r}^1$, $\mathbf{r}^2$, $\mathbf{r}^3$] where the three columns of R correspond to the normal, sliding and approach vectors resply.

Approach vector is aligned with the roll axis and points away from wrist. Consequently, it represents the direction in which the tool is pointing

Sliding vector is orthogonal to the approach vector and aligned with the open-close axis of the tool.

Yaw, Pitch and Roll motions are rotations about normal, sliding and approach vectors