Manipulator Kinematics

Lecture 2: Forward Kinematics

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Direct (Forward) Kinematics

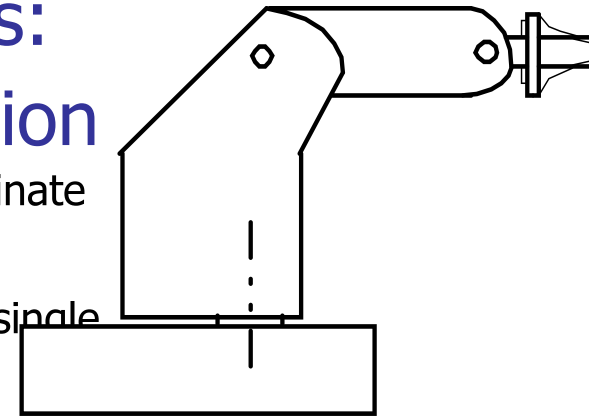
- In order to manipulate objects in space, it is required to control both the position and orientation of the tool/end effector in three- dimensional space.
- A relationship between the joint variables and the position and orientation of the tool is to be formulated.

Direct Kinematics Problem:

Given the vector of joint variables of a robotic manipulator, determine the position and orientation of the tool with respect to a co-ordinate frame attached to the robot base.

It is necessary to have a concise formulation of a general solution to the direct kinematics problem.

Assignment of Coordinate frames: Denavit-Hartenberg Representation

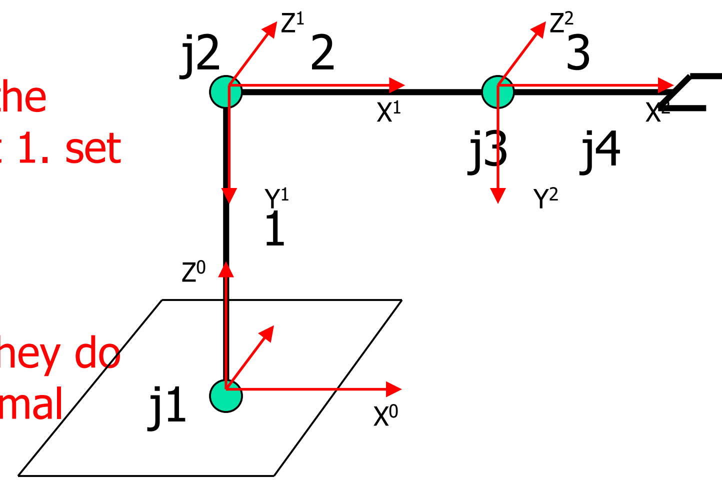


A systematic notation for assigning right-handed orthonormal coordinate frame.

Transformations between adjacent frames can be represented by a single standard 4x4 homogeneous coordinate transformation.

DH Algorithm

1. Number the joints from 1 to n starting with the base and ending with the tool yaw, pitch and roll in that order.
2. Assign a right-handed orthonormal coordinate frame L_0 to the robot base, making sure that Z^0 aligns with the axis of joint 1. set k=1
3. Align Z^k with the axis of joint k+1
4. Locate origin of L_k at the intersection of the Z^k and Z^{k-1}. If they do not intersect, use the intersection of Z^k with a common normal between Z^k and Z^{k-1}.
5. Select X^k to be orthogonal to both Z^k and Z^{k-1}. If Z^k and Z^{k-1} are parallel, point X^k away from Z^{k-1}.
6. Select y^k to form a right handed coordinate frame L_k.
7. Set k=k+1, If k<n, go to step 2; else continue



DH

- 8. Set the origin of L_n at the tool tip. Align Z^n with the approach vector, y^n with the sliding vector, and x^n with the normal vector of the tool. Set k=1

- 9. Locate point b_k at the intersection of x^k and z^{k-1} axes. If they do not intersect, use the intersection of x^k with a common normal between x^k and z^{k-1}.

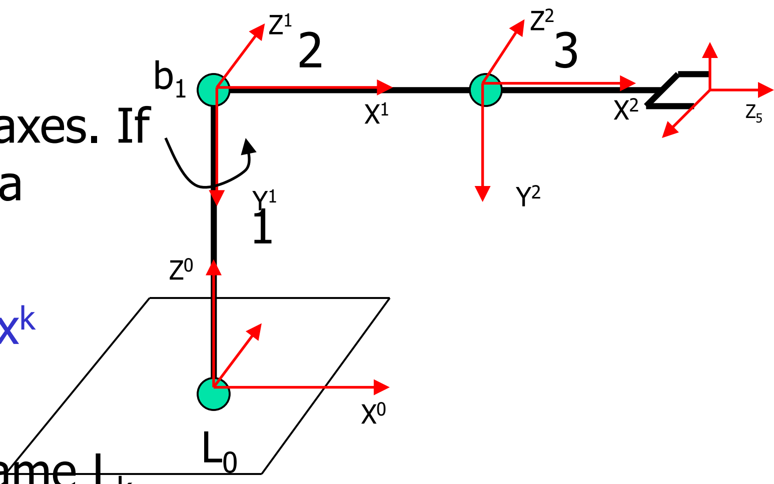
- 10 Compute θ_k as the angle of rotation from x^{k-1} to x^k measured about z^{k-1}

- 11 Compute d_k as the distance from the origin of frame L_{k-1} to point b_k measured along Z^{k-1}.

- 12 Compute a_k as the distance from point b_k to the origin of frame L_k measured along x^k.

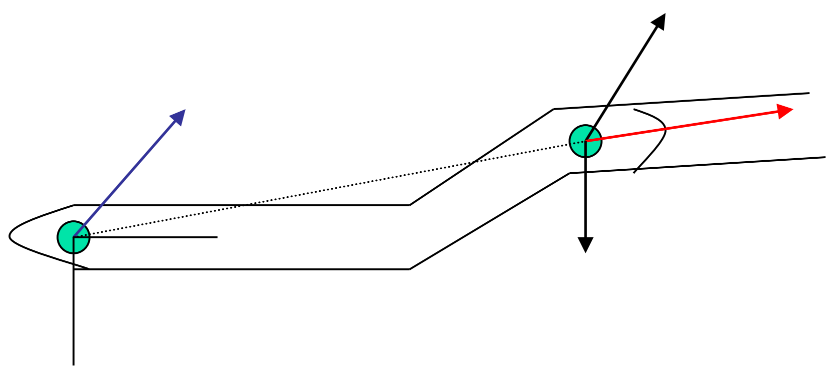
- 13 Compute α_k as the angle of rotation from Z^{k-1} to Z^k measured about x^k

- 14 set k=k+1. If k≤n, go to step 9; else stop



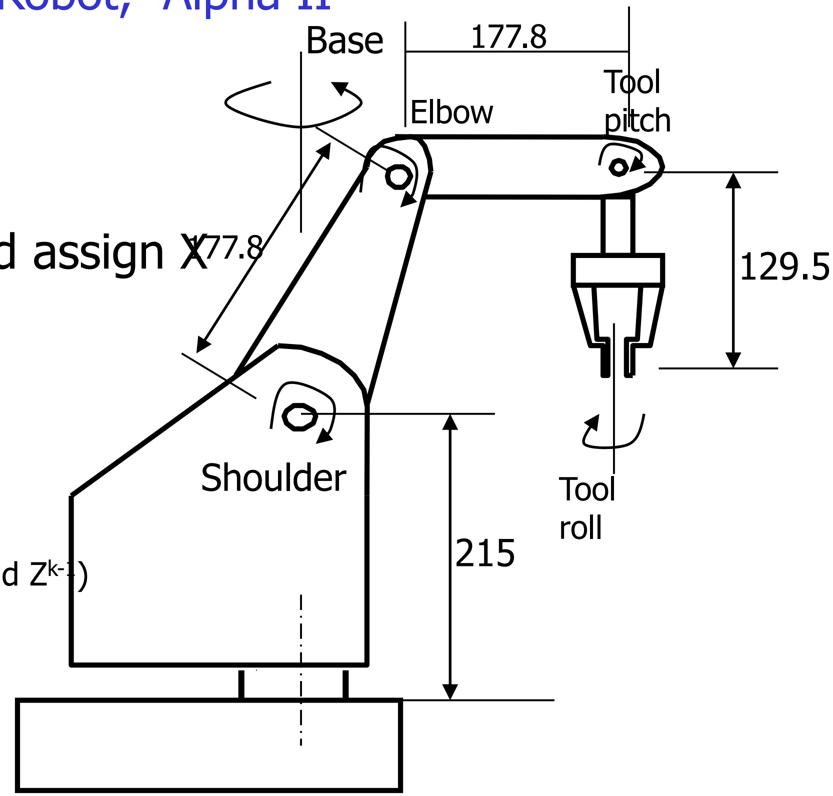
Notes

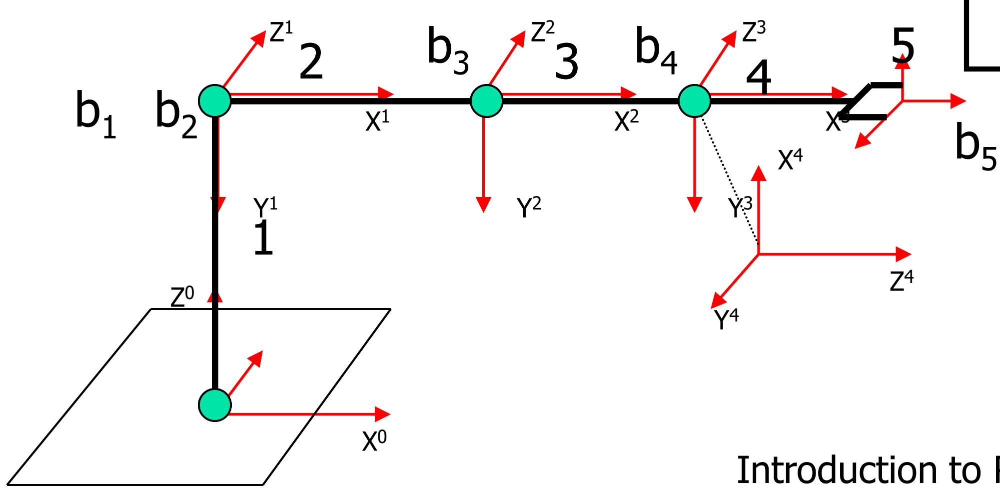
- In step 9, axis X^k should always intersect with axis Z^{k-1} when $k<n$
- DH algorithm is not unique; the directions of any of the Z axes could be reversed.



Example: Assign coordinate frames and determine kinematic parameters for a 5 axis articulated Robot, Alpha II

- Number the links and joints
- Base coordinate frame L_0.
- For K=1, align Z axis, locate origin and assign X and Y.
- For K=K+1, K<n, repeat above step
- Assign co-ordinate frame at tool tip
- For k=1 to n, Locate b_k. (intersection of x^k and Z^{k-1})
- Get Link and Joint Parameters

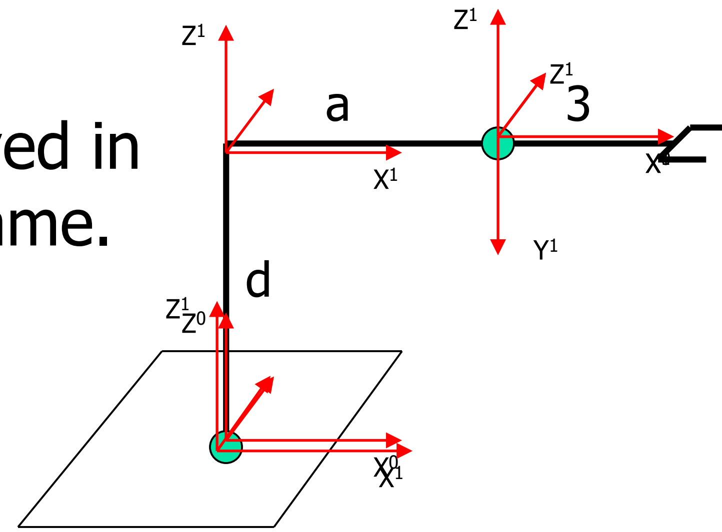




Axis	θ	d	a	α
1	Θ_1	215	0	-90
2	Θ_2	0	177.8	0
3	Θ_3	0	177.8	0
4	Θ_4	0	0	-90
5	Θ_5	129.5	0	0

Arm Matrix

- A homogeneous matrix that maps frame *k* coordinates to *k-1* coordinates



- Four fundamental operations are involved in making k-1 frame coincident with k frame.
  - Rotate L_{k-1} about z^{k-1} by θ_κ.
  - Translate L_{k-1} along z^{k-1} by d_κ.
  - Translate L_{k-1} along x^{k-1} by a_κ.
  - Rotate L_{k-1} about x^{k-1} by α_κ.

T = R (θ, z) *Trans* (d,z) *Trans* (a, x) R(α, x)

$$T_{k-1}^{k}(\theta_k, d_k, a_k, \alpha_k) = R(\theta_k, 3)Tran(d_k, 3)Tran(a_k, 1)R(\alpha_k, 1)$$

$$T - \text{Homogeneous Transformation matrix } T_{k-1:\textbf{Destination frame}}^{k:\textbf{source frame}}$$

DH Matrix

Link Coordinate Transformation

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

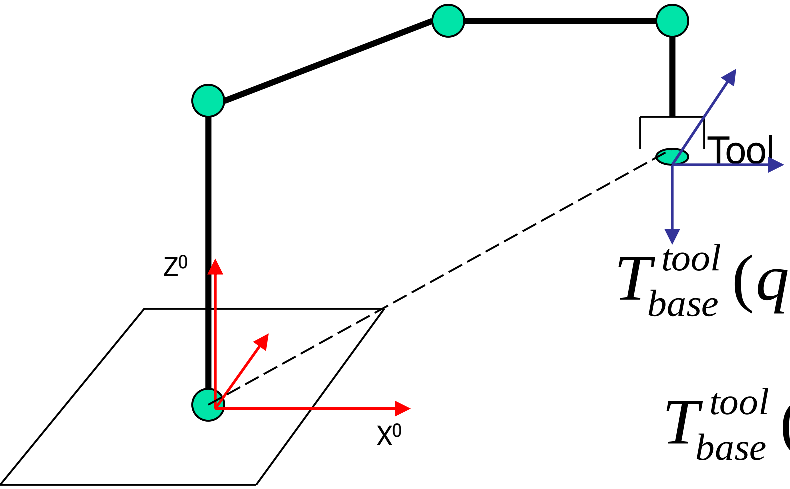
Inverse Link Coordinate Transformation

$$T_{k}^{k-1} = \begin{bmatrix} C\theta_k & S\theta_k & 0 & -a_k \\ -C\alpha_k S\theta_k & C\alpha_k C\theta_k & S\alpha_k & -d_k S\alpha_k \\ S\alpha_k S\theta_k & -S\alpha_k C\theta_k & C\alpha_k & -d_k C\alpha_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct (Forward) Kinematics Problem

Given the values of joint variables q_1, …, q_n, solve for the end-effector location (i.e., position and orientation) in the Cartesian space of the robot **base frame** $T_0^n(q_1, \ldots, q_n)$.

$$T_0^n(q) = T_0^1(q_1)T_1^2(q_2)T_2^3(q_3)\ldots\ldots T_{n-1}^n(q)$$



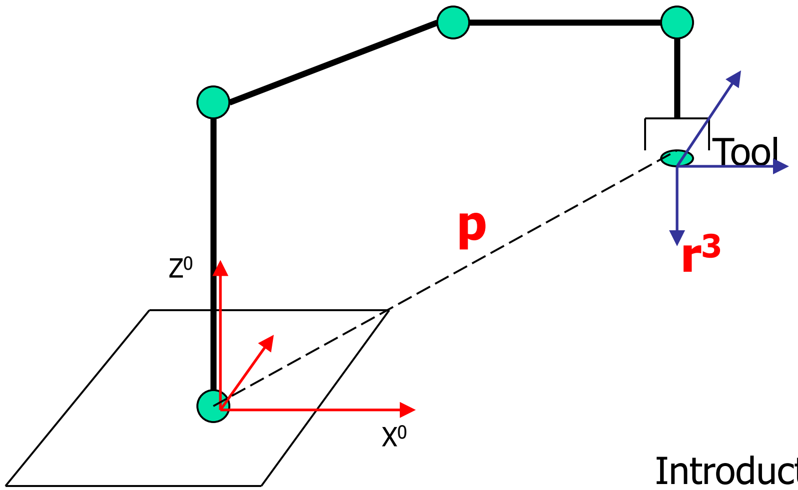
$$T_{base}^{tool}(q) = T_{base}^1(q_1)T_1^2(q_2)T_2^3(q_3)\ldots\ldots T_{n-1}^{tool}(q)$$

$$T_{base}^{tool}(q) = T_{base}^{wrist}(q_1, q_2, q_3)T_{wrist}^{tool}(q_4, q_5, q_6)$$

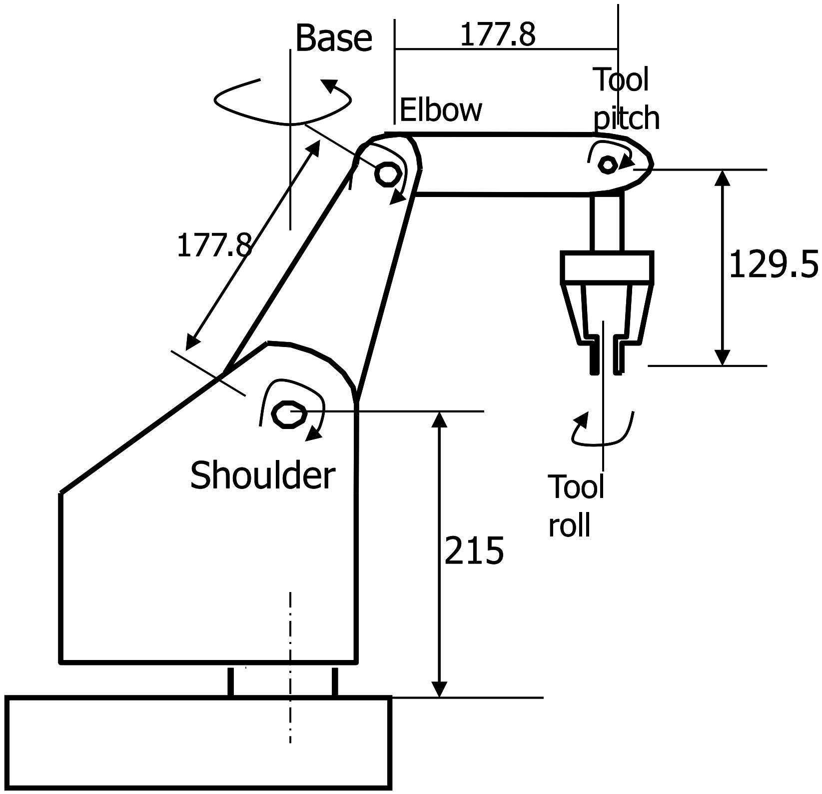
Arm Equation

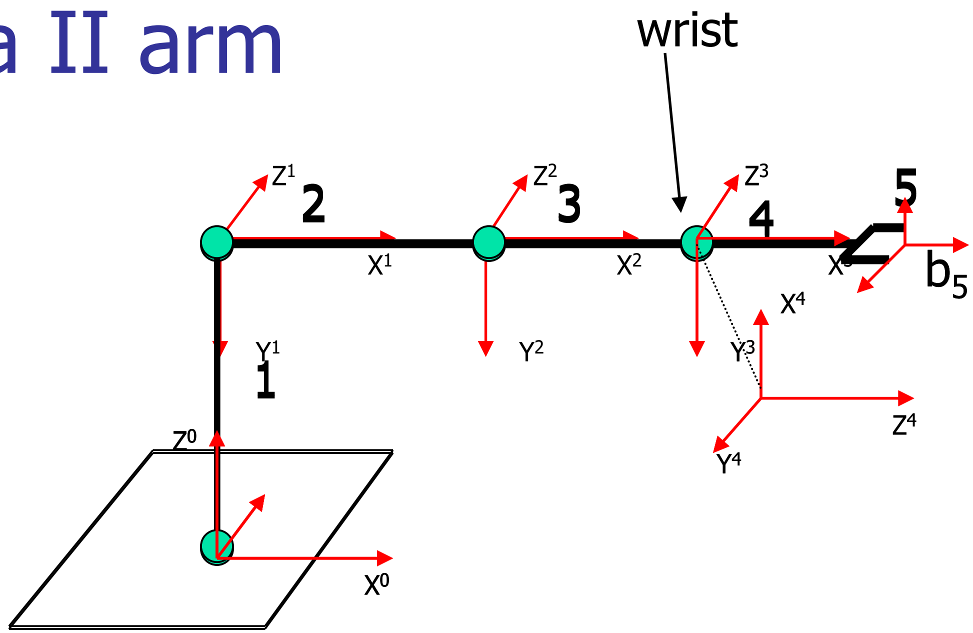
$$T_{base}^{tool}(q) = \left[\begin{array}{ccc|c} & R(q) & & p(q) \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

The 3x3 submatrix R(q) represents the tool orientation, 3x1 submatrix p(q) represents position of the tool. The three columns of R represents the direction of unit vectors of the tool frame wrt base frame.



Example- Alpha II arm





Axis	θ	d	a	α
1	Θ_1	215	0	-90
2	Θ_2	0	177.8	0
3	Θ_3	0	177.8	0
4	Θ_4	0	0	-90
5	Θ_5	129.5	0	0

$$T_{base}^{tool} = T_{base}^{wrist} T_{wrist}^{tool}$$

$$T_{base}^{wrist} = T_0^1 T_1^2 T_2^3; \qquad T_{wrist}^{tool} = T_3^4 T_4^5$$

Axis	θ	d	a	α
1	Θ_1	215	0	-90
2	Θ_2	0	177.8	0
3	Θ_3	0	177.8	0
4	Θ_4	0	0	-90
5	Θ_5	129.5	0	0

$$T_0^1 = \begin{bmatrix} C\theta_1 & -C\alpha_1 S\theta_1 & S\alpha_1 S\theta_1 & a_1 C\theta_1 \\ S\theta_1 & C\alpha_1 C\theta_1 & -S\alpha_1 C\theta_1 & a_1 S\theta_1 \\ 0 & S\alpha_1 & C\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{wrist} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3 C_3 \\ S_3 & C_3 & 0 & a_3 S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{wrist} = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & -S_1 & C_1(a_2 C_2 + a_3 C_{23}) \\ S_1 C_{23} & -S_1 S_{23} & C_1 & S_1(a_2 C_2 + a_3 C_{23}) \\ -S_{23} & -C_{23} & 0 & d_1 - a_2 S_2 - a_3 S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

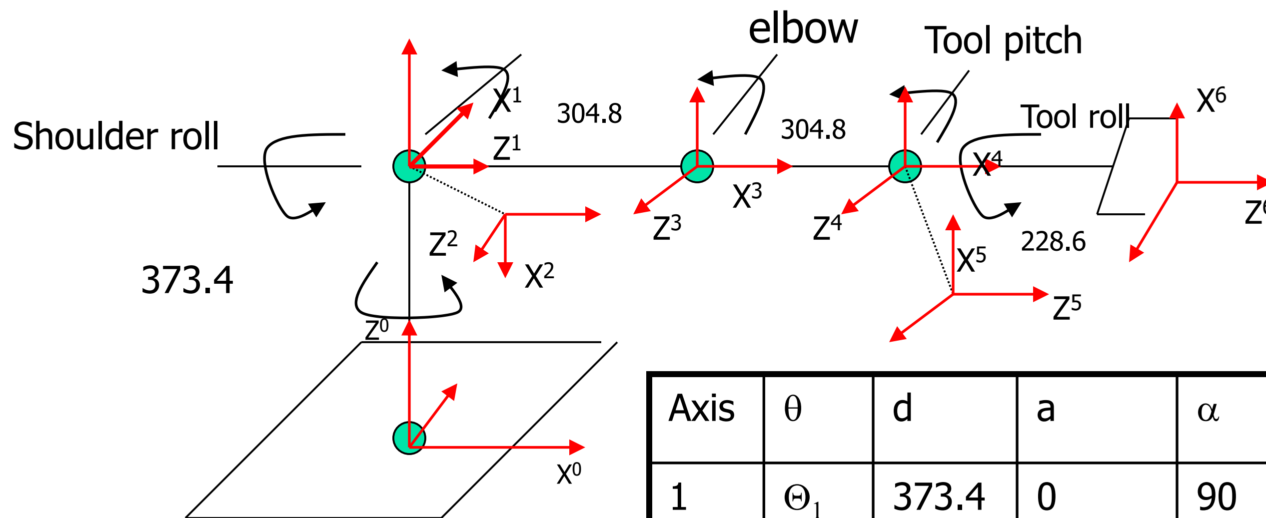
$$C_{23} = Cos(\theta_2 + \theta_3); S_{23} = Sin(\theta_2 + \theta_3)$$

$$T_{wrist}^{tool} = \begin{bmatrix} C_4 C_5 & -C_4 S_5 & -S_4 & -d_5 S_4 \\ S_4 C_5 & -S_4 S_5 & C_4 & d_5 C_4 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{tool} = \left[\begin{array}{ccc|c} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & -C_1S_{234} & C_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & -S_1S_{234} & S_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ -S_{234}C_5 & S_{234}S_5 & -C_{234} & 215 - 177.8S_2 - 177.8S_{23} - 129.5C_{234} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Forward Kinematics:
Example: Six-axis articulated Robot

Find the position and orientation of the tool at the soft home position shown below for the six-axis articulated arm, Intelledex 660T



Axis	θ	d	a	α
1	Θ_1	373.4	0	90
2	Θ_2	0	0	90
3	Θ_3	0	304.8	0
4	Θ_4	0	304.8	0
5	Θ_5	0	0	90
6	Θ_6	228.6	0	0

Step I: Assign Coordinate frame

Step II : Get DH parameters

Step III Arm matrix

At the soft home position Joint variables are [90, -90, 90, 0, 90,0]

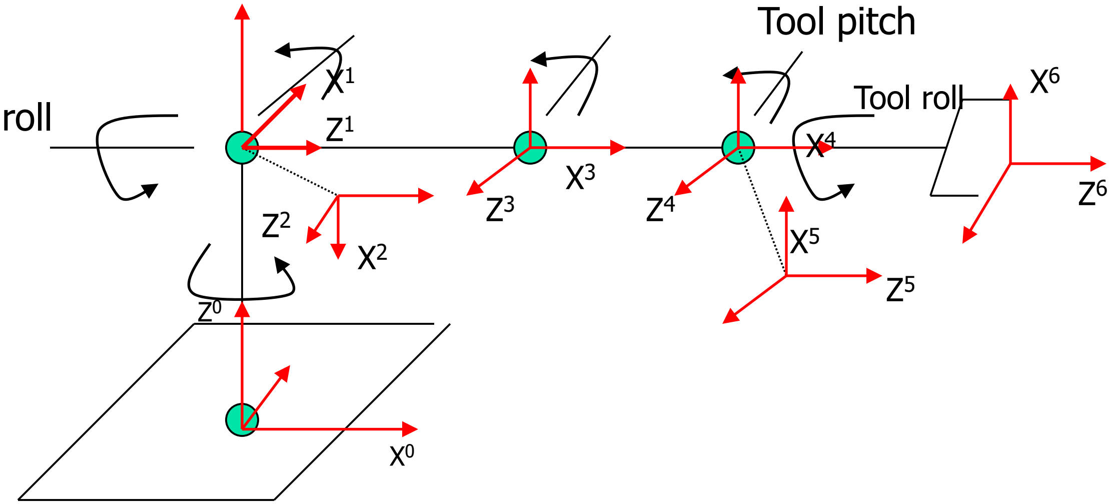
- We partition the arm matrix at third axis, elbow joint.

$$T_{base}^{elbow} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{elbow} = \begin{bmatrix} C_1C_2C_3 + S_1S_3 & -C_1C_2S_3 + S_1C_3 & C_1S_2 & (C_1C_2C_3 + S_1S_3)a_3 \\ S_1C_2C_3 - C_1S_3 & -S_1C_2S_3 - C_1C_3 & S_1S_2 & (S_1C_2C_3 - C_1S_3)a_3 \\ S_2C_3 & -S_2S_3 & -C_2 & d_1 + S_2C_3a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As a partial check of the expression, we can evaluate this at the soft home position where Joint variables are [90, -90, 90, 0, 90,0]

$$T_{base}^{Elbow}(\text{home}) = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_{elbow}^{tool} = \begin{bmatrix} C_{45}C_6 & -C_{45}S_6 & S_{45} & a_4C_4 + S_{45}d_6 \\ S_{45}C_6 & -S_{45}S_6 & -C_{45} & a_4S_4 - C_{45}d_6 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{base}^{tool} = T_{base}^{elbow} x T_{elbow}^{tool}$$

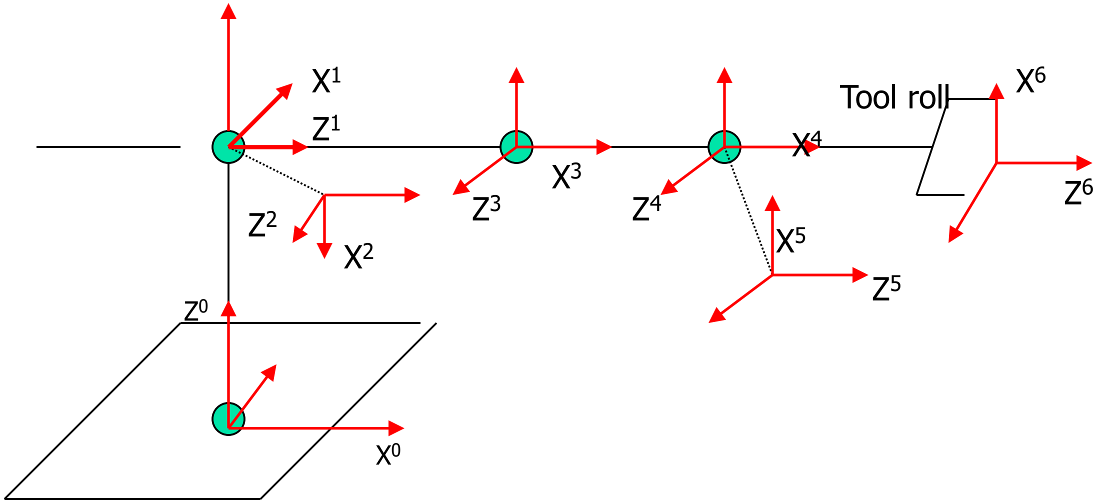
$$p(q) = \begin{bmatrix} C_1C_2(a_3C_3 + a_4C_{34} + d_6S_{345}) + S_1(a_3S_3 + a_4S_{34} - d_6C_{345}) \\ S_1C_2(a_3C_3 + a_4C_{34} + d_6S_{345}) - C_1(a_3S_3 + a_4S_{34} - d_6C_{345}) \\ d_1 + S_2(a_3C_3 + a_4C_{34} + d_6S_{345}) \end{bmatrix}$$

Position vector is independent of roll angle.

$$R = \begin{bmatrix} (C_1C_2C_{345} + S_1S_{345})C_6 + C_1S_2S_6 & C_1S_2C_6 - (C_1C_2C_{345} + S_1S_{345})S_6 & -S_1C_{345} + C_1C_2S_{345} \\ (S_1C_2C_{345} - S_1S_{345})C_6 + S_1S_2S_6 & S_1S_2C_6 - (S_1C_2C_{345} - C_1S_{345})S_6 & C_1C_{345} + S_1C_2S_{345} \\ S_2C_{345}C_6 - C_2S_6 & -C_2S_6 - S_2C_{345}S_6 & S_2S_{345} \\ & & \end{bmatrix}$$

Approach vector, r^3, is independent of roll angle.

As a partial check of the final expression, we can evaluate this at the soft home position where Joint variables are [90, -90, 90, 0, 90,0]



$$T_{base}^{tool}(\text{home}) = \left[\begin{array}{ccc|c} 0 & 0 & 1 & a_3 + a_4 + d_6 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Tutorial: 8th March
Rhino XR-3 Robot