STA 529 2.0 Data Mining

Dr Thiyanga S. Talagala

Classification and Regression Trees

Lecture 2

Classification and Regression Trees (CART)

Classification and Regression Trees (CART)

- Decision trees
- Supervised learning method
- Data driven method

Model

$$Y = f(X_1, X_2, ...X_n) + \epsilon$$

Goal: What is f?

How do we estimate f ?

Data-driven methods:

estimate f using observed data without making explicit assumptions about the functional form of f.

Parametric methods:

estimate f using observed data by making assumptions about the functional form of f.

Classification and Regression Trees

1. Classification tree - Outcome is categorical
2. Regression tree - Outcome is numeric

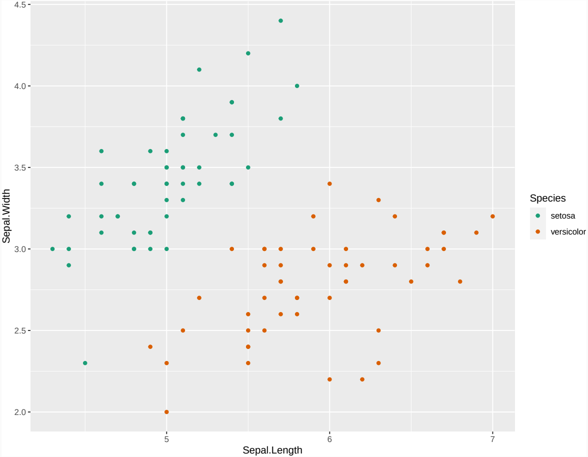
Classification and Regression Trees

- CART models work by partitioning the feature space into a number of simple rectangular regions, divided up by axis parallel **splits**.
- The **splits** are logical rules that split feature-space into two **non-overlapping** subregions.

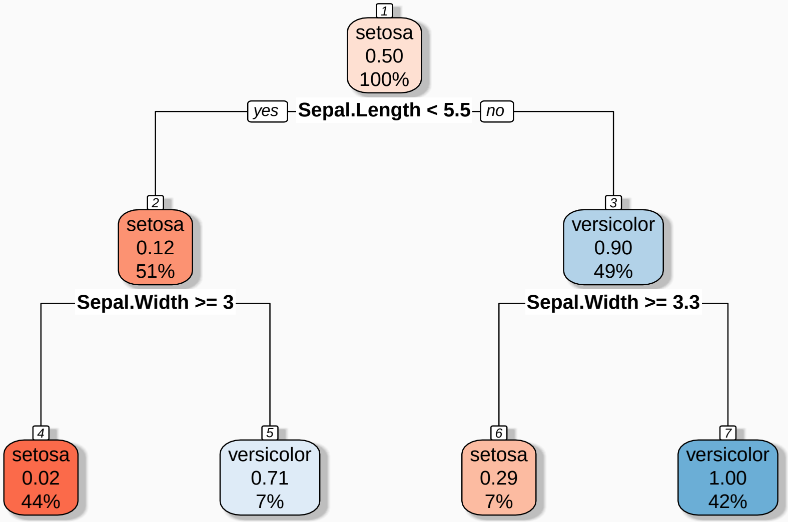
Example: Feature space

Features: Sepal Length, Sepal Width

Outcome: setosa/versicolor



Decision tree



Parts of a decision tree

- Root node
- Decision node
- Terminal node/ Leaf node (gives outputs/class assignments)
- Subtree

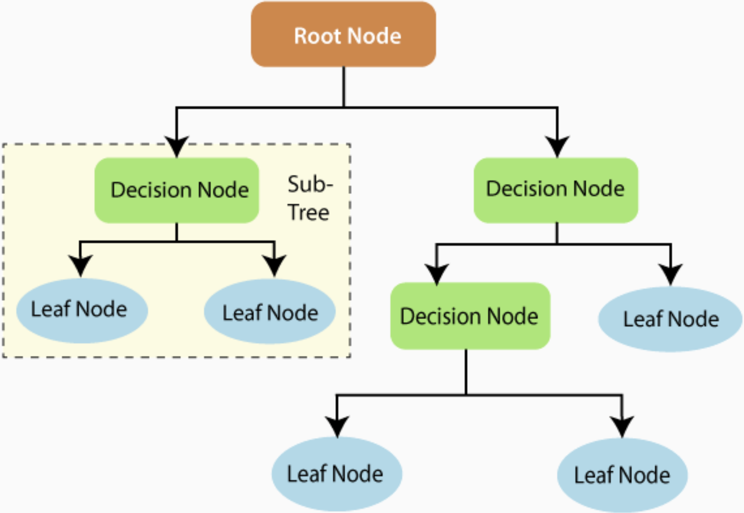
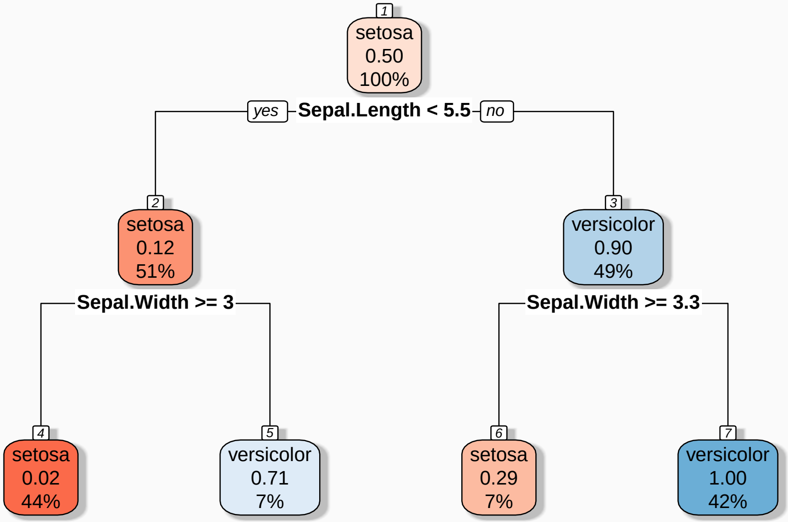
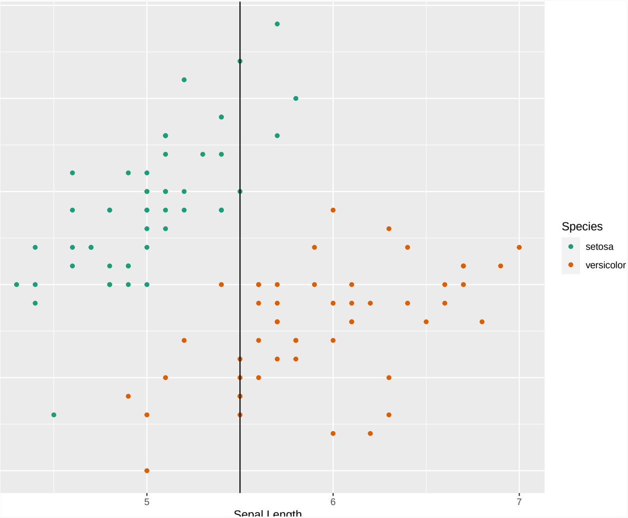


Image source:
`https://www.tutorialandexample.com/wp-content/uploads/2019/10/Decision-Trees-Root-Node.png`

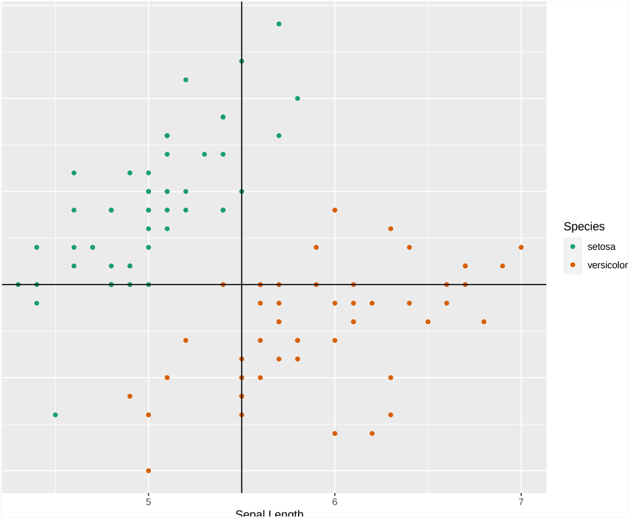
Decision tree



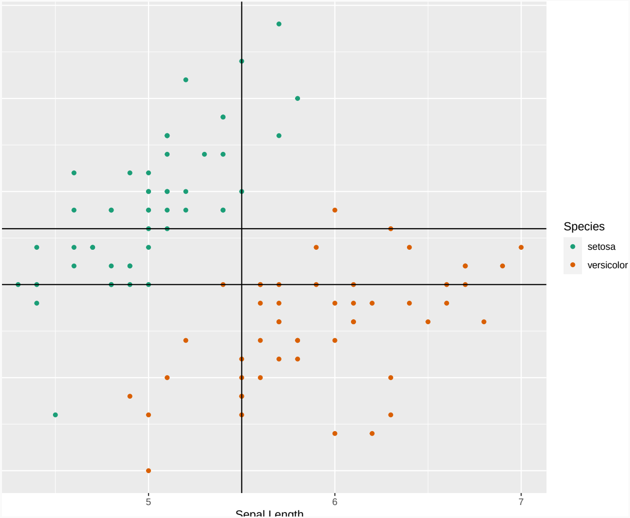
Root node split



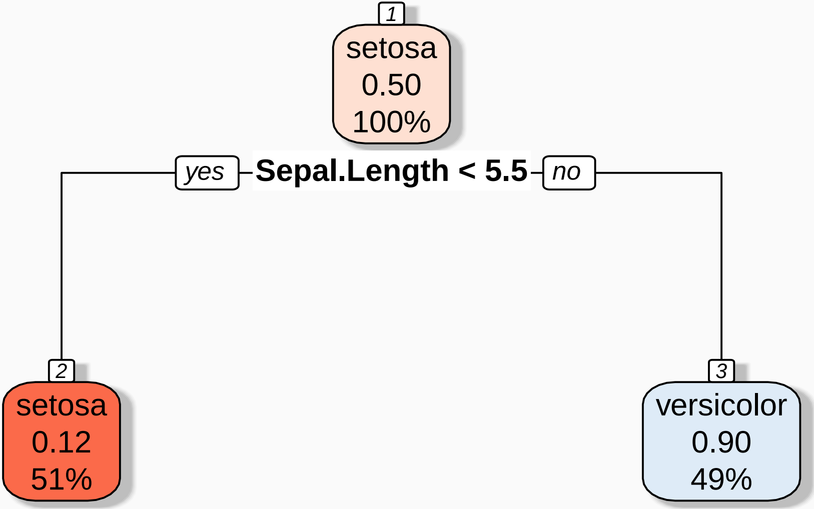
Root node split, Decision node split - 1



Root node split, Decision node splits



Shallow decision tree



Two key ideas underlying trees

- Recursive partitioning (for constructing the tree)
- Pruning (for cutting the tree back)
- Pruning is a useful strategy for avoiding over fitting.
- There are some alternative methods to avoid over fitting as well.

Leo Breiman

Key references

Breiman, L., J. Friedman, R. Olshen, and C. Stone, 1984: Classification and regression trees. Wadsworth Books, 358.

Breiman, L., 1996: Bagging predictors. Machine learning, 24 (2), 123–140.

Breiman, Leo (2001). "Random Forests". Machine Learning 45 (1): 5–32. `doi:10.1023/A`: 1010933404324

Constructing Classification Trees

Recursive Partitioning

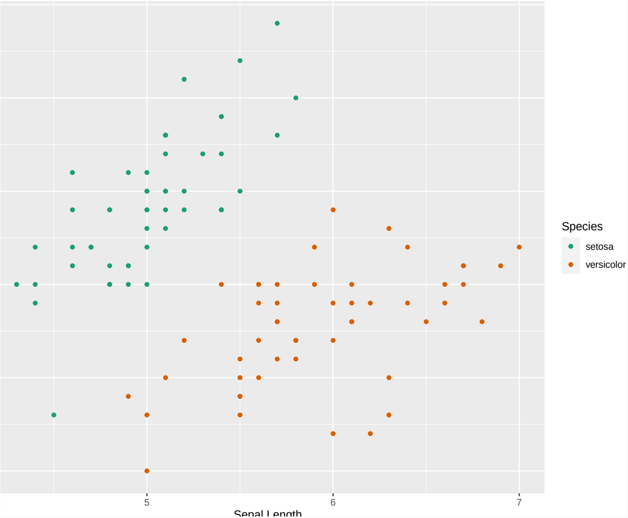
- Recursive partitioning splits P-dimensional feature space into nonoverlapping multidimensional rectangles.
- The division is accomplished recursively (i.e. operating on the results of prior division)

Main questions

- Splitting variable
  Which attribute/ feature should be placed at the root node?
  Which features will act as internal nodes?
- Splitting point
- Looking for a split that increases the homogeneity (or "pure" as possible) of the resulting subsets.

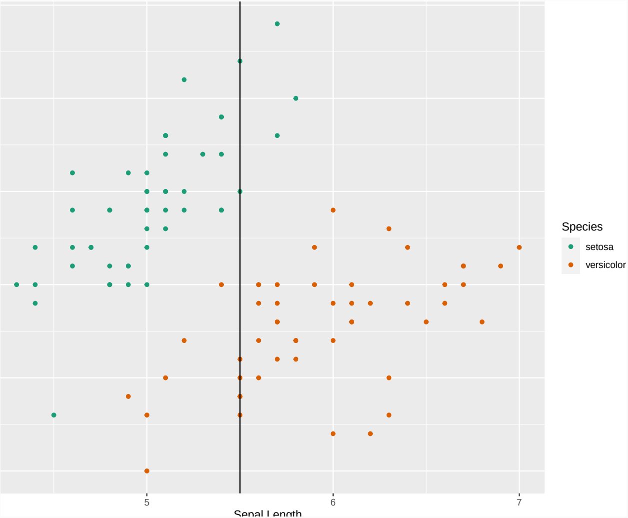
Example

split that increases the homogeneity



Example (cont.)

split that increases the homogeneity .



How does a decision tree determine the best split?

Decision tree uses entropy and information gain to select a feature which gives the best split.

Measures of Impurity

- An impurity measure is a heuristic for selection of the splitting criterion that best separates a given feature space.
- The two most popular measures
  - Gini index
  - Entropy measure

Gini index

Gini index for rectangle A is defined by

$$I(A) = 1 - \sum_{k=1}^{m} p_k^2$$

p_k - proportion of records in rectangle A that belong to class k

- Gini index takes value 0 when all the records belong to the same class.
- 1 denotes that the elements are randomly distributed across various classes.

Gini index (cont)

Gini index is at peak when $p_k = 0.5$

Example: Calculation

Entropy measure

$$entropy(A) = -\sum_{k=1}^{m} p_k log_2(p_k)$$