STA 331 2.0 Stochastic Processes

11. Birth-and-Death Process - important results (cont)

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Definition

A continuous parameter stationary Markov process is a stochastic process having the properties that

1. Each time it enters state i, the amount of time it spends in that state before making a transition into a different state is exponentially distributed (say with rate ν_i or mean $\frac{1}{\nu_i}$), and
2. When the process leaves state i, it enters state j with some probability, p_{ij} satisfying,

$$P_{ii} = 0 \text{ all } i$$
$$\sum_j P_{ij} = 1 \text{ all } i$$

Birth-and-death process

For birth and death process, let λ_i and μ_i be given by

$\lambda_i = q_{i,i+1}$ and $\mu_i = q_{i,i-1}$.

The values $\{\lambda_i, i \geq 0\}$ and $\{\mu_i, i \geq 0\}$ are called respectively the birth and death rate. Then

$$\nu_i = \lambda_i + \mu_i$$

Then $T_i \sim exp(\lambda_i + \mu_i)$.

Furthermore,

$$P_{i,i+1} = \frac{\lambda_i}{\lambda_i + \mu_i} = 1 - P_{i,i-1}$$

Examples of birth-and-death process

Example 1:

Suppose that life-time of a component of a machine is exponentially distributed with rate λ.

Let $X(t)$ be the state of the machine at time t.

$$X(t) = \begin{cases} 1, & \text{if the machine is operational at time t} \\ 0, & \text{if the machine is not operational at time t} \end{cases} \tag{1}$$

This is a continuous parameter discrete state Markov process with absorbing barrier state 0 (suppose that there are no repairs).