STA 506 2.0 Linear Regression Analysis

Lecture 13: Variable Selection and Model Building

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Contents

Recap

1. Simple linear regression
2. Multiple linear regression
3. Variable transformations
4. Detection and treatment of outliers: leverage and influence
5. Indicator variables

Qualitative variable with more than 2 levels

In general, a qualitative variable with k levels is represented by $k-1$ indicator variables, each taking the values 0 and 1.

```
   IQ         BMI headcir D1 D2
1  10      Normal    50.2  0  1
2  20      Normal    50.5  0  1
3 100       Obese    58.5  0  0
4  98       Obese    55.0  0  0
5 100 Underweight    54.9  1  0
6  11 Underweight    40.0  1  0
7  50 Underweight    48.5  1  0
8  70 Underweight    50.0  1  0
```

D_1	D_2	Description
1	0	observation is from underweight
0	1	observation is from normal
0	0	observation is from Obese

Your turn

Write the regression equations for the three levels.

```
   IQ headcir D1 D2
1  10    50.2  0  1
2  20    50.5  0  1
3 100    58.5  0  0
4  98    55.0  0  0
5 100    54.9  1  0
6  11    40.0  1  0
7  50    48.5  1  0
8  70    50.0  1  0
```

$$D_{1i} = \begin{cases} 1 & \text{if underweight} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$D_{2i} = \begin{cases} 1 & \text{if normal} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Let x_i be the head circumference

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 D_{1i} + \beta_3 D_{2i} + \epsilon_i$$

For underweight

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 + \epsilon_i$$

For normal

$$y_i = \beta_0 + \beta_1 x_i + \beta_3 + \epsilon_i$$

For overweight

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

Different in both intercept and slope

```
   IQ Gender  BMI
1  10   Male 20.2
2  20   Male 20.5
3 100   Male 18.5
4  98   Male 25.0
5 100 Female 24.9
6  11 Female 31.0
7  50 Female 18.5
8  70 Female 20.0
```

Indicator variable for `Gender`

$$D_i = \begin{cases} 1 & \text{if male} \\ 0 & \text{if female} \end{cases} \tag{3}$$

The choice of 0 and 1 to identify the levels of a qualitative variable is arbitrary.

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 D_i + \epsilon_i,$$

Regression line differ in intercept only

Regression equation for **males**, $D_i = 1$

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 + \epsilon_i,$$

Regression equation for **females**, $D_i = 0$

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

Regression line both differ in slope and intercept

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 D_i + \beta_3 x_i D_i + \epsilon_i,$$

Regression equation for **males**, $D_i = 1$

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 + \beta_3 x_i + \epsilon_i,$$

$$y_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) x_i + \epsilon_i,$$

Regression equation for **females**, $D_i = 0$

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

Variable Selection: Introduction

Variable selection: Finding an appropriate subset of regression for the model.

```
library(tidyverse)
realestate <- read.csv("real-estate.csv")
head(realestate)
```

```
  ID  Price Sqft Bedroom Bathroom Airconditioning Garage Pool YearBuild Quality
1  1 360000 3032       4        4               1      2    0      1972       2
2  2 340000 2058       4        2               1      2    0      1976       2
3  3 250000 1780       4        3               1      2    0      1980       2
4  4 205500 1638       4        2               1      2    0      1963       2
5  5 275500 2196       4        3               1      2    0      1968       2
6  6 248000 1966       4        3               1      5    1      1972       2
    Lot AdjHighway
1 22221          0
2 22912          0
3 21345          0
4 17342          0
5 21786          0
6 18902          0
```

```
glimpse(realestate)
```

```
Rows: 522
Columns: 12
$ ID              <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ...
$ Price           <int> 360000, 340000, 250000, 205500, 275500, 248000, 229...
$ Sqft            <int> 3032, 2058, 1780, 1638, 2196, 1966, 2216, 1597, 162...
$ Bedroom         <int> 4, 4, 4, 4, 4, 4, 3, 2, 3, 3, 7, 3, 5, 5, 3, 5, 2, ...
$ Bathroom        <int> 4, 2, 3, 2, 3, 3, 2, 1, 2, 3, 5, 4, 4, 4, 3, 5, 2, ...
$ Airconditioning <int> 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, ...
$ Garage          <int> 2, 2, 2, 2, 2, 5, 2, 1, 2, 1, 2, 3, 3, 2, 2, 2, 2, ...
$ Pool            <int> 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, ...
$ YearBuild       <int> 1972, 1976, 1980, 1963, 1968, 1972, 1972, 1955, 197...
$ Quality         <int> 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 1, 1, 1, 2, 2, 2, ...
$ Lot             <int> 22221, 22912, 21345, 17342, 21786, 18902, 18639, 22...
$ AdjHighway      <int> 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
```

```
summary(realestate)
```

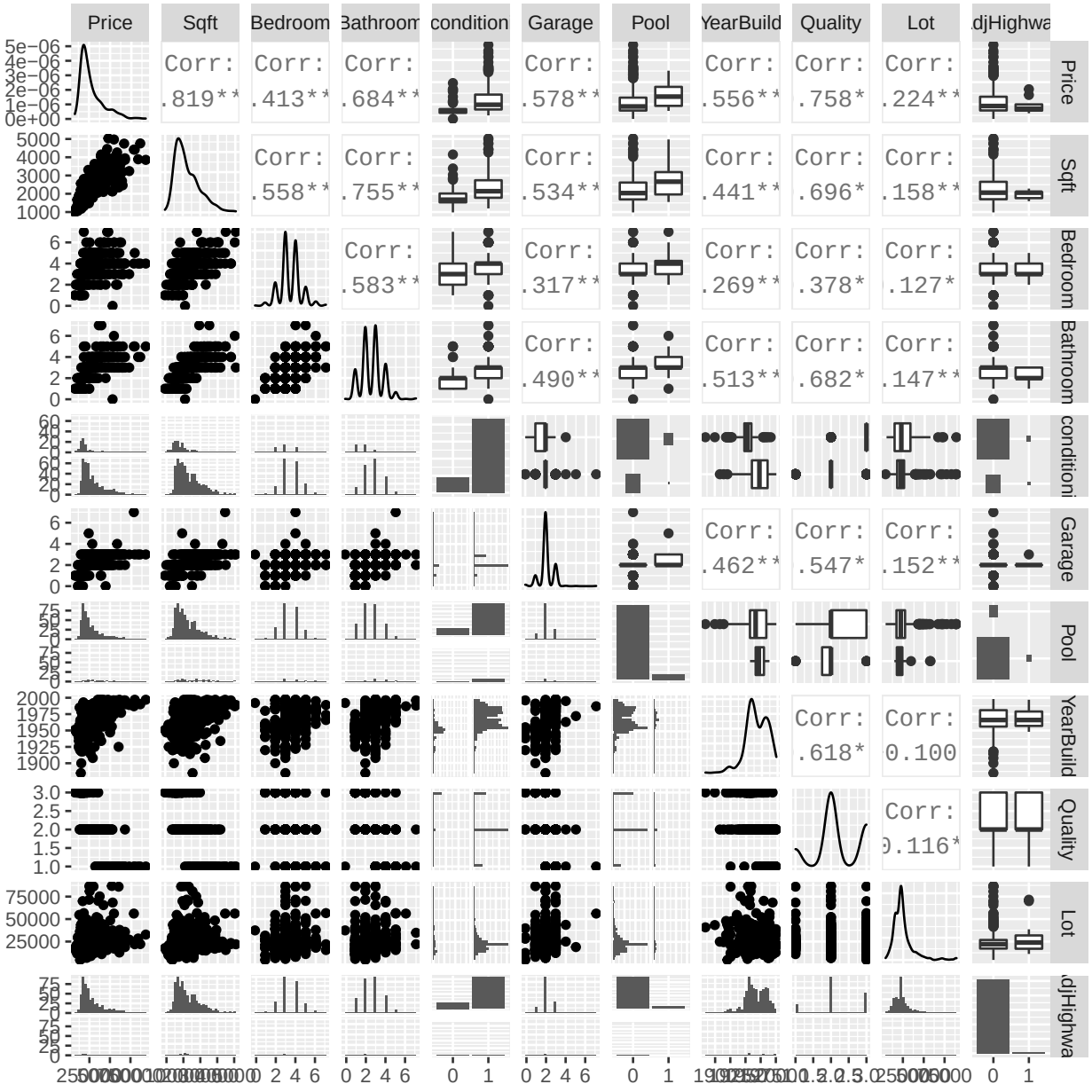
```
       ID              Price             Sqft         Bedroom     
 Min.   :  1.0   Min.   : 84000   Min.   : 980   Min.   :0.000  
 1st Qu.:131.2   1st Qu.:180000   1st Qu.:1701   1st Qu.:3.000  
 Median :261.5   Median :229900   Median :2061   Median :3.000  
 Mean   :261.5   Mean   :277894   Mean   :2261   Mean   :3.471  
 3rd Qu.:391.8   3rd Qu.:335000   3rd Qu.:2636   3rd Qu.:4.000  
 Max.   :522.0   Max.   :920000   Max.   :5032   Max.   :7.000  
    Bathroom     Airconditioning      Garage          Pool        
 Min.   :0.000   Min.   :0.0000   Min.   :0.0   Min.   :0.00000  
 1st Qu.:2.000   1st Qu.:1.0000   1st Qu.:2.0   1st Qu.:0.00000  
```

```
 Median :3.000   Median :1.0000   Median :2.0   Median :0.00000
 Mean   :2.642   Mean   :0.8314   Mean   :2.1   Mean   :0.06897
 3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.:2.0   3rd Qu.:0.00000
 Max.   :7.000   Max.   :1.0000   Max.   :7.0   Max.   :1.00000
   YearBuild       Quality           Lot          AdjHighway
 Min.   :1885   Min.   :1.000   Min.   : 4560   Min.   :0.00000
 1st Qu.:1956   1st Qu.:2.000   1st Qu.:17205   1st Qu.:0.00000
 Median :1966   Median :2.000   Median :22200   Median :0.00000
 Mean   :1967   Mean   :2.184   Mean   :24370   Mean   :0.02107
 3rd Qu.:1981   3rd Qu.:3.000   3rd Qu.:26787   3rd Qu.:0.00000
 Max.   :1998   Max.   :3.000   Max.   :86830   Max.   :1.00000
```

```
realestate$Airconditioning <- factor(realestate$Airconditioning)
realestate$Pool <- factor(realestate$Pool)
realestate$AdjHighway <- factor(realestate$AdjHighway)
summary(realestate)
```

```
       ID              Price             Sqft          Bedroom
 Min.   :  1.0   Min.   : 84000   Min.   : 980   Min.   :0.000
 1st Qu.:131.2   1st Qu.:180000   1st Qu.:1701   1st Qu.:3.000
 Median :261.5   Median :229900   Median :2061   Median :3.000
 Mean   :261.5   Mean   :277894   Mean   :2261   Mean   :3.471
 3rd Qu.:391.8   3rd Qu.:335000   3rd Qu.:2636   3rd Qu.:4.000
 Max.   :522.0   Max.   :920000   Max.   :5032   Max.   :7.000
    Bathroom     Airconditioning     Garage    Pool      YearBuild
 Min.   :0.000   0: 88           Min.   :0.0   0:486   Min.   :1885
 1st Qu.:2.000   1:434           1st Qu.:2.0   1: 36   1st Qu.:1956
 Median :3.000                   Median :2.0           Median :1966
 Mean   :2.642                   Mean   :2.1           Mean   :1967
 3rd Qu.:3.000                   3rd Qu.:2.0           3rd Qu.:1981
 Max.   :7.000                   Max.   :7.0           Max.   :1998
    Quality           Lot        AdjHighway
 Min.   :1.000   Min.   : 4560   0:511
 1st Qu.:2.000   1st Qu.:17205   1: 11
 Median :2.000   Median :22200
 Mean   :2.184   Mean   :24370
 3rd Qu.:3.000   3rd Qu.:26787
 Max.   :3.000   Max.   :86830
```

Correlation matrix



All possible regression

In-class

Intercept-only regression model

```
realty.lm.minimal <- lm(Price ~ 1, data=realestate)
realty.lm.minimal
```

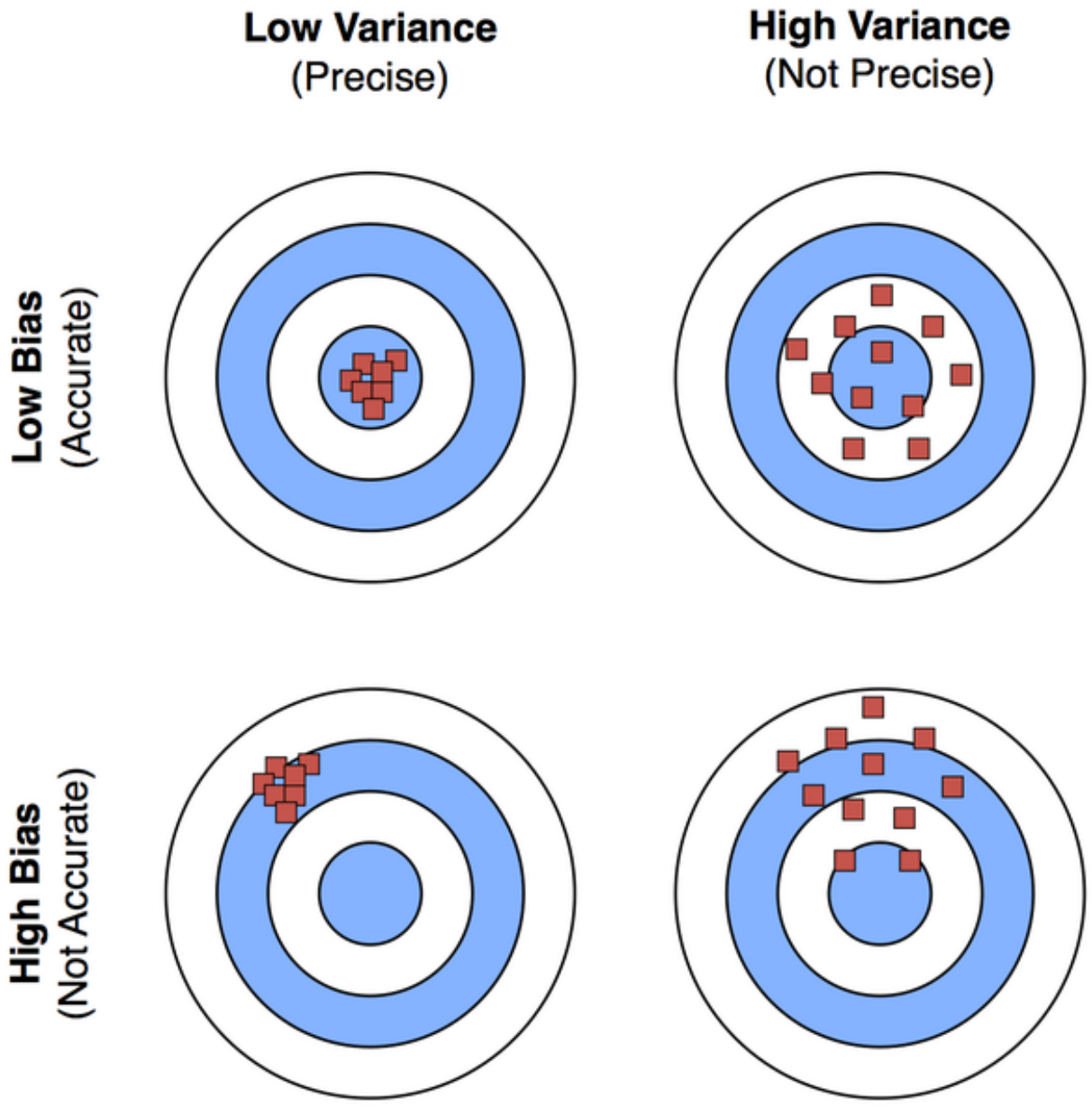
```
Call:
lm(formula = Price ~ 1, data = realestate)

Coefficients:
(Intercept)
     277894
```

Full model

```
realty.lm.all <- lm(Price ~ . , data=realestate.var)
realty.lm.all
```

```
Call:
lm(formula = Price ~ ., data = realestate.var)

Coefficients:
     (Intercept)             Sqft          Bedroom         Bathroom
      -2.390e+06         1.075e+02        -9.712e+03        -1.067e+02
Airconditioning1           Garage            Pool1         YearBuild
      -1.222e+04         1.732e+04         1.249e+04         1.279e+03
         Quality              Lot       AdjHighway1
      -5.390e+04         1.422e+00        -2.717e+04
```

We need model to include as few independent variables as possibles because the variance of the the predictions increases as the number of independent variables increases.



Computational Techniques for Variable Selection

1. All possible regression
2. Stepwise regression methods
   2.1 Forward selection
   2.2 Backward elimination
   2.3 Stepwise regression

Criteria for evaluating subset regression models

- lowest p-value
- highest adjusted R^2_{adj}
- lowest Mallow's C_p
- lowest AIC
- lowest residual mean square
- lowest score under cross-validation, etc.

Forward selection

- Starts with intercept-only regression model.
- Then we gradually add one more variable at a time (or add main effects first, then interactions).

Intercept-only regression model

```
realty.lm.minimal <- lm(Price ~ 1, data=realestate)
realty.lm.minimal
```

```
Call:
lm(formula = Price ~ 1, data = realestate)

Coefficients:
(Intercept)
     277894
```

Forward selection

```
step(realty.lm.minimal, scope=list(upper = realty.lm.all,
lower= realty.lm.minimal), direction="forward",
trace=0)
```

```
Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
 -2.233e+06    1.093e+02   -5.223e+04    1.191e+03    1.415e+00    1.665e+04
    Bedroom
 -1.007e+04
```

```
step(realty.lm.minimal, scope=list(upper = realty.lm.all,
lower= realty.lm.minimal), direction="forward",
trace=1)
```

```
Start:  AIC=12356.17
Price ~ 1

                  Df  Sum of Sq        RSS   AIC
+ Sqft             1 6.6555e+12 3.2554e+12 11777
+ Quality          1 5.6956e+12 4.2153e+12 11912
+ Bathroom         1 4.6326e+12 5.2783e+12 12029
+ Garage           1 3.3086e+12 6.6023e+12 12146
+ YearBuild        1 3.0585e+12 6.8524e+12 12166
+ Bedroom          1 1.6931e+12 8.2178e+12 12260
+ Airconditioning  1 8.2546e+11 9.0855e+12 12313
```

```
+ Lot              1 4.9804e+11 9.4129e+12 12331
+ Pool             1 2.1303e+11 9.6979e+12 12347
<none>                          9.9109e+12 12356
+ AdjHighway       1 2.5746e+10 9.8852e+12 12357

Step:  AIC=11777.02
Price ~ Sqft

                  Df  Sum of Sq        RSS   AIC
+ Quality          1 6.7926e+11 2.5762e+12 11657
+ YearBuild        1 4.6302e+11 2.7924e+12 11699
+ Garage           1 2.7313e+11 2.9823e+12 11733
+ Bathroom         1 9.6767e+10 3.1587e+12 11763
+ Lot              1 9.1880e+10 3.1635e+12 11764
+ Airconditioning  1 5.0865e+10 3.2046e+12 11771
+ Bedroom          1 2.7613e+10 3.2278e+12 11775
<none>                          3.2554e+12 11777
+ Pool             1 1.8642e+09 3.2536e+12 11779
+ AdjHighway       1 1.6494e+07 3.2554e+12 11779

Step:  AIC=11656.86
Price ~ Sqft + Quality

                  Df  Sum of Sq        RSS   AIC
+ YearBuild        1 1.0457e+11 2.4716e+12 11637
+ Garage           1 8.8087e+10 2.4881e+12 11641
+ Lot              1 8.7374e+10 2.4888e+12 11641
+ Bedroom          1 2.3350e+10 2.5528e+12 11654
<none>                          2.5762e+12 11657
+ Airconditioning  1 2.2920e+09 2.5739e+12 11658
+ Bathroom         1 1.4980e+09 2.5747e+12 11659
+ AdjHighway       1 8.5057e+08 2.5753e+12 11659
+ Pool             1 8.3092e+08 2.5753e+12 11659

Step:  AIC=11637.23
Price ~ Sqft + Quality + YearBuild

                  Df  Sum of Sq        RSS   AIC
+ Lot              1 1.4256e+11 2.3290e+12 11608
+ Garage           1 5.7571e+10 2.4140e+12 11627
+ Bedroom          1 2.7902e+10 2.4437e+12 11633
+ Airconditioning  1 1.6548e+10 2.4550e+12 11636
<none>                          2.4716e+12 11637
+ AdjHighway       1 2.0662e+09 2.4695e+12 11639
+ Pool             1 1.3587e+09 2.4702e+12 11639
+ Bathroom         1 3.3406e+08 2.4713e+12 11639

Step:  AIC=11608.22
Price ~ Sqft + Quality + YearBuild + Lot

                  Df  Sum of Sq        RSS   AIC
+ Garage           1 3.6990e+10 2.2920e+12 11602
+ Bedroom          1 3.5910e+10 2.2931e+12 11602
<none>                          2.3290e+12 11608
```

```
+ AdjHighway       1 7.1918e+09 2.3218e+12 11609
+ Airconditioning  1 7.0188e+09 2.3220e+12 11609
+ Pool              1 4.1055e+09 2.3249e+12 11609
+ Bathroom          1 2.5720e+09 2.3265e+12 11610

Step:  AIC=11601.86
Price ~ Sqft + Quality + YearBuild + Lot + Garage

                  Df  Sum of Sq        RSS   AIC
+ Bedroom          1 3.7251e+10 2.2548e+12 11595
+ Airconditioning  1 1.1182e+10 2.2809e+12 11601
<none>                          2.2920e+12 11602
+ AdjHighway       1 7.3877e+09 2.2847e+12 11602
+ Pool              1 3.2365e+09 2.2888e+12 11603
+ Bathroom          1 3.0575e+09 2.2890e+12 11603

Step:  AIC=11595.31
Price ~ Sqft + Quality + YearBuild + Lot + Garage + Bedroom

                  Df  Sum of Sq        RSS   AIC
<none>                          2.2548e+12 11595
+ AdjHighway       1 7444567926 2.2473e+12 11596
+ Airconditioning  1 6973094059 2.2478e+12 11596
+ Pool              1 4676597321 2.2501e+12 11596
+ Bathroom          1   35810304 2.2548e+12 11597


Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
 -2.233e+06    1.093e+02   -5.223e+04    1.191e+03    1.415e+00    1.665e+04
    Bedroom
 -1.007e+04
```

Backward elimination

- we start with the full model and gradually delete variables one at a time.

```
step(realty.lm.all, direction="backward", trace=0)
```

```
Call:
lm(formula = Price ~ Sqft + Bedroom + Garage + YearBuild + Quality +
    Lot, data = realestate.var)

Coefficients:
(Intercept)         Sqft      Bedroom       Garage    YearBuild      Quality
 -2.233e+06    1.093e+02   -1.007e+04    1.665e+04    1.191e+03   -5.223e+04
        Lot
  1.415e+00
```

```
step(realty.lm.all, direction="backward", trace=1)
```

```
Start:  AIC=11598.65
Price ~ Sqft + Bedroom + Bathroom + Airconditioning + Garage +
    Pool + YearBuild + Quality + Lot + AdjHighway

                  Df  Sum of Sq        RSS   AIC
- Bathroom         1 2.1979e+06 2.2348e+12 11597
- Pool             1 4.9762e+09 2.2397e+12 11598
- AdjHighway       1 7.7808e+09 2.2425e+12 11598
- Airconditioning  1 8.1526e+09 2.2429e+12 11599
<none>                          2.2348e+12 11599
- Bedroom          1 3.0812e+10 2.2656e+12 11604
- Garage           1 4.0949e+10 2.2757e+12 11606
- Lot              1 1.2533e+11 2.3601e+12 11625
- YearBuild        1 1.3792e+11 2.3727e+12 11628
- Quality          1 2.1663e+11 2.4514e+12 11645
- Sqft             1 9.9451e+11 3.2293e+12 11789

Step:  AIC=11596.65
Price ~ Sqft + Bedroom + Airconditioning + Garage + Pool + YearBuild +
    Quality + Lot + AdjHighway

                  Df  Sum of Sq        RSS   AIC
- Pool             1 4.9985e+09 2.2398e+12 11596
- AdjHighway       1 7.7816e+09 2.2426e+12 11596
- Airconditioning  1 8.1628e+09 2.2429e+12 11597
<none>                          2.2348e+12 11597
- Bedroom          1 3.4220e+10 2.2690e+12 11603
- Garage           1 4.0949e+10 2.2757e+12 11604
- Lot              1 1.2607e+11 2.3608e+12 11623
- YearBuild        1 1.4205e+11 2.3768e+12 11627
- Quality          1 2.2740e+11 2.4622e+12 11645
- Sqft             1 1.1742e+12 3.4089e+12 11815

Step:  AIC=11595.82
```

```
Price ~ Sqft + Bedroom + Airconditioning + Garage + YearBuild +
    Quality + Lot + AdjHighway

                  Df  Sum of Sq        RSS   AIC
- Airconditioning  1 7.5771e+09 2.2473e+12 11596
- AdjHighway       1 8.0486e+09 2.2478e+12 11596
<none>                          2.2398e+12 11596
- Bedroom          1 3.2942e+10 2.2727e+12 11601
- Garage           1 4.1888e+10 2.2817e+12 11604
- Lot              1 1.2320e+11 2.3630e+12 11622
- YearBuild        1 1.3949e+11 2.3793e+12 11625
- Quality          1 2.2897e+11 2.4687e+12 11645
- Sqft             1 1.1908e+12 3.4306e+12 11816

Step:  AIC=11595.58
Price ~ Sqft + Bedroom + Garage + YearBuild + Quality + Lot +
    AdjHighway

             Df  Sum of Sq        RSS   AIC
- AdjHighway  1 7.4446e+09 2.2548e+12 11595
<none>                     2.2473e+12 11596
- Bedroom     1 3.7308e+10 2.2847e+12 11602
- Garage      1 3.8532e+10 2.2859e+12 11602
- YearBuild   1 1.3231e+11 2.3797e+12 11623
- Lot         1 1.3423e+11 2.3816e+12 11624
- Quality     1 2.2142e+11 2.4688e+12 11643
- Sqft        1 1.2209e+12 3.4682e+12 11820

Step:  AIC=11595.31
Price ~ Sqft + Bedroom + Garage + YearBuild + Quality + Lot

            Df  Sum of Sq        RSS   AIC
<none>                    2.2548e+12 11595
- Bedroom    1 3.7251e+10 2.2920e+12 11602
- Garage     1 3.8331e+10 2.2931e+12 11602
- YearBuild  1 1.2861e+11 2.3834e+12 11622
- Lot        1 1.2923e+11 2.3840e+12 11622
- Quality    1 2.2231e+11 2.4771e+12 11642
- Sqft       1 1.2399e+12 3.4947e+12 11822



Call:
lm(formula = Price ~ Sqft + Bedroom + Garage + YearBuild + Quality +
    Lot, data = realestate.var)

Coefficients:
(Intercept)         Sqft      Bedroom       Garage    YearBuild      Quality
 -2.233e+06    1.093e+02   -1.007e+04    1.665e+04    1.191e+03   -5.223e+04
        Lot
  1.415e+00
```

Stepwise regression

```
step(realty.lm.minimal, scope=list(upper = realty.lm.all,
lower= realty.lm.minimal), direction="both", trace=0)
```

```
Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
 -2.233e+06    1.093e+02   -5.223e+04    1.191e+03    1.415e+00    1.665e+04
    Bedroom
 -1.007e+04
```

```
step(realty.lm.minimal, scope=list(upper = realty.lm.all,
lower= realty.lm.minimal), direction="both", trace=1)
```

```
Start:  AIC=12356.17
Price ~ 1

                  Df  Sum of Sq        RSS   AIC
+ Sqft             1 6.6555e+12 3.2554e+12 11777
+ Quality          1 5.6956e+12 4.2153e+12 11912
+ Bathroom         1 4.6326e+12 5.2783e+12 12029
+ Garage           1 3.3086e+12 6.6023e+12 12146
+ YearBuild        1 3.0585e+12 6.8524e+12 12166
+ Bedroom          1 1.6931e+12 8.2178e+12 12260
+ Airconditioning  1 8.2546e+11 9.0855e+12 12313
+ Lot              1 4.9804e+11 9.4129e+12 12331
+ Pool             1 2.1303e+11 9.6979e+12 12347
<none>                          9.9109e+12 12356
+ AdjHighway       1 2.5746e+10 9.8852e+12 12357

Step:  AIC=11777.02
Price ~ Sqft

                  Df  Sum of Sq        RSS   AIC
+ Quality          1 6.7926e+11 2.5762e+12 11657
+ YearBuild        1 4.6302e+11 2.7924e+12 11699
+ Garage           1 2.7313e+11 2.9823e+12 11733
+ Bathroom         1 9.6767e+10 3.1587e+12 11763
+ Lot              1 9.1880e+10 3.1635e+12 11764
+ Airconditioning  1 5.0865e+10 3.2046e+12 11771
+ Bedroom          1 2.7613e+10 3.2278e+12 11775
<none>                          3.2554e+12 11777
+ Pool             1 1.8642e+09 3.2536e+12 11779
+ AdjHighway       1 1.6494e+07 3.2554e+12 11779
- Sqft             1 6.6555e+12 9.9109e+12 12356

Step:  AIC=11656.86
```

```
Price ~ Sqft + Quality

                  Df  Sum of Sq        RSS   AIC
+ YearBuild        1 1.0457e+11 2.4716e+12 11637
+ Garage           1 8.8087e+10 2.4881e+12 11641
+ Lot              1 8.7374e+10 2.4888e+12 11641
+ Bedroom          1 2.3350e+10 2.5528e+12 11654
<none>                          2.5762e+12 11657
+ Airconditioning  1 2.2920e+09 2.5739e+12 11658
+ Bathroom         1 1.4980e+09 2.5747e+12 11659
+ AdjHighway       1 8.5057e+08 2.5753e+12 11659
+ Pool             1 8.3092e+08 2.5753e+12 11659
- Quality          1 6.7926e+11 3.2554e+12 11777
- Sqft             1 1.6391e+12 4.2153e+12 11912

Step:  AIC=11637.23
Price ~ Sqft + Quality + YearBuild

                  Df  Sum of Sq        RSS   AIC
+ Lot              1 1.4256e+11 2.3290e+12 11608
+ Garage           1 5.7571e+10 2.4140e+12 11627
+ Bedroom          1 2.7902e+10 2.4437e+12 11633
+ Airconditioning  1 1.6548e+10 2.4550e+12 11636
<none>                          2.4716e+12 11637
+ AdjHighway       1 2.0662e+09 2.4695e+12 11639
+ Pool             1 1.3587e+09 2.4702e+12 11639
+ Bathroom         1 3.3406e+08 2.4713e+12 11639
- YearBuild        1 1.0457e+11 2.5762e+12 11657
- Quality          1 3.2082e+11 2.7924e+12 11699
- Sqft             1 1.6214e+12 4.0930e+12 11898

Step:  AIC=11608.22
Price ~ Sqft + Quality + YearBuild + Lot

                  Df  Sum of Sq        RSS   AIC
+ Garage           1 3.6990e+10 2.2920e+12 11602
+ Bedroom          1 3.5910e+10 2.2931e+12 11602
<none>                          2.3290e+12 11608
+ AdjHighway       1 7.1918e+09 2.3218e+12 11609
+ Airconditioning  1 7.0188e+09 2.3220e+12 11609
+ Pool             1 4.1055e+09 2.3249e+12 11609
+ Bathroom         1 2.5720e+09 2.3265e+12 11610
- Lot              1 1.4256e+11 2.4716e+12 11637
- YearBuild        1 1.5976e+11 2.4888e+12 11641
- Quality          1 2.6818e+11 2.5972e+12 11663
- Sqft             1 1.4919e+12 3.8210e+12 11865

Step:  AIC=11601.86
Price ~ Sqft + Quality + YearBuild + Lot + Garage

                  Df  Sum of Sq        RSS   AIC
+ Bedroom          1 3.7251e+10 2.2548e+12 11595
+ Airconditioning  1 1.1182e+10 2.2809e+12 11601
<none>                          2.2920e+12 11602
```

```
+ AdjHighway       1 7.3877e+09 2.2847e+12 11602
+ Pool             1 3.2365e+09 2.2888e+12 11603
+ Bathroom         1 3.0575e+09 2.2890e+12 11603
- Garage           1 3.6990e+10 2.3290e+12 11608
- Lot              1 1.2198e+11 2.4140e+12 11627
- YearBuild        1 1.2203e+11 2.4141e+12 11627
- Quality          1 2.3090e+11 2.5229e+12 11650
- Sqft             1 1.2961e+12 3.5882e+12 11834

Step:  AIC=11595.31
Price ~ Sqft + Quality + YearBuild + Lot + Garage + Bedroom

                  Df  Sum of Sq        RSS   AIC
<none>                          2.2548e+12 11595
+ AdjHighway       1 7.4446e+09 2.2473e+12 11596
+ Airconditioning  1 6.9731e+09 2.2478e+12 11596
+ Pool             1 4.6766e+09 2.2501e+12 11596
+ Bathroom         1 3.5810e+07 2.2548e+12 11597
- Bedroom          1 3.7251e+10 2.2920e+12 11602
- Garage           1 3.8331e+10 2.2931e+12 11602
- YearBuild        1 1.2861e+11 2.3834e+12 11622
- Lot              1 1.2923e+11 2.3840e+12 11622
- Quality          1 2.2231e+11 2.4771e+12 11642
- Sqft             1 1.2399e+12 3.4947e+12 11822


Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
 -2.233e+06    1.093e+02   -5.223e+04    1.191e+03    1.415e+00    1.665e+04
    Bedroom
 -1.007e+04
```

In this example stepwise regression reaches the same answer as only doing forward selection.

Model adequacy checking

```
model1 <- lm(Price ~ Sqft + Quality + YearBuild + Lot + Garage + Bedroom, data=realestate)
model1
```

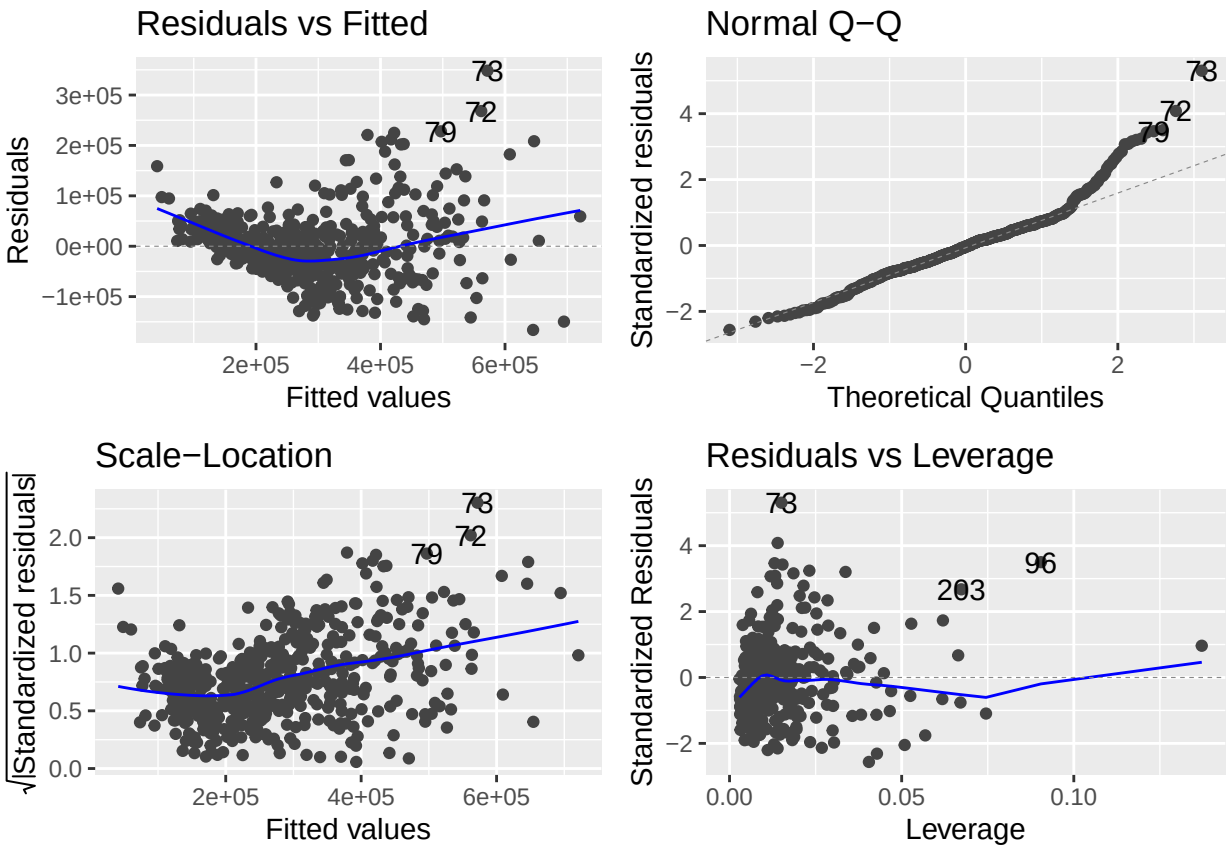
```
Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
 -2.233e+06    1.093e+02   -5.223e+04    1.191e+03    1.415e+00    1.665e+04
```

```
    Bedroom
 -1.007e+04
```

```
summary(model1)
```

```

Call:
lm(formula = Price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Residuals:
    Min      1Q  Median      3Q     Max
-166119  -41432   -2654   32273  348313

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.233e+06  4.392e+05  -5.084 5.18e-07 ***
Sqft         1.093e+02  6.496e+00  16.828  < 2e-16 ***
Quality     -5.223e+04  7.330e+03  -7.126 3.51e-12 ***
YearBuild    1.191e+03  2.198e+02   5.420 9.18e-08 ***
Lot          1.415e+00  2.604e-01   5.433 8.57e-08 ***
Garage       1.665e+04  5.626e+03   2.959  0.00323 **
Bedroom     -1.007e+04  3.454e+03  -2.917  0.00369 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 66170 on 515 degrees of freedom
Multiple R-squared:  0.7725,    Adjusted R-squared:  0.7698
F-statistic: 291.4 on 6 and 515 DF,  p-value: < 2.2e-16
```

```
library(ggfortify)
autoplot(model1)
```




```
realestate$log.price <- log(realestate$Price)
model2 <- lm(log.price ~ Sqft + Quality + YearBuild + Lot + Garage + Bedroom, data=realestate)
model2
```

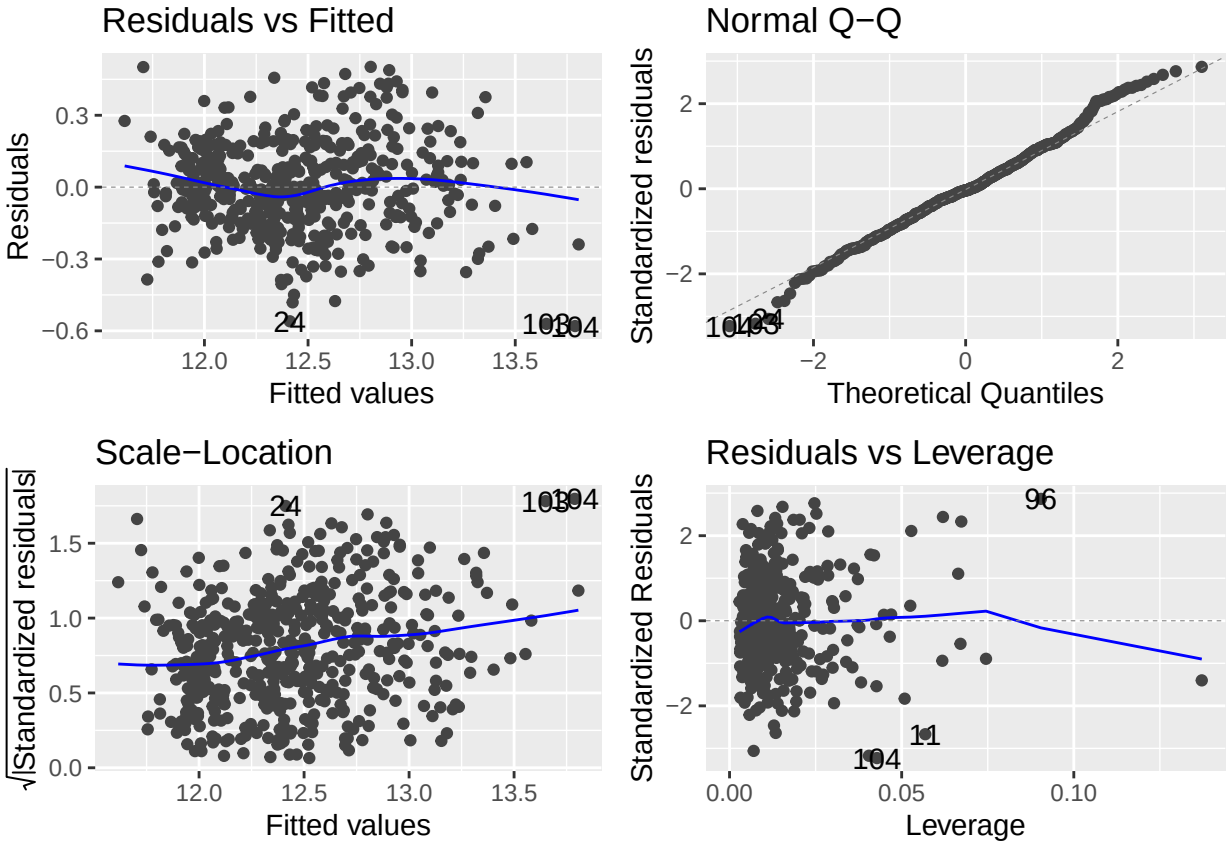
```
Call:
lm(formula = log.price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Coefficients:
(Intercept)         Sqft      Quality    YearBuild          Lot       Garage
  3.737e+00    3.112e-04   -1.782e-01    4.138e-03    4.978e-06    5.000e-02
    Bedroom
  5.045e-03
```

```
summary(model2)
```

```
Call:
lm(formula = log.price ~ Sqft + Quality + YearBuild + Lot + Garage +
    Bedroom, data = realestate)

Residuals:
     Min       1Q   Median       3Q      Max
-0.58012 -0.11594 -0.00685  0.10988  0.50210
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.737e+00  1.219e+00   3.065  0.00229 **
Sqft         3.112e-04  1.804e-05  17.253  < 2e-16 ***
Quality     -1.782e-01  2.035e-02  -8.755  < 2e-16 ***
YearBuild    4.138e-03  6.102e-04   6.781 3.27e-11 ***
Lot          4.978e-06  7.230e-07   6.885 1.69e-11 ***
Garage       5.000e-02  1.562e-02   3.201  0.00145 **
Bedroom      5.045e-03  9.588e-03   0.526  0.59902
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1837 on 515 degrees of freedom
Multiple R-squared:  0.821, Adjusted R-squared:  0.8189
F-statistic: 393.6 on 6 and 515 DF,  p-value: < 2.2e-16
```

```
library(ggfortify)
autoplot(model2)
```



Normality test

```
library(broom)
residout <- augment(model2)
residout
```

```
# A tibble: 522 x 13
   log.price  Sqft Quality YearBuild   Lot Garage Bedroom .fitted   .resid
       <dbl> <int>   <int>     <int> <int>  <int>   <int>   <dbl>    <dbl>
 1      12.8  3032       2      1972 22221      2       4    12.7  0.0784
 2      12.7  2058       2      1976 22912      2       4    12.4  0.304
 3      12.4  1780       2      1980 21345      2       4    12.4  0.0746
 4      12.2  1638       2      1963 17342      2       4    12.2  0.0130
 5      12.5  2196       2      1968 21786      2       4    12.4  0.0897
 6      12.4  1966       2      1972 18902      5       4    12.5 -0.0961
 7      12.3  2216       2      1972 18639      2       3    12.4 -0.0933
 8      11.9  1597       2      1955 22112      1       2    12.1 -0.220
 9      12.2  1622       3      1975 14321      2       3    12.1  0.114
10      12.0  1976       3      1918 32358      1       3    12.0  0.00225
# ... with 512 more rows, and 4 more variables: .std.resid <dbl>, .hat <dbl>,
#   .sigma <dbl>, .cooksd <dbl>
```

```
shapiro.test(residout$.resid)
```

```
    Shapiro-Wilk normality test

data:  residout$.resid
W = 0.99333, p-value = 0.02065
```

Use level of significance: 0.01

Note

In variable selection it is usually assumed that the correct functional specification of the regressors is known ($1/x$, $ln(Y)$), and that no outliers or influential observations are present. However, it practice these assumptions are rarely met. Hence, in practice we often use i) a particular variable selection strategy is employed, and then ii) the resulting model is checked for model adequacy, outliers, and influential cases and update the model accordingly.

Acknowledgement

Introduction to Linear Regression Analysis, Douglas C. Montgomery, Elizabeth A. Peck, G. Geoffrey Vining

Data: http://www.stat.cmu.edu/~cshalizi/mreg/15/hw/08/real-estate.csv