Model Questions

STA 506 2.0 Linear Regression Analysis

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Use 5% significance level for all tests.

Question 1

A chemical reaction is performed at different levels of temperature (Celsius) and the end product is weighed (g). The following results were obtained for the purpose of finding a regression model to represent the relationship of the two variables.

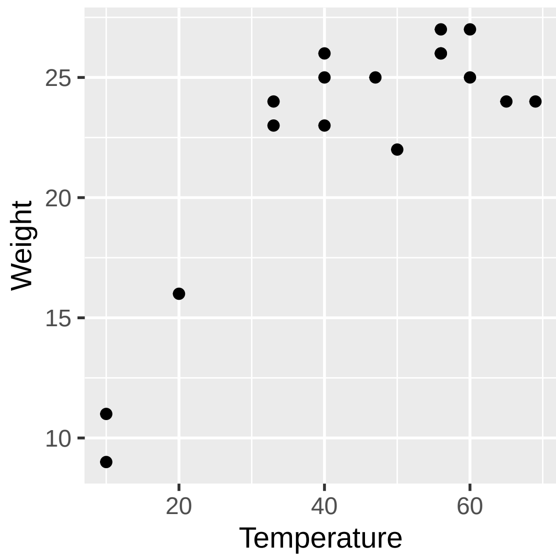
	Temperature	Weight
1	10	11
2	10	9
3	20	16
4	33	23
5	33	24
6	40	25
7	40	26
8	40	23
9	47	25
10	50	22
11	56	26
12	56	27
13	56	26
14	60	25
15	60	27
16	65	24
17	69	24

i) The two variables are supposed to have a linear relationship. Write the model you would fit to these data.

Answer

A regression analysis was performed with these data and the following outputs were obtained using R.

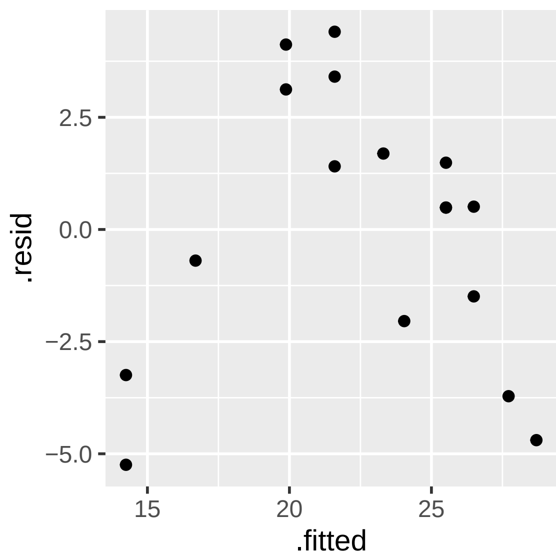
Output a



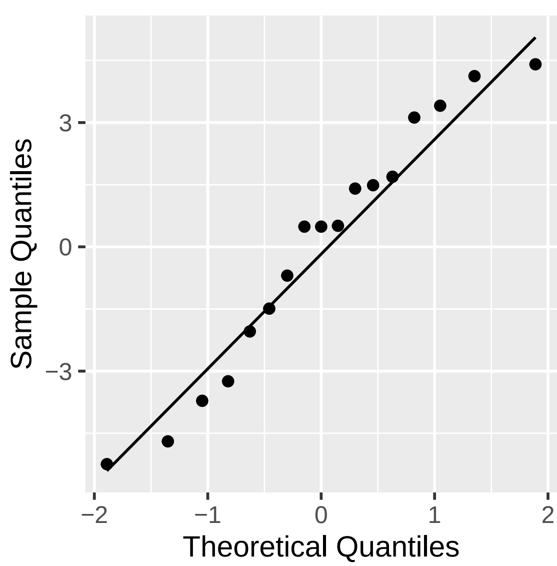
Output b

```
Call:
lm(formula = Weight ~ Temperature, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-5.2450 -2.0422  0.4882  1.6926  4.4071

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.79572    2.03828   5.787 3.58e-05 ***
Temperature  0.24493    0.04318   5.672 4.43e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.123 on 15 degrees of freedom
Multiple R-squared:  0.682, Adjusted R-squared:  0.6608
F-statistic: 32.18 on 1 and 15 DF,  p-value: 4.429e-05
```

Output c



Output d



Output e

```
    Shapiro-Wilk normality test

data:  fitmodel$.resid
W = 0.95278, p-value = 0.502
```

ii) Two undergraduates studying statistics were looking at this analysis.

(A) One said that the results strongly suggest that this model is highly significant and can be used for prediction purposes.

(B) The other said that the results show the fitted model is not appropriate for this case and this model cannot be used for prediction.

With whom would you agree? Justify your argument using each part ((a) to (e)) of the results given.

Answer

Question 2

In a soap production factory, there are two machines used for the production. Using 27 production runs; 15 of line 1 and 12 of line 2, the management wanted to find the relationship between the machine speed and the amount of scrap produced during the production process. To allow the two machines have different regression lines with different intercepts and slopes the following model was fitted with all 27 observations.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \epsilon$$

where,

X_1 is line speed and

$$X_2 = \begin{cases} 1 & \text{if line 1} \\ 0 & \text{if line 2} \end{cases} \tag{1}$$

i) Draw a sketch of the scatter plot which is expected with the above model.

Answer

Answer

ii) Write the model for each line.

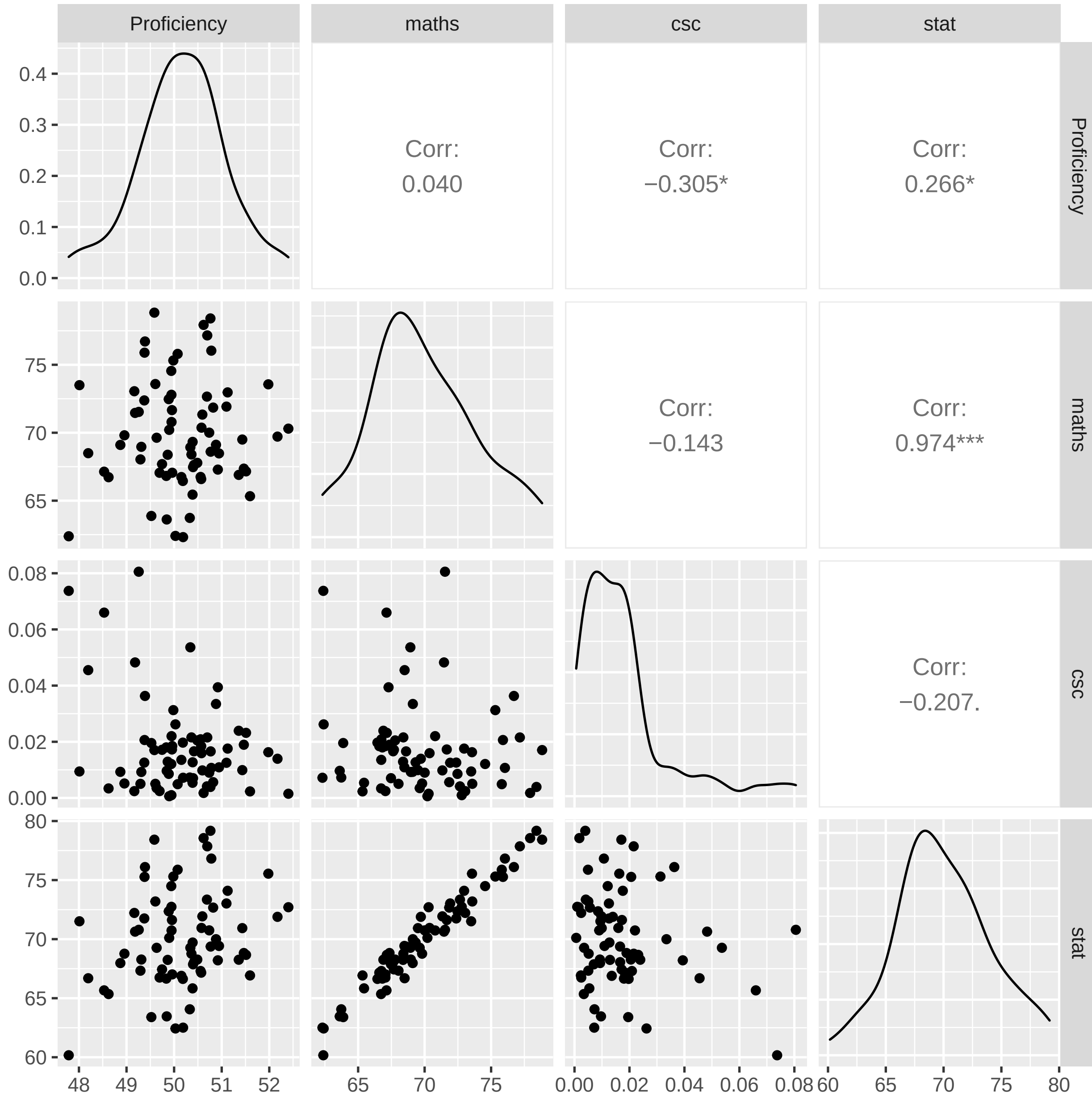
Answer

iii) Write the hypotheses that should be tested to find whether the two machines have the same regression model or not, i.e. whether both the intercept and the slope are the same of the two models you wrote in ii) in the above.

Answer

Question 3

A group of new graduates who have studied Statistics, Mathematics and Computer Science at the Faculty of Applied Sciences of University of Jayewardenepura joined a company. They were given three tests in the three subjects they have studied for the degree at the final interview at which they were selected for the job. After three months of a probationary period, their proficiency for the job was measured. The tests scores and the measure of proficiency were analysed to find a model to predict proficiency by the test scores. Some results are shown below.




```
model.sjp <- lm(Proficiency ~ maths + csc + stat, data=df)
summary(model.sjp)
```

```
Call:
lm(formula = Proficiency ~ maths + csc + stat, data = df)
```

```
Residuals:
       Min         1Q     Median         3Q        Max 
-1.136e-13  5.390e-16  2.112e-15  2.632e-15  9.808e-15 

Coefficients:
              Estimate Std. Error    t value Pr(>|t|)    
(Intercept)  5.000e+01  3.311e-14  1.510e+15   <2e-16 ***
maths       -1.000e+00  2.113e-15 -4.732e+14   <2e-16 ***
csc          1.647e-14  1.175e-13  1.400e-01    0.889    
stat         1.000e+00  2.062e-15  4.849e+14   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.51e-14 on 66 degrees of freedom
Multiple R-squared:      1, Adjusted R-squared:      1 
F-statistic: 8.644e+28 on 3 and 66 DF,  p-value: < 2.2e-16
```

```
car::vif(model.sjp)
```

```
    maths       csc      stat 
20.786453  1.123955 21.276288 
```

A statistician examined these results and claimed that "multicollinearity" has affected this model.

i) What is meant by multicollinearity?

Answer

Answer

ii) Do you agree with statistician claim. Justify your answer.

Answer

The following outputs are obtained using R

```
as.data.frame(augment(model.sjp))
```

```
   Proficiency    maths          csc     stat  .fitted       .resid       .hat
1     49.37355 72.37755 0.0125863639 71.75109 49.37355 2.842171e-14 0.03491457
2     50.18364 66.45027 0.0196940463 66.63391 50.18364 2.131628e-14 0.02523086
3     49.16437 73.05363 0.0024284454 72.21800 49.16437 2.842171e-14 0.06256608
4     51.59528 65.32951 0.0023299210 66.92479 51.59528 2.842171e-14 0.07669201
5     50.32951 63.73183 0.0072678104 64.06134 50.32951 2.842171e-14 0.06006372
6     49.17953 71.45723 0.0482494756 70.63676 49.17953 2.842171e-14 0.07585078
7     50.48743 67.78354 0.0204927009 68.27097 50.48743 3.552714e-14 0.02118640
8     50.73832 70.00553 0.0089947140 70.74385 50.73832 2.842171e-14 0.02170681
9     50.57578 70.37171 0.0159427916 70.94749 50.57578 2.842171e-14 0.01751163
10    49.69461 67.05240 0.0024507665 66.74701 49.69461 2.842171e-14 0.04671670
11    51.51178 67.15666 0.0231789188 68.66844 51.51178 2.131628e-14 0.06134208
12    50.38984 69.32411 0.0127004976 69.71395 50.38984 2.131628e-14 0.01638207
13    49.37876 75.89043 0.0206267258 75.26919 49.37876 3.552714e-14 0.06085543
14    47.78530 62.38217 0.0737322370 60.16747 47.78530 2.842171e-14 0.25137809
15    51.12493 72.96973 0.0175757195 74.09466 51.12493 2.842171e-14 0.04135050
16    49.95507 71.66475 0.0172540658 71.61982 49.95507 2.842171e-14 0.01821781
17    49.98381 75.31550 0.0312672529 75.29931 49.98381 2.842171e-14 0.05858375
18    50.94384 68.47908 0.0109124440 69.42292 50.94384 2.131628e-14 0.02748128
19    50.82122 71.85009 0.0056155579 72.67132 50.82122 2.842171e-14 0.02828862
20    50.59390 71.33549 0.0098079954 71.92940 50.59390 2.131628e-14 0.02064506
21    50.91898 67.28740 0.0394085876 68.20638 50.91898 2.842171e-14 0.06759207
22    50.78214 76.03934 0.0106982098 76.82148 50.78214 2.842171e-14 0.05587418
23    50.07456 75.80201 0.0049020065 75.87658 50.07456 2.842171e-14 0.05378780
24    48.01065 73.50107 0.0094310921 71.51172 48.01065 3.552714e-14 0.12589022
25    50.61983 77.93417 0.0017678771 78.55399 50.61983 2.842171e-14 0.08316856
26    49.94387 72.79243 0.0009906527 72.73630 49.94387 2.842171e-14 0.03894284
27    49.84420 63.61704 0.0096452077 63.46124 49.84420 2.842171e-14 0.06199928
28    48.52925 67.13367 0.0659822142 65.66292 48.52925 3.552714e-14 0.15392031
29    49.52185 63.87694 0.0195552018 63.39879 49.52185 2.842171e-14 0.05478978
30    50.41794 67.63300 0.0166135493 68.05094 50.41794 2.131628e-14 0.02027931
31    51.35868 66.89817 0.0239214224 68.25685 51.35868 2.131628e-14 0.05575867
32    49.89721 70.21058 0.0006211421 70.10779 49.89721 3.552714e-14 0.03525760
33    50.38767 65.44539 0.0054001692 65.83306 50.38767 2.842171e-14 0.04441475
34    49.94619 70.79014 0.0220077988 70.73634 49.94619 2.131628e-14 0.01688088
35    48.62294 66.72708 0.0033918392 65.35002 48.62294 3.552714e-14 0.09649255
36    49.58501 78.83644 0.0170454313 78.42144 49.58501 3.552714e-14 0.09788394
37    49.60571 73.58354 0.0050290156 73.18925 49.60571 3.552714e-14 0.04402388
38    49.94069 74.55087 0.0120869051 74.49156 49.94069 3.552714e-14 0.03723970
39    51.10003 71.92093 0.0125257115 73.02095 51.10003 2.131628e-14 0.03288907
40    50.76318 78.41088 0.0039171242 79.17406 50.76318 2.842171e-14 0.09001817
41    49.83548 66.82132 0.0179980190 66.65679 49.83548 2.842171e-14 0.02484526
42    49.74664 67.69178 0.0171374484 67.43841 49.74664 2.842171e-14 0.02205663
43    50.69696 77.16141 0.0215376941 77.85837 50.69696 2.842171e-14 0.07572514
44    50.55666 66.74652 0.0208850892 67.30318 50.55666 2.131628e-14 0.02736433
45    49.31124 68.96310 0.0092440233 68.27434 49.31124 2.131628e-14 0.03744683
46    49.29250 68.03596 0.0050213833 67.32847 49.29250 3.552714e-14 0.04921764
47    50.36458 68.40004 0.0215520776 68.76462 50.36458 3.552714e-14 0.01826358
48    50.76853 68.60443 0.0165759298 69.37297 50.76853 2.842171e-14 0.02250110
49    49.88765 72.47094 0.0085695716 72.35860 49.88765 2.842171e-14 0.02702832
```

```
50    50.88111 69.11335 0.0334638734 69.99446 50.88111 2.842171e-14 0.04677216
51    50.39811 67.47021 0.0070373741 67.86832 50.39811 2.842171e-14 0.02738920
52    49.38797 76.71519 0.0363128761 76.10317 49.38797 2.842171e-14 0.09128065
53    50.34112 68.92710 0.0536298170 69.26822 50.34112 2.842171e-14 0.09704183
54    48.87064 69.10222 0.0092971559 67.97285 48.87064 2.842171e-14 0.05691727
55    51.43302 69.49905 0.0099102942 70.93207 51.43302 3.552714e-14 0.04220150
56    51.98040 73.56333 0.0162899301 75.54373 51.98040 2.131628e-14 0.08826138
57    49.63278 69.63218 0.0034977763 69.26496 49.63278 2.842171e-14 0.03618049
58    48.95587 69.81183 0.0051574643 68.76769 48.95587 2.842171e-14 0.05991494
59    50.56972 66.59170 0.0184322711 67.16142 50.56972 2.842171e-14 0.02772283
60    49.86495 68.37865 0.0129031294 68.24359 49.86495 2.131628e-14 0.02052014
61    52.40162 70.30080 0.0014945680 72.70242 52.40162 2.842171e-14 0.10042059
62    49.96076 67.05553 0.0184696109 67.01629 49.96076 2.842171e-14 0.02226123
63    50.68974 72.65748 0.0041210709 73.34722 50.68974 2.842171e-14 0.03095501
64    50.02800 62.40803 0.0261997808 62.43603 50.02800 2.842171e-14 0.06744995
65    49.25673 71.53279 0.0805468791 70.78952 49.25673 2.131628e-14 0.24356832
66    50.18879 62.31775 0.0071855355 62.50654 50.18879 2.842171e-14 0.08013045
67    48.19504 68.49512 0.0455064886 66.69016 48.19504 3.552714e-14 0.09760958
68    51.46555 67.35860 0.0189471903 68.82416 51.46555 2.131628e-14 0.05324077
69    50.15325 66.73953 0.0135561374 66.89278 50.15325 2.842171e-14 0.02530942
70    52.17261 69.71552 0.0139501082 71.88813 52.17261 2.131628e-14 0.08633564
         .sigma      .cooksd  .std.resid
1  5.066616e-15 5.307226e-01 -7.66025591
2  1.495837e-14 1.418752e-02 -1.48070338
3  1.521186e-14 1.915585e-04  0.10714738
4  1.518271e-14 5.486408e-03  0.51401085
5  1.521056e-14 3.642169e-04  0.15099172
6  1.521170e-14 2.645126e-04  0.11353877
7  1.517002e-14 2.023748e-03  0.61154678
8  1.521048e-14 1.300986e-04  0.15314500
9  1.521041e-14 1.074192e-04  0.15526421
10 1.521178e-14 1.498162e-04  0.11058196
11 1.520254e-14 1.509239e-03 -0.30393684
12 1.520122e-14 4.323106e-04 -0.32222317
13 1.516498e-14 6.764687e-03  0.64620513
14 1.521308e-14 7.768606e-05  0.03042069
15 1.520839e-14 4.483794e-04  0.20391182
16 1.521086e-14 9.380469e-05  0.14220069
17 1.520991e-14 4.417104e-04  0.16850045
18 1.520107e-14 7.424018e-04 -0.32417535
19 1.521063e-14 1.613325e-04  0.14888595
20 1.520242e-14 4.921791e-04 -0.30560012
21 1.521086e-14 3.658557e-04  0.14208232
22 1.521041e-14 3.564197e-04  0.15521017
23 1.520987e-14 4.086890e-04  0.16958165
24 1.516675e-14 1.448465e-02  0.63426528
25 1.520932e-14 7.617036e-04  0.18326864
26 1.520990e-14 2.888309e-04  0.16885449
27 1.521065e-14 3.634170e-04  0.14829990
28 1.517214e-14 1.617741e-02  0.59640621
29 1.521269e-14 6.245641e-05  0.06564973
30 1.520024e-14 5.810753e-04 -0.33509719
31 1.520036e-14 1.642139e-03 -0.33351870
32 1.516954e-14 3.455356e-03  0.61497247
```

```
33 1.521074e-14 2.464007e-04  0.14562036
34 1.520029e-14 4.802367e-04 -0.33447465
35 1.517435e-14 8.986237e-03  0.58014681
36 1.517076e-14 9.972055e-03  0.60631430
37 1.517203e-14 4.106020e-03  0.59720009
38 1.516258e-14 4.238810e-03  0.66207624
39 1.520212e-14 8.160685e-04 -0.30981731
40 1.520334e-14 2.111800e-03  0.29221843
41 1.521179e-14 7.709561e-05  0.11001700
42 1.521118e-14 9.816836e-05  0.13194807
43 1.521033e-14 5.074447e-04  0.15740006
44 1.520153e-14 7.110164e-04 -0.31794536
45 1.519811e-14 1.271458e-03 -0.36156458
46 1.517006e-14 4.835531e-03  0.61126880
47 1.516948e-14 1.761129e-03  0.61536115
48 1.521091e-14 1.135505e-04  0.14046909
49 1.520973e-14 2.081507e-04  0.17312488
50 1.521028e-14 3.094770e-04  0.15883595
51 1.521119e-14 1.217094e-04  0.13148371
52 1.520998e-14 6.996276e-04  0.16691255
53 1.521073e-14 5.727841e-04  0.14600906
54 1.521204e-14 1.501848e-04  0.09976906
55 1.516389e-14 4.703669e-03  0.65346379
56 1.520540e-14 1.635025e-03 -0.25992139
57 1.521133e-14 1.516222e-04  0.12710776
58 1.521132e-14 2.577378e-04  0.12718476
59 1.521042e-14 1.711108e-04  0.15493337
60 1.519883e-14 6.520898e-04 -0.35285094
61 1.520653e-14 1.612393e-03  0.24036644
62 1.521181e-14 6.822809e-05  0.10948345
63 1.521026e-14 2.026733e-04  0.15930696
64 1.521123e-14 3.072524e-04  0.13035356
65 1.519558e-14 1.229282e-02 -0.39077805
66 1.521245e-14 1.402815e-04  0.08025915
67 1.517021e-14 1.006996e-02  0.61023202
68 1.520041e-14 1.557687e-03 -0.33286468
69 1.521147e-14 9.662335e-05  0.12200098
70 1.520324e-14 2.038639e-03 -0.29376393
```

iii) Are there any observations that have high leverage values? If so, what are the observation numbers.

Answer

Answer

iv) Are there any observations that are outliers? If so, what are the sample observation numbers?

Answer

Answer

Question 4

it is required to study the relationship between age (X) and girth (Y) of teak trees growing in a plantation. Note that girth is the diameter of the tree (in inches) measured at 5 inches above the ground. The girth of the trees and the ages (in years) have been recorded from a sample of 25 trees. Assume that the scatterplot of the data clearly shows a linear relationship between the two variables with an intercept.

i) Write the simple linear regression model that you would be fitted for the above variables. Define all terms in it and state any assumptions regarding the model.

Answer

ii) Later it was suggested that a linear model goes through the origin is suitable for this situation. Write the new model using the usual notation.

Answer

iii) The estimated regression model in part (ii) satisfied all of the assumptions regarding the error term. Sketch the residual plot vs fitted values and Q-Q normality plot of residuals.

Answer

Question 5

An experiment was conducted to determine the influence of sulphide concentration (X_1) on the whiteness of rayon (Y). The results obtained through R are given below.

```
x1 <- rnorm(15, mean=40)
y <- 13 + (2*x1) + rnorm(15)
df5 <- data.frame(x1=x1, Y=y)
mod5 <- lm(Y ~ x1, data=df5)
summary(mod5)
```

```
Call:
lm(formula = Y ~ x1, data = df5)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.69929 -0.48179  0.02163  0.66530  1.31226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   21.718     12.780   1.699    0.113    
x1             1.786      0.321   5.563 9.19e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9087 on 13 degrees of freedom
Multiple R-squared:  0.7042,    Adjusted R-squared:  0.6814 
F-statistic: 30.94 on 1 and 13 DF,  p-value: 9.185e-05
```

```
anova(mod5)
```

```
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1 25.550 25.5497  30.944 9.185e-05 ***
Residuals 13 10.734  0.8257                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

i) Construct the ANOVA table using the above results.

Answer

Answer

ii) Write the hypothesis to be tested in the ANOVA in part i.

ANOVA table

Source of variation	DF	Sum of squares (SS)	Mean Square (MS)	F	p-value
Regression	1	SSM = $\sum_{i=1}^{n}(\hat{Y}_i - \bar{y})^2$	MSR = $\frac{SSM}{1}$	$F* = \frac{MSR}{MSE}$	$P(F > F*)$
Residual error	$n-2$	SSE = $\sum_{i=1}^{n}(y_i - \hat{Y}_i)^2$	MSE = $\frac{SSE}{(n-2)}$		
Total	$n-1$	SST = $\sum_{i=1}^{n}(y_i - \bar{y})^2$			

Hypotheses

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

The p -value of this test is the same as the p -value of the t-test for $H_0 : \beta_1 = 0$, this only happens in simple linear regression.

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Answer

Answer

iii) What is your decision about the fitted model.

Answer