école normale supérieure paris–saclay

Optimal Transport applied to Generative Adversarial Networks

Deep Learning Project

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Outline

1. Introduction
2. Principle of the Generative Adversarial Network GAN
3. Wasserstein GAN
4. Difference between GAN and WGAN
5. Implementation
6. Some results
7. Conclusion

Introduction

- **Problem:** Lack of data in some fields (medicine, genetics, ...) for training

  $\longrightarrow$ Neuroimaging studies in 2017 and 2018 with median sample size = 12
- **Solution:** Generation of new data with same distribution

Principle of GAN (1)

Adversarial : generator and discriminator **competing** with each other

- **Generator G** turns noise *z* into an imitation of the data to fool the discriminator
- **Discriminator D** tries to identify real instances from fake generated samples

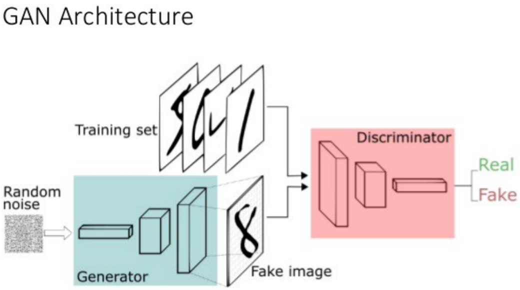


Figure: Principle of GAN

Principle of GAN (2)

The objective is dual :

$$\min_G \max_D \mathbb{E}_{x\sim p} \log D(x) - \mathbb{E}_{z\sim\mathcal{N}(0,\mathcal{I})} \log(1 - D(G(z))$$

which is approximated by :

$$\min_G \max_D \frac{1}{|\mathcal{T}|} \sum_{x\in\mathcal{T}} \log D(x) + \frac{1}{N_z} \sum_{z\sim\mathcal{N}(0,\mathcal{I})} \log(1 - D(G(z)))$$

$\longrightarrow$ Roughly equals to the **Jensen-Shannon divergence**

Wasserstein GAN

- **Different metric :** 1 - Wasserstein distance between two probability measures α, β

$$\mathcal{W}_1(\alpha, \beta) := \min_{\pi \in \Pi(\alpha,\beta)} \mathbb{E}_{(x,y)\sim\pi}(||x - y||)$$

Kantorovich-Rubinstein duality gives :

$$\mathcal{W}_1(\alpha, \beta) = \sup_{||f||_L \leqslant 1} \mathbb{E}_{x\sim\alpha}[f(x)] - \mathbb{E}_{x\sim\beta}[f(x)] \quad (1)$$

Wasserstein GAN (2)

The objective is dual :

$$\min_{\theta} \mathcal{W}_1(p, p_\theta) = \min_{\theta} \max_{||f||_L \leqslant 1} \mathbb{E}_{x\sim p}(f(x)) - \mathbb{E}_{y\sim p_\theta}(f(y))$$

which is approximated by :

$$\min_{\theta} \max_{w s.t. ||f_w||_L \leqslant 1} \frac{1}{|\mathcal{T}|} \sum_{x \in \mathcal{T}} f_w(x) - \frac{1}{N_z} \sum_{z \sim \mathcal{N}(0,\mathcal{I})} f_w(g_\theta(z))$$

f is called the critic

Difference between GAN and WGAN

- GAN : Jensen-Shannon divergence

  $\longrightarrow$ Loss discontinuity

  $\longrightarrow$ Training instability

- WGAN : Wasserstein distance

  $\longrightarrow$ Loss smoothness

  $\longrightarrow$ Improved Training stability

WGAN - Lipschitz constraint

Problem: How to enforce $||f_w||_L \leqslant 1$?

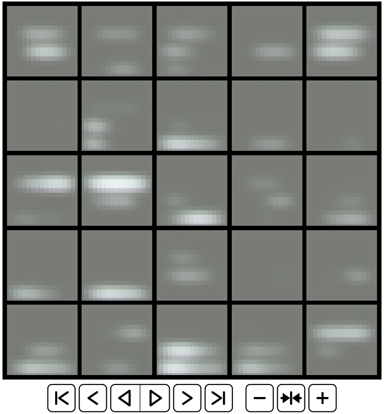
- **Solution :** Weight clipping

  $\longrightarrow$ Naive brute force

- **Improved solution :** Gradient Penalty term in the objective :

$$\lambda_{gp} \frac{1}{T} \sum_{x \in T} (||\nabla f_w(x)|| - 1)^2$$

Embedded Animation



Implementation

Algorithm 1 WGAN Pseudo Code Algorithm

for *iter* to N_{epochs} **do**
  for k_{critic} steps **do**
    Sample minibatch of m samples $\{z^{(1)}, \ldots, z^{(m)}\}$;
    Sample minibatch of m samples $\{x^{(1)}, \ldots, x^{(m)}\}$;
    Update parameters w for the critic f_w by stochastic gradient ascent
    $\nabla_\omega = \frac{\partial}{\partial w}\left[\frac{1}{m}\sum_i \left(f_w(x^{(i)}) - f_w(g_\theta(z^{(i)}))\right)\right];$
    $w \leftarrow \text{clip}(w, -c, c);$
  end for
  Sample minibatch of m samples $\{z^{(1)}, \ldots, z^{(m)}\}$;
  Update parameters θ for the generator g_θ by stochastic gradient descent
  $\nabla_\theta = \frac{\partial}{\partial \theta}\left[\frac{1}{m}\sum_i f_w(x^i) - \frac{1}{m}\sum_i f_w(g_\theta(z^i))\right];$
end for

Some results

MNIST dataset

Standard GAN

WGAN

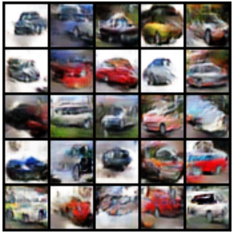
WGAN-GP

Some results

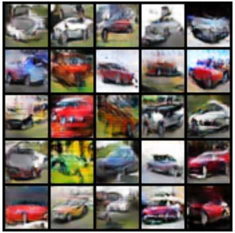
CIFAR dataset

Standard GAN

WGAN



WGAN-GP



Conclusion

- GANs: Promising results but instability, mode collapse
- WGANs : Introduction of Optimal Transport with ML
- To be studied: Sample quality metric, hyper parameter tuning

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References

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[2] M. Arjovsky, S. Chintala, and L. Bottou. Wasserstein gan

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Thank You

for Listening.