Additionaly, optimal (primal and dual) solutions need to be feasible:

$$\begin{cases} \|X^T\mu^*\|_\infty \leq \lambda \\ Xw^* - y = r^* \\ r^* = -\mu^* \quad \text{(from Q1)} \\ \forall i \in [\![1;d]\!],\ w_i^*(\lambda - (X^T\mu^*)_i) = 0 \end{cases} \iff \begin{cases} \|X^T\mu^*\|_\infty \leq \lambda \\ Xw^* = y - \mu^* \quad (*) \\ w_i^*(\lambda - (X^T\mu^*)_i) = 0 \end{cases}$$

To recover w^* from $(*)$ we would like to invert X:

$$\begin{pmatrix} \boxed{x_1} & \cdots & \boxed{x_d} \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_p \\ 0 \\ \vdots \\ 0 \end{pmatrix} = y - \mu^*$$

If we sort w s.t. $\forall i \in [\![1;p]\!],\ w_i \neq 0$, we can remove
$\forall i > p,\ w_i = 0$.

from X the redundant columns: from $\boxed{x_{p+1}}$ to $\boxed{x_d}$.

Then we only keep $\tilde{X} = \begin{pmatrix} \boxed{x_1} & \cdots & \boxed{x_p} \end{pmatrix}$, which we might

invert to recover w^* from $*$:

$$w^* = \tilde{X}^{-1}(y - \mu^*).$$