RMSC 4007
Risk Management with Derivatives Concepts
Department of Statistics, The Chinese University of Hong Kong

Investigation on Auto-callable Reverse Convertible with Memory Coupon
FINAL REPORT

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Abstract

This project is going to compare the simulated price results of a complex structured product: *Auto-callable Reverse Convertible with Memory Coupon*, with the quoted market price, using three different pricing models. The associated risks are then analyzed, followed by possible methods proposed to manage the underlying risks. Some limitations about the project will be discussed in the last section.

1 Description of product

The following table only shows useful information of the product. For the full version of the term sheet, please refer to the 'References' section [1].

Useful Information of Term Sheet

Name	*Auto-callable Reverse Convertible*	
Issuer	*Credit Suisse AG, Zurich, acting through its Nassau Branch, Bahamas*	
Product Currency	*USD*	
Number of Underlying	*3*	
Underlying	*1*	*EURO STOXX 50 Index (SX5E)*
	2	*Swiss Market Index (SMI)*
	3	*FTSE 100 Index (UKX)*
Initial Level	*SX5E*	*3,607.77*
	SMI	*9,269.86*
	UKX	*7,516.17*

Issue Date	*24/10/2017*	
Last Trading Date	*21/10/2020*	
Final Fixing Date	*21/10/2020*	
Final Redemption Date	*26/10/2020*	
Denomination	*USD1,000*	
Minimum Trading Lot	*USD1,000*	
Issue Price	*100% * Denomination*	
Strike	*SX5E*	*2705.8275*
	SMI	*6952.395*
	UKX	*5637.1275*

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1. DESCRIPTION OF PRODUCT

The values of ***Initial Level*** and ***Strike*** are fixed at 17/10/2017 by the issuer. Note that the values of ***Strike*** are exactly equal to 75% of the ***Initial Level*** of three underlying correspondingly.

Payoff structure

Payout Observation Date	*Payout Threshold (applicable to all 3 underlying)*	*Payout Dates*
19/1/2018	75%	24/1/2018
19/4/2018	75%	24/4/2018
19/7/2018	75%	24/7/2018
19/10/2018	75%	24/10/2018
21/1/2019	75%	24/1/2019
17/4/2019	75%	24/4/2019
19/7/2019	75%	24/7/2019
21/10/2019	75%	24/10/2019
21/1/2020	75%	24/1/2020
21/4/2020	75%	24/4/2020
21/7/2020	75%	24/7/2020
21/10/2020	75%	26/10/2020

Trigger Observation Dates	*Trigger Barrier (applicable to all 3 underlying)*	*Trigger Redemption Dates*
19/4/2018	100%	24/4/2018
19/7/2018	100%	24/7/2018
19/10/2018	100%	24/10/2018
21/1/2019	100%	24/1/2019
17/4/2019	100%	24/4/2019
19/7/2019	100%	24/7/2019
21/10/2019	100%	24/10/2019
21/1/2020	100%	24/1/2020
21/4/2020	100%	24/4/2020
21/7/2020	100%	24/7/2020
19/4/2018	100%	24/4/2018
19/7/2018	100%	24/7/2018

1. *Coupon paying*

If the closing level of each Underlying on the relevant ***Payout Observation Date*** is at or above its ***Payout Threshold***, a payout amount of $\frac{5.20\%\ p.a}{4} \times USD\$1000$ will be paid to the investor on the corresponding ***Payout Dates***. Otherwise, no payout will be given.

2. *Memory function*

If the condition mentioned in the last paragraph is fulfilled on a certain ***Payout Observation Date***, coupons which has not been paid to the investor before on the past ***Payout Date***, will also be paid altogether with the coupon paid this time, on the related ***Payout Date***. For example, supposed it is the third Payout Observation Date now, and coupons were not paid during the first two time, then the payout amount this time will be $\frac{5.20\%\ p.a}{4} \times USD\$1000 \times 3$.

3. *Autocall*

If the closing level of each Underlying on a ***Trigger Observation Date*** is at or above its ***Trigger Barrier***, the Product is redeemed on the related ***Trigger Redemption Date*** with USD 1,000. Note that the related payout (coupons) on that observation date will also be paid to the investor. Otherwise, nothing will happen.

4. *Final redemption*

On the ***Final Redemption Date*** specified on the term sheet above, the investor may obtain a ***Final Redemption Amoun***t, plus the sum of the ***Payout Amounts (coupons)*** due on the Payout Dates.

The ***Final Redemption Amount*** is defined as follows:

- If the Final Level of each Underlying is at or above its Strike: **Denomination**
- If the Final Level of at least one Underlying is below its Strike: $\frac{USD\$1000 \times \textit{final level of Worst-Performing Underlying}}{\textit{Its corresponding strike}}$

where Worst-Performing Underlying = underlying having the lowest $\left(\frac{Final\ level}{Strike}\right)$ *ratio.*

2 Incentive for investors

Optimal payoff, for any length of holding period, can be achieved if all underlying lie between 75% and 100% of their initial level during the holding period. Thus, the product suits for investors who are speculating on all three indices lying within this range, and provides the advantage of leveraging where investors are not required to pay the huge full amount of the three indices.

The Reverse-Convertible characteristic of product provides the following advantages:

1. Provides a relatively high coupon rate ($\frac{5.20\%\ p.a}{4}$), which is attractive during a low interest rate environment
2. Favorable in the case of a market with high volatility:

2.1 Even the underlying drops below 75%, Memory Function provides investors with a hope that when the underlying bounces back, he/she can get back the coupon(s) not paid previously.

2.2 When the underlying moves above 100%, early-redemption can at least guarantee the investors getting back the amount of Denomination.

The product is also equivalent as shorting multiple At-the-money down-and-in put options.

3 Pricing models

In this project, we are going to price the product with three models: 1. traditional Black-Scholes Model 2. Heston Stochastic Volatility (SV) model and 3. Heston SV model with Jump Diffusion incorporated.

The reason of employing Heston SV model is to obtain a better fit to the volatility smile observed in market. We then try to combine Heston SV model and Jump Diffusion model together, with the aim of accounting for rare events happened in countries related to underlying, as well as discontinued sample paths observed. Performance will be examined in later section.

Black-Scholes Model

We define the followings (which are also applicable to all dynamics shown below):

$S_{(1,2,3)}$	EURO STOXX 50 Index (SX5E), Swiss Market Index (SMI), FTSE 100 Index (UKX)
$F_{(1,2,3)}$	EUR/USD, CHF/USD, GBP/USD
$\mathrm{Q}^{(1,2,3)}$	Risk neutral probability measure under Euro market, Swiss market, UK market
$\mathbb{Q}$	Risk neutral probability measure under US market
$r_{(1,2,3)}$	Euro, Swiss, UK interest rate
r	US interest rate

Under the physical probability measure $\mathbb{P}$,

$$\begin{cases} \dfrac{dS_j}{S_j} = \mu_j^S dt + \sigma_j^S dW_j^{\mathrm{P}}(t) \\ \dfrac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+3}^{\mathrm{P}}(t) \end{cases} \quad \text{for } j = 1,2,3 \qquad \text{and } \mathbb{E}\left(dW_a^{\mathrm{P}} dW_b^{\mathrm{P}}\right) = \rho_{a,b}\, dt \quad \text{for } \begin{cases} a = 1,2,3,4,5,6 \\ b = 1,2,3,4,5,6 \end{cases}$$

Using standard change of measure, from $\mathbb{P}$ to $\mathbb{Q}^{(j)}$,

$$\begin{cases} \dfrac{dS_j}{S_j} = r_j dt + \sigma_j^S dW_j^{\mathbb{Q}^{(j)}}(t) & \textit{for } j = 1,2,3 \\ \dfrac{dF_j}{F_j} = (r - r_j)dt + \sigma_j^F dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

and finally from $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

$$\frac{dS_j}{S_j} = (r_j - \rho_{j,\,j+3}\,\sigma_j^S \sigma_j^F)dt + \sigma_j^S dW_j^{\mathbb{Q}}(t)$$

Heston Stochastic Volatility (SV) Model

Under the physical probability measure $\mathbb{P}$,

$$\textit{for } j = 1,2,3 \quad \begin{cases} \dfrac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_j^S} dW_j^P(t) \\ dV_j^S = \kappa_j(\theta_j - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^P(t) \\ \dfrac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+6}^P(t) \end{cases}$$

and $\mathbb{E}(dW_a^{\mathrm{P}} dW_b^{\mathrm{P}}) = \rho_{a,b}\, dt$

for $\begin{cases} a = 1,2,\ldots,8,9 \\ b = 1,2,\ldots,8,9 \end{cases}$

κ = mean-reverting rate of V^S

θ = long-run average V^S

Using standard change of measure, from $\mathbb{P}$ to $\mathbb{Q}^{(j)}$,

$$\textit{for } j = 1,2,3 \quad \begin{cases} \dfrac{dS_j}{S_j} = r_j dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t) \\ dV_j^S = \kappa_j^*(\theta_j^* - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \\ \dfrac{dF_j}{F_j} = (r - r_j)dt + \sigma_j^F dW_{j+6}^{\mathbb{Q}}(t) \end{cases}$$

$\kappa_j^* = \kappa_j + c_j$

$\theta_j^* = \dfrac{\kappa_j \theta_j}{\kappa_j + c_j}$

$c_j \in \mathbb{R}$

for j = 1,2,3

and finally from $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

$$\textit{for } j = 1,2,3 \quad \begin{cases} \dfrac{dS_j}{S_j} = (r_j - \rho_{j,\,j+6}\, \sigma_j^F \sqrt{V_j^S})dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t) \\ dV_j^S = \left[\kappa_j^*(\theta_j^* - V_j^S) - (\rho_{j,\,j+3})(\rho_{j,\,j+6})(\sigma_j^F \sigma_j^V)\sqrt{V_j^S} \right] + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

Heston Stochastic Volatility Model with Jump Diffusion

Under the physical probability measure $\mathbb{P}$,

$$\textit{for } j = 1,2,3 \quad \begin{cases} \dfrac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_j^S} dW_j^P(t) + \boldsymbol{J_j\, dN_j(t)} \\ \qquad = \mu_j^S dt + \sqrt{V_j^S} dW_j^{\mathrm{P}}(t) + \boldsymbol{(e^{Y_j} - 1)\, dN_j(t)} \\ dV_j^S = \kappa_j(\theta_j - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^P(t) \\ \dfrac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+6}^P(t) \end{cases}$$

We assume the following Independency:

- $dN_a \perp\!\!\!\perp dN_b$
- $J_a \perp\!\!\!\perp J_b$
- $dN_a \perp\!\!\!\perp dW_b$
- $Y_j \sim N(m_j, v_j)$
- $\mathbb{E}(dN_j^{\mathrm{P}}) = \lambda_j dt$

Using standard change of measure, from $\mathbb{P}$ to $\mathbb{Q}^{(j)}$,

for j = 1,2,3

$$\begin{cases} \frac{dS_j}{S_j} = \left\{ \boldsymbol{r_j - \lambda_j \left[\mathbb{E}\left(e^{Y_j}\right) - 1\right]} \right\} dt + \sqrt{V_j^S}\, dW_j^{\mathbb{Q}^{(j)}}(t) + \boldsymbol{(e^{Y_j} - 1) dN_j(t)} \\ dV_j^S = \kappa_j^*\left(\theta_j^* - V_j^S\right) dt + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \\ \frac{dF_j}{F_j} = (r - r_j) dt + \sigma_j^F\, dW_{j+6}^{\mathbb{Q}}(t) \end{cases}$$

κ_j^*, θ_j^* same as above

and finally from $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

for j = 1,2,3

$$\begin{cases} \frac{dS_j}{S_j} = \left\{ \boldsymbol{r_j - \lambda_j \left[\mathbb{E}\left(e^{Y_j}\right) - 1\right]} - \rho_{j,\, j+6}\, \sigma_j^F \sqrt{V_j^S} \right\} dt + \sqrt{V_j^S}\, dW_j^{\mathbb{Q}}(t) + \boldsymbol{\left(e^{Y_j} - 1\right) dN_j(t)} \\ dV_j^S = \left[\kappa_j^*\left(\theta_j^* - V_j^S\right) - \left(\rho_{j,\, j+3}\right)\left(\rho_{j,\, j+6}\right)\left(\sigma_j^F \sigma_j^V\right) \sqrt{V_j^S} \right] + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

The detailed proves for the derivation of formulas above will be shown in the Appendix, along with the correlation structure ($\rho_{a,b}$).

For both Heston and Heston with jump models, we have assumed the risk premium of index's variance is proportional to the variance itself, i.e. $risk\ \ premium = c_j V_j^S, where\ \ c_j \in \mathbb{R}$

In Heston with Jump Diffusion model, we express the Jump as $\boldsymbol{(e^{Y_j} - 1) dN_j(t)}$ instead of simply $\boldsymbol{(Y_j) dN_j(t)}$, the reason will be shown in Appendix [a], with reference to [2]. For simplicity we have further assumed that Y_j follows normal distribution with mean m_j and variance v_j, as in Merton's original paper (Merton(1976)) [3].

4 Parameters calibration/estimation

Calibration

We have implemented the ***Differential Evolution (DE) Algorithm*** to obtain the best set of parameters for Heston and Heston with Jump models, where the parameters needed to be calibrated are listed as follows:

Heston SV model	$\{\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4}\}$, $\{\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5}\}$,	$\{\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}\}$
Heston SV model with Jump Diffusion	$\{\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4}\}$, $\{\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5}\}$, $\{\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}\}$	$\boldsymbol{\lambda_j,\ m_j,\ v_j}$ $(for\ j = 1,2,3)$

DE algorithm greatly reduces the chance of staying at a local minimum, furthermore it copes with a discontinuous cost function. For settings we have chosen NP = 75, no. of generations = 1500, CR = 0.5, F = 0.8 as suggested by Vollrath and Wendland (2009) [4]. The cost function is defined as follows:

$$\sum_{i=1}^{N} \left(w_i \left(c_i^{mkt} - c_i^{model} \right)^2 \right)$$

$$\begin{aligned} where\ \ c_i^{mkt} &= market\ observed\ call\ price \\ c_i^{model} &= call\ price\ calculated\ by\ model\ (obtained\ \ by\ \ IFFT) \\ w_i &= weighting\ (Equal) \end{aligned}$$

For each index, around 40 standard European calls with approximately 2 years maturity are available and we have obtained their mid-price and implied volatility from Bloomberg for calibration. For simplicity, we have performed the calibration for each index separately. After an investigation on their respective volatility smile (figures are available in the corresponding excel files), we suspected that the smile for SX5E and SMI could be fully explained by the Heston model alone, as there were no irregularities shown on the volatility smile. Thus, as a comparison, we have decided to also consider a situation that the jump diffusion is only included in the dynamics of UKX, i.e. we have set all $\lambda_j, m_j, v_j \ \ for \ \ j = 1,2$ to 0. The tables below show the calibration result for both Heston and Heston with jump models, and Feller condition is satisfied in Heston model:

Heston	κ_j^*	θ_j^*	σ_j^V	$V_j^S(0)$	$\rho_{j,j+3}$
SX5E (j=1)	0.55893	0.06543	0.26796	0.00115	-0.57148
SMI (j=2)	1.30631	0.02579	0.24243	0.01999	-0.70398
UKX (j=3)	1.23484	0.02599	0.25325	0.01953	-0.66184

Heston+Jump	κ_j^*	θ_j^*	σ_j^V	$V_j^S(0)$	$\rho_{j,j+3}$	λ_j	m_j	v_j
SX5E (j=1)	19.95531	0.01717	0.30519	0.00546	-0.99074	0.01504	-2.19378	0.90609
SMI (j=2)	19.34681	0.01049	0.40454	0.05663	-0.96687	0.08218	-0.31372	0.36849
UKX (j=3)	5.07212	0.01125	0.10706	0.02536	-0.62628	0.02211	-1.47102	0.02415

Heston+Jump	κ_j^*	θ_j^*	σ_j^V	$V_j^S(0)$	$\rho_{j,j+3}$	λ_j	m_j	v_j
SX5E (j=1)	0.55893	0.06543	0.26796	0.00115	-0.57148	**0**	**0**	**0**
SMI (j=2)	1.30631	0.02579	0.24243	0.01999	-0.70398	**0**	**0**	**0**
UKX (j=3)	5.07212	0.01125	0.10706	0.02536	-0.62628	0.02211	-1.47102	0.02415

Estimation

Firstly, the initial index prices on 19/11/2018 for all 3 indices are obtained from Bloomberg terminal, where

$$S_1(0) = 3160.33, \qquad S_2(0) = 8812.61, \qquad S_3(0) = 7000.89$$

Secondly, the 2-year interest rates for respective countries are also extracted from Bloomberg, where the 2-year government bond yield for Euro area, Switzerland, United Kingdom and United States are chosen:

$$r_1 = -0.619\%, \qquad r_2 = -0.723\%, \qquad r_3 = 0.687\%, \qquad r = 2.779\%$$

For the Black-Scholes model, we choose to use the volatility index (VIX) for each index and exchange rate as a proxy for their corresponding volatilities, where:

$$\sigma_1^S = 18.00\%, \qquad \sigma_2^S = 16.72\%, \qquad \sigma_3^S = 16.32\%,$$

$$\sigma_1^F = 7.70\%, \qquad \sigma_2^F = 6.72\%, \qquad \sigma_3^F = 13.45\%$$

For all models in this project, the correlations between indices, the correlations between exchange rates and the cross correlations between indices and exchange rates could be found by historical estimation (steps are shown in related Excel files), where

$$\rho_{1,2} = 0.78724, \rho_{1,3} = 0.74052, \rho_{2,3} = 0.70554, \rho_{7,8} = 0.75447, \rho_{7,9} = 0.60728, \rho_{8,9} = 0.49501$$

$$\rho_{1,7} = 0.10702, \rho_{2,8} = 0.25440, \rho_{3,9} = 0.28148$$

Lastly, for Heston and Heston with jump model, correlations between either the indices or exchange rates and volatilities could be found by the formulas below as suggested by Szimayer, Dimitroff and Lorenz (2009) [5]:

$$\text{For } \ i \neq \text{j}, \frac{dS_i(t)dV_j^S(t)}{\sqrt{\left(dS_i(t)\right)^2\left(dV_j^S(t)\right)^2}} = \rho_{i,j}\rho_j; \quad \frac{dF_i(t)dV_j^S(t)}{\sqrt{\left(dF_i(t)\right)^2\left(dV_j^S(t)\right)^2}} = \rho_{i,j}\rho_j$$

5 Simulation of product price

In Black-Scholes model, direct Monte-Carlo simulation is used. For detailed steps, please referred to the VBA code in the related Excel file. While for the remaining two models, direct simulation is no longer possible due to the complicated dynamics of volatility, the modified procedure is as follows:

1. Generate 6 Independent Normal Random Variables and perform Cholesky Decomposition
2. Discretize the SDE of V_j^S as aforementioned, to obtain the following approximation for $\widehat{V_j^S}$:
$$\widehat{V_j^S}(t+\Delta) = \widehat{V_j^S}(t) + \left[\kappa_j^*\left(\theta_j^* - \widehat{V_j^S}(t)\right) - (\rho_{j,\,j+3})(\rho_{j,\,j+6})(\sigma_j^F\sigma_j^V)\sqrt{\widehat{V_j^S(t)}}\right]\Delta + \sigma_j^V\sqrt{\widehat{V_j^S}(t)}Z_V\sqrt{\Delta}$$
3. Employ the ***Full Truncation scheme***: $\widehat{V_j^S}^* = \max(\widehat{V_j^S}, 0)$
4. Use the $\widehat{V_j^S}^*$ obtained in step 3 to simulate $\widehat{S_j}$ at each time step
   4.1. If Jump added, then also perform the following to simulate S
   - Generate 3 independent Poisson Random Variables for jump frequency
   - Generate another 3 independent Normal Random Variables for jump size
5. Repeat step 1 to 4 for m times, where m = defined time step = 1/253

Full Truncation scheme

One major obstacle of simulation under Heston model with traditional Euler Scheme is that negative volatility may be generated during simulation. Thus, a possible solution is to employ other scheme when generating the volatility. In this project, we opt for the ***Full Truncation scheme:*** $\widehat{V_j^S}^* = \max(\widehat{V_j^S}, 0)$, rather than the ***Quadratic-exponential (QE) discretization scheme*** which is proved to have better performance. The reason of abandoning the QE scheme will be further explained in the 'Limitations' section [X].

Result Analysis

In the simulation results shown below, we use 10,000 sample paths for each simulation, and 30 simulations for each model to obtain the statistics of simulated product price.

The following table shows the simulation result of different models:

	Mean of simulated price	Standard deviation of simulated price	Difference from market mid-price in percentage
Market mid-price	948	N/A	N/A
Black-Scholes Model	991.25401	7.69029	4.56266%
Heston Model	916.74316	11.5495517	-3.29714%
Heston with Jump Diffusion	731.7037	1.61413	-22.81607%
Heston for SX5E and SMI + Heston with Jump for UKX	874.5258	1.57917	-7.75044%

As we could see from the table, the simulated product price under Heston model achieved the smallest percentage difference when compared to the market mid-price. A possible explanation is the Heston Model handles the problem of volatility smile in a better way. Therefore, the model acts as a better proxy for the market than the standard Black-Scholes model which assumes constant volatility for all three indices.

However, we can see that the simulated price using Heston with Jump model deviates significantly from the market price for over 20%. Even if we include the Jump-Diffusion only for UKX, a difference of around 7% can still be observed. One possible explanation is that actually all 3 indices are already well explained by a 2-factor model, where Heston model is in fact an example of 2-factor model. The 2 factors here are respectively the wiener processes in the index dynamics and the variance dynamics. The preliminary evidence to support this claim could be seen in the calibration result, we observed that when the Jump-Diffusion processes are introduced to the Heston model, the parameter sets for both SX5E and SMI are quite extreme, with $\kappa \sim 19$ and $\sigma_j^V > 0.95$. Even the parameter set for UKX is acceptable, the simulated product price is still much lower than the market mid-price, possibly due to the more-frequent hit to the barrier of 75% and 100% introducing by adding Jump, causing a smaller sum of payoffs. Thus, it could be a hint that a 3-factor model (i.e. Heston with jump) actually over parametrizes the dynamics of indices and a 2-factor model is already enough for all 3 indices.

Merton Jump-Diffusion Model

Thus, based on this hypothesis, it is natural to also consider ***Merton Jump-Diffusion model*** for all 3 indices as it is also a 2 factor model, with the first factor being the Wiener process and second one being the Poisson process. The following table shows the calibration result as well as the pricing result using Merton Jump-Diffusion model, note that the percentage difference this time (-2.77146%) is even smaller than Heston model:

Merton Jump	σ_j^S	λ_j	m_j	v_j
SX5E (j=1)	0.05903	0.34547	0.23183	0.32911
SMI (j=2)	0.05381	0.34901	0.20479	0.31851
UKX (j=3)	0.13162	0.28995	0.05645	0.15122

	Mean of simulated price	Standard deviation of simulated price	Difference from market mid-price in percentage
Market mid-price	948	N/A	N/A
Merton Jump Diffusion Model	921.72657	1.66714	**-2.77146%**
Heston Model	916.74316	11.54955	-3.29714%

In conclusion, among all models we have considered for the time being, the Merton Jump-Diffusion Model is the best model in reflecting the market most accurately, and it caters for the problem of discontinuous path caused by rare events and the **problem of volatility smile** (further explained on 'Conclusion' part).

6 Risk Analysis

There are mainly three types of risk associated with the product: Currency Risk, Credit Risk, Model Risk.

Currency Risk

This Quanto Product is associated with currency risk from the dynamics of indices in different models: Black-Scholes Model: [X]; Heston Model [X], Heston with Jump Model: [X].
All three dynamics of indices involve the correlations between underlying and its corresponding foreign exchange rate ($\rho_{a,b}$) and variances of foreign exchange rate, constituting the components of currency risk for the product.

Credit Risk

By looking at the credit ratings on debts issued by Credit Suisse, which are slightly lower than the industry average, one can argue that the credit risk brought the issuer is not negligible.

Debts issued by Credit Suisse	Moody's	S&P
Rating on Senior Unsecured Debt	Baa2	BBB+
Rating on Subordinated Debt	Baa3	BBB+

In the previous section, even the closest simulated prices generated by Merton jump diffusion model still exhibit a small difference with the market price. It provides us a hint that the market price may already be incorporating factor of credit risk. However, concrete validation of the argument requires further investigation since the computation of Default Probability and valuation adjustment of counterparty credit risk are not the main focus of this project.

Model Risk

Black-Scholes model assumes constant interest rate and constant volatility, where the latter violates the observed volatility smile, and the normal assumption of this model does not incorporate the fat tail distribution observed in the stock price market data.

In Heston Model, parameters are sensitive to initial data set and it still assumes continuous sample path of stock price, while in the Heston with Jump Model, issue of over-parametrized might occurred. Independent jump sizes and Poisson processes are assumed in this model as well, and the distribution of jump sizes needs not to be following Lognormal distribution.

7 Managing Risk

Hedging

Perfect hedging for such a complicated structured product would be extremely difficult and we are even not certain whether the partial derivatives of product price function i.e. the Greeks (Δ,V,Θ,Γ), exists as the function might be discontinuous.

As for replicating cash flow of product, we suggest using cash-or-nothing put option which pay fixed amount of money $S_j(t) < K$ to replicate coupon payments, but there exist possibility of over-hedging and introducing additional credit risk. As for replicating the final redemption payments, standard European put option can be used, while the additional transaction cost will be a potential issue.

Even if we can perfectly replicate our targeted product using other structured product(s) available in market, such as corridor variance swap, new risk and cost introduced may further complicate the situation and hence much harder to analysis.

Parameter sensitivity analysis

Apart from hedging, one could also analyse the impact on product price by adjusting the parameters in our pricing model, i.e. analysing the sensitivity of product price to certain parameters. Referencing the concept of elasticity in economics, we define $sensitivity = \left|\frac{\Delta P/P}{\Delta\varphi/\varphi}\right|$ $, where\ \ \varphi\ \ represents\ \ parameter.$

The tables below show the result of Black-Scholes model and Heston model (for detailed steps please refer to corresponding excel files). From the tables, we see that the product price is quite sensitive to change in all 3 index prices and all 3 index's volatilities. Furthermore, in the case of Heston model, we see that the product price exhibits certain degree of sensitivity to all parameters, thus verifying our claim above that this model is very sensitive to initial data.

Black-Scholes Model

	S1	S2	S3	sigma_S1	sigma_S2	sigma_S3
Sensitivity	0.10837	0.02166	0.01523	0.03486	0.00840	0.01642
	r1	**r2**	**r3**			
Sensitivity	0.00000	0.00000	0.00000			

Heston Model

	kappa1	kappa2	kappa3	theta1	theta2	theta3
Sensitivity	0.02390	0.00126	0.00159	0.03245	0.01321	0.01583
	V1(0)	**V2(0)**	**V3(0)**	**sigma_v1**	**sigma_v2**	**sigma_v3**
Sensitivity	0.00096	0.00838	0.00864	0.04380	0.03054	0.03214

8 Conclusion

To summarize, we believe that two-factor models like Merton Jump-Diffusion Model and Heston Model perform well in pricing our structured product, with both models being able to generate an implied volatility smile, as the Merton Jump-Diffusion Model actually randomized the volatility component with Poisson shocks (Chan and Wong, 2013) [2]. Furthermore, Merton Jump-Diffusion Model stands out from Heston Model a little bit, since it handles the problem of discontinuous path caused by rare events, where Heston Model is unable to do so.

We also believe that it is hard to perform a perfect hedge for the product. One way out is to analyze how sensitive the product price will be when the parameters of models change. The product price is sensitive to Index Price and Index Volatility, as well as all parameters in Heston Model.

9 Limitations

In this section, we are going to cover what we are unable to perform in this project due to time constraint, theoretical difficulties, or due to our limit of knowledge.

Model assumption

Similar to the section of Model Risk, we are concerning about whether the distribution of jump sizes will change over time or not. Secondly, independency or correlation in Poisson process and jump size still requires further investigation.

Calibration

During Calibration, we are in fact trying to fit our model to the market implied volatility smile. Thus, a rational choice would be to obtain a set of standard European call/put options with the same maturity but with different strikes. As the time-to-maturity of our product is around 2 years, we should also collect calls with maturity 2 years for calibration. However, difficulty arises since the liquidity of 2 year calls on different strikes is not high enough, which results in a narrow spectrum of strike if only calls with maturity 2 years are collected. Thus, to compensate this liquidity issue, we have gathered both 1 year and 2 year calls and mixed them together for calibration and the detail selection of data could be viewed in the corresponding calibration excel files.

Simulation

As mentioned above, we have abandoned the QE scheme in our project and adopted the Full truncation scheme.

The reason is that we are not certain about the distribution of V_j^S under $\mathbb{Q}$-world. Originally, the QE scheme is designed based on the fact that when V is large it is proportional to Noncentral chi-squared Random Variable and when V is small it is proportional to an ordinary Central chi-squared Random Variable. However, in our model, there is an extra $\sqrt{V_j^S}$ term inside the coefficient of dt, so we are not certain about the distribution of V_j^S anymore. For simplicity, we decide to abandon the QE scheme at the end and use the full truncation scheme to ensure the simulation procedure is correct.

Furthermore, due to the heavy computation introduced by the full truncation scheme, we are only abled to simulate 30 times for each model, so we could at least make a fair comparison among the models.

Appendices

The derivation of Heston Model (From P to Q(j), From Q(j) to Q)

From $\mathbb{P} \to \mathbb{Q}^{(j)}$, for S_j SDE

Consider $d(e^{-r_j t} S_j)$,

$$d(e^{-r_j t} S_j) = \left(-r_j e^{-r_j t} S_j + (\mu_j^S S_j) e^{-r_j t} + 0\right) dt + (\sqrt{V_j^S} S_j) e^{-r_j t} dW_j^{\mathbb{P}}(t)$$

$$\Rightarrow \frac{d(e^{-r_j t} S_j)}{e^{-r_j t} S_j} = (\mu_j^S - r_j) dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t)$$

We know that under $\mathbb{Q}^{(j)}$ world, discounted index price, i.e. $(e^{-r_j t} S_j)$ is a martingale.

So, by Girsanov Theorem, the θ that connects $dW_j^{\mathbb{P}}(t)$ and $dW_j^{\mathbb{Q}^{(j)}}$ and satisfies the above condition is as follow:

$$dW_j^{\mathbb{Q}^{(j)}}(t) = dW_j^{\mathbb{P}}(t) - \theta dt, \text{ where } \theta = \frac{r_j - \mu_j^S}{\sqrt{V_j^S}} = \text{ Market price of risk.}$$

i.e. θ is chosen to eliminate the dt term.

$\therefore$ Under $\mathbb{Q}^{(j)}$,

$$\frac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_j^S} \left(dW_j^{\mathbb{Q}^{(j)}}(t) + \left(\frac{r_j - \mu_j^S}{\sqrt{V_j^S}} \right) dt \right)$$

$$\Rightarrow \frac{dS_j}{S_j} = r_j dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t)$$

In $\mathbb{Q}^{(j)}$,

$$\begin{cases} \frac{dS_j}{S_j} = r_j dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t) \\ dV_j = \kappa_j^*(\theta_j^* - V_j)dt + \sigma_j^V \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j+3)}}(t) \quad \text{, for } j = 1, 2, 3 \\ \frac{dF_j}{F_j} = (r - r_j)dt + \sigma_j^F dW_j^{\mathbb{Q}}(t) \end{cases}$$

$\therefore$ By Cholesky decomposition:

$$dW_{j+3}^{\mathbb{Q}^{(j)}}(t) = \rho_{(j,j+3)} dW_j^{\mathbb{Q}^{(j)}}(t) + \sqrt{1 - \rho_{(j,j+3)}^2} dW_\perp^{\mathbb{Q}^{(j)}}(t)$$

$$dW_j^{\mathbb{Q}^{(j)}}(t) = \rho_{(j,j+6)} dW_{j+6}^{\mathbb{Q}^{(j)}}(t) + \sqrt{1 - \rho_{(j,j+6)}^2} dW_\perp^{\mathbb{Q}^{(j)}}(t)$$

and Girsanov Theorem: choose $\theta = \sigma_j^F$,

$$dW_{j+6}^{\mathbb{Q}^{(j)}}(t) = dW_{j+6}^{\mathbb{Q}} - \sigma_j^F dt$$

We get: $dW_j^{\mathbb{Q}^{(j)}}(t) = \rho_{(j,j+6)} \left[dW_{j+6}^{\mathbb{Q}}(t) - \sigma_j^F dt\right] + \sqrt{1 - \rho_{(j,j+6)}^2} dW_\perp^{\mathbb{Q}}(t)$

$$\Rightarrow dW_j^{\mathbb{Q}}(t) = dW_j^{\mathbb{Q}}(t) - \rho_{(j,j+6)} \sigma_j^F dt$$

and $dW_{j+3}^{\mathbb{Q}^{(j)}}(t) = \rho_{(j,j+3)} \left[dW_j^{\mathbb{Q}}(t) - \rho_{(j,j+6)} \sigma_j^F dt\right] + \sqrt{1 - \rho_{(j,j+3)}^2} dW_\perp^{\mathbb{Q}}(t)$

$$\Rightarrow dW_{j+3}^{\mathbb{Q}^{(j)}}(t) = dW_{j+3}^{\mathbb{Q}}(t) - \rho_{(j,j+3)} \rho_{(j,j+6)} \sigma_j^F dt$$

So, in $\mathbb{Q}$, for $j = 1, 2, 3$

$$\begin{cases} \frac{dS_j}{S_j} = (r - \rho_{(j,j+6)} \sigma_j^F \sqrt{V_j^S})dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t) \\ dV_j = (\kappa_j^*(\theta_j^* - V_j^S) - \rho_{(j,j+3)} \rho_{(j,j+6)} \sigma_j^F \sigma_j^V \sqrt{V_j^S})dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

The derivation of Heston With Jump Model

$$\frac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_S^j} dW_j^{\mathbb{P}}(t) + J_j dN_j(t)$$

$$= \mu_j^S dt + \sqrt{V_S^j} dW_j^{\mathbb{P}}(t) + e^{Y_j - 1} dN_j(t)$$

$$dV_j = \kappa_j(\theta_j - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{P}}(t)$$

$$\frac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+6}^F(t)$$

$$where \quad \mathbb{E}(dW_a dW_n) = \rho_{a,b} dt$$
$$dN \perp\!\!\!\perp dN, dN \perp\!\!\!\perp dW, J \perp\!\!\!\perp J \quad with\ Y_j \sim N(m_j, v_j), \mathbb{E}(dN_j) = \lambda_j dt$$

Justification of changing from P to Q(j)

$$f = lnS_j(t)$$

$$df = dlnS_j(t)$$
$$= (\mu_j^S S_j)\left(\frac{1}{S_j}\right)dt + \frac{1}{2}\left(-\frac{1}{S_j}\right)(V_j^S S_j^2)dt + \left(\sqrt{V_j^S} S_j\right)\left(\frac{1}{S_j}\right)dW_j^{\mathbb{P}}(t) + [lnS_t - lnS_{t^-}]dN_j(t)$$

$$\Rightarrow dlnS_j = \left(\mu_j^S - \frac{V_j^S}{2}\right)dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + ln\left(\frac{S_t}{S_{t^-}}\right)dN_j(t)$$

Consider a time point t where a jump just occurred,

$$\because S_t - S_{t^-} = \left(e^{Y_j} - 1\right)S_{t^-}$$
$$S_t = S_{t^-} e^{Y_j}$$
$$\Rightarrow ln\frac{S_t}{S_{t^-}} = Y_j$$

$$\therefore dlnS_j = \left(\mu_j^S - \frac{V_j^S}{2}\right)dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + Y_j dN_j(t)$$

$$\text{Under } \mathbb{Q}^{(j)},\ dlnS_j = \left(\mu_j^* - \frac{V_j^S}{2}\right)dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t) + Y_j dN_j(t)$$

$$\Rightarrow ln\frac{S_t}{S_0} = \left(\mu_j^* - \frac{V_j^S}{2}\right)dt + \left(-\int_0^t \frac{V_j^S}{2} ds + \int_0^t \sqrt{V_j^S} dW_j^{\mathbb{Q}(j)}(t)\right) + \sum_{k=1}^{N(t)} Y_j(k)$$

$$\Rightarrow S_t = S_0 exp\left(\mu_j^* t + \left(-\int_0^t \frac{V_j^S}{2} ds + \int_0^t \sqrt{V_j^S} dW_j^{\mathbb{Q}(j)}(t)\right) + \sum_{k=1}^{N(t)} Y_j(k)\right)$$

$$\Rightarrow \mathbb{E}_0^{\mathbb{Q}^{(j)}}(S_t) = S_0 exp\left(\mu_j^* t\right) \mathbb{E}_0^{\mathbb{Q}^{(j)}}\left[exp\left(-\int_0^t \frac{V_j^S}{2} ds + \int_0^t \sqrt{V_j^S} dW_j^{\mathbb{Q}(j)}(t)\right)\right] \mathbb{E}_0^{\mathbb{Q}^{(j)}}\left[exp\left(\sum_{k=1}^{N(t)} Y_j(k)\right)\right]$$

By Girsanov Theorem,

$$\mathbb{E}_0^{\mathbb{Q}^{(j)}}\left[exp\left(-\int_0^t \frac{V_j^S}{2} ds + \int_0^t \sqrt{V_j^S} dW_j^{\mathbb{Q}(j)}(t)\right)\right] = \mathbb{E}_0^{\mathbb{Q}^*}(1) = 1$$

$$\Rightarrow \mathbb{E}_0^{\mathbb{Q}^{(j)}}(S_t) = S_0 exp\left(\mu_j^* t\right) exp\left(\lambda t \mathbb{E}\left(\left(e^Y\right) - 1\right)\right)$$

By Martingale condition, under $\mathbb{Q}^{(j)}$ world

$$RHS = S_0 exp\left(\mu_j^* t\right) exp\left(\lambda_j t \mathbb{E}\left(\left(e^{Y_j}\right) - 1\right)\right) = S_0 exp\left(r_j t\right)$$

$$\Rightarrow \mu_j^* = r_j - \lambda_j\left(\mathbb{E}\left(e^{Y_j} - 1\right)\right)$$

$\therefore$ under $\mathbb{Q}^{(j)}$world, for $j = 1, 2, 3$

$$\begin{cases} \frac{dS_j}{S_j} = \left[r_j - \lambda_j\left(\mathbb{E}\left(e^{Y_j} - 1\right)\right)\right] dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + \left(e^{Y_j} - 1\right) dN_j(t) \\ dV_j = \kappa_j^*\left(\theta_j^* - V_j^S\right) dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}(j)}(t) \\ \frac{dF_j}{F_j} = \left(r - r_j\right) dt + \sigma_j^F dW_{j+6}^{\mathbb{Q}(j)}(t) \end{cases}$$

$\mathbb{Q}^{(j)} \to \mathbb{Q}$

$$\begin{cases} \frac{dS_j}{S_j} = \left[r_j - \lambda_j\left(\mathbb{E}\left(e^{Y_j} - 1\right)\right) - \rho_{(j,j+6)\sigma_j^F}\sqrt{V_j^S}\right] dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + \left(e^{Y_j} - 1\right) dN_j(t) \\ dV_j = \left[\kappa_j^*\left(\theta_j^* - V_j^S\right) - \rho_{(j,j+3)}\rho_{(j,j+6)}\sigma_j^F \sigma_j^V \sqrt{V_j^S}\right] dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}(j)}(t) \end{cases}$$

Characteristic function of log-asset price of Heston Model

$$\Phi_j(x_j, v_j, t) = exp\{i\phi(x_j) + A(t,T) + B(t,T)V_j\}$$

where $A(t,T) = ir_j\phi(T-t) + \frac{\kappa_j^*\theta_j^*}{(\sigma_j^V)^2}\left[(\kappa_j^* - i\phi\rho_{(j,j+3)}\sigma_j^V + d)(T-t) - 2ln\left(\frac{1-ge^{d(T-t)}}{1-g}\right)\right]$,

$$B(t,T) = \left(\frac{\kappa_j^* - i\phi\rho_{(j,j+3)}\sigma_j^V + d}{(\sigma_j^V)^2}\right)\left(\frac{1-e^{d(T-t)}}{1-ge^{d(T-t)}}\right), \quad g = \frac{\kappa_j^* - i\phi\rho_{(j,j+3)}\sigma_j^V + d}{\kappa_j^* - i\phi\rho_{(j,j+3)}\sigma_j^V - d},$$

$d = \sqrt{(\kappa_j^* - i\phi\rho_{(j,j+3)}\sigma_j^V)^2 + (\sigma_j^V)^2(i\phi - \phi^2)}$, where $i = \sqrt{-1}$

Characteristics function of log-asset price of Heston With Jump Model

Set $X_j(T) = lnS_j(T)$

$$\Phi_j(x, v, t; \phi) = \mathbb{E}_t^{\mathbb{Q}^{(j)}}\{e^{i\phi X_j(T)}\} \quad , x = lnS_0, v = V_j(0)$$

By previous page, we already know that, under $\mathbb{Q}^{(j)}$:

$$dlnS_j = \left(r_j - \frac{V_j^S}{2} - \lambda_j\left(\mathbb{E}\left(e^{Y_j}\right) - 1\right)\right)dt + \sqrt{V_j^S}dW_j^{\mathbb{Q}^{(j)}}(t) + Y_j dN_j(t)$$

So, $lnS_j(T)$

$$= \left[lnS_j(t) + \int_t^T \left(r_j - \frac{V_j^S}{2}\right)ds + \int_t^T \sqrt{V_j^S}dW_j^{\mathbb{Q}^{(j)}}(t)\right] + \left(-\int_T^t \lambda_j\left(\mathbb{E}\left(e^{Y_j}\right) - 1\right)ds + \sum_{k=1}^{N_j(t)} Y_j(k)\right)$$

$$\therefore \Phi_j = \mathbb{E}_t^{\mathbb{Q}^{(j)}}\{e^{i\phi X_T}\} = exp\left(i\phi x + A(t,T) + B(t,T)v\right)\mathbb{E}_t^{\mathbb{Q}^{(j)}}\{e^{i\phi\left(-\int_t^T \lambda_j\left(\mathbb{E}\left(e^{Y_j}\right)-1\right)ds + \sum_{k=1}^{N_j(T)} Y_j(k)\right)}\}$$

$$= exp\left[i\phi x + A(t,T) + B(t,T)v - i\phi\lambda_j(\mathbb{E}^{\mathbb{Q}^{(j)}}(e_j^Y) - 1)(T-t) + \lambda_j(T-t)(\mathbb{E}^{\mathbb{Q}^{(j)}}(e^{Y_j\phi i}) - 1)\right]$$

where $A(t,T), B(t,T)$ is found in Heston's characteristics function

$$\mathbb{E}^{\mathbb{Q}^{(j)}}\left(e^{Y_j}\right) = e^{m_j + \frac{V_j}{2}}$$

$$\mathbb{E}^{\mathbb{Q}^{(j)}}\left(e^{i\phi Y_j}\right) = e^{i\mu\phi - \frac{\phi^2\sigma^2}{2}}$$

Analytical solution of Call price by Inverse Fourier Transform

$$C_j(S,V,t) = e^{-rT} e^{-\alpha lnK} \mathcal{F}^{-1}_{\xi,k}\left[\frac{\Phi_j(\phi = \xi - (\alpha+1)i)}{\alpha^2 + \alpha - \xi^2 + i(2\alpha+1)\xi}\right]$$

$\mathcal{F}^{-1}_{\xi,k}(...)$ denotes inverse fourier transform with respect to $k = lnK$

Black-Scholes Model Setting

$$\begin{cases} \frac{dS_j}{S_j} = \mu_j^S dt + \sigma_j^S dW_j^{\mathbb{P}}(t) \\ \frac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+3}^{\mathbb{P}}(t) \end{cases}, \text{ for } j = 1,2,3$$

where $\mathbb{E}(dW_a dW_b) = \rho_{(a,b)} dt$

$$\begin{cases} \frac{dS_j}{S_j} = r_j dt + \sigma_j^S dW_j^{\mathbb{Q}^{(j)}}(t) \\ \frac{dF_j}{F_j} = (r - r_j) dt + \sigma_j^F dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \end{cases}, \text{ for } j = 1,2,3$$

where $r =$ U.S. interest rate,
$r_1 =$ euro interest rate, $r_2 =$ Swiss interest rate, $r_3 =$ UK interest rate

$$F_1 = \frac{\text{EUR}}{\text{USD}}, F_2 = \frac{\text{CHF}}{\text{USD}}, F_3 = \frac{\text{GBP}}{\text{USD}}$$

From $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$,

$$\frac{dS_j}{S_j} = (r_j - \rho_{(j,j+3)} \sigma_j^S \sigma_j^F) dt + \sigma_j^S dW_j^{\mathbb{Q}}(t) \text{ for } j = 1,2,3$$

Under $\mathbb{Q}^{(j)}$:

$$\begin{cases} \frac{dS_j}{S_j} = r_j dt + \sigma_j^S dW_j^{\mathbb{Q}^{(j)}}(t) \\ \frac{dF_j}{F_j} = (r - r_j) dt + \sigma_j^F dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \end{cases}, \text{ for } j = 1,2,3$$

where $\mathbb{E}(dW_a dW_b) = \rho_{(a,b)} dt$, for $a, b = 1$ to 9

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Division of Labour

All major decisions are made together by all group members. The major responsibilities of members are listed as follows:

Kenneth, CHENG Tsun Him:
Group Leader, Proof and derivation of formula/dynamics, coding
Anson, LEUNG Chi Shun:
Proof and derivation of formula/dynamics, pricing simulation, risk analysis
Edward, CHAN Chi Fung:
Building PowerPoint and Final report, error checking of theories/ideas, risk managing
YIP Kam Pak:
Parameters calibration, limitations, appendices