RMSC4007 Project Presentation

Investigation on
Auto-callable Reverse Convertible with Memory Coupon

Issued by: CREDIT SUISSE
ISIN: *CH0384867401*

Group 1
Kenneth, CHENG Tsun Him
Anson, LEUNG Chi Shun
Edward, CHAN Chi Fung
YIP Kam Pak

Main Objective

- Price a structural Quanto product (3 underlyings)
  1. Black-Scholes Model
  2. Heston Stochastic Volatility Model

     (to fit volatility smile)
  3. Heston SV x Jump-diffusion Model

     (rare events like Brexit, discontinuous sample path)

- Compare the simulation results with quoted market price

- Analyse the associated risks

Contents

Description (term sheet)

Product Currency		USD
Number of Underlying		3
Underlying	1	EURO STOXX 50 Index (SX5E)
	2	Swiss Market Index (SMI)
	3	FTSE 100 Index (UKX)
Issue Date		24/10/2017
Last Trading Date		21/10/2020
Final Fixing Date		21/10/2020
Final Redemption Date		26/10/2020
Denomination	USD	1000
Minimum Trading Lot	USD	1000
Issue Price		100% * Denomination
Initial Level (Fix @ 17/10/2017)	SX5E	3,607.77
	SMI	9,269.86
	UKX	7,516.17
Strike (75% of Initial)	SX5E	2705.8275
	SMI	6952.395
	UKX	5637.1275

Local currency

Foreign Equity index

Valuation date = 19/11/2018
∴Maturity (T-t) ≈ 2 years

USD$1000, 2000, 3000…..

Predetermined

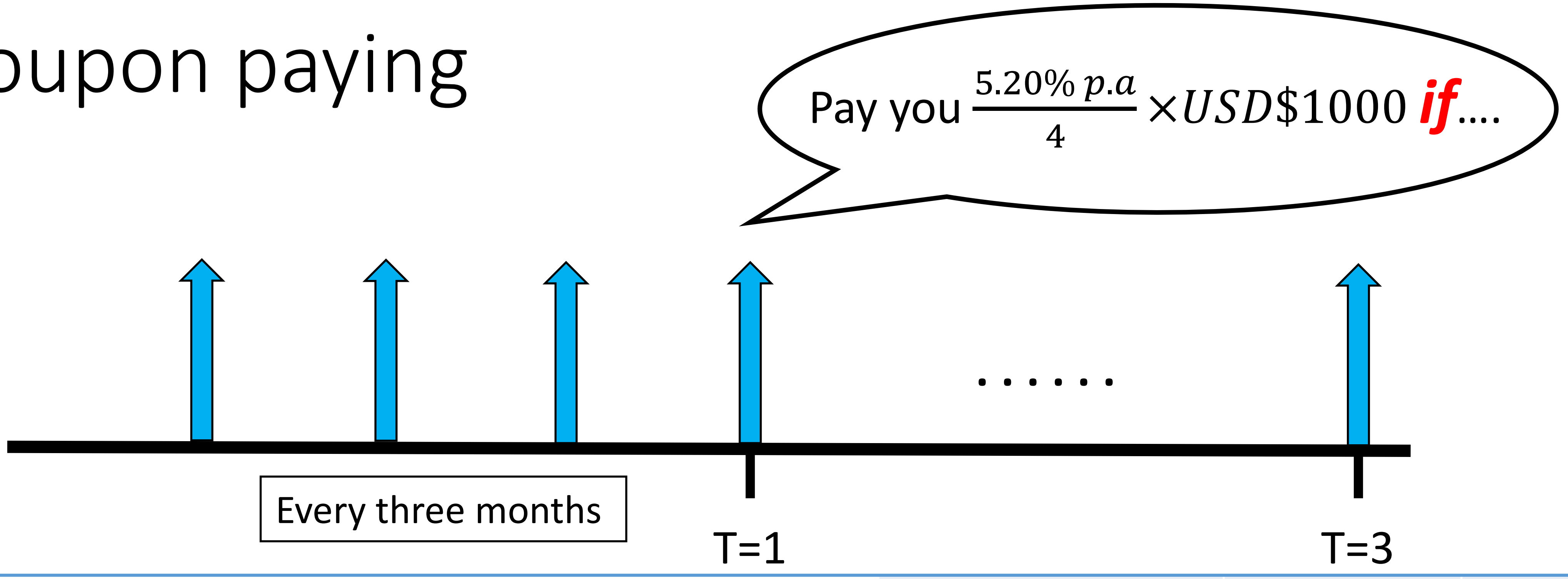
Special features

1. Coupon paying

2. Autocall

Path-dependency

1. Coupon paying

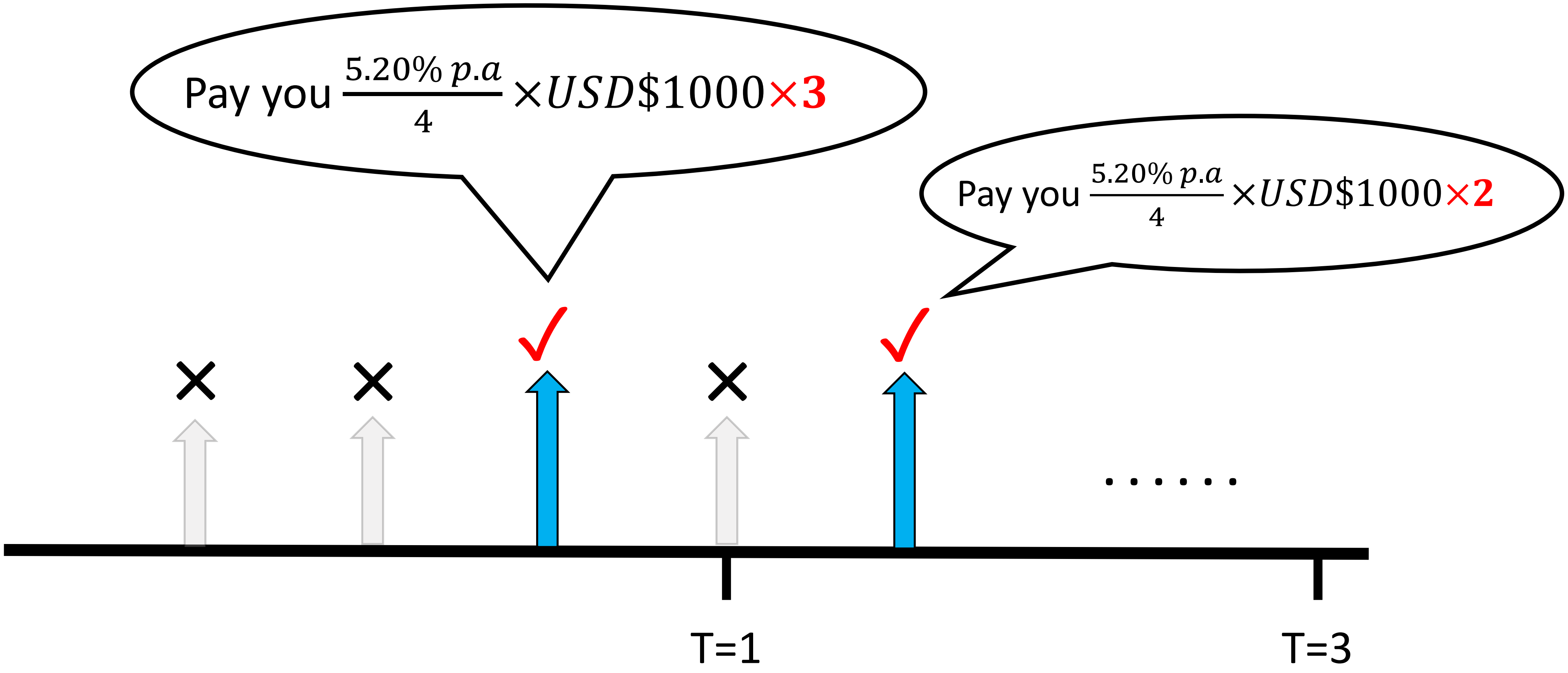


- If closing level for EACH underlying **≥ threshold**
  - EURO STOXX 50 Index (SX5E) ≥ 75% of initial level
    Swiss Market Index (SMI) ≥ 75% of initial level
    FTSE 100 Index (UKX) ≥ 75% of initial level

- Otherwise: No coupon is paid.

Payout Threshold (% to the Initial Level)	19/1/2018	75%
	19/4/2018	75%
	19/7/2018	75%
	19/10/2018	75%
	21/1/2019	75%
	17/4/2019	75%
	19/7/2019	75%
	21/10/2019	75%
	21/1/2020	75%
	21/4/2020	75%
	21/7/2020	75%
	21/10/2020	75%

1. Coupon paying – Memory function



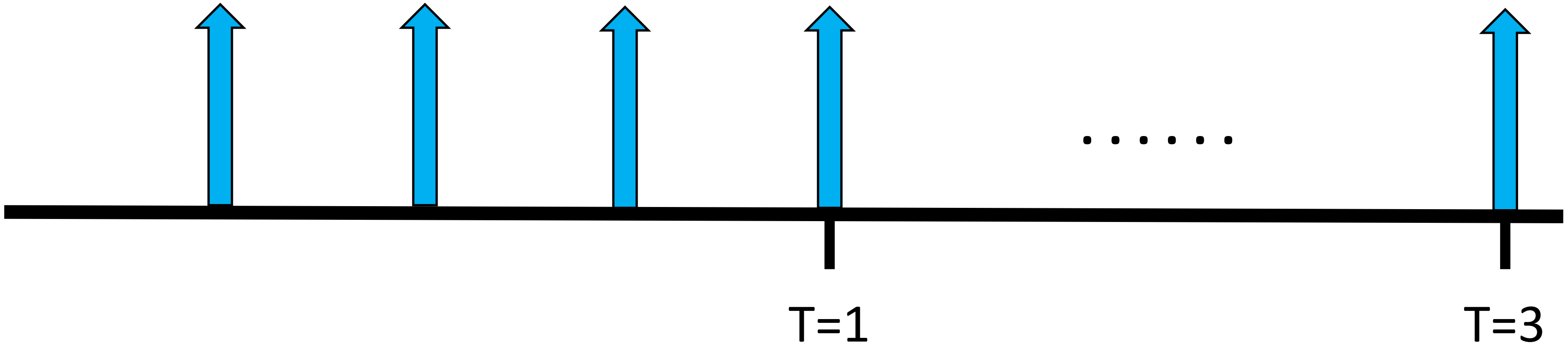
Special features

1. Coupon paying

2. Autocall

2. Autocall feature

Pay you
1. $USD\$1000+$
2. Payout needed **+end** ***if....***



- If closing level for EACH underlying **≥ barrier**
  - EURO STOXX 50 Index (SX5E) ≥ 100% of initial level
    Swiss Market Index (SMI) ≥ 100% of initial level
    FTSE 100 Index (UKX) ≥ 100% of initial level

- Otherwise: nothing happen

same as coupon

Trigger Barrier
(In % to the Initial Level)

19/4/2018	100%
19/7/2018	100%
19/10/2018	100%
21/1/2019	100%
17/4/2019	100%
19/7/2019	100%
21/10/2019	100%
21/1/2020	100%
21/4/2020	100%
21/7/2020	100%

Final redemption logic

Terms used in final redemption

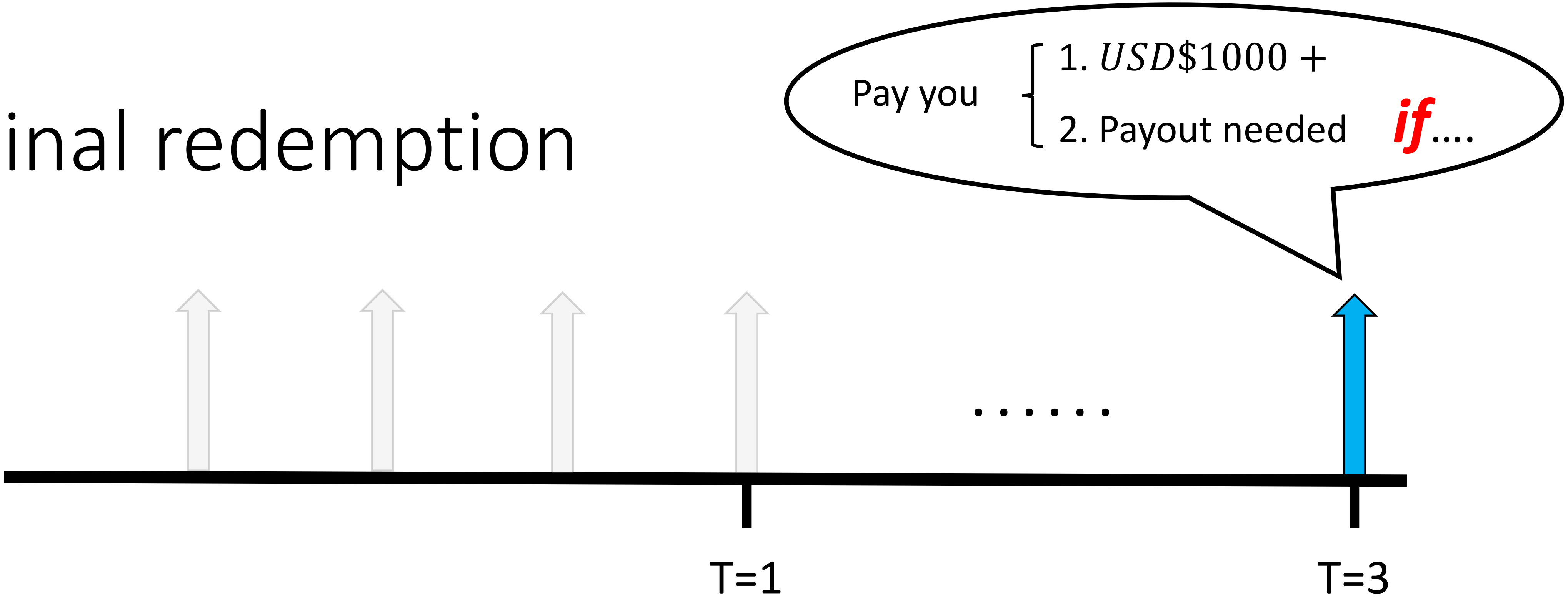
- *First define…*

Final Level	100% of closing level on *final fixing date*
Worst Performing Underlying	underlying having the lowest $\left(\frac{Final\ level}{Strike}\right)$ ratio

Strike (75% of Initial)	SX5E	2705.8275
	SMI	6952.395
	UKX	5637.1275

Predetermined

Final redemption



- If *final level* for EACH underlying **≥ its strike (=75% of initial)**
  - EURO STOXX 50 Index (SX5E) ≥ 2705.8275
    Swiss Market Index (SMI) ≥ 6952.395
    FTSE 100 Index (UKX) ≥ 5637.1275

Strike	SX5E	*2705.8275*
	SMI	*6952.395*
	UKX	*5637.1275*

- Otherwise: pay you $\dfrac{USD\$1000 \times \textit{final level of Worst–Performing Underlying}}{\textit{its corresponding strike}}$

Coupon payout	> 75% of initial level → GOOD
Autocall barrier	> 100% of initial level → BAD
Final redemption	> 75% of initial level → GOOD

- *What are we betting on?*
  - If $(75\% \leq underlying\ level \leq 100\%)$ all the time

    → *Happy~~~~~~~~~*

- *What are the incentives to trade this product?*

  1. For investors who are speculating on all 3 indices, all fall between 75%-100% of initial level
  2. Leveraging

- *What are the incentives to trade this product? (continued)*

3. *Reverse convertible*:

   - Favorable for **low interest rate environment**

     - $\frac{5.20\%\ p.a}{4}$

   - Favorable for **market of high volatility**

     Down: Hope to bounce back, Memory Function

     Up: Early redemption: get back capital

   - Increase the protection for investor:

     - alternative to shorting ATM down-and-in put options

Pricing models adopted

1. Black-Scholes model

2. Heston Stochastic Volatility *(SV)* model

3. Heston *SV* model × Jump-diffusion model

1. Black-Scholes Model

S_1 = EURO STOXX 50 Index (SX5E)
S_2 = Swiss Market Index (SMI)
S_3 = FTSE 100 Index (UKX)

F_1 = EUR/USD
F_2 = CHF/USD
F_3 = GBP/USD

BS Model in **physical world**

- Under $\mathbb{P}$:

$$\begin{cases} \dfrac{dS_j}{S_j} = \mu_j^S dt + \sigma_j^S dW_j^{\mathbb{P}}(t) \\ \dfrac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+3}^{\mathbb{P}}(t) \end{cases} \qquad \textit{for } j = 1,2,3$$

and $\mathbb{E}\left(dW_a^{\mathbb{P}} dW_b^{\mathbb{P}}\right) = \rho_{a,b}\, dt$ *for* $\begin{cases} a = 1,2,3,4,5,6 \\ b = 1,2,3,4,5,6 \end{cases}$

BS Model in **foreign world**

- Change of measure:

$\mathbb{Q}^{(1)}$ = risk-neutral world under Euro market
$\mathbb{Q}^{(2)}$ = …… under Swiss market
$\mathbb{Q}^{(3)}$ = …… under UK market

r_1 = Euro interest rate (Taken from yield curve)
r_2 = Swiss interest rate (Taken from govt. bond yield)
r_3 = UK interest rate (Taken from govt. bond yield)
r = U.S. interest rate (Taken from govt. bond yield)

BS Model in **foreign world**

- From $\mathbb{P}$ to $\mathbb{Q}^{(j)}$:

$$\begin{cases} \dfrac{dS_j}{S_j} = r_j dt + \sigma_j^S dW_j^{\mathbb{Q}^{(j)}}(t) \\ \dfrac{dF_j}{F_j} = (r - r_j)dt + \sigma_j^F dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

for j = 1,2,3

(Proof is shown in the appendix of report)

BS Model in **local world**

- Change of measure: $\boxed{\mathbb{Q} = \text{ risk-neutral world under U.S. market}}$

- From $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

$$\frac{dS_j}{S_j} = (r_j - \rho_{j,j+3}\, \sigma_j^S \sigma_j^F)dt + \sigma_j^S dW_j^{\mathbb{Q}}(t) \qquad \textit{for } j = 1,2,3$$

(Proof is shown in the appendix of report)

Correlation structure in BS Model

	dW_1	dW_2	dW_3	dW_4	dW_5	dW_6
dW_1	dt					
dW_2	$\rho_{1,2}$	dt				
dW_3	$\rho_{1,3}$	$\rho_{2,3}$	dt			
dW_4	$\rho_{1,4}$	$\rho_{2,4}$	$\rho_{3,4}$	dt		
dW_5	$\rho_{1,5}$	$\rho_{2,5}$	$\rho_{3,5}$	$\rho_{4,5}$	dt	
dW_6	$\rho_{1,6}$	$\rho_{2,6}$	$\rho_{3,6}$	$\rho_{4,6}$	$\rho_{5,6}$	dt

Source of parameters

Parameter types	Estimated by Historical data	Calibrated by option data	Extract from Bloomberg	Imply by others
Volatilities	$\sigma_1^S, \sigma_2^S, \sigma_3^S,$ $\sigma_1^F, \sigma_2^F, \sigma_3^F$	N/A	N/A	N/A
Correlations	$\rho_{1,2}, \rho_{1,3}, \rho_{1,4},$ $\rho_{2,3}, \rho_{2,5}, \rho_{3,6},$ $\rho_{4,5}, \rho_{4,6}, \rho_{5,6}$	N/A	N/A	$\rho_{1,5}, \rho_{1,6}, \rho_{2,4},$ $\rho_{2,6}, \rho_{3,4}, \rho_{3,5},$
Initial values	N/A	N/A	$S_1(0), S_2(0), S_3(0),$ $F_1(0), F_2(0), F_3(0),$ r_1, r_2, r_3, r	N/A

Pricing models adopted

1. Black-Scholes model

2. **Heston Stochastic Volatility *(SV)* model**

3. Heston *SV* model × Jump-diffusion model

2. Heston Stochastic Volatility *(SV)* model

S_1 = EURO STOXX 50 Index (SX5E)
S_2 = Swiss Market Index (SMI)
S_3 = FTSE 100 Index (UKX)

$\mathbb{Q}^{(1)}$ = risk-neutral world under Euro market
$\mathbb{Q}^{(2)}$ = ... under Swiss market
$\mathbb{Q}^{(3)}$ = ... under UK market

F_1 = EUR/USD
F_2 = CHF/USD
F_3 = GBP/USD

r_1 = Euro interest rate (Taken from yield curve)
r_2 = Swiss interest rate (Taken from govt. bond yield)
r_3 = UK interest rate (Taken from govt. bond yield)
r = U.S. interest rate (Taken from govt. bond yield)

Heston Model in **physical world**

- Under $\mathbb{P}$:

$$\begin{cases} \dfrac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) \\ dV_j^S = \kappa_j(\theta_j - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{P}}(t) \\ \dfrac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+6}^{\mathbb{P}}(t) \end{cases}$$

for j = 1,2,3

κ	= mean-reverting rate of V^S
θ	= long-run average V^S
σ^V	= volatility of V^S

and $\mathbb{E}\left(dW_a^{\mathbb{P}} dW_b^{\mathbb{P}}\right) = \rho_{a,b}\, dt$ *for* $\begin{cases} a = 1,2,\ldots,8,9 \\ b = 1,2,\ldots,8,9 \end{cases}$

Heston Model in **foreign world**

- From $\mathbb{P}$ to $\mathbb{Q}^{(j)}$:

for j = 1,2,3

$$\begin{cases} \dfrac{dS_j}{S_j} = r_j dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t) \\ dV_j^S = \kappa_j^* \left(\theta_j^* - V_j^S\right) dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \\ \dfrac{dF_j}{F_j} = (r - r_j) dt + \sigma_j^F dW_{j+6}^{\mathbb{Q}}(t) \end{cases} ,$$

$$\boxed{\begin{aligned} &\kappa_j^* = \kappa_j + c_j \\ &\theta_j^* = \frac{\kappa_j \theta_j}{\kappa_j + c_j} \\ &c_j \in \mathbb{R} \end{aligned}}$$

for j = 1,2,3

(Proof is shown in the appendix of report)

Heston Model in **local world**

- From $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

$$\begin{cases} \dfrac{dS_j}{S_j} = (r_j - \rho_{j,j+6}\,\sigma_j^F\sqrt{V_j^S})dt + \sqrt{V_j^S}dW_j^{\mathbb{Q}}(t) \\ dV_j^S = \left[\kappa_j^*(\theta_j^* - V_j^S) - (\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F\sigma_j^V)\sqrt{V_j^S}\right]dt \\ \qquad + \sigma_j^V\sqrt{V_j^S}\,dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

for j = 1,2,3

(Proof is shown in the appendix of report)

Correlation structure in Heston Model

	dW_1	dW_2	dW_3	dW_4	dW_5	dW_6	dW_7	dW_8	dW_9
dW_1	dt								
dW_2	$\rho_{1,2}$	dt							
dW_3	$\rho_{1,3}$	$\rho_{2,3}$	dt						
dW_4	$\rho_{1,4}$	$\rho_{2,4}$	$\rho_{3,4}$	dt					
dW_5	$\rho_{1,5}$	$\rho_{2,5}$	$\rho_{3,5}$	$\rho_{4,5}$	dt				
dW_6	$\rho_{1,6}$	$\rho_{2,6}$	$\rho_{3,6}$	$\rho_{4,6}$	$\rho_{5,6}$	dt			
dW_7	$\rho_{1,7}$	$\rho_{2,7}$	$\rho_{3,7}$	$\rho_{4,7}$	$\rho_{5,7}$	$\rho_{6,7}$	dt		
dW_8	$\rho_{1,8}$	$\rho_{2,8}$	$\rho_{3,8}$	$\rho_{4,8}$	$\rho_{5,8}$	$\rho_{6,8}$	$\rho_{7,8}$	dt	
dW_9	$\rho_{1,9}$	$\rho_{2,9}$	$\rho_{3,9}$	$\rho_{4,9}$	$\rho_{5,9}$	$\rho_{6,9}$	$\rho_{7,9}$	$\rho_{8,9}$	dt

Source of parameters

Parameter types	Estimated by Historical data	Calibrated by option data	Extract from Bloomberg	Imply by others
Volatilities	$\sigma_1^F, \sigma_2^F, \sigma_3^F$	N/A	N/A	N/A
Heston's Parameters	N/A	$\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4};$ $\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5};$ $\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}$	N/A	N/A
Other Correlations $(\rho_{\text{a},b})$	(1,2), (1,3), (1,7), (2,3), (2,8), (3,9), (7,8), (7,9), (8,9)	N/A	N/A	(2,4), (3,4), (1,5), (3,5), (4,5), (1,6), (2,6), (4,6), (5,6), (2,7), (3,7), (4,7), (5,7), (6,7), (1,8), (3,8), (4,8), (5,5), (6,5), (1,9), (2,9), (4,9), (5,9), (6,9)
Initial values	N/A	N/A	$S_1(0), S_2(0), S_3(0),$ $F_1(0), F_2(0), F_3(0),$ r_1, r_2, r_3, r	N/A

Pricing models adopted

1. Black-Scholes model

2. Heston Stochastic Volatility *(SV)* model

3. Heston *SV* model × Jump-diffusion model

Heston x Jump Model in **physical world**

- Under $\mathbb{P}$:

Lognormal Jump size

for j = 1,2,3

$$\begin{cases} \frac{dS_j}{S_j} = \mu_j^S dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + \boldsymbol{J_j\, dN_j(t)} \\ \quad = \mu_j^S dt + \sqrt{V_j^S} dW_j^{\mathbb{P}}(t) + \boldsymbol{\left(e^{Y_j} - 1\right) dN_j(t)} \\ dV_j^S = \kappa_j(\theta_j - V_j^S)dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{P}}(t) \\ \frac{dF_j}{F_j} = \mu_j^F dt + \sigma_j^F dW_{j+6}^{\mathbb{P}}(t) \end{cases}$$

and $\mathbb{E}\left(dW_a^{\mathbb{P}} dW_b^{\mathbb{P}}\right) = \rho_{a,b}\, dt$ for $\begin{cases} a = 1,2,\dots,8,9 \\ b = 1,2,\dots,8,9 \end{cases}$

Independency:

- $dN_a \perp\!\!\!\perp dN_b$
- $J_a \perp\!\!\!\perp J_b$
- $dN_a \perp\!\!\!\perp dW_b^{\mathbb{P}}$

- $Y_j \sim N(m_j, v_j)$
- $\mathbb{E}\left(dN_j^{\mathbb{P}}\right) = \lambda_j dt$

Heston x Jump Model in **foreign world**

- From $\mathbb{P}$ to $\mathbb{Q}^{(j)}$:

for j = 1,2,3

$$\begin{cases} \dfrac{dS_j}{S_j} = \left\{ r_j - \lambda_j \left[\mathbb{E}\left(e^{Y_j}\right) - 1 \right] \right\} dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}^{(j)}}(t) \\ \qquad + \left(e^{Y_j} - 1\right) dN_j(t) \\ dV_j^S = \kappa_j^* \left(\theta_j^* - V_j^S\right) dt + \sigma_j^V \sqrt{V_j^S} dW_{j+3}^{\mathbb{Q}^{(j)}}(t) \\ \dfrac{dF_j}{F_j} = (r - r_j) dt + \sigma_j^F dW_{j+6}^{\mathbb{Q}}(t) \end{cases} ,$$

$$\boxed{\begin{aligned} \kappa_j^* &= \kappa_j + c_j \\ \theta_j^* &= \frac{\kappa_j \theta_j}{\kappa_j + c_j} \\ c_j &\in \mathbb{R} \end{aligned}}$$

for j = 1,2,3

(Proof is shown in the appendix of report)

Heston x Jump Model in **local world**

- From $\mathbb{Q}^{(j)}$ to $\mathbb{Q}$:

for j = 1,2,3

$$\begin{cases} \dfrac{dS_j}{S_j} = \left\{ r_j - \lambda_j \left[\mathbb{E}\left(e^{Y_j}\right) - 1\right] - \rho_{j,j+6}\, \sigma_j^F \sqrt{V_j^S} \right\} dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t) \\ \qquad + \left(e^{Y_j} - 1\right) dN_j(t) \\ dV_j^S = \left[\kappa_j^*\left(\theta_j^* - V_j^S\right) - \left(\rho_{j,j+3}\right)\left(\rho_{j,j+6}\right)\left(\sigma_j^F \sigma_j^V\right)\sqrt{V_j^S} \right] dt \\ \qquad + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

(Proof is shown in the appendix of report)

Source of parameters

Others are same with Heston

Parameter types	Estimated by Historical data	Calibrated by option data	Extract from Bloomberg	Imply by others
Volatilities	$\sigma_1^F, \sigma_2^F, \sigma_3^F$	N/A	N/A	N/A
Heston's Parameters	N/A	$\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4};$ $\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5};$ $\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}$	N/A	N/A
Jump size-related	N/A	$\boldsymbol{\lambda_j, m_j, v_j}$ $(for\ j = 1,2,3)$	N/A	N/A
Other Correlations ($\rho_{a,b}$)	(1,2), (1,3), (1,7), (2,3), (2,8), (3,9), (7,8), (7,9), (8,9)	N/A	N/A	(2,4), (3,4), (1,5), (3,5), (4,5), (1,6), (2,6), (4,6), (5,6), (2,7), (3,7), (4,7), (5,7), (6,7), (1,8), (3,8), (4,8), (5,5), (6,5), (1,9), (2,9), (4,9), (5,9), (6,9)
Initial values	N/A	N/A	$S_1(0), S_2(0), S_3(0), F_1(0), F_2(0), F_3(0), r_1, r_2, r_3, r$	N/A

Calibration of model parameters

Heston:

Parameter types	Historical data	Calibration by option data	Extract from Bloomberg	Imply by others
Volatilities	σ_1^F, σ_2^F, σ_3^F	N/A	N/A	N/A
Heston's Parameters	N/A	$\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4}$; $\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5}$; $\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}$	N/A	N/A
Other Correlations ($\rho_{a,b}$)	(1,2), (1,3), (2,3), (2,8), (3,9), (7,8), (7,9), (8,9)	N/A	N/A	(2,4), (3,4), (1,5), (3,5), (4,5), (1,6), (2,6), (4,6), (5,6), (2,7), (3,7), (4,7), (5,7), (6,7), (1,8), (3,8), (4,8), (5,5), (6,5), (1,9), (2,9), (4,9), (5,9), (6,9)
Initial values	N/A	N/A	$S_1(0), S_2(0), S_3(0), F_1(0), F_2(0), F_3(0), r_1, r_2, r_3, r$	N/A

Heston x Jump:

Parameter types	Historical data	Calibration by option data	Extract from Bloomberg	Imply by others
Volatilities	σ_1^F, σ_2^F, σ_3^F	N/A	N/A	N/A
Heston's Parameters	N/A	$\kappa_1^*, \theta_1^*, \sigma_1^V, V_1^S(0), \rho_{1,4}$; $\kappa_2^*, \theta_2^*, \sigma_2^V, V_2^S(0), \rho_{2,5}$; $\kappa_3^*, \theta_3^*, \sigma_3^V, V_3^S(0), \rho_{3,6}$	N/A	N/A
Jump size-related	N/A	$\boldsymbol{\lambda_j, m_j, v_j}$ $(for\ j = 1,2,3)$	N/A	N/A
Other Correlations ($\rho_{a,b}$)	(1,2), (1,3), (2,3), (2,8), (3,9), (7,8), (7,9), (8,9)	N/A	N/A	(2,4), (3,4), (1,5), (3,5), (4,5), (1,6), (2,6), (4,6), (5,6), (2,7), (3,7), (4,7), (5,7), (6,7), (1,8), (3,8), (4,8), (5,5), (6,5), (1,9), (2,9), (4,9), (5,9), (6,9)
Initial values	N/A	N/A	$S_1(0), S_2(0), S_3(0), F_1(0), F_2(0), F_3(0), r_1, r_2, r_3, r$	N/A

***'Differential Evolution'* (DE) algorithm**

Minimizing $\sum \boldsymbol{weight} * \left(\boldsymbol{call\ price}_{\boldsymbol{mkt}} - \boldsymbol{call\ \widehat{price}}_{\boldsymbol{model}}\right)^2$

How our model differs from the truth

Obtain the best set of parameters $(\kappa_1^*, \theta_1^*, \sigma_1^V \ldots)$

Differential Evolution (DE)

- Evolves a class of parameters in parallel at each generation
- Obtain the parameter set that minimize the cost function
- Similar to the concept of ‘Natural selection’

For the j^{th} generation…

1. *Select initial population (Random / Best from last gen.)*
2. *Mutation (Basis for the “Evolution”)*
3. *Crossover of parameters into individual*
4. *Select individuals for next generation (Compare cost function)*

Why we choose DE algorithm

- Avoid staying at the local minimum
- Stable speed for standard setting (NP=75, Gen=1500, CR=0.5, F=0.8)
- Cost function may not be continuous:

$$\min_{\Xi} \sum_{i=1}^{N} (w_i (c_i^{mkt} - c_i^{model})^2)$$

$$Where\ \Xi = Parameter\ set\ \{\kappa_{\mathrm{j}}^{*}, \theta_{\mathrm{j}}^{*}, \sigma_{\mathrm{j}}^{V}, V_{\mathrm{j}}^{\mathrm{S}}(0), \rho_{(j,j+3)}\}$$

Cost function

Using calls of ≈50 different strikes

$$\sum_{i=1}^{N}\left(w_i\left(c_i^{mkt} - c_i^{model}\right)^2\right)$$

where c_i^{mkt} = *market observed call price*
c_i^{model} = *call price calculated by model (IFFT)*
w_i = *weighting (Vega)*

(Proof is shown in the appendix of report)

---

e.g.

- $c^{Heston\ model}(S,V,t) = e^{-rT}e^{-\alpha k}\mathcal{F}_{\xi,k}^{-1}\left\{\frac{\phi_H\left(\log S, V, t;\ \varphi=\xi-(\alpha+1)i\right)}{\alpha^2+\alpha-\xi^2+i(2\alpha+1)\xi}\right\}$

By inverse FFT

where $\phi_H(x,v,t;\varphi)$ = ***characteristic function*** *of* lnS_t

$$= exp\{i\varphi x + A(t,T) + B(t,T)v\}$$

Calibration result of Heston model

	κ_j^*	θ_j^*	σ_j^V	$V_j^S(0)$	$\rho_{(j,j+3)}$
SX5E (j=1)	0.363477	0.0745492	0.530653	0.0218171	-0.405048
SMI (j=2)	0.535387	0.0654437	0.999361	0.0308053	-0.229081
UKX (j=3)	0.485086	0.104248	0.996793	0.0337472	-0.432254

- Feller condition: $2\kappa_j^*\theta_j^* > {\sigma_j^V}^2$ *(~~negative variance~~)*
  - ➤ **Not satisfied….**
- Use *'Full Truncation Scheme'* to sim V: $\hat{V}(t) = max\{\hat{V}(t), 0\}$
  - ➤ Ensuring no negative variance
- Not satisfying Feller condition → **possibly better fit**

Calibration result of Heston x Jump model

	κ_j^*	θ_j^*	σ_j^V	$V_j^S(0)$	$\rho_{(j,j+3)}$	λ_j	m_j	v_j
SX5E (j=1)	0.000439	0.59683	0.098254	0.359915	0.645909	0.033407	-886.227	0.081417
SMI (j=2)	0.000105	0.827242	0.071353	0.616089	0.848338	0.034248	-379.826	0.914164
UKX (j=3)	0.052035	-0.36572	0.056172	0.509112	0.952581	0.045038	-806.159	0.666227

- The equivalence of "Feller condition" in this case is unknown

- Use *'Full Truncation Scheme'* to sim V: $\hat{V}(t) = max\{\hat{V}(t), 0\}$
  - ➤ Ensuring no negative variance

Simulation of product price

Simulation procedure (BS)

1. Generate 3 independent Normal R.V.s and perform Cholesky decomposition

2. Exactly simulate the stock prices for the 3 indices on each payout/trigger observation day

3. Check on each <u>*payout observation day*</u> whether all three indices are above 75%*Initial level
   a) if yes and memory function = j (where j $\geq$ 0) then
      i. payout given at payout day = (0.052/4)*(denomination)*(j+1) and set memory = 0
      ii. Discounted payout = exp(-rt)*payout
   b) if no then
      i. no coupon given and memory = memory + 1

4. Check on each *trigger observation day* whether all the three indices are above 100%*Initial level
   a) if yes and memory function = j (where j $\geq$ 0) then
      i. Redemption at redemption day =denomination + (0.052/4)*(denomination)*(j+1) and **PATH END**
      ii. Discounted redemption = exp(-rt)*redemption
   b) if no then continue to check other dates

5. On *final redemption date*, check whether all three indices are above 75%*Initial level
   a) If yes, final redeem = denomination
   b) If no, final redeem = (denomination)*(final level of worst underlying)/(corresponding strike)
   c) Discounted final redemption = exp(-rt)*(final redeem)

6. Store the discounted total payout = payout (i),

   discounted final/early redemption = redemption (i)

7. Repeat step 1 to 7 from i = 1 to 10,000

8. Estimated price $= \sum \{\text{payout(i)} + \text{redemption (i)}\}/10{,}000$

Simulation procedure (Heston/ Jump added)

1. Generate 3 independent Normal R.V.s and perform Cholesky decomposition
2. Exactly simulate the stock prices for the 3 indices on each payout/trigger observation day
3. Check on each payout observation day whether all three indices are above 75%*Initial level
   a) if yes and ... function = j (where j $\geq$ 0) then
      ... 2/4)*(denomination)*(j+1) and set memory = 0
      ... and memory = memory + 1

Not possible anymore

4. Check on each trigger observation day whether all the three indices are above 100%*Initial level
   a) if yes and memory function = j (where j $\geq$ 0) then
      i. Redemption at redemption day =denomination + (0.052/4)*(denomination)*(j+1) and **PATH END**
      ii. Discounted redemption = exp(-rt)*redemption
   b) if no then continue to check other dates
5. On final redemption date, check whether all three indices are above 75%*Initial level
   a) If yes, final redeem = denomination
   b) If no, final redeem = (denomination)*(final level of worst underlying)/(corresponding strike)
   c) Discounted final redemption = exp(-rt)*(final redeem)
6. Store the discounted total payout = payout(i), discounted (final) redemption = redemption (i)
7. Repeat step 1 to 7 from i = 1 to 10,000
8. Estimated price = ∑{payout(i) + redemption (i)}/10,000

Simulation of V and S (Heston / Jump added)

1. Generate 6 Normal R.V.s and perform Cholesky decomposition
2. Discretize the SDE of V as aforementioned
3. Employ the **full truncation scheme [V* = max(V,0)]**

   Explain in the coming slides
4. Use the V* obtained to simulate S at each time step
   a. If Jump added, also do the following to simulate S
      - Generate 3 independent Poisson R.V.s for jump frequency
      - Generate another 3 independent Normal R.V.s for jump size
5. Repeat step 1 to 4 for m times (m = defined time step)

Stochastic volatility: problems encountered

Recall: Heston Model under $\mathbb{Q}$ (local world) :

$$\begin{cases} \dfrac{dS_j}{S_j} = \left(r_j - \rho_{j,j+6}\,\sigma_j^F \sqrt{V_j^S}\right) dt + \sqrt{V_j^S}\, dW_j^{\mathbb{Q}}(t) \\ dV_j^S = \left[\, \kappa_j^*(\theta_j^* - V_j^S) - (\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F \sigma_j^V)\sqrt{V_j^S}\,\right] dt \\ \qquad\quad + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

for j = 1,2,3

Stochastic volatility: problems encountered

Recall: Heston Model under $\mathbb{Q}$ (local world) :

$$\begin{cases} \dfrac{dS_j}{S_j} = \left(r_j - \rho_{j,j+6}\,\sigma_j^F \sqrt{V_j^S}\right) dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t) \\ dV_j^S = \left[\kappa_j^*(\theta_j^* - V_j^S) - (\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F\sigma_j^V)\sqrt{V_j^S}\right] dt \\ \qquad + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t) \end{cases}$$

for j = 1,2,3

V_j^S need to stay non-negative

Stochastic volatility: problems encountered

- Discrete-time approximation for $\widehat{V_j^S}$:

$$\widehat{V_j^S}(t+\Delta) = \widehat{V_j^S}(t) + \left[\kappa_j^* \left(\theta_j^* - \widehat{V_j^S}(t) \right) - \left(\rho_{j,j+3}\right)\left(\rho_{j,j+6}\right)\left(\sigma_j^F \sigma_j^V\right) \sqrt{\widehat{V_j^S(t)}} \right] \Delta$$

$$+\sigma_j^V \sqrt{\widehat{V_j^S}(t)} Z_V \sqrt{\Delta}$$

Stochastic volatility: problems encountered

- Discrete-time approximation for $\widehat{V_j^S}$:

$$\widehat{V_j^S}(t+\Delta) = \widehat{V_j^S}(t) + \left[\kappa_j^*\left(\theta_j^* - \widehat{V_j^S}(t)\right) - (\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F \sigma_j^V)\sqrt{\widehat{V_j^S}(t)}\right]\Delta$$

$$+\sigma_j^V \sqrt{\widehat{V_j^S}(t)}\,Z_V\sqrt{\Delta}$$

If large NEGATIVE value is generated…

Stochastic volatility: problems encountered

- Discrete-time approximation for $\widehat{V_j^S}$:

$$\widehat{V_j^S}(t+\Delta) = \widehat{V_j^S}(t) + \left[\kappa_j^*\left(\theta_j^* - \widehat{V_j^S}(t)\right) - (\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F\sigma_j^V)\sqrt{\widehat{V_j^S}(t)}\right]\Delta$$

$$+\sigma_j^V\sqrt{\widehat{V_j^S}(t)}Z_V\sqrt{\Delta}$$

May then be Negative!!! If large NEGATIVE value is generated…

‘Full Truncation’ scheme

- Traditional Euler scheme *(Cholesky decomposition)*:

$$Z_S = \Psi^{-1}(U_1), \qquad Z_V = \rho Z_S + \sqrt{1-\rho^2}\,\Psi^{-1}(U_2)$$

➤ *can generate NEGATIVE* $\boldsymbol{Z_V}$ *!!!*

- Full Truncation scheme:

$$\hat{V}(t) = \max\{\hat{V}(t), 0\}$$

For both Heston / Heston x Jump model

Other scheme…?

- Quadratic-exponential (QE) discretization scheme, *Andersen (2008)*:

  ➤ Outperforms the normal truncation scheme **in all cases** tested[1] !!!!!!!!!

  ➤ Maintains pricing accuracy even when the parameters are extreme[1]

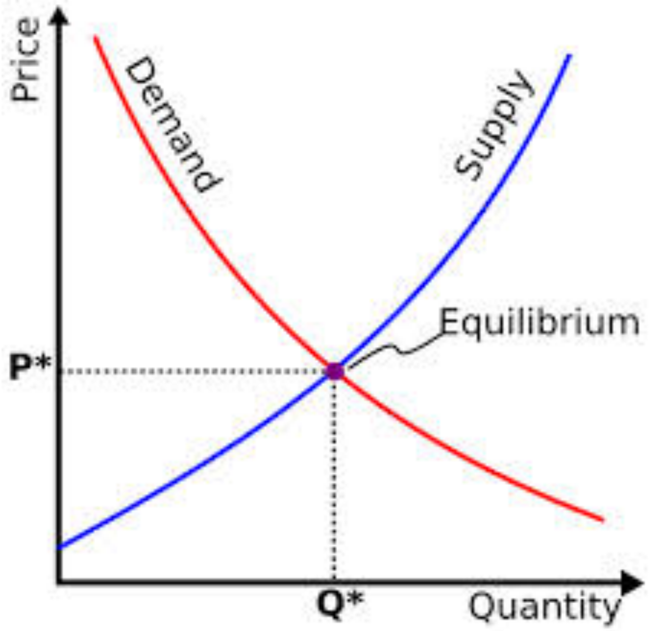
We will further explained it in ‘Limitations’ part

[1] N.H. Chan and H.Y. Wong (2013). Handbook of Financial Risk Management: Simulations and Case Studies, Wiley, New York.

Comparing pricing result of different models

	No. of simulation	Mean of price	Sd of price	$\lvert price^{market} - price^{model} \rvert$
Black-Scholes Model	100	USD $986.5	4.212	USD $38.5
Heston Model	100	USD $828.4	3.914	USD $119.6
Heston x Jump Model	100	USD $805.0	3.703	USD $143

- Mid bid-ask price as @ 19/11/2018 = USD $948
- Closest = BS Model
- Market fails to consider SV + jump event



Risk involved

1. Currency risk
2. Credit risk
3. Model risk

Currency Risk

e.g. ρ between underlying and foreign exchange rate

$$BS: \frac{dS_j}{S_j} = (r_j - \boxed{\rho_{j,j+3}\, \sigma_j^S \sigma_j^F})dt + \sigma_j^S dW_j^{\mathbb{Q}}(t)$$

$$Heston: \frac{dS_j}{S_j} = (r_j - \boxed{\rho_{j,j+6}\, \sigma_j^F \sqrt{V_j^S}})dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t)$$

$$\begin{matrix} Heston \\ X \\ Jump \end{matrix} : \frac{dS_j}{S_j} = \left\{ r_j - \lambda_j \left[\mathbb{E}\left(e^{Y_j}\right) - 1 \right] - \boxed{\rho_{j,j+6}\, \sigma_j^F \sqrt{V_j^S}} \right\} dt + \sqrt{V_j^S} dW_j^{\mathbb{Q}}(t) + \left(e^{Y_j} - 1\right) dN_j(t)$$

Credit risk

Debts issued by Credit Suisse	Moody’s	S&P
Rating on Senior Unsecured Debt	Baa2	BBB+
Rating on Subordinated Debt	Baa3	BBB+

- Credit rating of issuer
- Slight lower than industry average
- Further explained in ‘Limitations’ part

Model risk

- **<u>Black-Scholes model</u>**:
  - ➢ Constant volatility (Violate with the observed volatility smile)
  - ➢ Constant interest rate
  - ➢ Normal assumption (Size of tail)
- **<u>Heston model</u>:**
  - ➢ Parameters are sensitive to initial data set
  - ➢ Still assumed continuous sample path
- **<u>Heston x Jump Model</u>:**
  - ➢ Over-parametrized
  - ➢ Assumed independent jump sizes & Poisson processes
  - ➢ Size of tail for Jump sizes (may not be Gaussian)

Managing Risk

1.Hedging

2. Value-at-Risk (VaR)

3.Parameter Sensitivity Analysis

Hedging

- Greeks (Δ,V,Θ,Γ)
  - ➢Partial derivatives exist?

Pay you fixed amount of money IF $S_i(t) < K$

- Replicating cash flow
  - ➢Early redemption: Not a concern
  - ➢Coupon payments: Cash-or-nothing put (Over-hedge? Credit risk?)
  - ➢Final redemption: Standard European put (Transaction cost?)

- Even if perfect replication by structural product(s)
  - ➢More risk(s)/cost is introduced

Value-at-Risk (VaR)

- Assumed initial investment = USD 1,000

	99% Absolute VaR
BS model	USD 340.3

Suppose…

- Restrict 99% Absolute VaR of portfolio = USD 50,000
- Maximum holding unit = $\lfloor 50{,}000/340.3 \rfloor$ = 146 units

Parameter Sensitivity (BS model)

- Analyze Sensitivity = $\left| \frac{\Delta Price / Price}{\Delta\Theta / \Theta} \right|$ for each parameter in Θ

	S_1	S_2	S_3
Sensitivity	0.114	0.000714	0.0253

	σ_1	σ_2	σ_3
Sensitivity	0.0376	0.00865	0.0217

	r_1	r_2	r_3
Sensitivity	≈ 0	≈ 0	≈ 0

Parameter Sensitivity (Heston)

	κ_1	κ_2	κ_3
Sensitivity	0.0197	0.00860	0.0119

	θ_1	θ_2	θ_3
Sensitivity	0.0199	0.0154	0.0266

	σ_{v1}	σ_{v2}	σ_{v3}
Sensitivity	0.00596	0.00132	0.00742

	V_{01}	V_{02}	V_{03}
Sensitivity	0.00944	0.00601	0.0104

Parameter Sensitivity (Heston x Jump)

	λ_1	λ_2	λ_3
Sensitivity	0.0295	0.0352	0.0206

	m_1	m_2	m_3
Sensitivity	≈ 0	≈ 0	≈ 0

	v_1	v_2	v_3
Sensitivity	≈ 0	≈ 0	≈ 0

Conclusion

- Market consensus $\approx$ Black-Scholes price
- Neglect stochastic volatility (maybe Pure Jump diffusion…?)
- Hard to perform perfect hedging
- Sensitive to ΔS, ΔV^S,λ

Limitations

1. Model assumption
2. Calibration
3. Simulation
4. Credit Risk

Limitation in model assumption…

- *Will the distribution of jump sizes change over time?*
- **Independency:**
  - $dN_a \perp\!\!\!\perp dN_b$
  - $J_a \perp\!\!\!\perp J_b$
  - $dN_a \perp\!\!\!\perp dW_b^{\mathbb{P}}$

Limitation in calibration…

1. We want to fit the volatility smile well, given (T-t) $\approx$ 2 :
   - *Hard to find call options with the **EXACT same maturity** of our product, for calibration*
   - $\therefore$*Only calls with **T= 1,2** are used in calibration*
   - *When maturity (T-t) decreases → recalibration is needed*

2. Not feasible to obtain <u>a single set of parameters</u> which fits the volatility surface well
   - *We can only separately obtain one set for one specific day, using different value of T*
   - *Then combine all smiles to obtain the surface*
   - *Theoretically possible but **EXTREMELY** time consuming*

Why we gave up on QE scheme…

- *QE scheme discovered/required that:*

For $dV_j^S = \left[\,\kappa_j^*\left(\theta_j^* - V_j^S\right)\,\right]dt + \sigma_j^V\sqrt{V_j^S}\,dW_{j+3}^{\mathbb{Q}}(t)$

$$\begin{cases} \textbf{For sufficiently large } \widehat{V_j^S}(t): \\ \widehat{V_j^S}(t+\Delta) \propto \text{non-central chi-square random variable}^1 \\ \\ \textbf{For a small } \widehat{V_j^S}(t): \\ \widehat{V_j^S}(t+\Delta) \sim \text{ordinary central chi-square}^1 \end{cases}$$

[1] N.H. Chan and H.Y. Wong (2013). Handbook of Financial Risk Management: Simulations and Case Studies, Wiley, New York.

Why we gave up on QE scheme…

$$dV_j^S = \left[\kappa_j^*(\theta_j^* - V_j^S) - \boxed{(\rho_{j,j+3})(\rho_{j,j+6})(\sigma_j^F \sigma_j^V)\sqrt{V_j^S}}\right] dt + \sigma_j^V \sqrt{V_j^S}\, dW_{j+3}^{\mathbb{Q}}(t)$$

- an extra $\sqrt{\widehat{V_j^S}}$ appears in the coefficient of dt
- Unsure about the distribution of $\widehat{V_j^S}$
- So we find an alternative…

Limitation in simulation…

- Full Truncation scheme:

$$\hat{V}(t) = max\{\hat{V}(t), 0\}$$

  - ➢ *Fail to reflect the true distribution of $V(t)$ when it approaches zero*[1]
  - ➢ *Computational time* ↑ ↑ ↑
  - ➢ *Further studies needed…*

[1] N.H. Chan and H.Y. Wong (2013). Handbook of Financial Risk Management: Simulations and Case Studies, Wiley, New York.

Limitation in credit risk involved…

- <u>Credit Risk</u>:
  - ➢ Actually can calculate '<u>Default Probability</u>' (DP) of Credit Suisse…

    - not the main focus of this project
  - ➢ <u>Lower medium rating</u>
    - ➤ May be because of Credit Suisse needing more funds
    - ➤ Raise the risk premium
    - ➤ Thus, rating institutions gave Credit Suisse low rating

Thank you!