Astrodynamics HW 3

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1.

$$\mathbf{r}_{in} = 7000km$$

Kepler's 3rd law

$$P^2 = \frac{4\pi^2}{\mu}a^3$$

$$\mu = 398600.4418km^3/s^2$$
$$P_{fin} = 12hours \times 60min \times 60sec$$

In [17]:
```
Rin = 7000;
```

In [16]:
```
mu = 398600.4418;
```

In [13]:
```
Pfin = 12*3600;
```

In [14]:
```
Rfin = cbrt(mu*Pfin^2/4/pi^2);
```

$$\mathbf{r}_{fin} = 26610.2228km$$

In [15]:
```
Atrans = (Rin + Rfin)/2;
```

$$a_{trans} = \frac{r_{in} + r_{fin}}{2}$$
$$= 16805.1114km$$

In [16]:
```
e = 1-(Rin/Atrans);
```

using vis-viva equation

$$\Delta V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

by vis-viva equation

$$\mathbf{V}_{in} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{in}}}, \mathbf{V}_{fin} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{fin}}}$$

$$\mathbf{V}_{trans,a} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}}, \mathbf{V}_{trans,b} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

$$\Delta\mathbf{V}_a = \sqrt{\frac{\mu}{\mathbf{r}_{in}}} - \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

$$\Delta\mathbf{V}_b = \sqrt{\frac{\mu}{\mathbf{r}_{fin}}} - \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

In [10]:
```
Vin = sqrt((2mu/Rin)-(mu/Rin));
```

In [44]:
```
Vfin = sqrt((2*mu/Rfin)-(mu/Rfin));
```

In [45]:
```
Vtrans_a = sqrt((2mu/Rin)-(mu/Atrans));
```

In [46]:
```
Vtrans_b = sqrt((2mu/Rfin)-(mu/Atrans));
```

In [50]:
```
deltaVa = Vtrans_a - Vin;
```

In [52]:
```
deltaVb = Vfin - Vtrans_b;
```

$$\begin{cases} \Delta V_a = 1.9496 km/s \\ \Delta V_b = 1.3724 km/s \end{cases}$$

In [53]:
```
deltaV = deltaVa + deltaVb;
```

$$\begin{aligned} \Delta V &= \Delta V_a + \Delta V_b \\ &= 3.3220 km/s \end{aligned}$$

2.

$$Period = 318 min$$

In [14]:
```
P = 318*60;
```

In [18]:
```
mu = 398600.4418;
```

In [28]:
```
a = cbrt(P^2*mu/4/pi^2);
```

In [56]:
```
incl = 10*2pi/360;
```

In identical equatorial plane, we can do inclination-only maneuver on two nodes which is ascending and descending nodes.

$$P^2 = \frac{4\pi^2}{\mu}a^3$$

$$a = 15433km, e = 0.5$$

In [5]:
```
e = 0.5;
```

In [7]:
```
nu = 45/360*2pi;
```

In [36]:
```
nu2 = (180+45)/360*2pi;
```

In [29]:
```
r = a*(1-e^2)/(1+0.5*cos(nu));
```

In [41]:
```
r2 = a*(1-e^2)/(1+0.5*cos(nu2));
```

In [30]:
```
a*(1-e^2);
```

$r = \frac{a(1-e^2)}{1+ecos\nu}(km)$
$= \frac{15432.8071(1-0.5^2)}{1+0.5cos\nu}(km)$
$= \frac{11574.6053}{1+0.5cos\nu}(km)$

by vis-viva equation

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$
$$= \sqrt{\frac{\mu}{r}(2 - \frac{1-e^2}{1+0.5cos\nu})}$$

In [45]:
```
Vasc = sqrt((mu/r)*(2-((1-e^2)/(1+0.5cos(nu)))));
```

In [46]:
```
Vdesc = sqrt((mu/r2)*(2-((1-e^2)/(1+0.5cos(nu2)))));
```

at descending node,

$$V_{descending} = 4.3239km/s$$

at ascending node,

$$V_{ascending} = 8.2096km/s$$

$$\Delta \overrightarrow{V_{i,only}} = \overrightarrow{V_{fin}} - \overrightarrow{V_{in}}$$

In [62]:
```
cosfpa1 = sqrt((1-e^2)*a^2/r/(2a-r));
```

In [63]:
```
cosfpa2 = sqrt((1-e^2)*a^2/r2/(2a-r2));
```

In [61]:
```
delV_a = 2cosfpa1*Vasc*sin(incl/2);
```

In [64]:
```
delV_d = 2cosfpa2*Vdesc*sin(incl/2);
```

1)ascending node

$sin(5^\circ) = \frac{\Delta V_{i,only,ascending}/2}{2cos(\phi_{fpa})V_{ascending}}$
$\Rightarrow \Delta V_{i,only,ascending} = 2cos(\phi_{fpa})V_{ascending} sin(5^\circ)$
$= 1.3846km/s$

2)descending node

$sin(5^\circ) = \frac{\Delta V_{i,only,descending}/2}{2cos(\phi_{fpa})V_{descending}}$
$\Rightarrow \Delta V_{i,only,descending} = 2cos(\phi_{fpa})V_{descending} sin(5^\circ)$
$= 0.6613km/s$

3.

$$altitude_{sat1} = 191.344km$$
$$altitude_{GEO} = 35781.35km$$

Note. the initial and final orbits are circular, whereas the transfer orbits are elliptical.

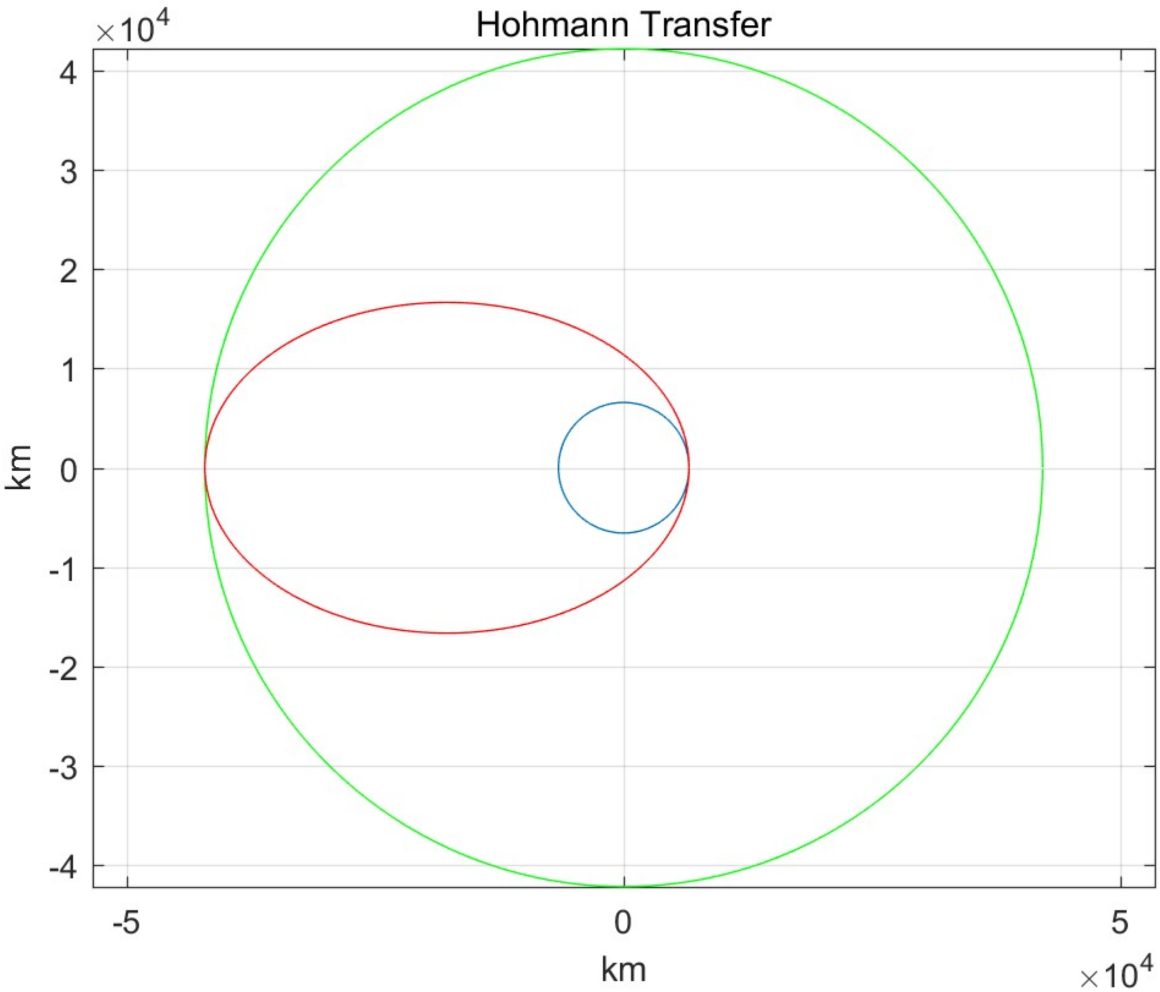
For the One-tangent burn transfer, it is given as follows.

$\nu_{trnans,b} = 160^\circ$

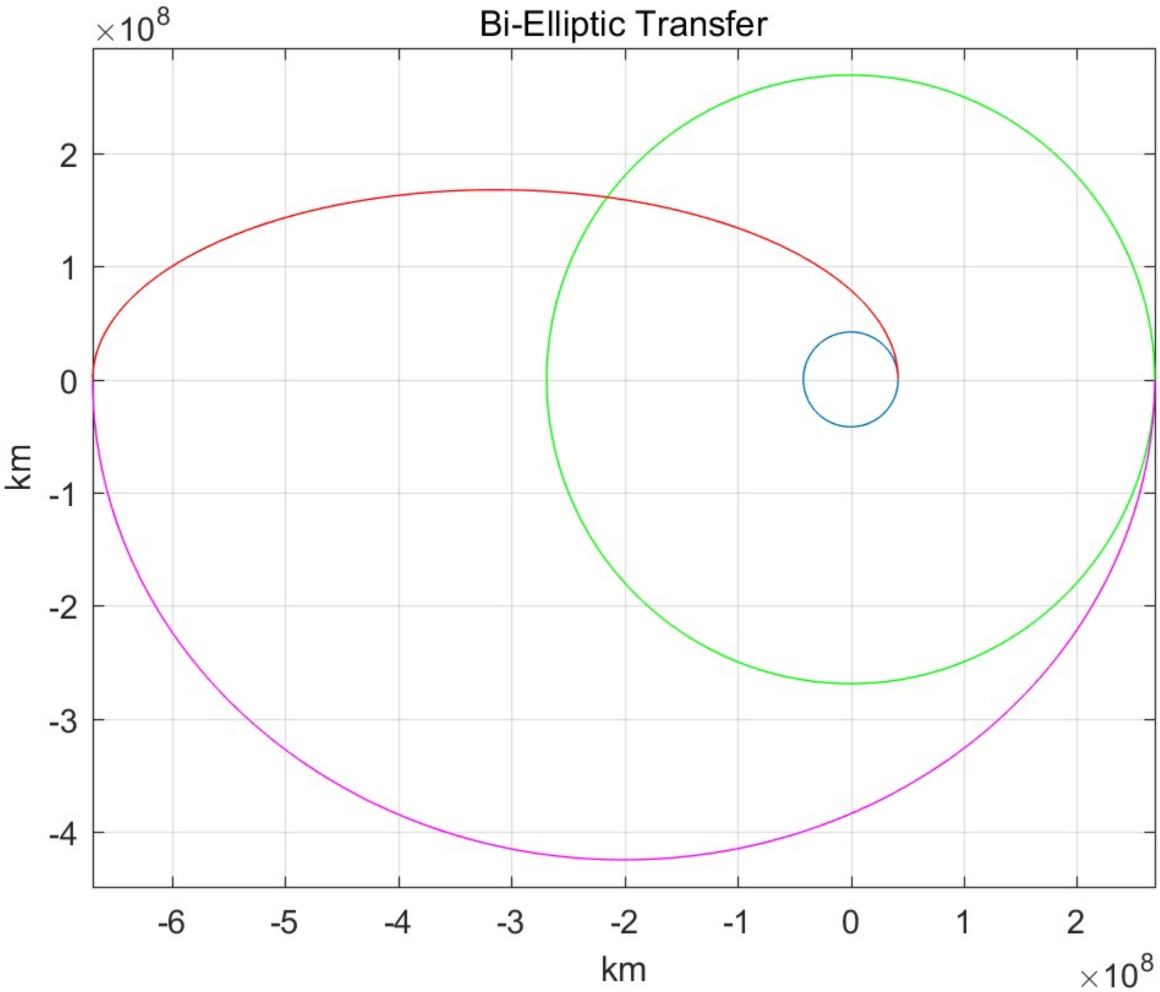
the total change in velocity, transfer time and comparing the results from each transfer.

hohmann(191.344+re, 35781.35+re,0,0,0,pi)

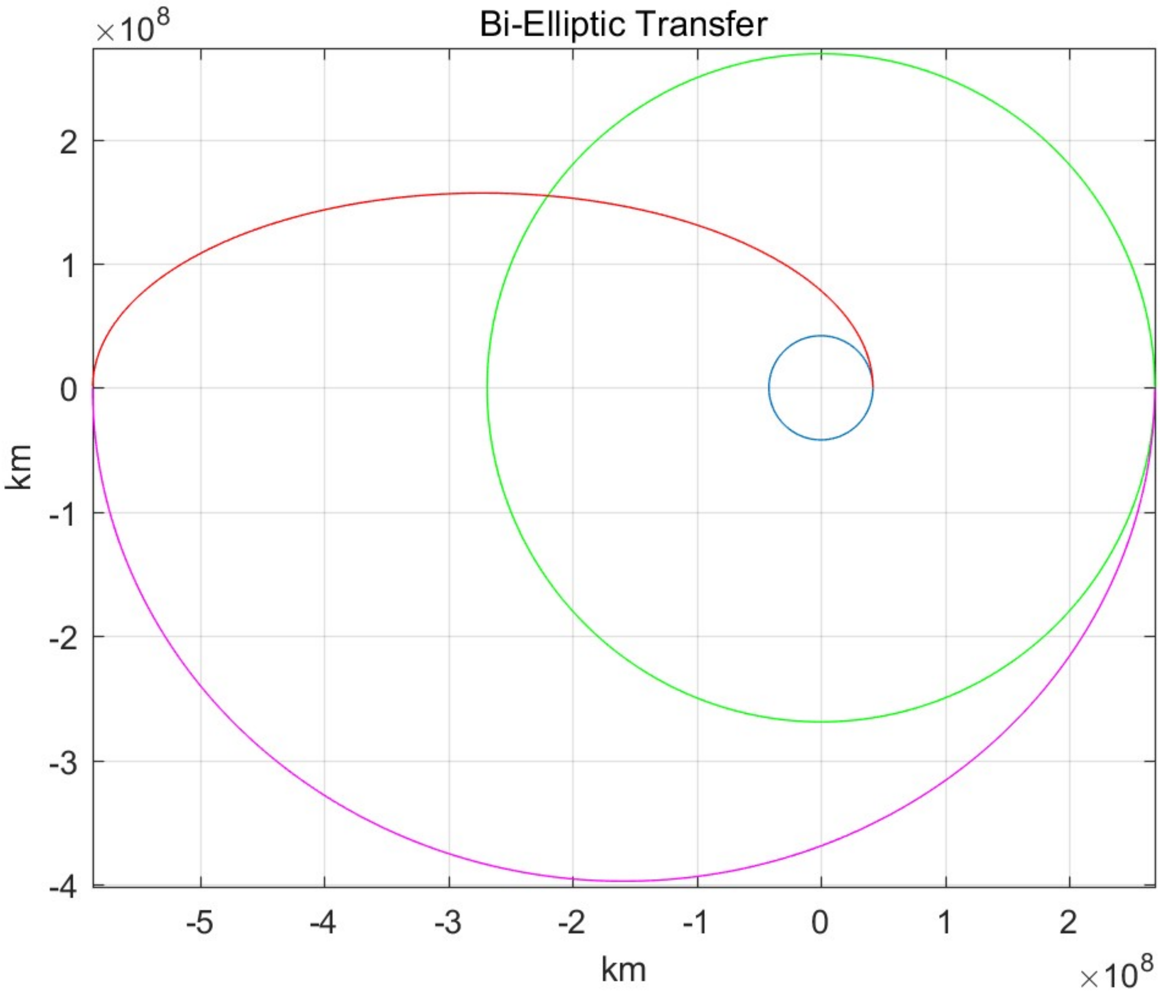
$$\begin{cases} \Delta V_a = V_{trans,a} - V_{initial} \\ \Delta V_b = V_{final} - V_{trans,b} \end{cases}$$
$$\Delta V_{total} = \Delta V_a + \Delta V_b$$



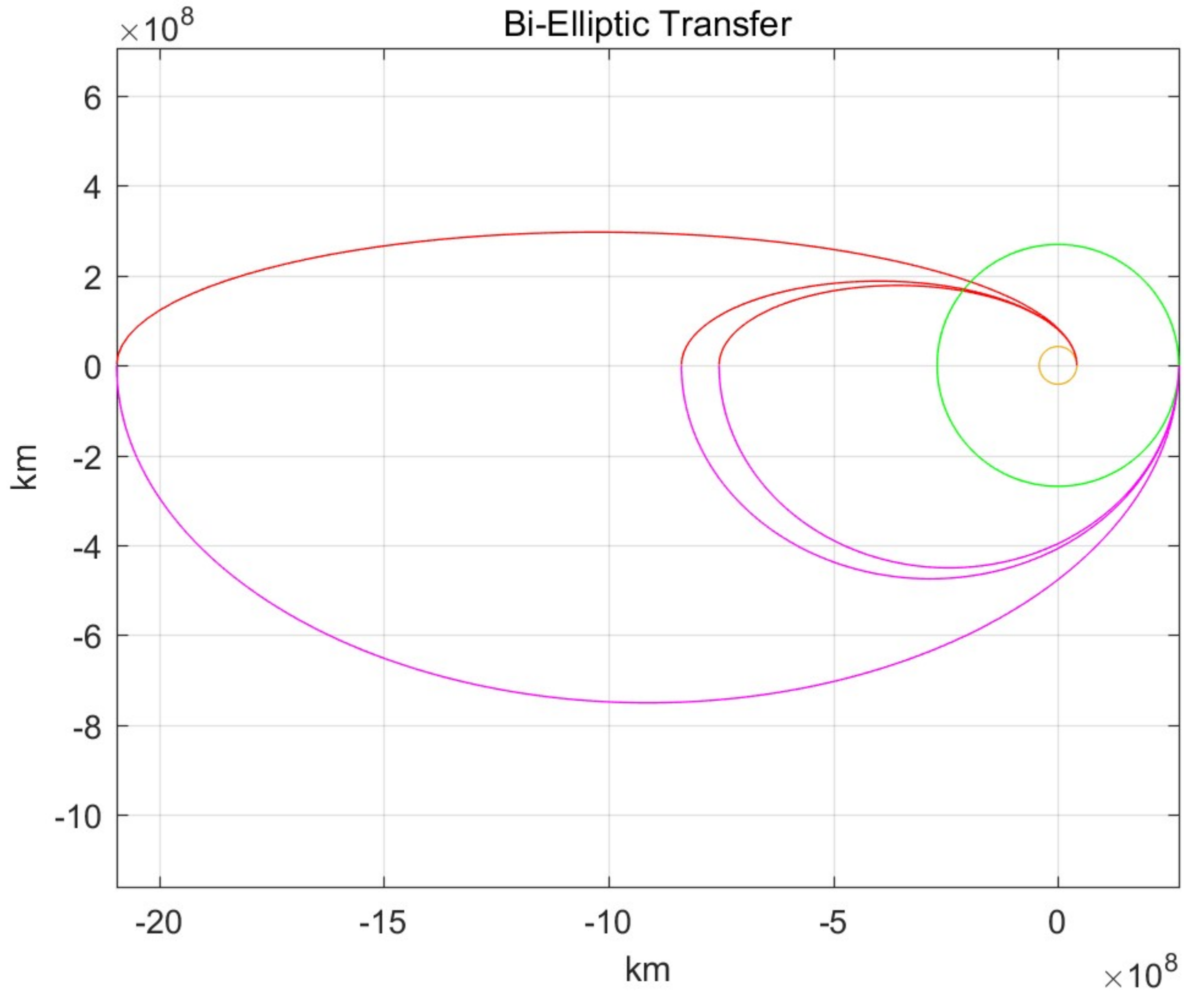
biellip(191.344+re,105112,35781.35+re,0, 0, 0, pi)



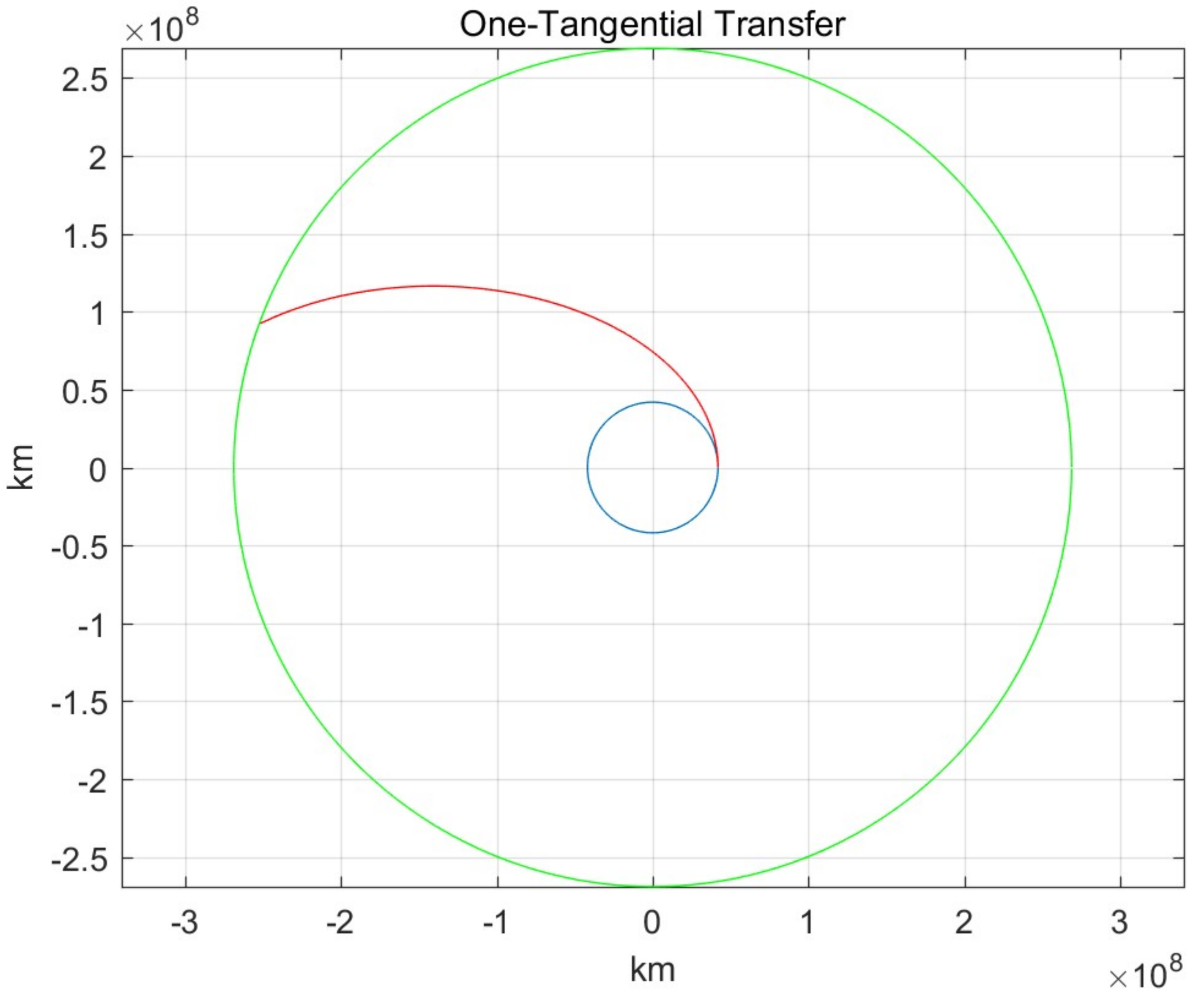
biellip(191.344+re,91973,35781.35+re,0, 0, 0, pi)



biellip(191.344+re,328475,35781.35+re,0, 0, 0, pi)



onetang(191.344+re,35781.35+re,0,0,0,160/180*pi)


One-Tangential Transfer
×10^8
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
km
-3
-2
-1
0
1
2
3
km
×10^8

In [ ]: