

# Astrodynamics HW 3

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1.

$$\mathbf{r}_{in} = 7000km$$

## Kepler's 3rd law

$$P^2 = \frac{4\pi^2}{\mu} a^3$$

$$\mu = 398600.4418km^3/s^2$$

$$P_{fin} = 12hours \times 60min \times 60sec$$

In [3]: Rin = 7000;

In [4]: mu = 398600.4418;

In [5]: Pfin = 12\*3600;

In [6]: Rfin = cbrr(mu\*Pfin^2/4/pi^2)

Out[6]: 26610.222805310135

$$\mathbf{r}_{fin} = 26610.2228km$$

In [7]: Atrans = (Rin + Rfin)/2

Out[7]: 16805.111402655068

$$a_{trans} = \frac{r_{in} + r_{fin}}{2}$$

$$= 16805.1114km$$

In [8]: e = 1-(Rin/Atrans)

Out[8]: 0.5834600656741818

## using vis-viva equation

$$\Delta V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

by vis-viva equation

$$\mathbf{V}_{in} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{in}}}, \mathbf{V}_{fin} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{fin}}}$$

$$\mathbf{V}_{trans,a} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}}, \mathbf{V}_{trans,b} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

$$\Delta \mathbf{V}_a = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}} - \sqrt{\frac{\mu}{\mathbf{r}_{in}}}$$

$$\Delta \mathbf{V}_b = \sqrt{\frac{\mu}{\mathbf{r}_{fin}}} - \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

```
In [9]: Vin = sqrt((2mu/Rin)-(mu/Rin))
```

```
Out[9]: 7.546053290107541
```

```
In [10]: Vfin = sqrt((2*mu/Rfin)-(mu/Rfin))
```

```
Out[10]: 3.870300022016199
```

```
In [11]: Vtrans_a = sqrt((2mu/Rin)-(mu/Atrans))
```

```
Out[11]: 9.49562216234698
```

```
In [12]: Vtrans_b = sqrt((2mu/Rfin)-(mu/Atrans))
```

```
Out[12]: 2.4978879591780316
```

```
In [13]: deltaVa = Vtrans_a - Vin
```

```
Out[13]: 1.9495688722394382
```

```
In [14]: deltaVb = Vfin - Vtrans_b
```

```
Out[14]: 1.3724120628381673
```

$$\begin{cases} \Delta V_a = 1.9496 km/s \\ \Delta V_b = 1.3724 km/s \end{cases}$$

```
In [15]: deltaV = deltaVa + deltaVb
```

```
Out[15]: 3.3219809350776055
```

$$\begin{aligned}\Delta V &= \Delta V_a + \Delta V_b \\ &= 3.3220 km/s\end{aligned}$$

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2.

$$Period = 318min$$

In [17]: `P = 318*60;`

In [18]: `mu = 398600.4418;`

In [19]: `a = cbrt(P^2*mu/4/pi^2)`

Out[19]: 15432.807055900254

In [20]: `incl = 10*2pi/360`

Out[20]: 0.17453292519943295

In identical equatorial plane, we can do inclination-only maneuver on two nodes which is ascending and descending nodes.

$$P^2 = \frac{4\pi^2}{\mu} a^3$$

$$a = 15433km, e = 0.5$$

In [21]: `e = 0.5;`

In [22]: `nu = 45/360*2pi;`

In [23]: `nu2 = (180+45)/360*2pi;`

In [24]: `r = a*(1-e^2)/(1+0.5*cos(nu))`

Out[24]: 8551.273538498503

In [25]: `r2 = a*(1-e^2)/(1+0.5*cos(nu2))`

Out[25]: 17904.96712875908

In [26]: `a*(1-e^2)`

Out[26]: 11574.60529192519

$$\begin{aligned}
 r &= \frac{a(1-e^2)}{1+e\cos\nu} (km) \\
 &= \frac{15432.8071(1-0.5^2)}{1+0.5\cos\nu} (km) \\
 &= \frac{11574.6053}{1+0.5\cos\nu} (km)
 \end{aligned}$$

by vis-viva equation

$$\begin{aligned}
 v &= \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \\
 &= \sqrt{\frac{\mu}{r} \left(2 - \frac{1-e^2}{1+0.5\cos\nu}\right)}
 \end{aligned}$$

```
In [27]: Vasc = sqrt((mu/r)*(2-((1-e^2)/(1+0.5cos(nu)))))
```

```
Out[27]: 8.209619970055103
```

```
In [28]: Vdesc = sqrt((mu/r2)*(2-((1-e^2)/(1+0.5cos(nu2)))))
```

```
Out[28]: 4.323873716051834
```

at descending node,

$$V_{descending} = 4.3239 km/s$$

at ascending node,

$$V_{ascending} = 8.2096 km/s$$

$$\Delta \vec{V}_{i,only} = \vec{V}_{fin} - \vec{V}_{in}$$

```
In [29]: cosfpa1 = sqrt(((1-e^2)*a^2/r/(2a-r)))
```

```
Out[29]: 0.9675382212353983
```

```
In [30]: cosfpa2 = sqrt(((1-e^2)*a^2/r2/(2a-r2)))
```

```
Out[30]: 0.8773551979613604
```

```
In [31]: delV_a = 2cosfpa1*Vasc*sin(incl/2)
```

```
Out[31]: 1.3845772389062325
```

```
In [32]: delV_d = 2cosfpa2*Vdesc*sin(incl/2)
```

```
Out[32]: 0.6612633589284205
```

1) ascending node

$$\begin{aligned}\sin(5^\circ) &= \frac{\Delta V_{i,only,ascending}/2}{2\cos(\phi_{fpa})V_{ascending}} \\ \Rightarrow \Delta V_{i,only,ascending} &= 2\cos(\phi_{fpa})V_{ascending}\sin(5^\circ) \\ &= 1.3846\text{km/s}\end{aligned}$$

2)descending node

$$\begin{aligned}\sin(5^\circ) &= \frac{\Delta V_{i,only,descending}/2}{2\cos(\phi_{fpa})V_{descending}} \\ \Rightarrow \Delta V_{i,only,descending} &= 2\cos(\phi_{fpa})V_{descending}\sin(5^\circ) \\ &= 0.6613\text{km/s}\end{aligned}$$

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3.

$$\begin{aligned}\text{altitude}_{sat1} &= 191.344\text{km} \\ \text{altitude}_{GEO} &= 35781.35\text{km}\end{aligned}$$

Note. the initial and final orbits are circular, whereas the transfer orbits are elliptical.

For the One-tangent burn transfer, it is given as follows.

$$\nu_{trans,b} = 160^\circ$$

the total change in velocity, transfer time and comparing the results from each transfer.

$$[\text{delA}, \text{delB}, \text{dttu}] = \text{hohmann}(191.344 + \text{re}, 35781.35 + \text{re}, 0, 0, 0, \text{pi})$$

$$\begin{cases} \Delta V_a = V_{trans,a} - V_{initial} \\ \Delta V_b = V_{final} - V_{trans,b} \\ \Delta V_{total} = \Delta V_a + \Delta V_b \end{cases}$$

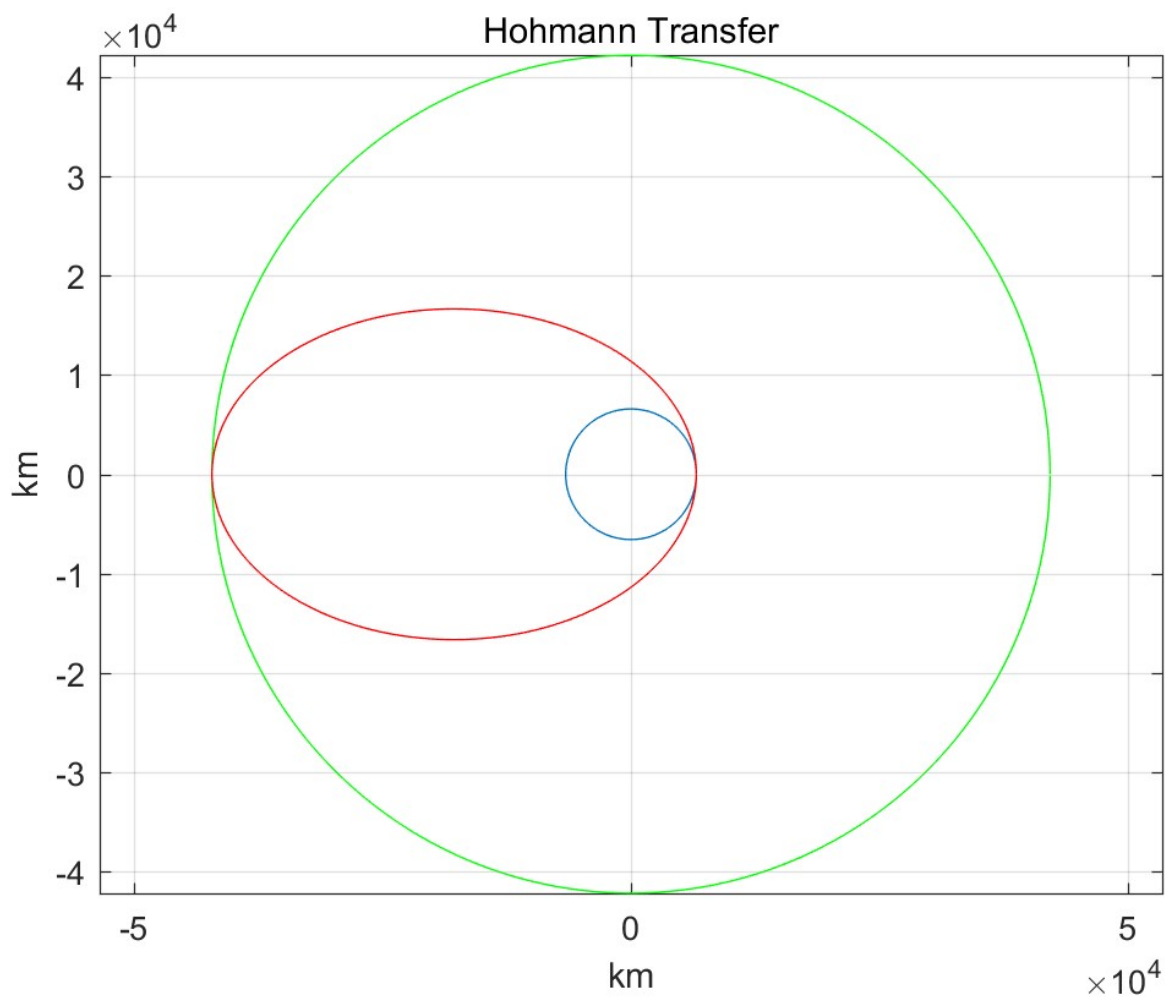
$$V_{trans,a} - V_{initial} = 0.0308\text{km/s}$$

$$V_{final} - V_{trans,b} = 0.0185\text{km/s}$$

$$\Delta V_{total} = 0.0493\text{km/s}$$

$$\tau_{trans} = 9639568133.3624\text{sec}$$

$$= 160659468.8894\text{min}$$



```
[delva,delvb,delvc,dttu] = biellip(191.344+re,105112,35781.35+re,0,0,0,pi)
```

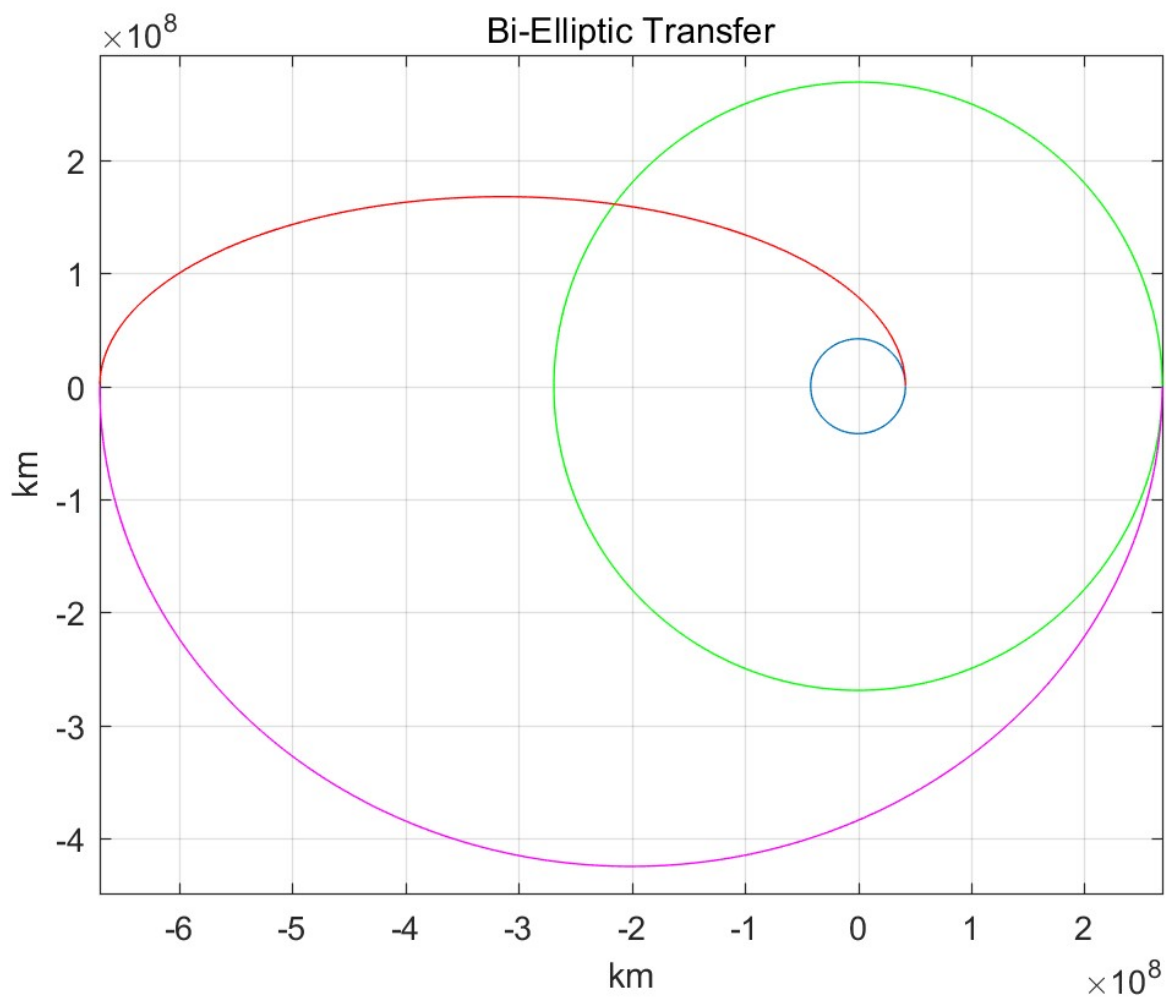
$$\begin{cases} \Delta V_a = V_{trans1,a} - V_{initial} \\ \Delta V_b = V_{trans2,b} - V_{trans1,b} \\ \Delta V_c = V_{final} - V_{trans2,c} \end{cases}$$

$$\Delta V_{total} = \Delta V_a + \Delta V_b + \Delta V_c$$

$$\begin{cases} \Delta V_a = 0.0362816 km/s \\ \Delta V_b = 0.0100867 km/s \\ \Delta V_c = 0.007499 km/s \end{cases}$$

$$\Delta V_{total} = 0.05387 km/s$$

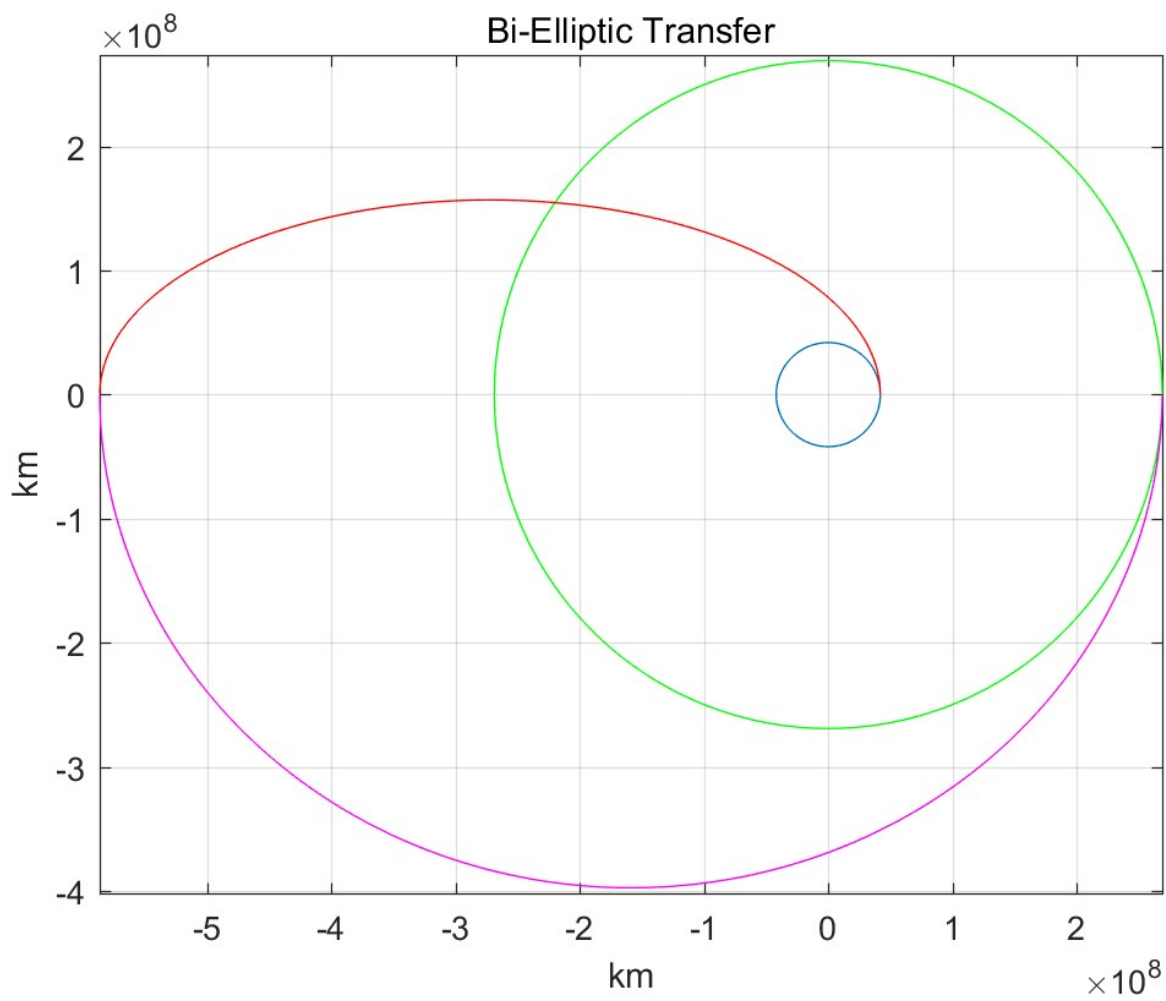
$$\begin{cases} \tau_1 = 557438875.4811 min \\ \tau_2 = 844118685.6927 min \\ \tau_{total} = \tau_1 + \tau_2 = 1401557561.1738 min \end{cases}$$



```
[delva,delvb,delvc,dtu] = biellip(191.344+re,91973,35781.35+re,0, 0, 0, pi)
```

$$\begin{cases} \Delta V_a = 0.03572 km/s \\ \Delta V_b = 0.01115 km/s \\ \Delta V_c = 0.006586 km/s \\ \Delta V_{total} = 0.05346 km/s \end{cases}$$

$$\begin{cases} \tau_1 = 462019966.8668 min \\ \tau_2 = 733713435.5272 min \\ \tau_{total} = \tau_1 + \tau_2 = 1195733402.3940 min \end{cases}$$



```
[delva,delvb,delvc,dttu] = biellip(191.344+re,328475,35781.35+re,0, 0, 0, pi)
```

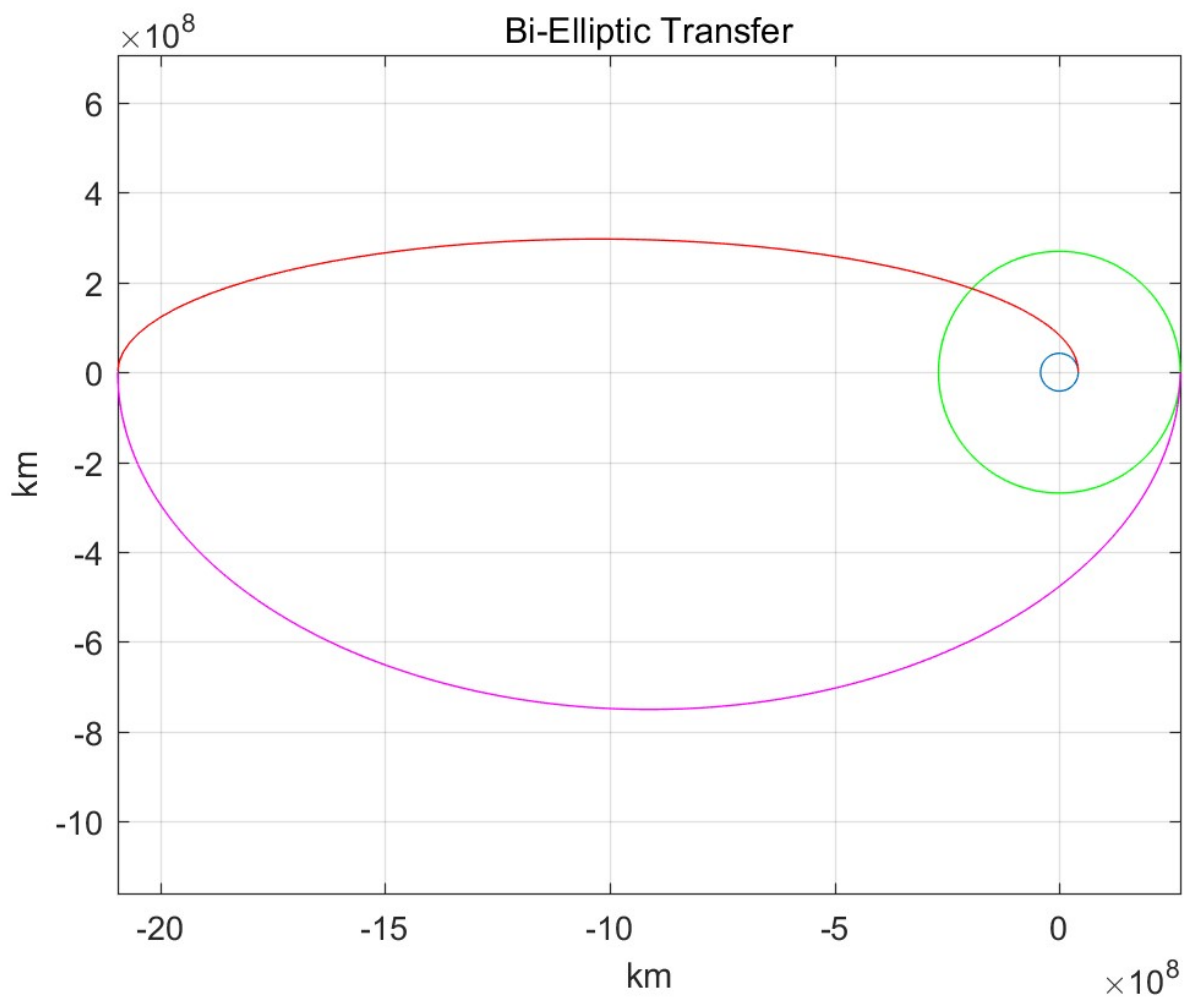
$$\begin{cases} \Delta V_a = 0.03904 km/s \\ \Delta V_b = 0.003848 km/s \\ \Delta V_c = 0.01276 km/s \end{cases}$$

$$\Delta V_{total} = 0.05565 km/s$$

$$\begin{cases} \tau_1 = 2896537856.1201 min \\ \tau_2 = 3370111325.1706 min \end{cases}$$

$$\tau_{total} = \tau_1 + \tau_2 = 6266649181.2906 min$$





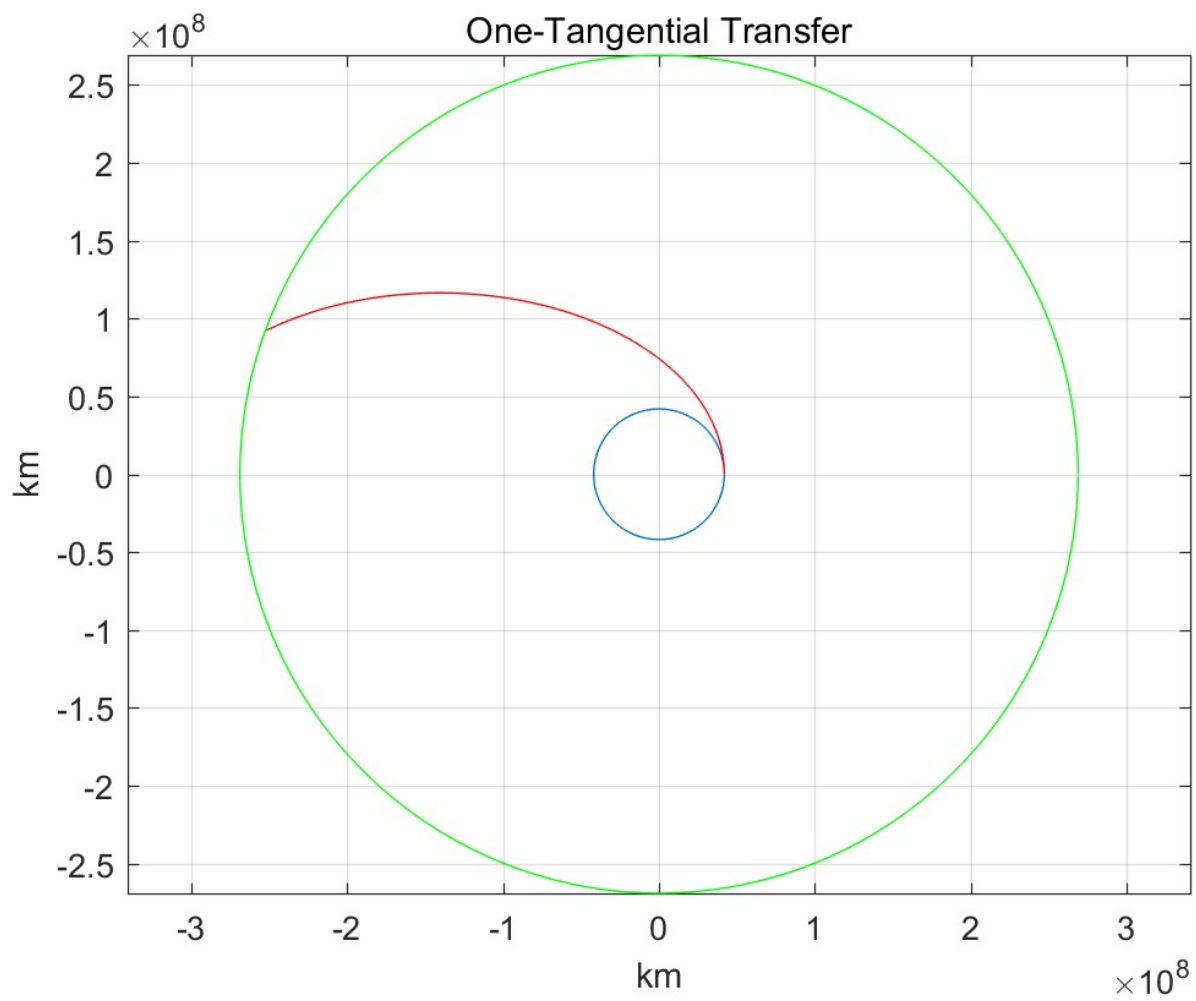
`[delva,delvb,dttu,etran, atran ] = onetang(191.344+re,35781.35+re,0,0,0,160/180*pi)`

$$\begin{cases} \Delta V_a = 0.03225 \text{ km/s} \\ \Delta V_b = 0.01053 \text{ km/s} \end{cases}$$

$$\Delta V_{total} = 0.04278 \text{ km/s}$$

$$\tau_{trans} = 105668295.4356 \text{ min}$$

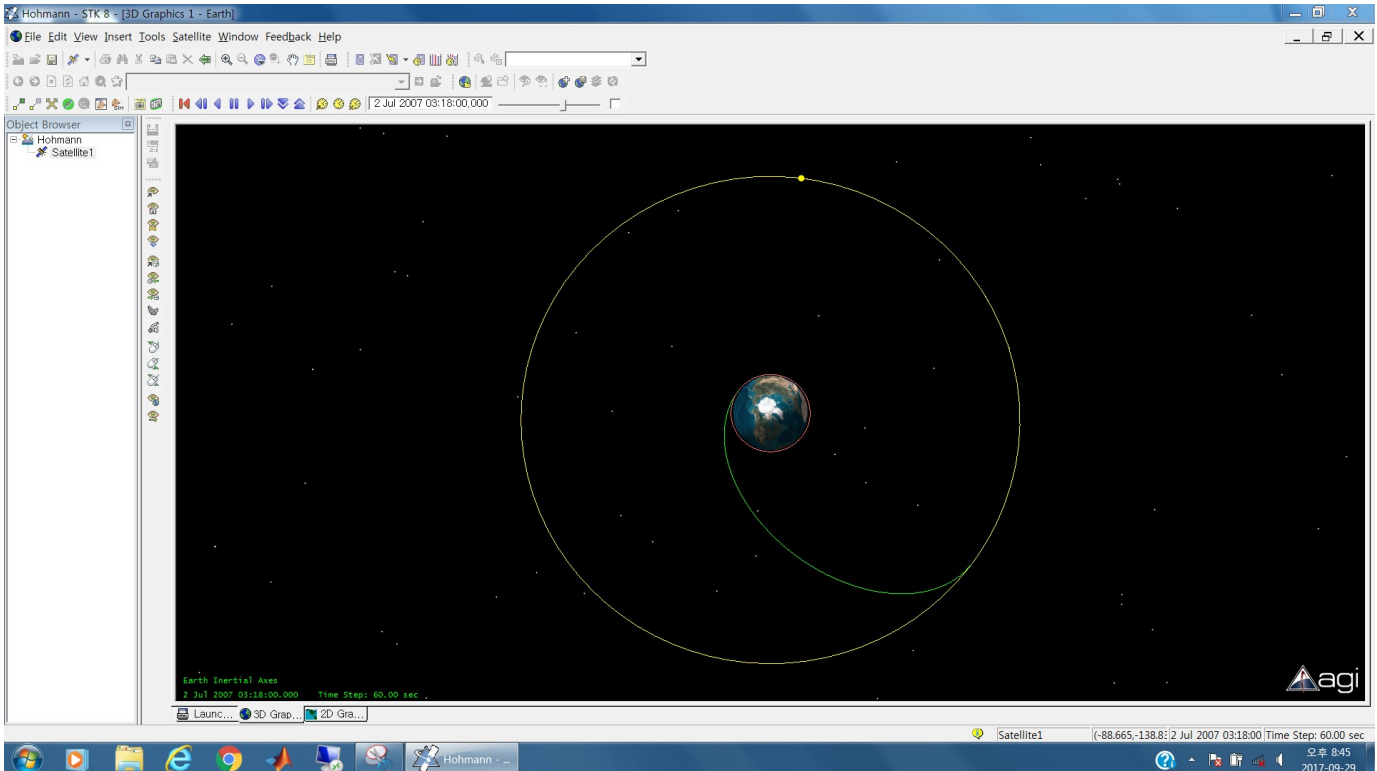
$$= 6340097726.1365 \text{ sec}$$



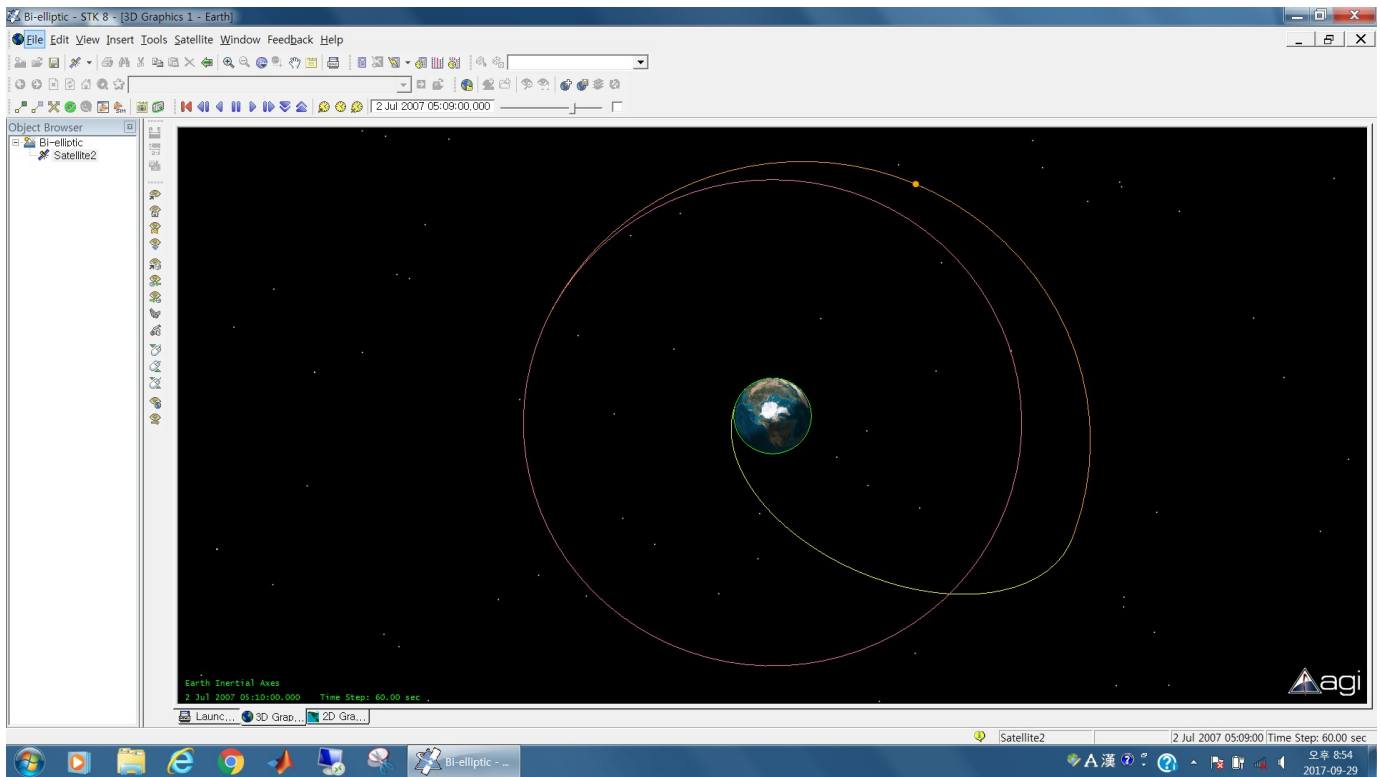
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4.

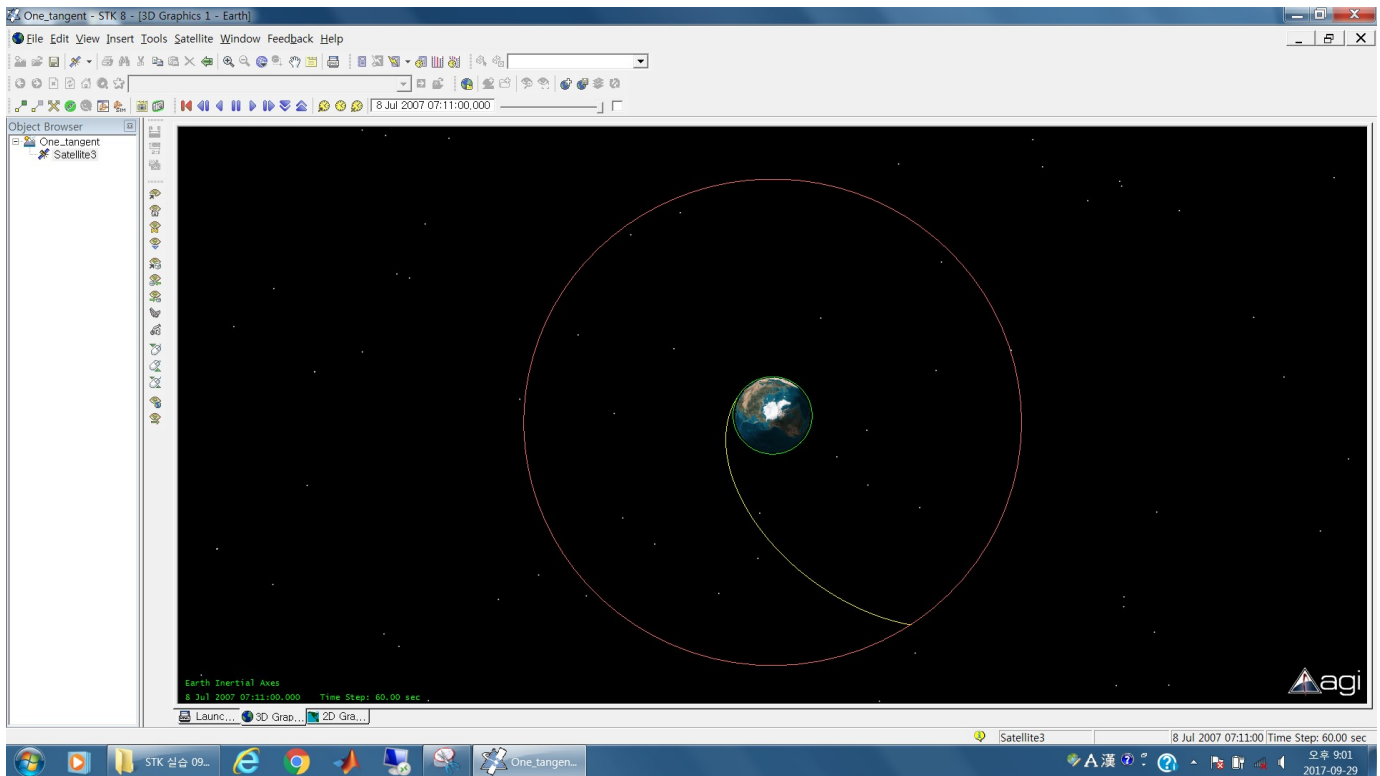
### Hohmann transfer



### Bi-elliptic transfer



### One-tangential burn



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