Astrodynamics HW 3

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1.

$$\mathbf{r}_{in} = 7000km$$

Kepler's 3rd law

$$P^2 = \frac{4\pi^2}{\mu}a^3$$

$$\mu = 398600.4418km^3/s^2$$
$$P_{fin} = 12hours \times 60min \times 60sec$$

In [37]:
```
Rin = 7000;
```

In [38]:
```
mu = 398600.4418;
```

In [5]:
```
Pfin = 12*3600;
```

In [6]:
```
Rfin = cbrt(mu*Pfin^2/4/pi^2)
```

Out[6]: 26610.222805310135

$$\mathbf{r}_{fin} = 26610.2228km$$

In [7]:
```
Atrans = (Rin + Rfin)/2
```

Out[7]: 16805.111402655068

$$a_{trans} = \frac{r_{in} + r_{fin}}{2}$$
$$= 16805.1114km$$

In [8]:
```
e = 1-(Rin/Atrans)
```

Out[8]: 0.5834600656741818

using vis-viva equation

$$\Delta V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

by vis-viva equation

$$\mathbf{V}_{in} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{in}}}, \mathbf{V}_{fin} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{fin}}}$$

$$\mathbf{V}_{trans,a} = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}}, \mathbf{V}_{trans,b} = \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

$$\Delta\mathbf{V}_a = \sqrt{\frac{2\mu}{\mathbf{r}_{in}} - \frac{\mu}{\mathbf{a}_{trans}}} - \sqrt{\frac{\mu}{\mathbf{r}_{in}}}$$

$$\Delta\mathbf{V}_b = \sqrt{\frac{\mu}{\mathbf{r}_{fin}}} - \sqrt{\frac{2\mu}{\mathbf{r}_{fin}} - \frac{\mu}{\mathbf{a}_{trans}}}$$

In [9]:
```
Vin = sqrt((2mu/Rin)-(mu/Rin))
```
Out[9]: 7.546053290107541

In [10]:
```
Vfin = sqrt((2*mu/Rfin)-(mu/Rfin))
```
Out[10]: 3.870300022016199

In [11]:
```
Vtrans_a = sqrt((2mu/Rin)-(mu/Atrans))
```
Out[11]: 9.49562216234698

In [12]:
```
Vtrans_b = sqrt((2mu/Rfin)-(mu/Atrans))
```
Out[12]: 2.4978879591780316

In [13]:
```
deltaVa = Vtrans_a - Vin
```
Out[13]: 1.9495688722394382

In [14]:
```
deltaVb = Vfin - Vtrans_b
```
Out[14]: 1.3724120628381673

$$\begin{cases} \Delta V_a = 1.9496 km/s \\ \Delta V_b = 1.3724 km/s \end{cases}$$

In [15]:
```
deltaV = deltaVa + deltaVb
```
Out[15]: 3.3219809350776055

$$\Delta V = \Delta V_a + \Delta V_b$$
$$= 3.3220km/s$$

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2.

$$Period = 318min$$

In [17]:
```
P = 318*60;
```

In [18]:
```
mu = 398600.4418;
```

In [19]:
```
a = cbrt(P^2*mu/4/pi^2)
```

Out[19]: 15432.807055900254

In [20]:
```
incl = 10*2pi/360
```

Out[20]: 0.17453292519943295

In identical equatorial plane, we can do inclination-only maneuver on two nodes which is ascending and descending nodes.

$$P^2 = \frac{4\pi^2}{\mu}a^3$$

$$a = 15433km, e = 0.5$$

In [21]:
```
e = 0.5;
```

In [22]:
```
nu = 45/360*2pi;
```

In [23]:
```
nu2 = (180+45)/360*2pi;
```

In [24]:
```
r = a*(1-e^2)/(1+0.5*cos(nu))
```

Out[24]: 8551.273538498503

In [25]:
```
r2 = a*(1-e^2)/(1+0.5*cos(nu2))
```

Out[25]: 17904.96712875908

In [26]:
```
a*(1-e^2)
```

Out[26]: 11574.60529192519

$r = \frac{a(1-e^2)}{1+ecos\nu}(km)$
$= \frac{15432.8071(1-0.5^2)}{1+0.5cos\nu}(km)$
$= \frac{11574.6053}{1+0.5cos\nu}(km)$

by vis-viva equation

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$
$$= \sqrt{\frac{\mu}{r}(2 - \frac{1-e^2}{1+0.5cos\nu})}$$

In [27]:
```
Vasc = sqrt((mu/r)*(2-((1-e^2)/(1+0.5cos(nu)))))
```
Out[27]: 8.209619970055103

In [28]:
```
Vdesc = sqrt((mu/r2)*(2-((1-e^2)/(1+0.5cos(nu2)))))
```
Out[28]: 4.323873716051834

at descending node,

$$V_{descending} = 4.3239km/s$$

at ascending node,

$$V_{ascending} = 8.2096km/s$$

$$\Delta \overrightarrow{V_{i,only}} = \overrightarrow{V_{fin}} - \overrightarrow{V_{in}}$$

In [29]:
```
cosfpa1 = sqrt((1-e^2)*a^2/r/(2a-r))
```
Out[29]: 0.9675382212353983

In [30]:
```
cosfpa2 = sqrt((1-e^2)*a^2/r2/(2a-r2))
```
Out[30]: 0.8773551979613604

In [31]:
```
delV_a = 2cosfpa1*Vasc*sin(incl/2)
```
Out[31]: 1.3845772389062325

In [32]:
```
delV_d = 2cosfpa2*Vdesc*sin(incl/2)
```
Out[32]: 0.6612633589284205

1)ascending node

$sin(5^\circ) = \frac{\Delta V_{i,only,ascending}/2}{2cos(\phi_{fpa})V_{ascending}}$
$\Rightarrow \Delta V_{i,only,ascending} = 2cos(\phi_{fpa})V_{ascending}sin(5^\circ)$
$= 1.3846km/s$

2)descending node

$sin(5^\circ) = \frac{\Delta V_{i,only,descending}/2}{2cos(\phi_{fpa})V_{descending}}$
$\Rightarrow \Delta V_{i,only,descending} = 2cos(\phi_{fpa})V_{descending}sin(5^\circ)$
$= 0.6613km/s$

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3.

$$altitude_{sat1} = 191.344km$$
$$altitude_{GEO} = 35781.35km$$

Note. the initial and final orbits are circular, whereas the transfer orbits are elliptical.

For the One-tangent burn transfer, it is given as follows.

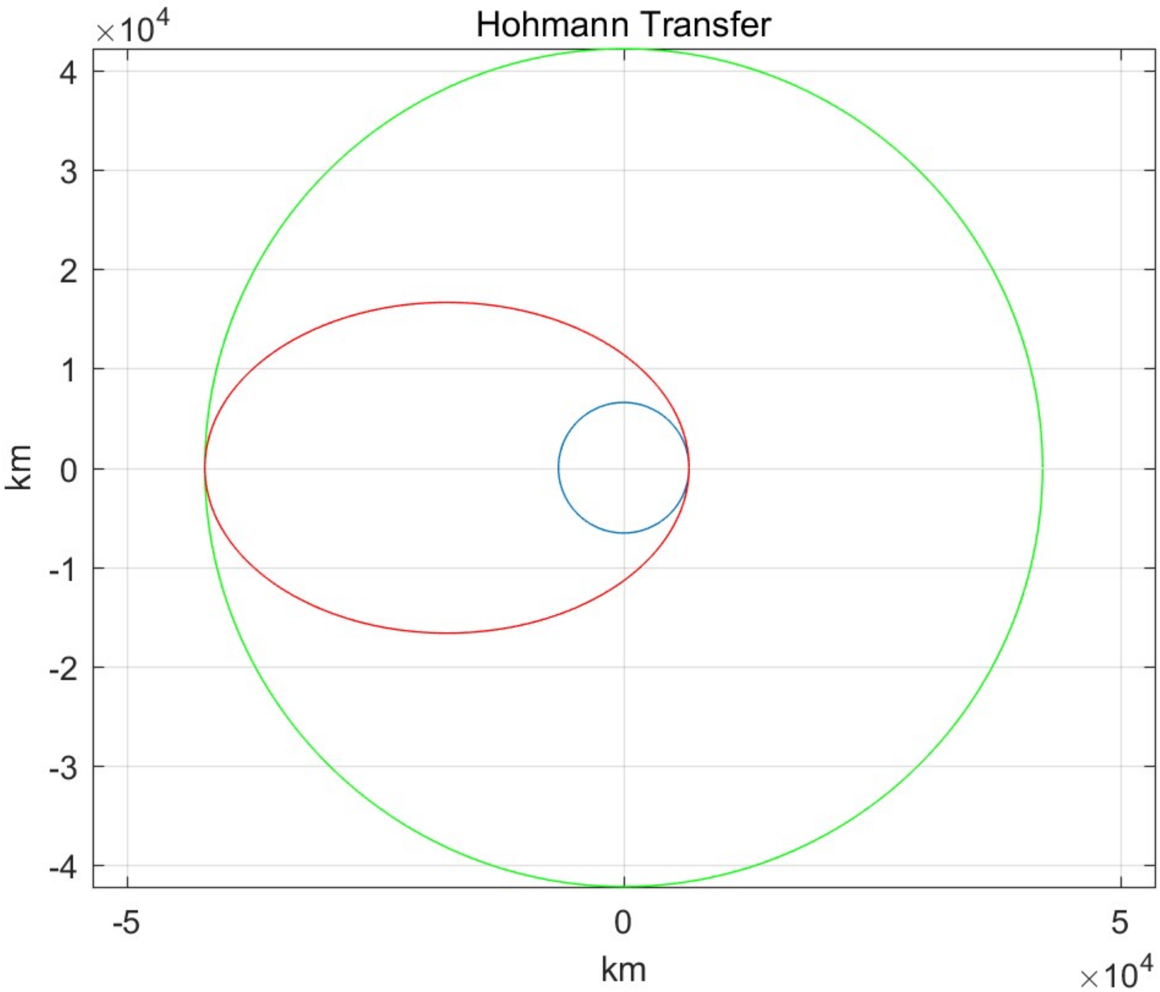
$\nu_{trnans,b} = 160^\circ$

the total change in velocity, transfer time and comparing the results from each transfer.

[delA, delB, dttu] = hohmann(191.344 + re, 35781.35 + re, 0, 0, 0, pi)

$$\begin{cases} \Delta V_a = V_{trans,a} - V_{initial} \\ \Delta V_b = V_{final} - V_{trans,b} \end{cases}$$
$$\Delta V_{total} = \Delta V_a + \Delta V_b$$

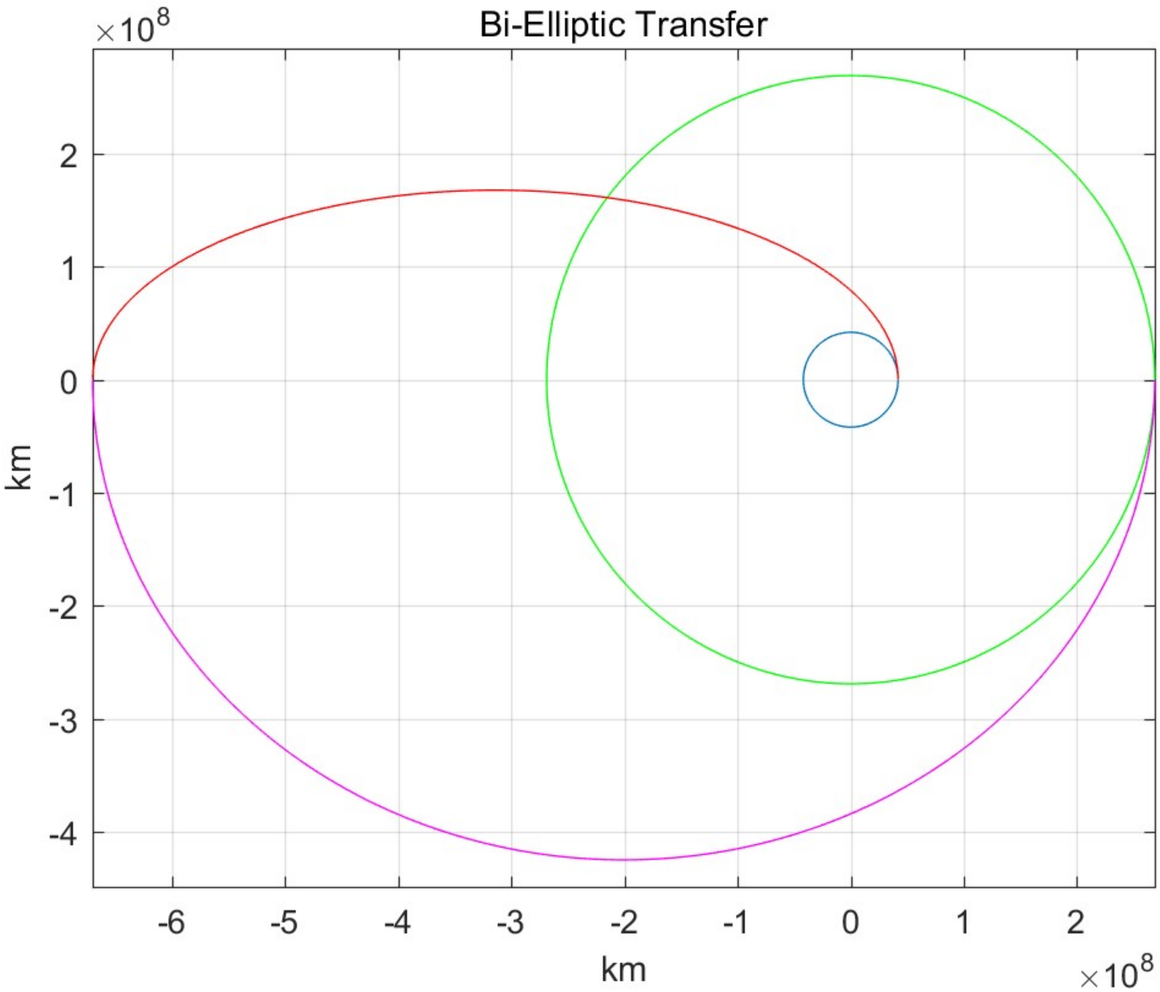
$$V_{trans,a} - V_{initial} = 0.0308km/s$$
$$V_{final} - V_{trans,b} = 0.0185km/s$$
$$\Delta V_{total} = 0.0493km/s$$
$$\tau_{trans} = 9639568133.3624sec$$
$$= 160659468.8894min$$



[delva,delvb,delvc,dttu] = biellip(191.344+re,105112,35781.35+re,0, 0, 0, pi)

$$\begin{cases} \Delta V_a = V_{trans1,a} - V_{initial} \\ \Delta V_b = V_{trans2,b} - V_{trans1,b} \\ \Delta V_c = V_{final} - V_{trans2,c} \end{cases}$$
$$\Delta V_{total} = \Delta V_a + \Delta V_b + \Delta V_c$$

$$\begin{cases} \Delta V_a = 0.0362816 km/s \\ \Delta V_b = 0.0100867 km/s \\ \Delta V_c = 0.007499 km/s \end{cases}$$
$$\Delta V_{total} = 0.05387 km/s$$
$$\begin{cases} \tau_1 = 557438875.4811 min \\ \tau_2 = 844118685.6927 min \\ \tau_{total} = \tau_1 + \tau_2 = 1401557561.1738 min \end{cases}$$

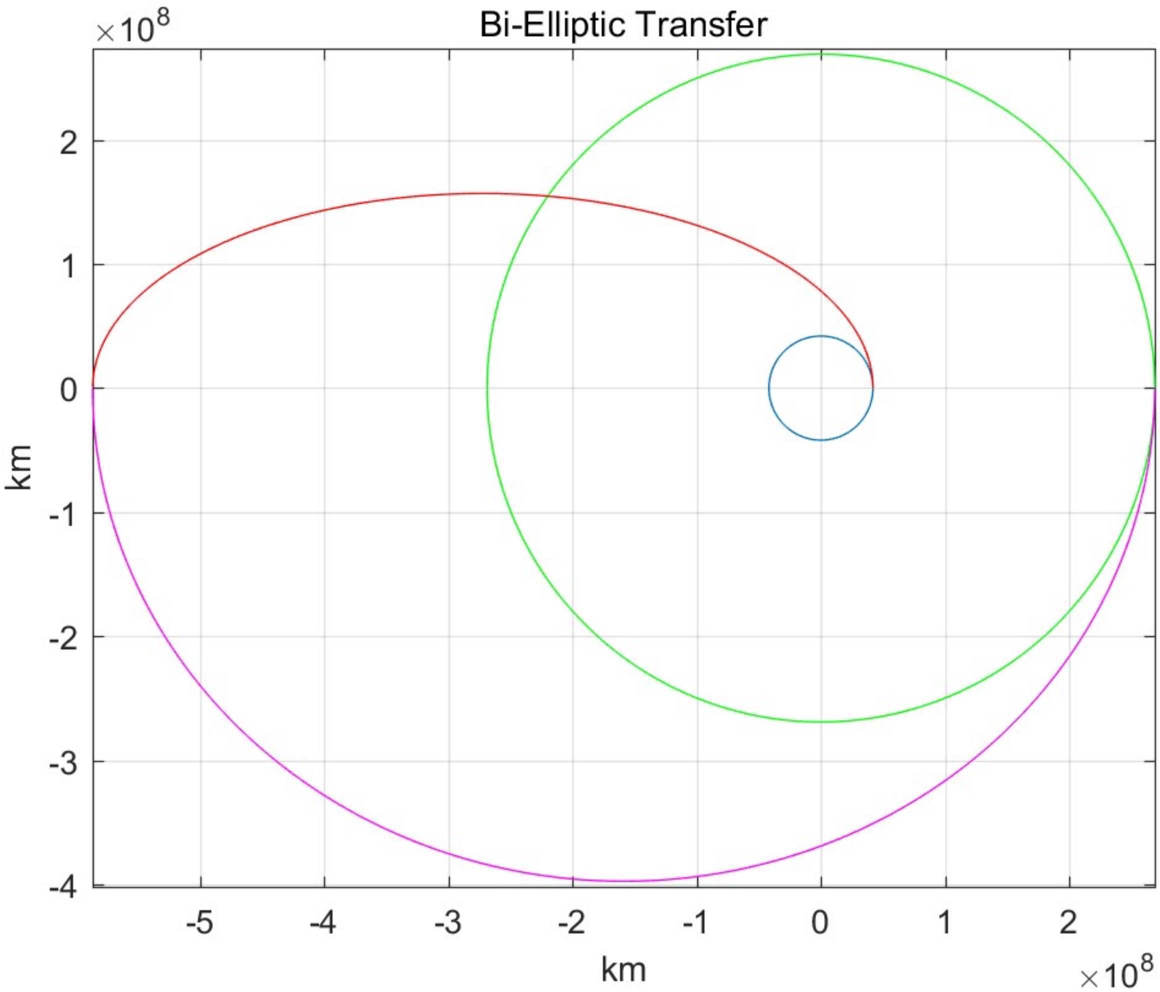



```
[delva,delvb,delvc,dttu ] = biellip(191.344+re,91973,35781.35+re,0, 0, 0, pi)
```

$$\begin{cases} \Delta V_a = 0.03572 km/s \\ \Delta V_b = 0.01115 km/s \\ \Delta V_c = 0.006586 km/s \end{cases}$$

$$\Delta V_{total} = 0.05346 km/s$$

$$\begin{cases} \tau_1 = 462019966.8668 min \\ \tau_2 = 733713435.5272 min \\ \tau_{total} = \tau_1 + \tau_2 = 1195733402.3940 min \end{cases}$$

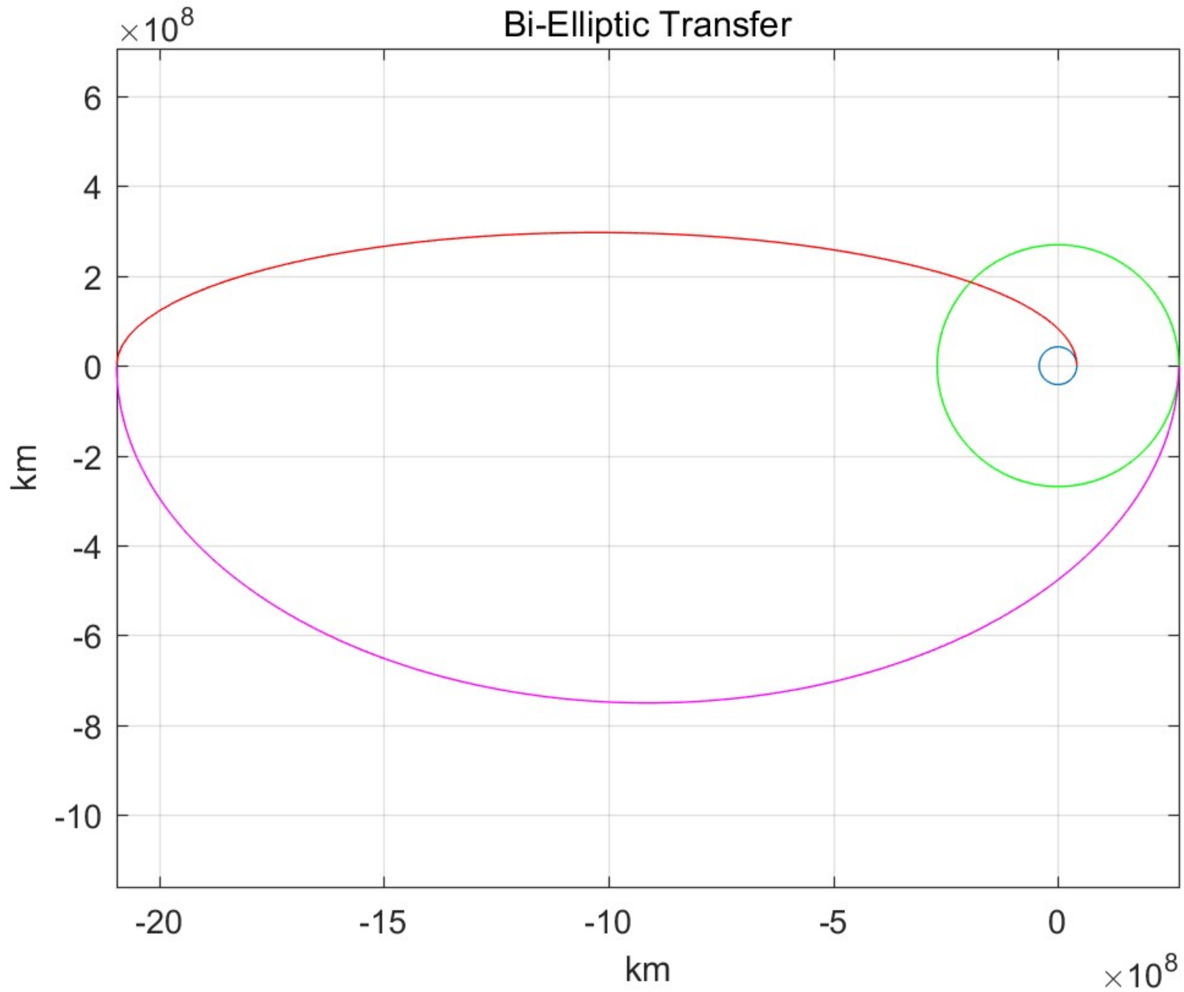


[delva,delvb,delvc,dttu] = biellip(191.344+re,328475,35781.35+re,0, 0, 0, pi)

$$\begin{cases} \Delta V_a = 0.03904 km/s \\ \Delta V_b = 0.003848 km/s \\ \Delta V_c = 0.01276 km/s \end{cases}$$

$$\Delta V_{total} = 0.05565 km/s$$

$$\begin{cases} \tau_1 = 2896537856.1201 min \\ \tau_2 = 3370111325.1706 min \\ \tau_{total} = \tau_1 + \tau_2 = 6266649181.2906 min \end{cases}$$



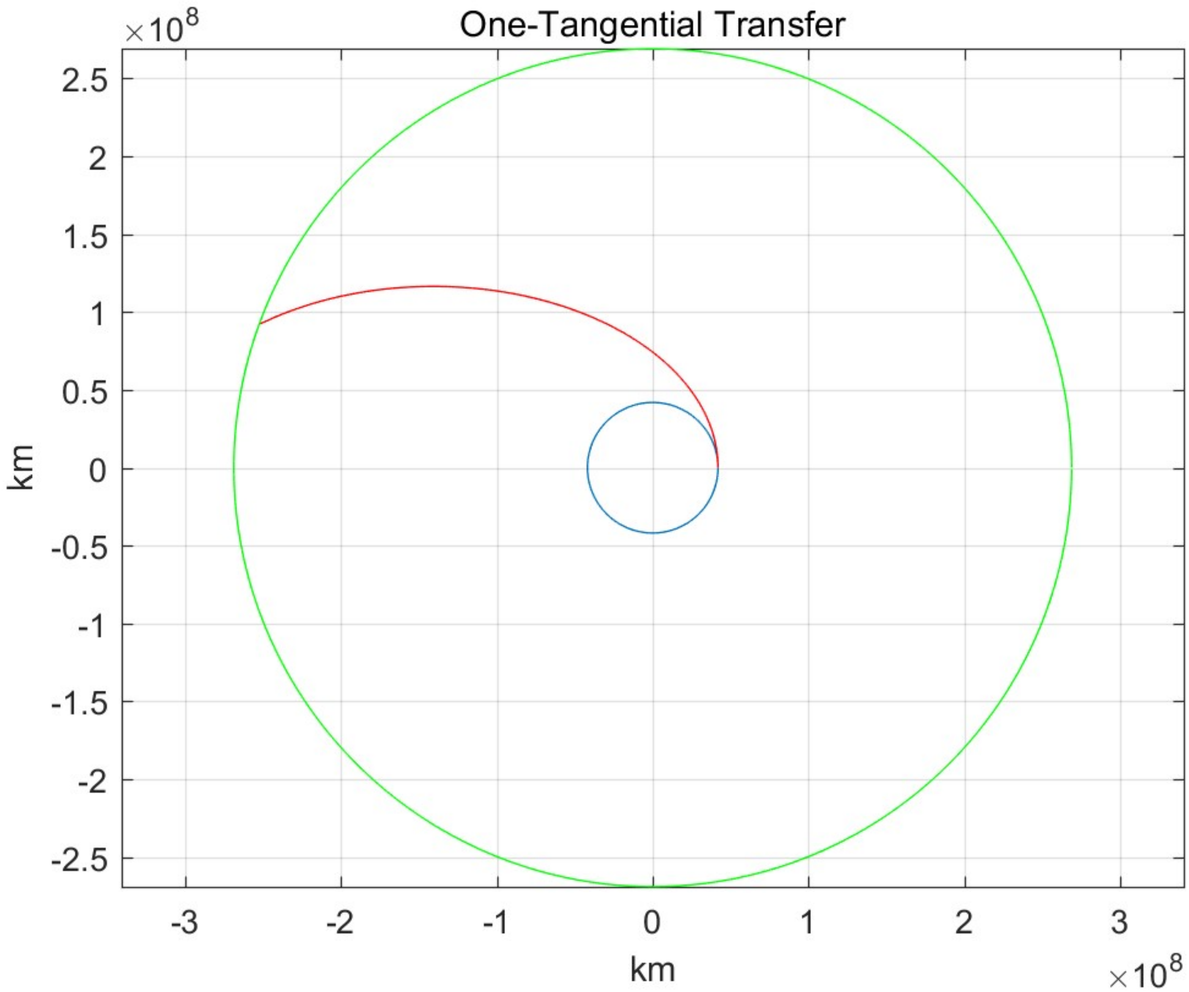
[delva,delvb,dttu,etran, atran] = onetang(191.344+re,35781.35+re,0,0,0,160/180*pi)

$$\begin{cases} \Delta V_a = 0.03225 km/s \\ \Delta V_b = 0.01053 km/s \end{cases}$$

$$\Delta V_{total} = 0.04278 km/s$$

$$\tau_{trans} = 105668295.4356 min$$

$$= 6340097726.1365 sec$$



Compare the results from each transfer

$$\begin{cases} \Delta V_{hohmann} = 0.0493 km/s \\ \Delta V_{bi-elliptic, R\star 16} = 0.05387 km/s \\ \Delta V_{bi-elliptic, R\star 14} = 0.05346 km/s \\ \Delta V_{bi-elliptic, R\star 50} = 0.05565 km/s \\ \Delta V_{One-tangential} = 0.04278 km/s \end{cases}$$
$$\Delta V_{superb} = 0.04278 km/s (One - tangentialburn)$$

$$\begin{cases} \tau_{hohmann} = 160659468.8894 min \\ \tau_{bi-elliptic, R\star 16} = 1401557561.1738 min \\ \tau_{bi-elliptic, R\star 14} = 1195733402.3940 min \\ \tau_{bi-elliptic, R\star 50} = 6266649181.2906 min \\ \tau_{One-tangential} = 105668295.4356 min \end{cases}$$
$$\tau_{superb} = 105668295.4356 min (One - tangentialburn)$$

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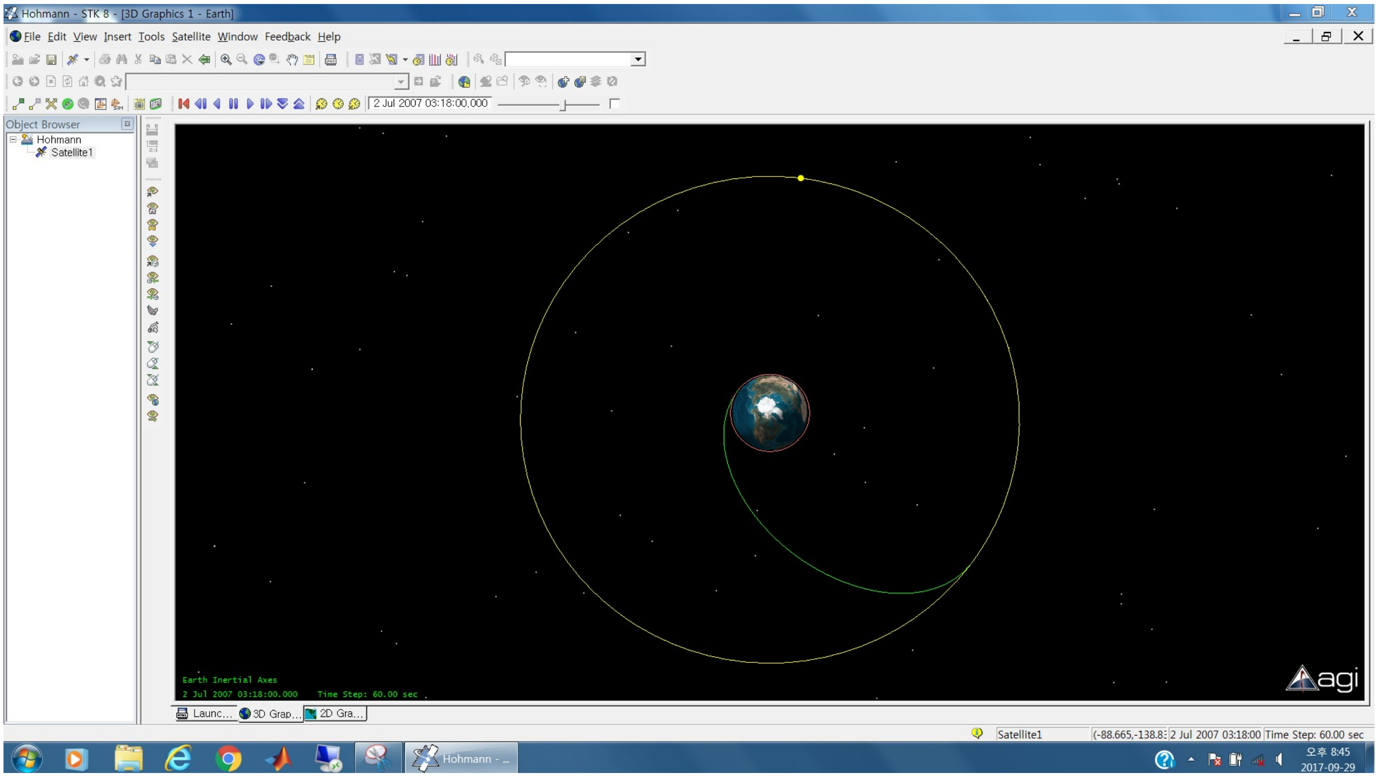
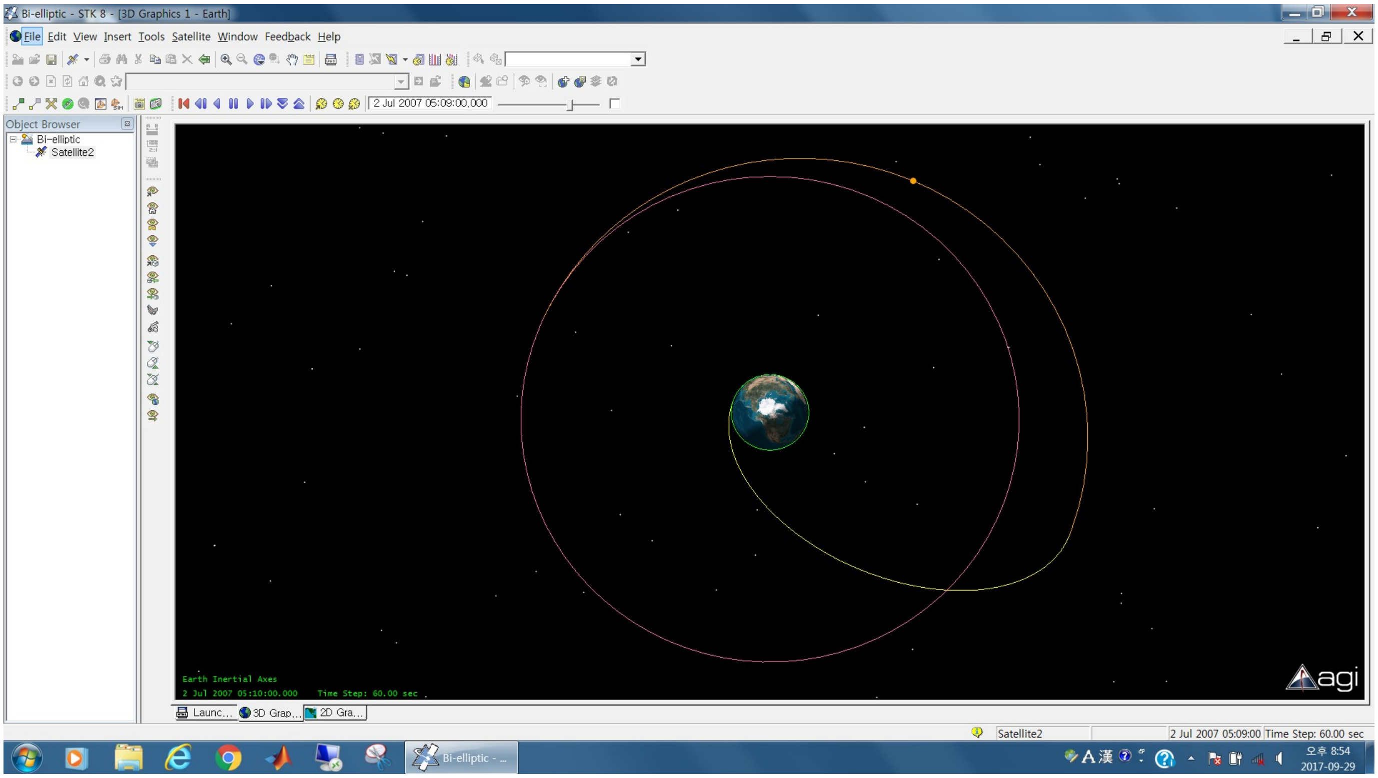
4.

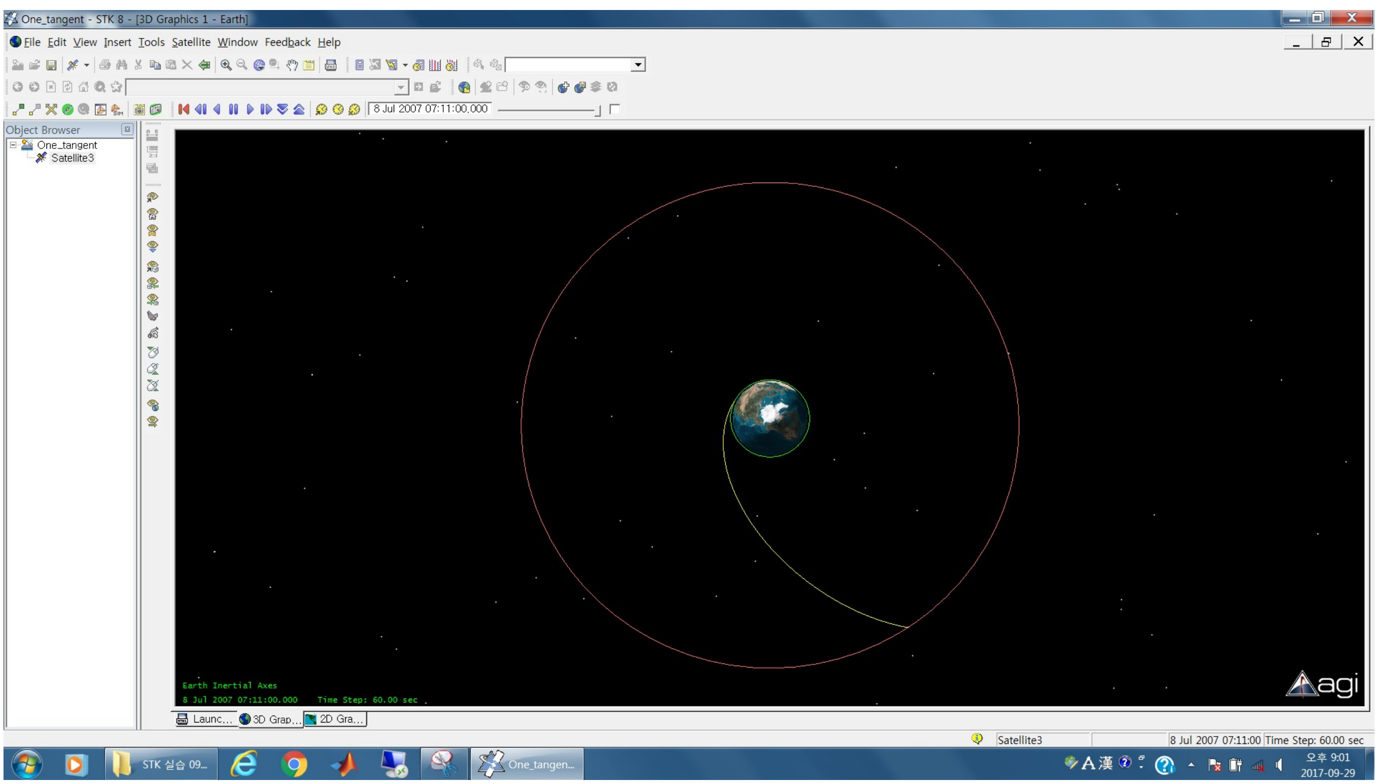
Hohmann transfer



Bi-elliptic transfer



One-tangential burn



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