TRƯỜNG ĐẠI HỌC BÁCH KHOA HÀ NỘI
VIỆN CÔNG NGHỆ THÔNG TIN VÀ TRUYỀN THÔNG

Data structure and Algorithm

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Course outline

Chapter 1. Fundamentals

Chapter 2. Algorithmic paradigms

Chapter 3. Basic data structures

Chapter 4. Tree

Chapter 5. Sorting

Chapter 6. Searching

Chapter 7. Graph

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Chapter 3. Basic data structures

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2.1. Array

- Imagine that we have 100 scores. We need to read them, process them and print them. We must also keep these 100 scores in memory for the duration of the program. We can define a hundred variables, each with a different name, as shown in Figure1.

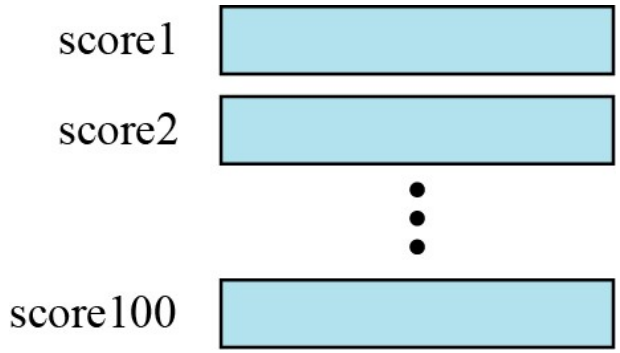


Figure 1 A hundred individual variables

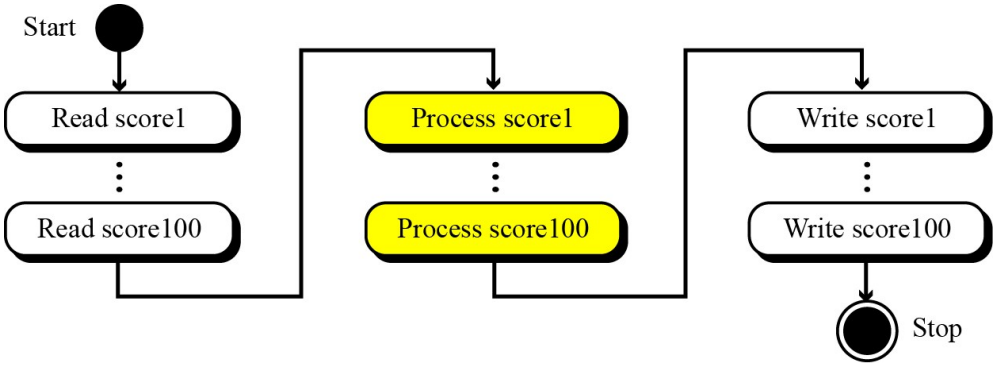


Figure 2 Processing individual variables

- But having 100 different names creates other problems. We need 100 references to read them, 100 references to process them and 100 references to write them. Figure 2 shows a diagram that illustrates this problem.

2.1. Array

- An array is a sequenced collection of elements, normally of the same data type, although some programming languages accept arrays in which elements are of different types.
- We can refer to the elements in the array as the first element, the second element and so forth, until we get to the last element.

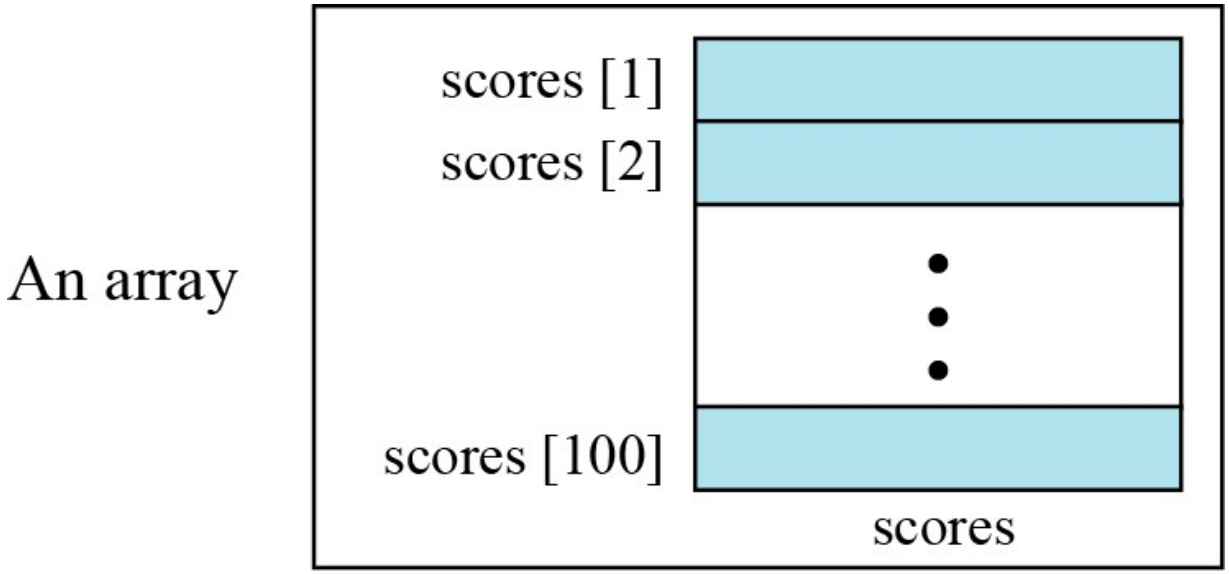
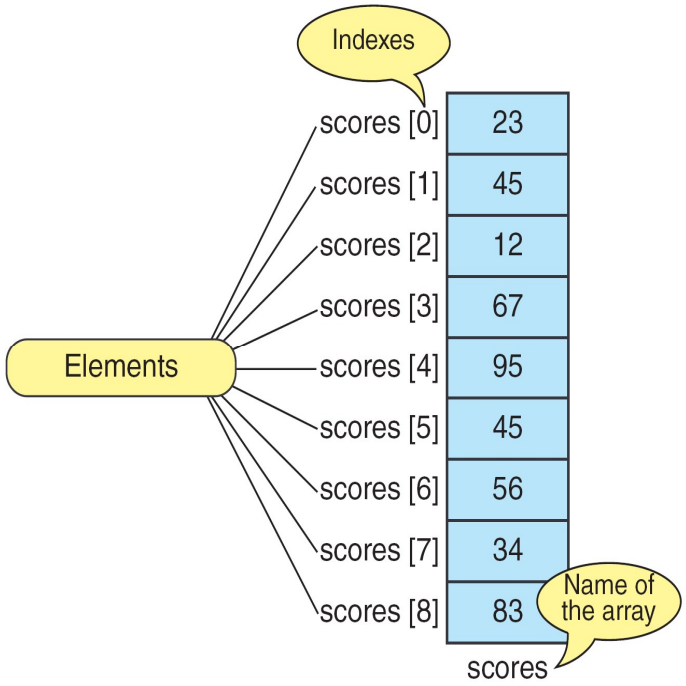


Figure 3. Arrays with indexes

2.1. Array

Basic definitions

- An array is a fixed size sequential collection of elements of identical types.
- Array: a set of pairs (index and value)
  - data structure: for each index, there is a value associated with that index.
  - representation: implemented by using consecutive memory.
- In C/C++/Java: the element in an array are indexed by the integers 0 to *n*- 1, where *n* is the size of the array



Declaring an one-dimensional array

To declare an array, we need to specify its data type, the array's identifier and the size:

```
type arrayName [arraySize];
```

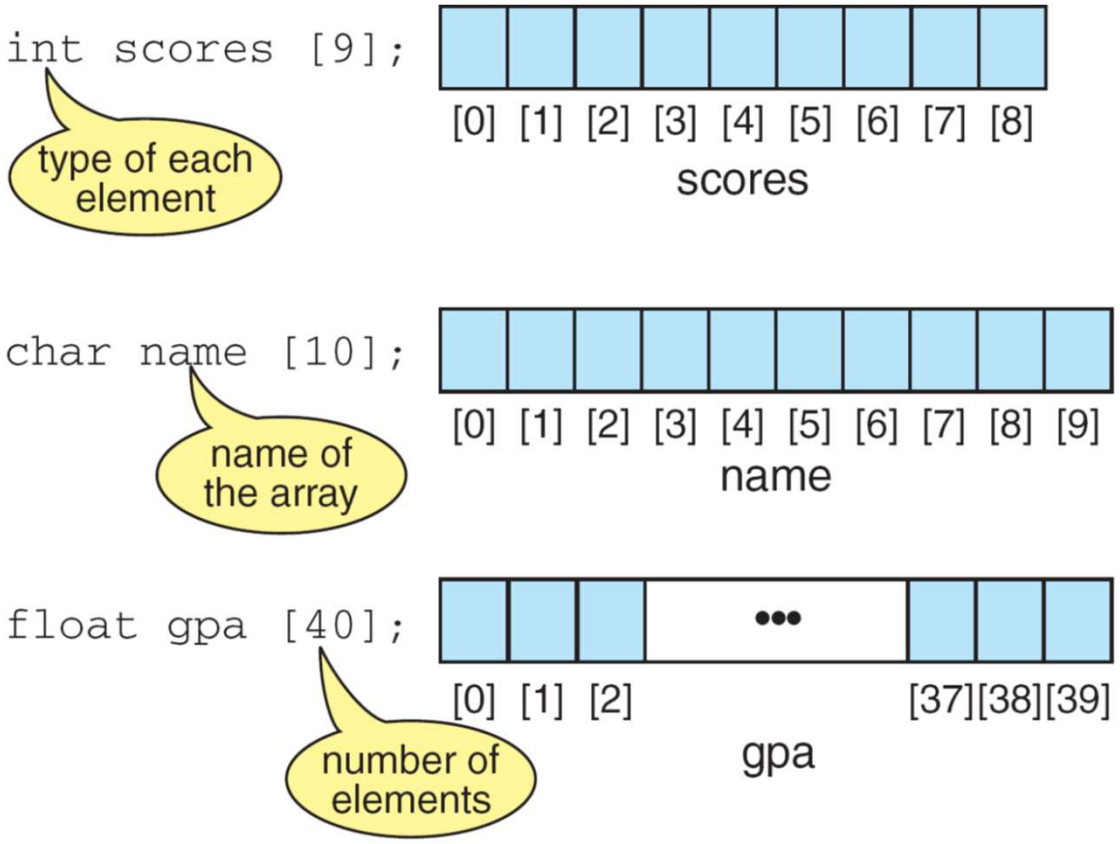
Example: declare `int A[5];`

to create an array A having 5 elements of integer type (4 bytes for each element)

- The **arraySize** can be a constant (for fixed length arrays) or a variable (for variable-length arrays)
  - Example:
```
double A[10];
int n;
double A[n];
```

- Before using an array (even if it is a variable-length array), we must declare and initialize it!

Declaration example: fixed length array

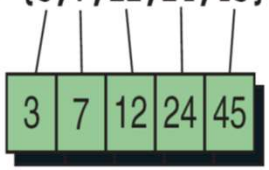


Declaring an one-dimensional array

- We can initialize fixed-length array elements when we define an array.
- If we initialize fewer values than the length of the array, C assigns zeroes to the remaining elements.

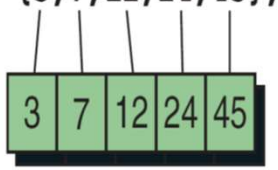
(a) Basic Initialization

```
int numbers[5] = {3,7,12,24,45};
```



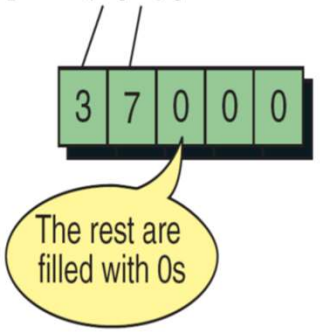
(b) Initialization without Size

```
int numbers[ ] = {3,7,12,24,45};
```



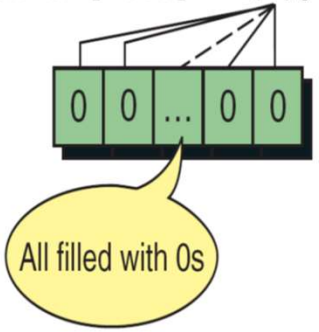
(c) Partial Initialization

```
int numbers[5] = {3,7};
```



(d) Initialization to All Zeros

```
int lotsOfNumbers [1000] = {0};
```

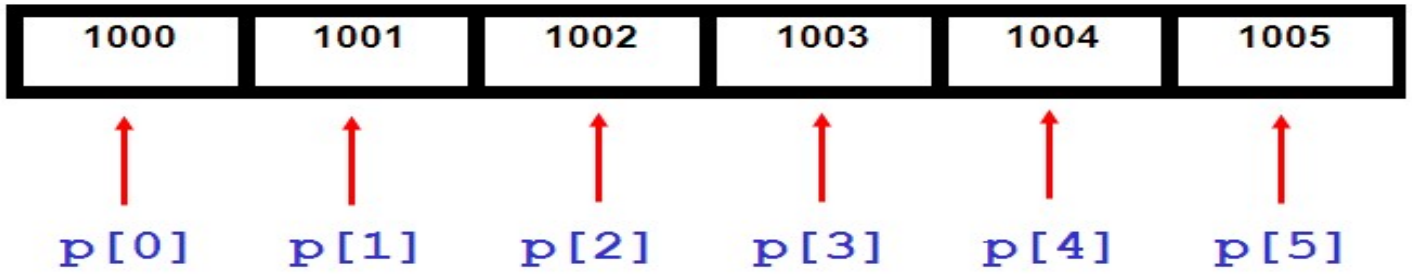


Declaring an Array

- So what is actually going on when you set up an array?

Memory:

- Each element is held in the next location along in memory
- Essentially what the computer is doing is looking at the **FIRST** address (which is **pointed** to by the variable **p)** and then just counts along.



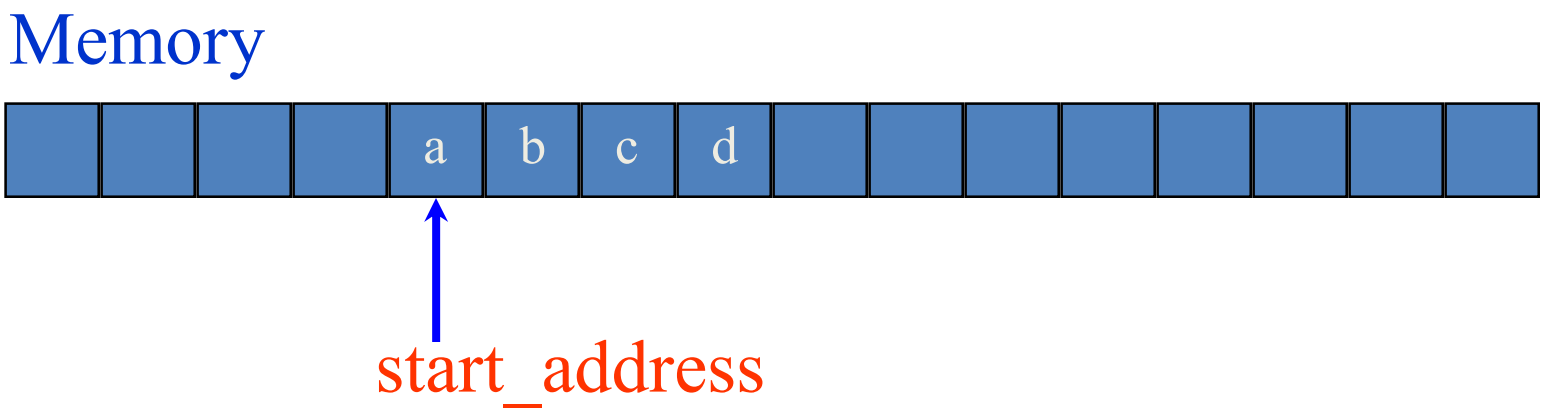
Accessing Elements

To access an array's element, we need to provide an integral value to identify the index we want to access.

- We can do this using a constant: `scores[0];`
- We can also use a variable:

```
for(i = 0; i < 9; i++)
    scoresSum += scores[i];
```

1D Array Representation in C



Example: 1-dimensional array x = [a, b, c, d]

- Map into contiguous memory locations
- Location($x[i]$) = start_address + W*i

where

- start_address: the address of the first element in the array
- W: size of each element in the array

Example

Write a C program that gives the address of each element of an 1D array:

```
#include <stdio.h>
int main()
{    int A[ ] = {5, 10, 12, 15, 4};
     int rows=5;
     /* print the address of 1D array by using pointer  */
    int *ptr = A;
    printf("Address    Contents\n");
    for (int i=0; i < rows; i++)
         printf("%8u %5d\n", ptr+i, *(ptr+i));
}
```

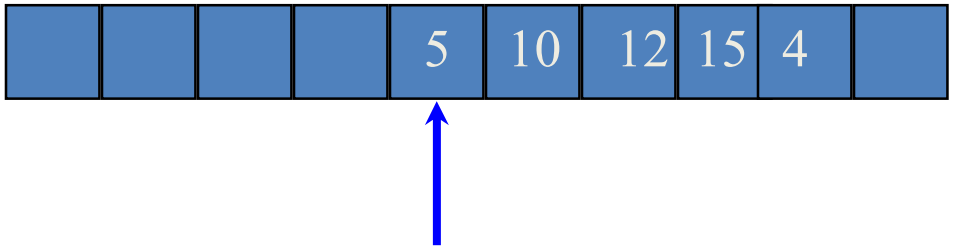
Result in DevC
(sizeof(int)=4)

```
Address    Contents
 6487536      5
 6487540     10
 6487544     12
 6487548     15
 6487552      4
```

ptr+i : address of element A[i]
*(ptr+i) : content of element A[i]

Memory

Location(A[i]) = start_address + W*i



start_address=6487536

Result in turboC
(sizeof(int)=2)

```
Address    Contents
   65516      5
   65518     10
   65520     12
   65522     15
   65524      4
```

Arrays in C

```
int list[5], *plist[5];
```

list[5]: five integers
list[0], list[1], list[2], list[3], list[4]
*plist[5]: five pointers to integers
plist[0], plist[1], plist[2], plist[3], plist[4]

Implementation of 1-D array

list[0]	start address = α
list[1]	α + sizeof(int)
list[2]	α + 2*sizeof(int)
list[3]	α + 3*sizeof(int)
list[4]	α + 4*size(int)

- Compare `int *list1` and `int list2[5]`:
  - Same: list1 and list2 are pointers.
  - Difference: list2 reserves five locations.
  - Notations:
    - list2 : a pointer to list2[0]
    - (list2 + i) : a pointer to list2[i] (&list2[i])
    - *(list2 + i) : content of list2[i]

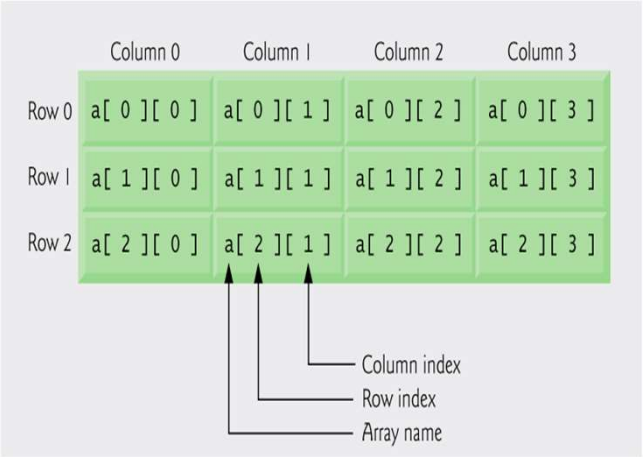
Declaring two-dimensional array

- How to declare:

```
<element-type> <arrayName> [size1][size2];
```

Example: `double a[3][4];`

may be shown as a table



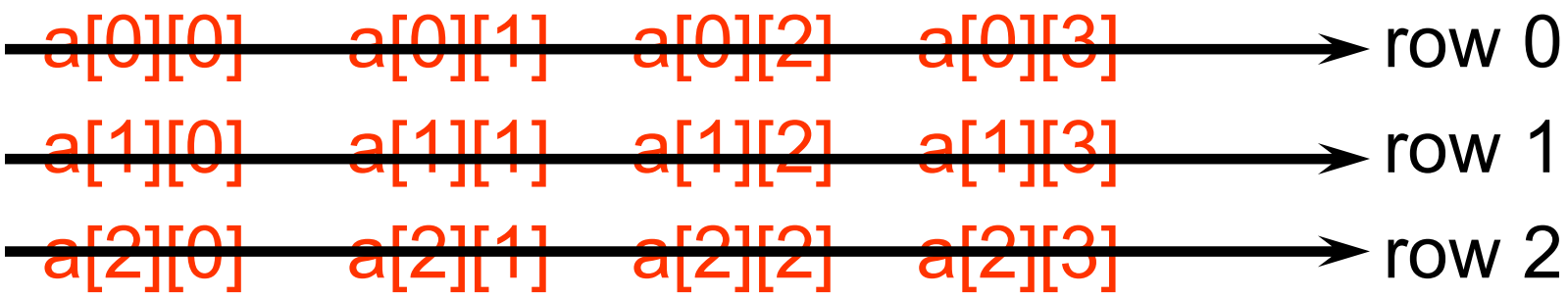
- Using the two-dimensional array initializer

Example: `int a[3][4] = {1,2,3,4,5,6,7,8,9,10,11,12};`

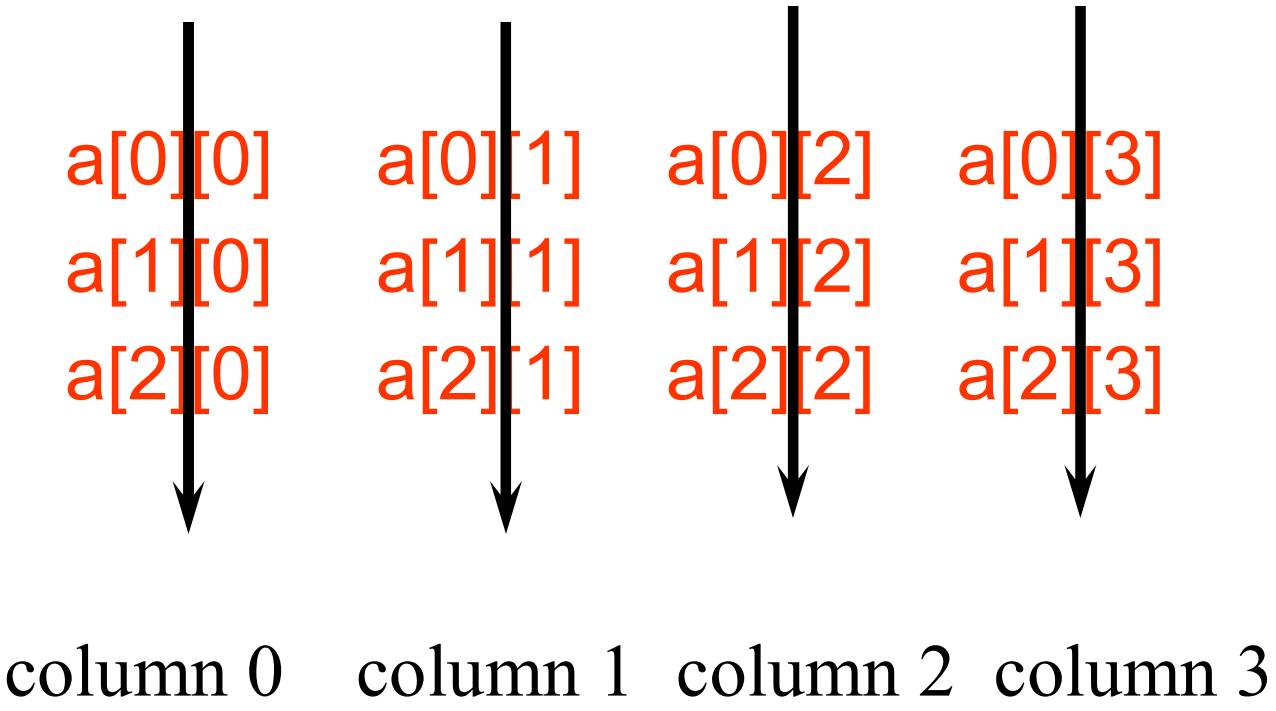
- Access to element of array: a[2] [1];

a[0][0] = 1	a[0][1]=2	a[0][2]=3	a[0][3]=4
a[1][0] = 5	a[1][1]=6	a[1][2]=7	a[1][3]=8
a[2][0] = 9	a[2][1]=10	a[2][2]=11	a[2][3]=12

Rows of a 2D Array

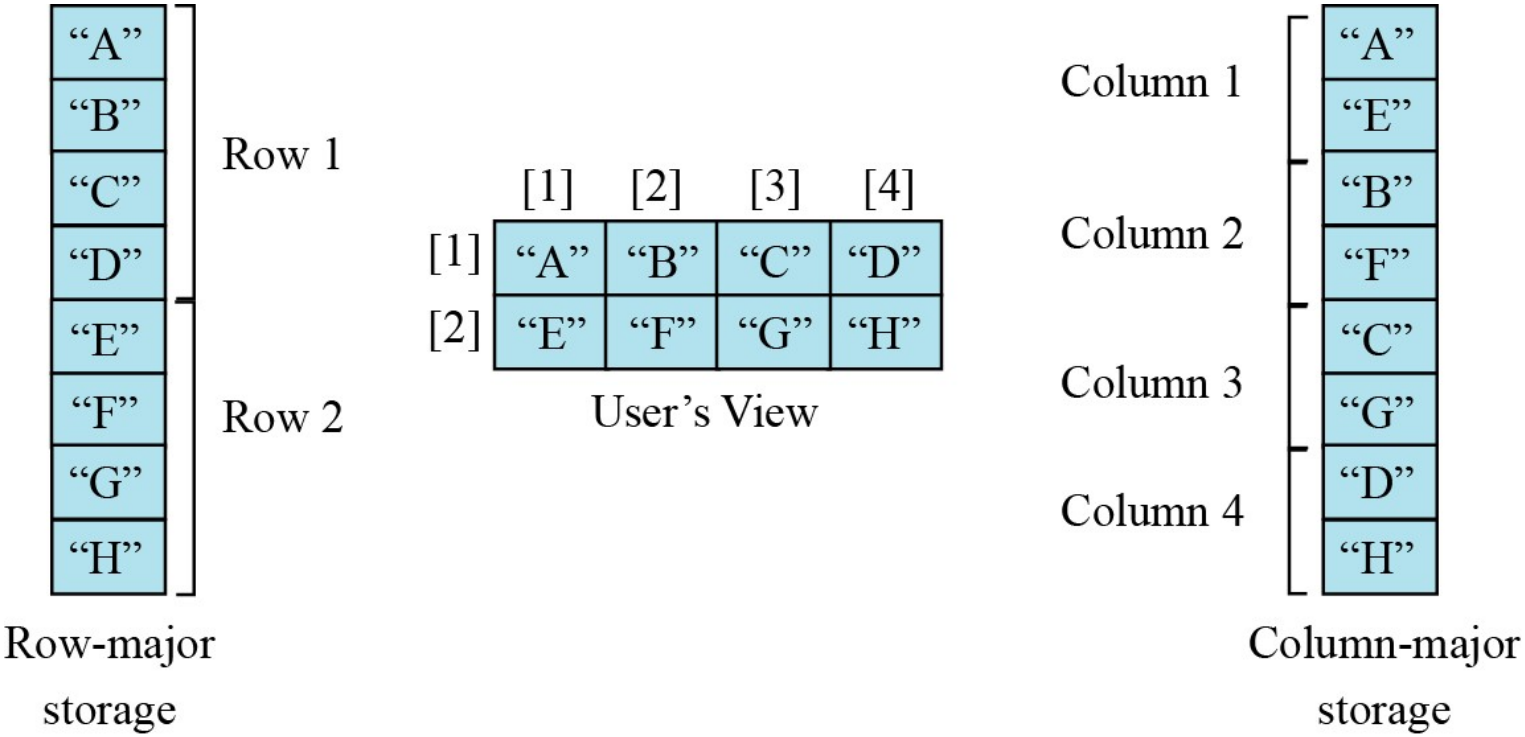


Columns of a 2D Array



Representation of Arrays

- Multidimensional arrays are usually implemented by one dimensional array via either row major order or column major order.

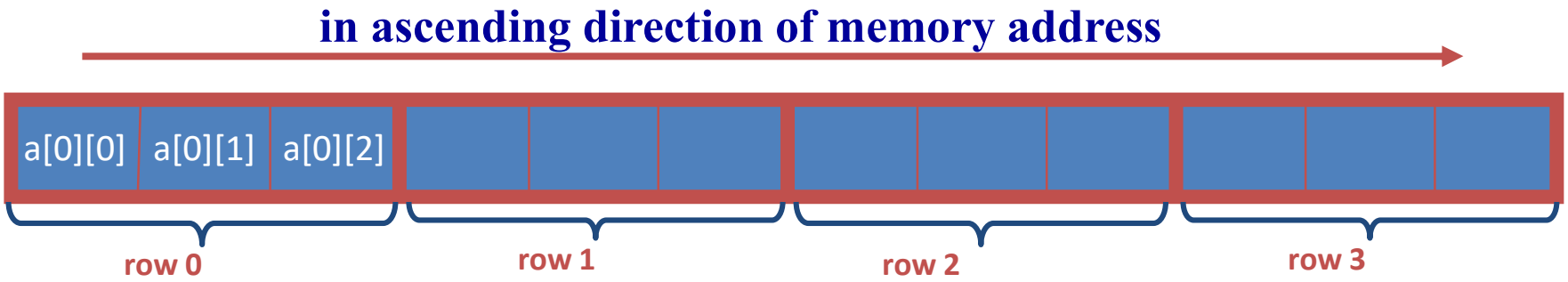


Row-Major Mapping (e.g. Pascal, C/C++)

- Row- major order is a method of representing multi-dimensional array in sequential memory. In this method, elements of an array are arranged sequentially row by row. Thus, elements of the first row occupies the first set of memory locations reserved for the array, elements of the second row occupies the next set of memory and so on.

Elements of Row 0	Elements of Row 1	Elements of Row 2	....	Elements of Row i	.....

- Example: int a[4][3]



- Example 3 x 4 array: a b c d
  e f g h
  i j k l

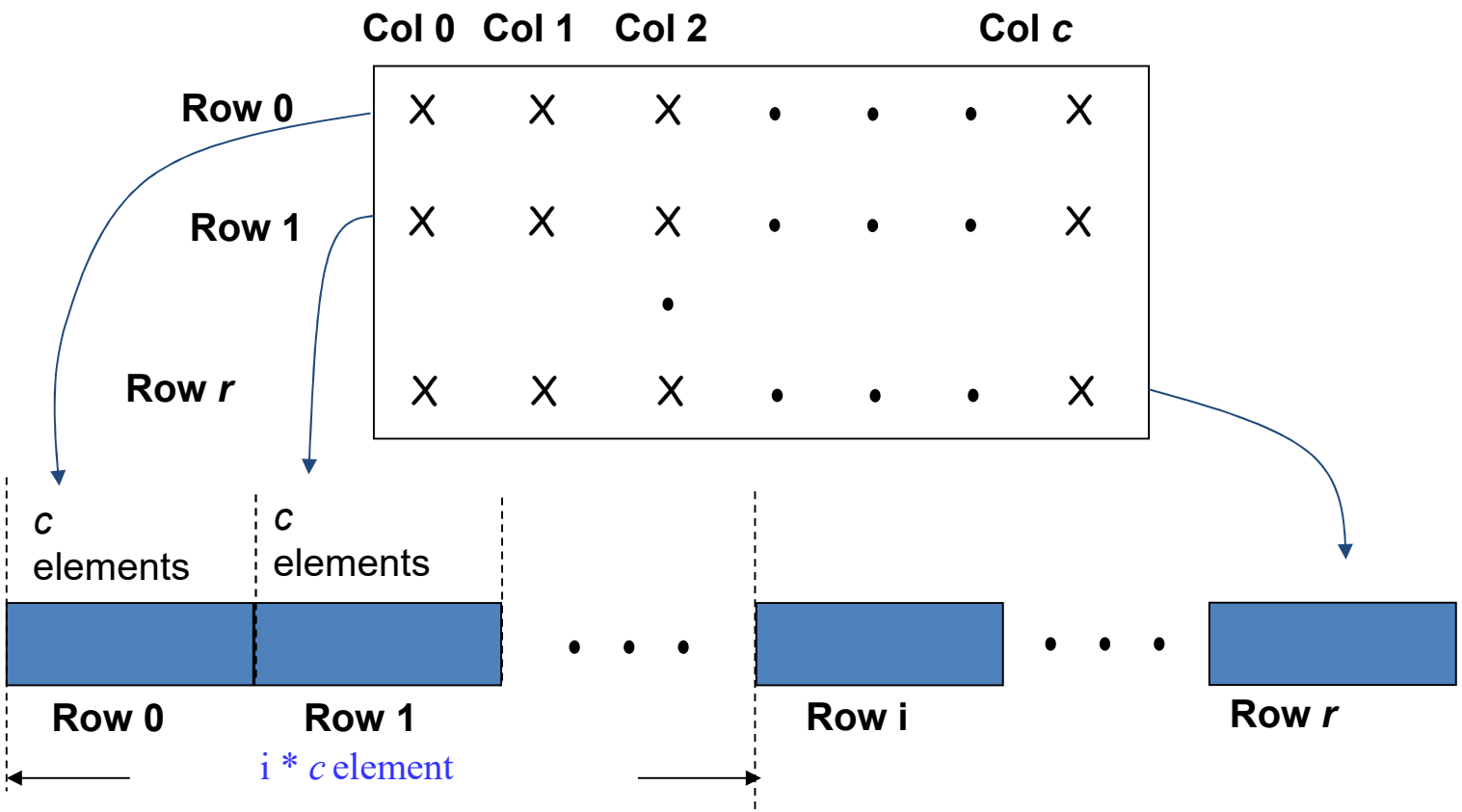
Convert into 1D array **Y** by collecting elements by rows:

- Within a row elements are collected from left to right.
- Rows are collected from top to bottom.

Thus, we get **Y**[ ] =

{a, b, c, d, e, f, g, h, i, j, k, l}

Two Dimensional Array Row Major Order



Column-Major Mapping (e.g. Matlab, Fortran)

- In this method, elements of an array are arranged sequentially column by column. Thus, elements of the first column occupies the first set of memory locations reserved for the array, elements of the second column occupies the next set of memory and so on.

- Example 3 x 4 array:

a b c d

e f g h

i j k l

Convert into 1D array **Y** by collecting elements by columns.

- Within a column elements are collected from top to bottom.
- Columns are collected from left to right.

Thus, we get **Y**[ ] =

{a, e, i, b, f, j, c, g, k, d, h, l}

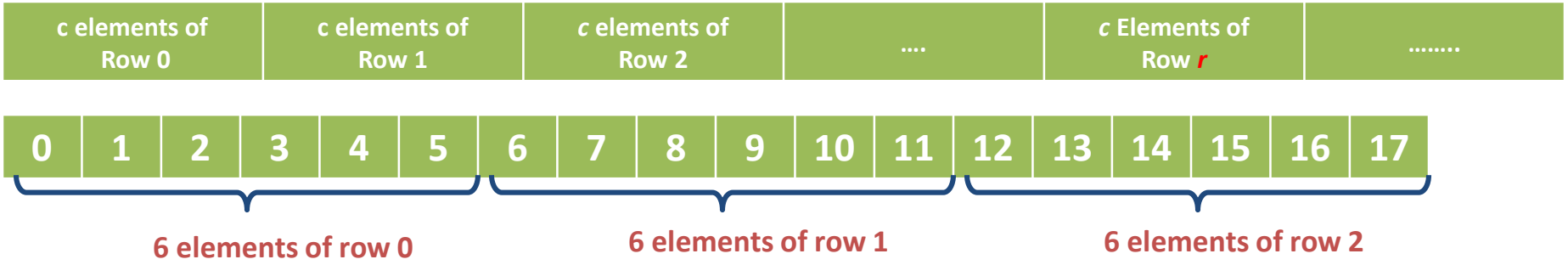
Row- and Column-Major Mappings

2D array: *r* rows, *c* columns

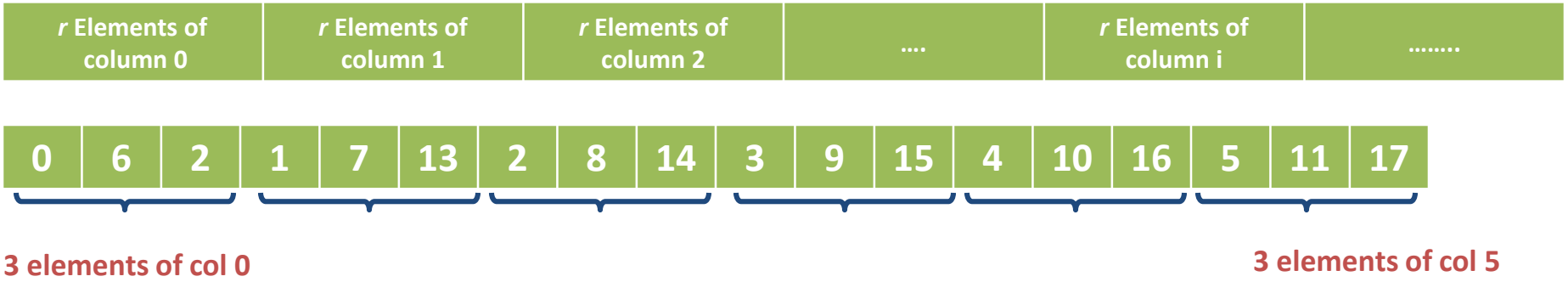
Example: int a[3][6]={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17};

a[0][0]=0	a[0][1]=1	a[0][2]=2	a[0][3]=3	a[0][4]=4	a[0][5]=5
a[1][0]=6	a[1][1]=7	a[1][2]=8	a[1][3]=9	a[1][4]=10	a[1][5]=11
a[2][0]=12	a[2][1]=13	a[2][2]=14	a[2][3]=15	a[2][4]=16	a[2][5]=17

Memory: row-major order



Memory: column-major order



Locating Element x[i][j]: row-major order

- Assume x:
  - has ***r*** rows and ***c*** columns (thus, each row has ***c*** elements)

- Locating element $x[i][j]$:
  - i rows to the left of row i ➔ so $i*c$ elements to the left of $x[i][0]$
  - $x[i][j]$ is mapped to position: $i*c + j$ of the 1D array
  - The location of element $x[i][j]$:

  Location($x[i][j]$) =start_address + W ($i*c + j$)

  Where
  - start_address: the address of the first element ($x[0][0]$) in the array
  - W: is the size of each element
  - c: number of columns in the array
  - r: number of rows in the array

Example

Write a C program that gives the address of each element of a 2D array:

```
#include <stdio.h>
int main()
{   int  a[3] [4] = {1,2,3,4,5,6,7,8,9,10,11,12};
    int rows=3, cols =4;
    /* print the address of 2D array by using pointer  */
    int *ptr = a;
    printf("Address    Contents\n");
    for (int i=0; i < rows; i++)
       for (int j=0; j < cols; j++)
        printf("%8u %5d\n", ptr +((i*cols)+j), *(ptr+ ((i*cols)+j)) );}
```

Result in DevC

(sizeof(int)=4)

```
Address    Contents
6487488        1
6487492        2
6487496        3
6487500        4
6487504        5
6487508        6
6487512        7
6487516        8
6487520        9
6487524       10
6487528       11
6487532       12
```

Memory

Location(a[i][j]) = start_address + W*[(i*cols) + j]

Example row-major order: Memory allocation for 2D array (type int)

- Address (location) of elements in 2D array:

  int a[4][3]

a[0][0]	address = α
a[0][1]	α + 1*sizeof(int)
a[0][2]	α + 2*sizeof(int)
a[1][0]	α + 3*sizeof(int)
a[1][1]	α + 4*sizeof(int)
a[1][2]	α + 5*sizeof(int)
a[2][0]	α + 6*sizeof(int)

. . .

General: Declare

int a[m][n];

- Assume: the address of the first element (a[0][0]) is α.
- Then, the address of element a[i][j] is:

  α + (i*n + j)*sizeof(int)

Locating Element x[i][j]: column-major order

- Assume x:

  - has ***r*** rows and ***c*** columns (thus, each column has ***r*** elements)
- Locating element $x[i][j]$:
  - j columns to the left of column j ➔ so $j*r$ elements to the left of $x[0][j]$
  - $x[i][j]$ is mapped to position: $j*r + i$ of the 1D array
  - The location of element $x[i][j]$:

    Location($x[i][j]$) = start_address + W ($j*r + i$)

    Where
    - start_address: the address of the first element in the array
    - W: is the size of each element
    - c: number of columns in the array
    - r: number of rows in the array

Example: array : `int a[3][4]={1,2,3,4,5,6,7,8,9,10,11,12};`

Determine the address of the element a[1][2] if start_address = 6487488

Example 1: Row- and Column-Major Mappings

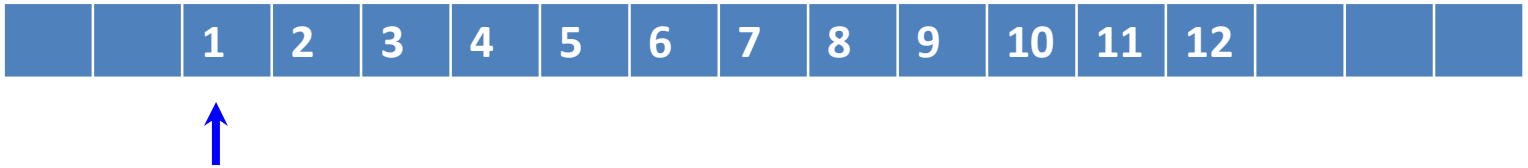
2D array:

int a[3][4] = {1,2,3,4,5,6,7,8,9,10,11,12};

a[0][0] = 1	a[0][1]=2	a[0][2]=3	a[0][3]=4
a[1][0] = 5	a[1][1]=6	a[1][2]=7	a[1][3]=8
a[2][0] = 9	a[2][1]=10	a[2][2]=11	a[2][3]=12

Memory: row-major order

Location(a[i][j]) = start_address + W*[(i*cols) + j]



start_address=6487488

Location(a[1][2]) = ?

Memory: column-major order

Location(a[i][j]) = start_address + W*[(j*rows) + i]

Location(a[1][2]) = ?

start_address=6487488

Example 2: Row- and Column-Major Mappings

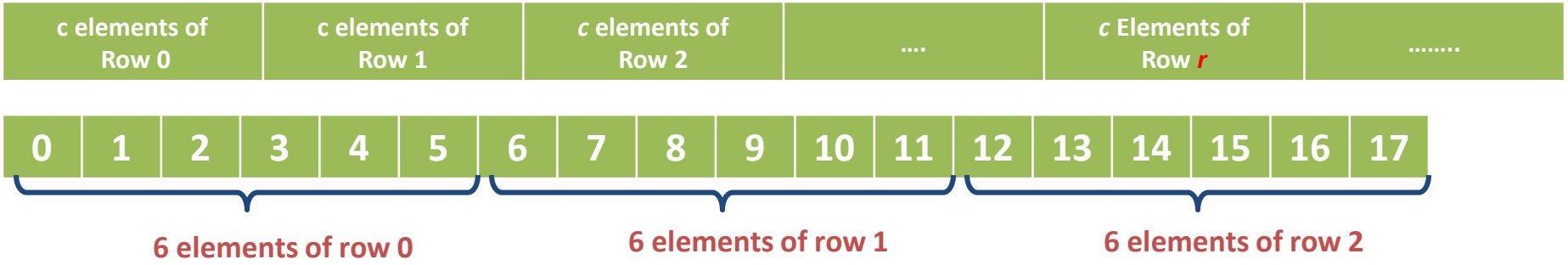
2D array: *r* rows, *c* columns

Example: int a[3][6]={0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17};

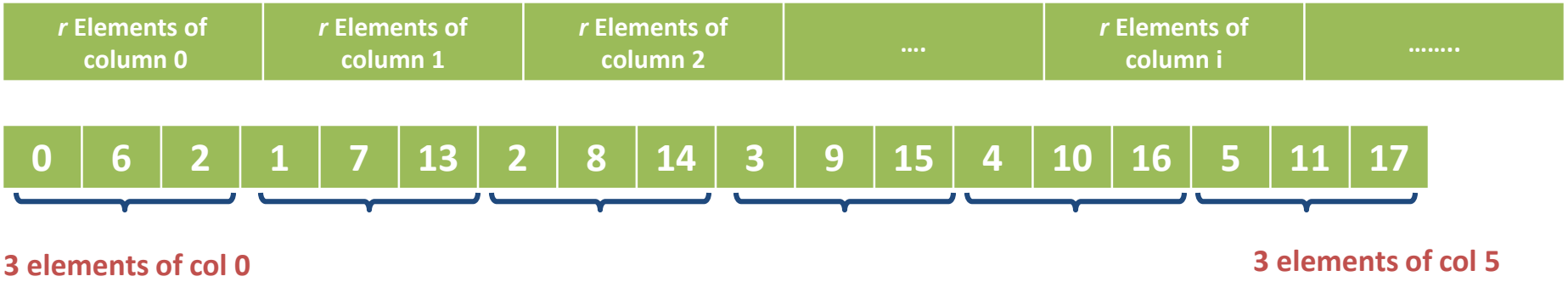
a[0][0]=0	a[0][1]=1	a[0][2]=2	a[0][3]=3	a[0][4]=4	a[0][5]=5
a[1][0]=6	a[1][1]=7	a[1][2]=8	a[1][3]=9	a[1][4]=10	a[1][5]=11
a[2][0]=12	a[2][1]=13	a[2][2]=14	a[2][3]=15	a[2][4]=16	a[2][5]=17

Memory: row-major order

start_address = 1000 ➔ Location(a[1][4]) = ?



Memory: column-major order



Operations on the array

- The common operations on arrays are searching, insertion, deletion, retrieval and traversal.

Example: Given an array *S* consists of *n* integers: S[0], S[1], …, S[*n*-1]

- **Search operation**: search a value ***key*** whether appears in the array S or not

  `function Search(S, key)` returns true if ***key*** appears in S; false otherwise
- **Retrieval operation**: get the value of the element at index ***i*** of the array S

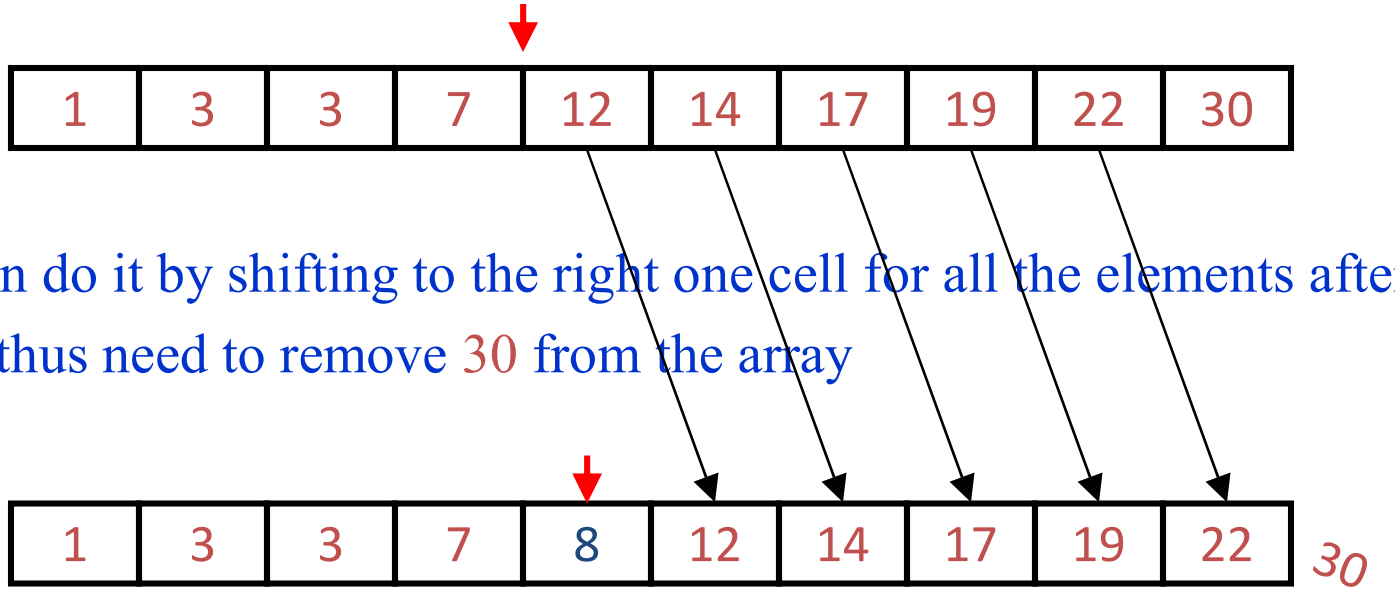
  `function Retrieve(S, i)`: returns the value S[i] if 0 <= *i* <= *n*-1
- **Traversal operation**: print the value of all elements in the array S

  `function PrintArray(S, n)`
- **Insert operation**: insert a value ***key*** into the array S
- **Delete operation**: delete the element at index ***i*** of the array S

- Although searching, retrieval and traversal of an array is an easy job, insertion and deletion is time consuming. The elements need to be shifted down before insertion and shifted up after deletion.

Inserting an element into an array

- Assume we need to insert 8 into an array already be sorted in ascending order:

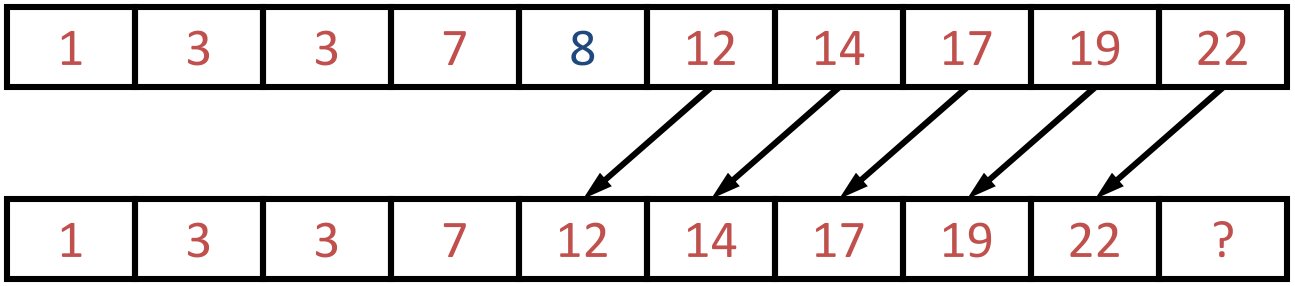


- We can do it by shifting to the right one cell for all the elements after the mark
  - It thus need to remove 30 from the array

• Moving all elements of the array is a slow operation (requires linear time O(n) where n is the size of array)

Deleting an element from an array

- In order to delete an element, we need to shift to the left all previous elements



- Delete operation is a slow operation.
- Regular implementation of this operation is undesirable.
- Delete operation makes the last element free
  - How we could mark the last element of the array being free?
    - We need variable to store the size of the array

    Example: variable size is used to store the size of the array. Before deletion, size = 10. After deletion, we need to update the value of size: size = 10 – 1 = 9

Operations on the array

Thinking about the operations discussed in the previous section gives a clue to the application of arrays. If we have a list in which a lot of insertions and deletions are expected after the original list has been created, we should not use an array. An array is more suitable when the number of deletions and insertions is small, but a lot of searching and retrieval activities are expected.

An array is a suitable structure when a small number of insertions and deletions are required, but a lot of searching and retrieval is needed.

Represent Matrices based on arrays

- ***m x n* matrix** is a table with *m* rows and *n* columns, but numbering begins at 1 rather than 0.
- *M(i,j)* denotes the element in row *i* and column *j*.
- Common matrix operations
  - transpose
  - addition
  - Multiplication

	col 1	col 2	col 3	col 4
row 1	7	2	0	9
row 2	0	1	0	5
row 3	6	4	2	0
row 4	8	2	7	3
row 5	1	4	9	6

- Shortcomings Of Using A 2D Array For A Matrix:
  - Indexes are off by 1.
  - C arrays do not support matrix operations such as add, transpose, multiply, and so on.
    - Suppose that x and y are 2D arrays. Can't do x + y, x –y, x * y, etc. in C.

    ➔We need to develop functions to support all matrix operations.

Sparse Matrix

	col 1	col 2	col 3
row 1	-27	3	4
row 2	6	82	-2
row 3	109	-64	11
row 4	12	8	9
row 5	48	27	47

5*3

15/15

	col1	col2	col3	col4	col5	col6
row1	15	0	0	22	0	−15
row2	0	11	3	0	0	0
row3	0	0	0	−6	0	0
row4	0	0	0	0	0	0
row5	91	0	0	0	0	0
row6	0	0	28	0	0	0

6*6

8/36

sparse matrix

data structure?

Sparse Matrix

(1) Represented by a two-dimensional array (e.g. `int M[6][6];`)

- Sparse matrix wastes space.

(2) Each element is characterized by <row, col, value>.

- The terms in A should be ordered based on <row, col>

	col1	col2	col3	col4	col5	col6
row1	15	0	0	22	0	−15
row2	0	11	3	0	0	0
row3	0	0	0	−6	0	0
row4	0	0	0	0	0	0
row5	91	0	0	0	0	0
row6	0	0	28	0	0	0

6*6

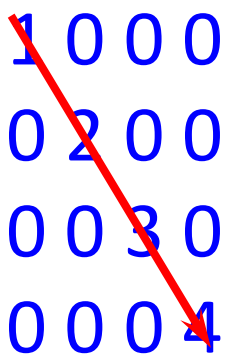
	row	*col*	*value*
A [0]	1	1	15
[1]	1	4	22
[2]	1	6	-15
[3]	2	2	11
[4]	2	3	3
[5]	3	4	-6
[6]	5	1	91
[7]	6	2	28

```
A[0][0] = 1;
A[0][1] = 1;
A[0][2] = 15
```

```
A[5][0] = 3;
A[5][1] = 4;
A[5][2] = -6;
```

```
/* Array representation of sparse matrix
  //[ ][0] represents row
  //[ ][1] represents col
  //[ ][2] represents value */
   int MAX = 8; //number of elements != 0 in sparse matrix
   int A[MAX][3];
```

Diagonal Matrix



- An ***n* x *n*** matrix in which all nonzero terms are on the diagonal.
  - M(i, j) is on diagonal iff $i = j$
  - number of diagonal elements in an n x n matrix is n
  - non diagonal elements are zero
- Store diagonal only vs n^2 whole:
  - `int M[5];`
  - `int M[5][5];`

Triangular Matrix

1 0 0 0

2 3 0 0

4 5 6 0

7 8 9 10

- An n x n matrix in which all nonzero terms are either on or below the diagonal.
  - M(i,j) is part of lower triangle iff $i >= j$
  - Number of elements in lower triangle is

$$1 + 2 + \ldots + n = n(n+1)/2$$

- Store n^2 whole vs only the lower triangle:
  - Store n^2 whole:
    - Use 2D array A[n][n]
  - Store only the lower triangle by:
    - Option 1: Map lower triangular into a 1D array
    - Option 2: Irregular 2D array

Option 1: Map Lower Triangular Array into a 1D array

Use row-major order, but omit terms that are not part of the lower triangle.

For the matrix

1 0 0 0
2 3 0 0
4 5 6 0
7 8 9 10

we get 1D array:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

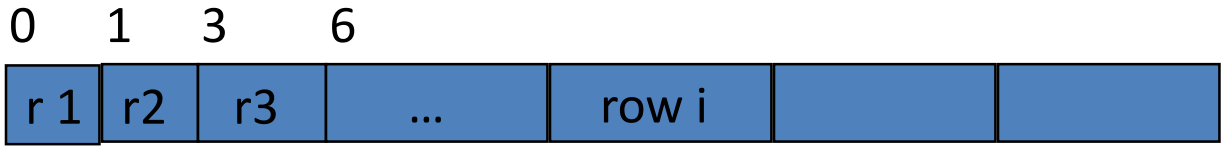
Index of element M(i,j) in 1D array

For the matrix M_{4x4}

1 0 0 0 → Row 1
2 3 0 0
4 5 6 0
7 8 9 10

we get 1D array:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10



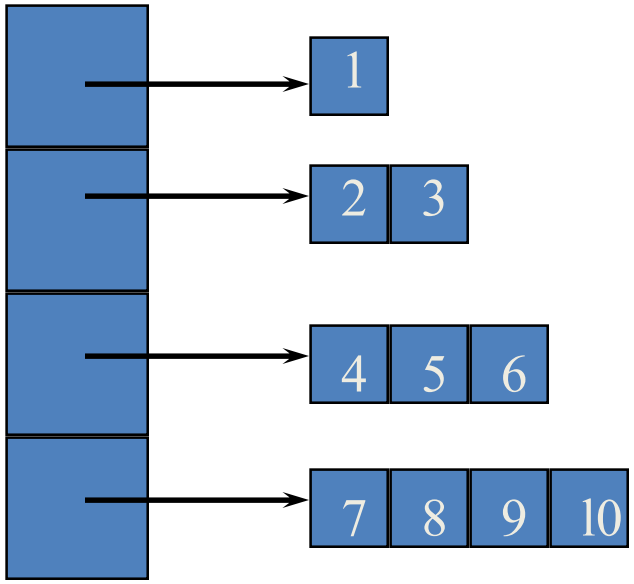
- Order is: row 1, row 2, row 3, …
  - Row *i* is preceded by rows 1, 2, …, *i*-1
- Size of row *i* (number of elements in row *i*) is *i*
- Number of elements that precede row *i* is

  $1 + 2 + 3 + \ldots + i\text{-}1 = i(i\text{-}1)/2$

➔ So element M(*i*, *j*) is at position $i(i\text{-}1)/2 + j\text{-}1$ of the 1D array

  - Example: M(3, 2) is at position $3(3\text{-}1)/2 + 2\text{-}1 = 4$

Option 2: Map Lower Triangular Array into Irregular 2D Arrays



Store only the lower triangle:

1 0 0 0

2 3 0 0

4 5 6 0

7 8 9 10

Irregular 2-D array: the length of rows is not required to be the same.

Creating and Using Irregular 2D Arrays

// STEP 1: declare a two-dimensional array variable

```
int ** iArray = new int*[numberOfRows];
```

OR:

```
 int ** iArray;
 malloc(iArray, numberOfRows*sizeof(*iArray));
```

//STEP 2: allocate the desired number of rows

// now allocate space for elements in each row

```
for (int i = 0; i < numberOfRows; i++)
    iArray[i] = new int [length[i]];
```

OR:

```
for (int i = 0; i < numberOfRows; i++)
        malloc(iArray[i], length[i]*sizeof(int));
```

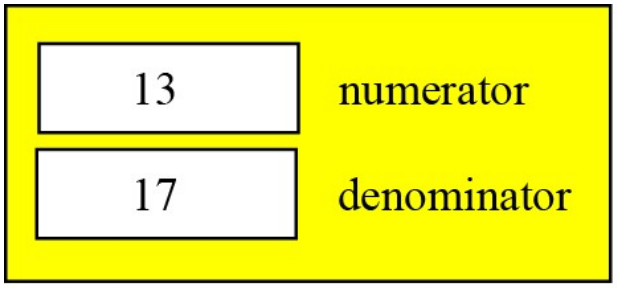
// STEP 3: use the array like any regular array:

```
iArray[2][3] = 5;
iArray[4][6] = iArray[2][3]+2;
iArray[1][1] += 3;
```

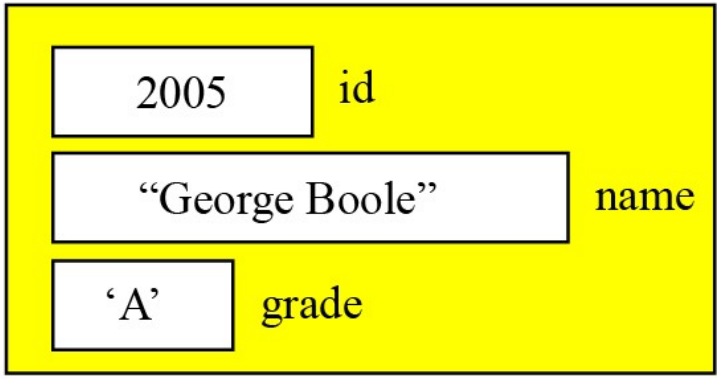
Contents

2.2. Record

- A record is a collection of related elements, possibly of different types, having a single name.
- Each element in a record is called a field:
  - A field has a type and exists in memory.
  - Fields can be assigned values, which in turn can be accessed for selection or manipulation.
- Example: Figure below contains two examples of records.
  - The first example: fraction has two fields: numerator and denominator, both of which are integers.
  - The second example: student has three fields (id, name, grade) made up of three different types.



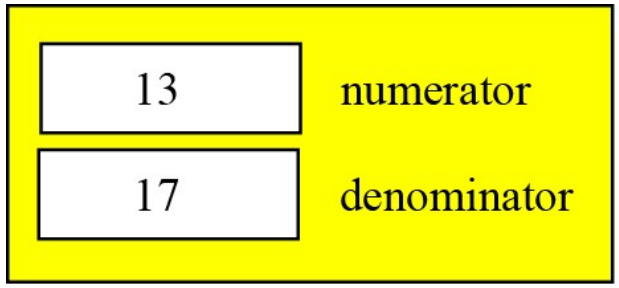
fraction



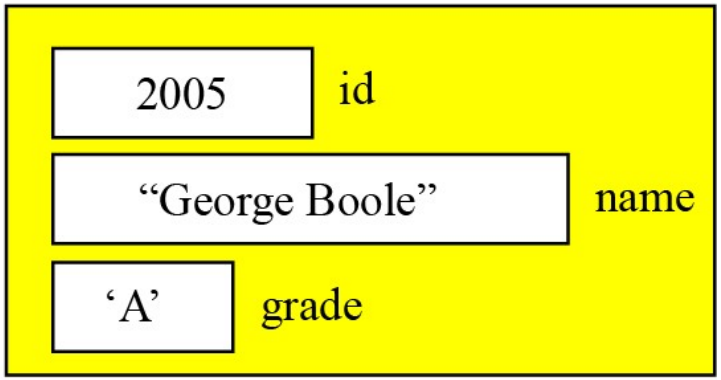
student

2.2. Record

- Example:



fraction



student

```
struct  {
    int numerator;
    int denominator;
} fraction;
fraction.numerator = 13;
fraction.denominator = 17;
```

```
struct  {
    int id;
    char* name;
    char grade;
} student;
student.id = 2005;
student.name = "George Boole";
student.grade = 'A';
```

Record name vs. field name

Just like in an array, we have two types of identifier in a record:

- the name of the record, and
- the name of each individual field inside the record.

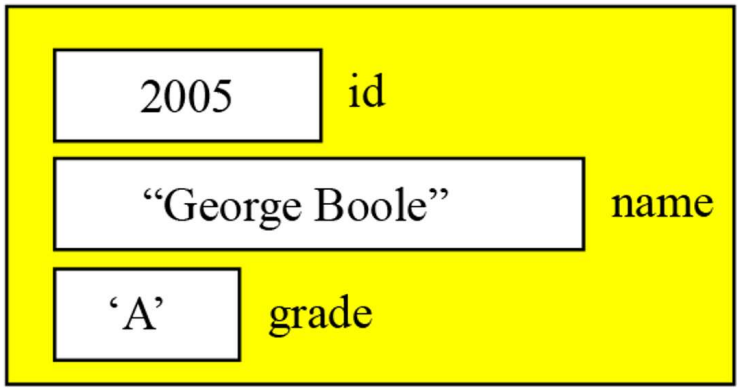
The name of the record is the name of the whole structure, while the name of each field allows us to refer to that field.

Example: in the student record:

- the name of the record is student,
- the name of the fields are student.id, student.name and student.grade.

Most programming languages use a *period* (.) to separate the name of the structure (record) from the name of its components (fields).

```
struct  {
    int id;
    char* name;
   char grade;
} student;
student.id = 2005;
student.name = "George Boole";
student.grade = 'A';
```



student

Comparison of records and arrays

We can compare an array with a record. This helps us to understand when we should use an array and when to use a record:

- An array defines a combination of elements, while a record defines the identifiable parts of an element.
- For example, an array can define a class of students (40 students), but a record defines different attributes of a student, such as id, name or grade.
- Array of records: If we need to define a combination of elements and at the same time some attributes of each element, we can use an array of records. For example, in a class of 30 students, we can have an array of 30 records, each record representing a student.

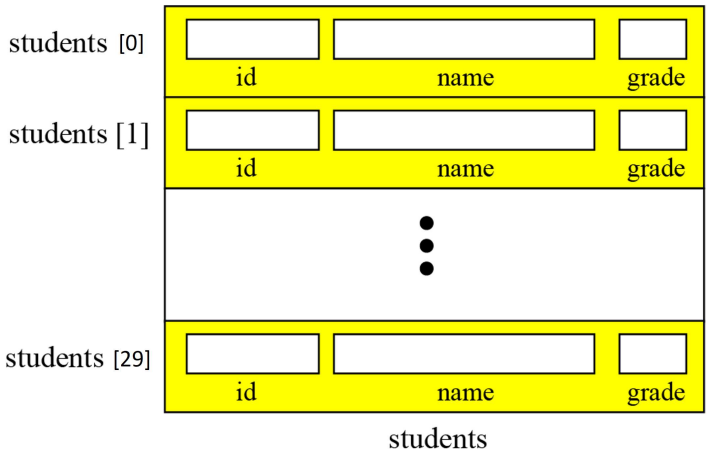


Figure 1. Array of records

```
struct  {
    int id;
    char* name;
    char grade;
} students[30];
```

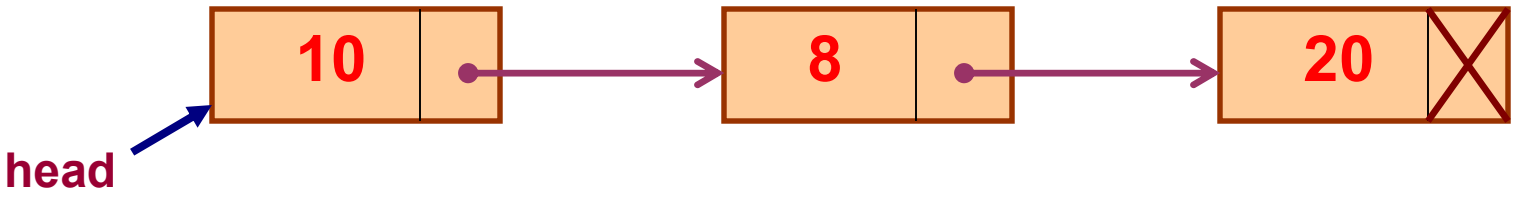
```
students[0].id = 1001;
students[0].name = "J.Aron";
students[0].grade = 'A';

students[1].id = 1002;
students[1].name = "F.Bush";
students[1].grade = 'F';
…..
students[29].id = 3021;
students[29].name = "M Blair";
students[29].grade = 'B';
```

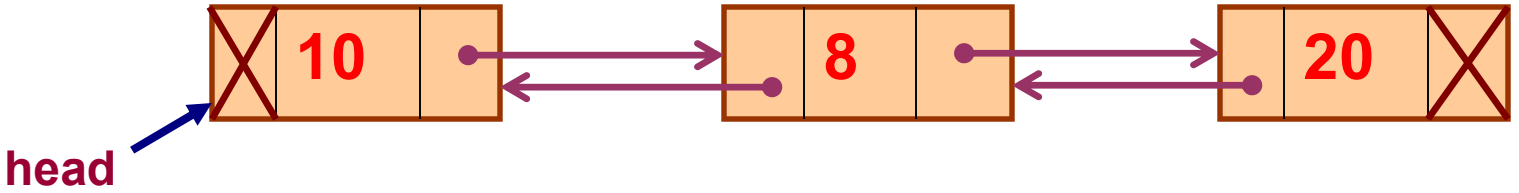
Contents

2.3. Linked list

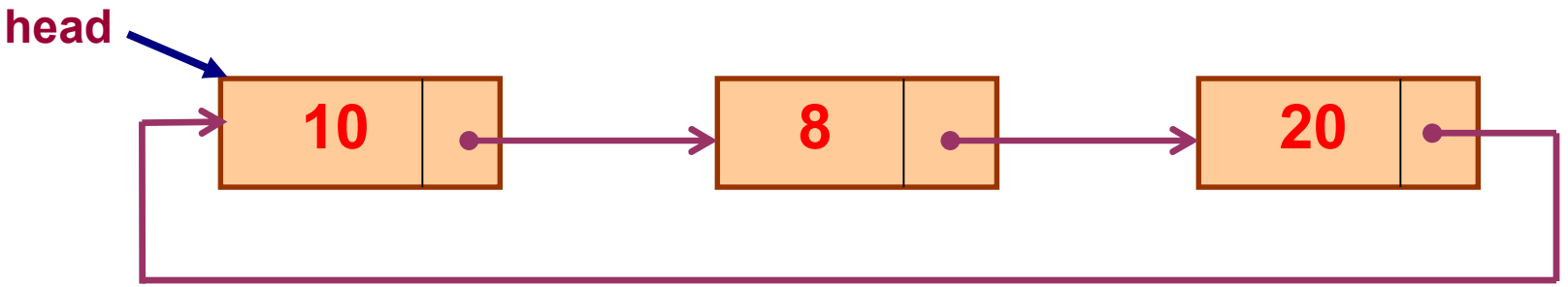
- Singly linked list



- Doubly linked list

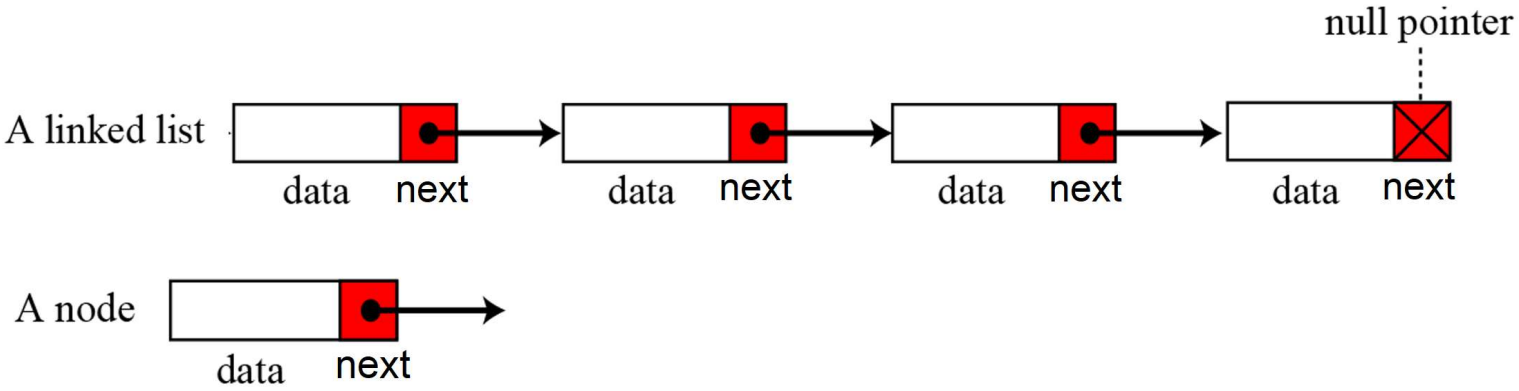


- Circular linked list

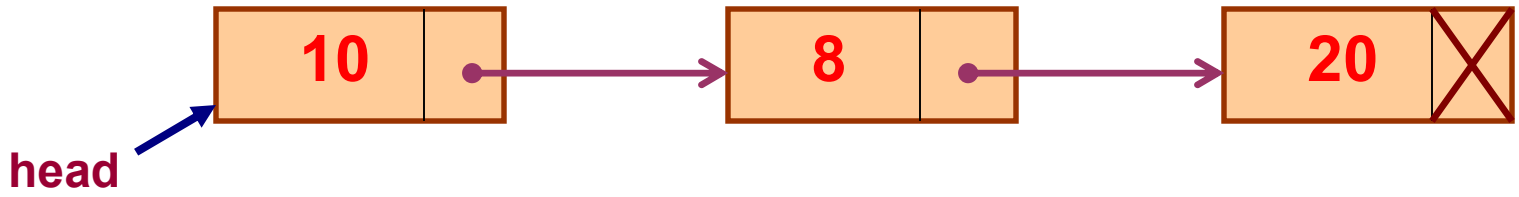


Singly Linked list

- A singly linked list is a sequences of nodes, each node contains 2 parts: data and reference (address) to the next node.
- Example: Figure shows a singly linked list contains four nodes:



- Keeping track of a singly linked list:
  - Must know the pointer to the first element of the list (called *start*, *head*, etc.)
  - If head is NULL, the singly linked list is empty



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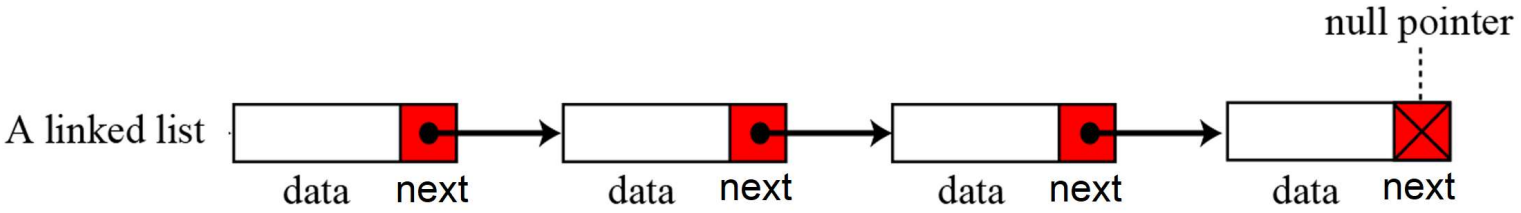

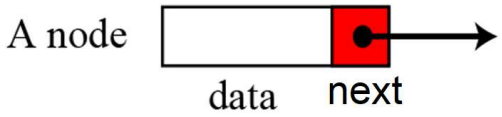

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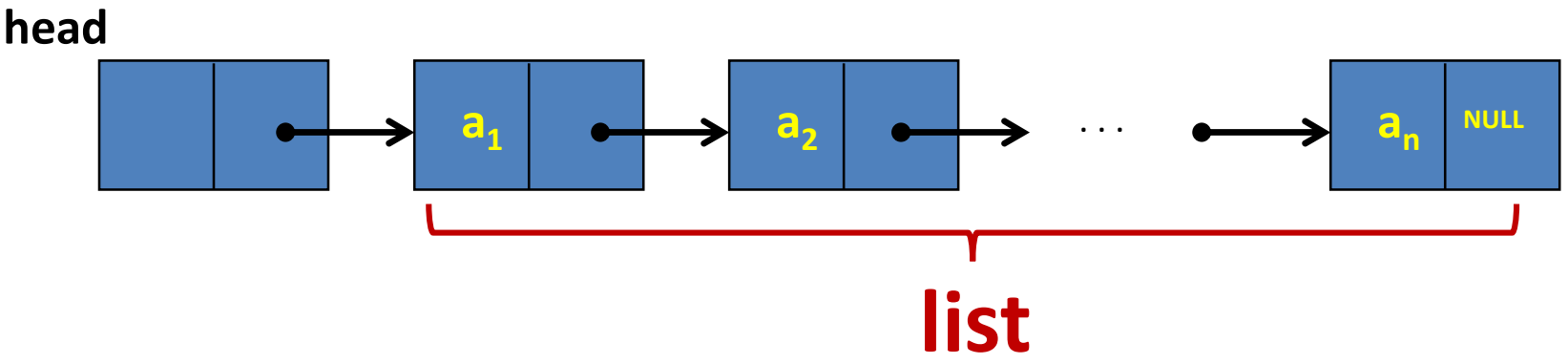


Singly Linked list

- Before further discussion of singly linked lists, we need to explain the notation we use in the figures. We show the connection between two nodes using a line. One end of the line has an arrowhead, the other end has a solid circle.

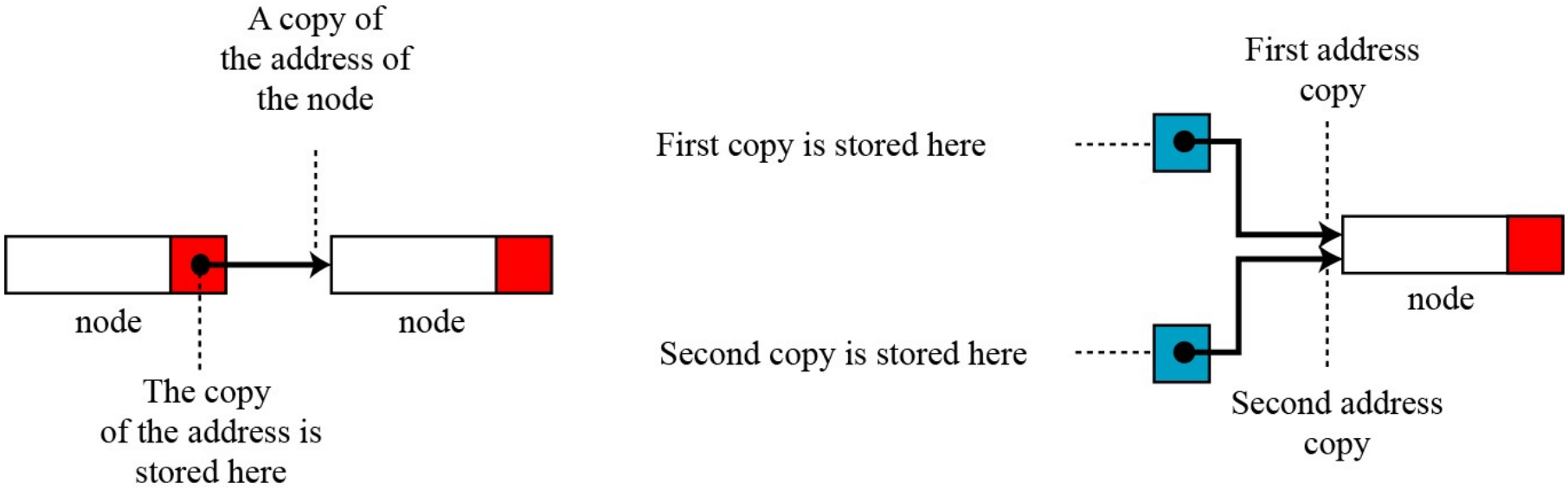
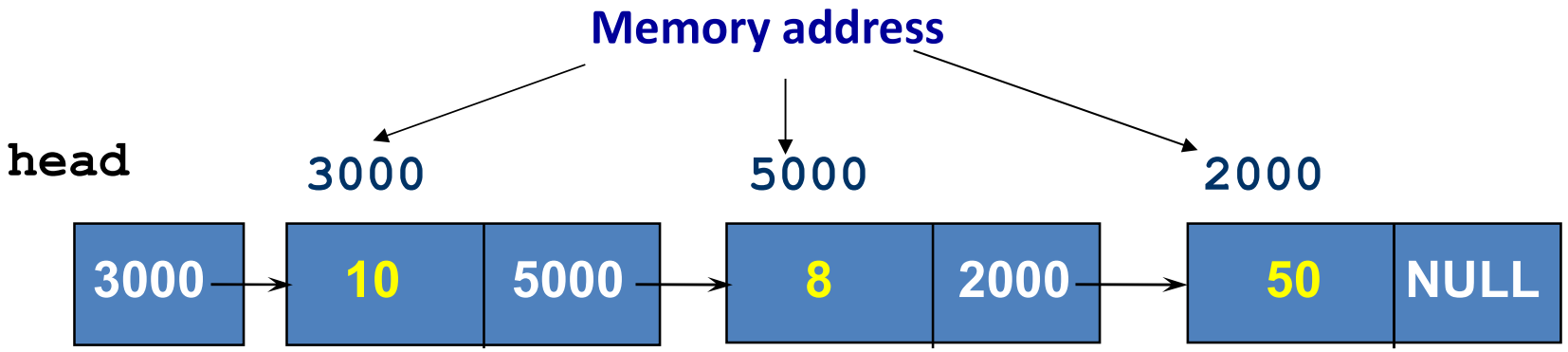
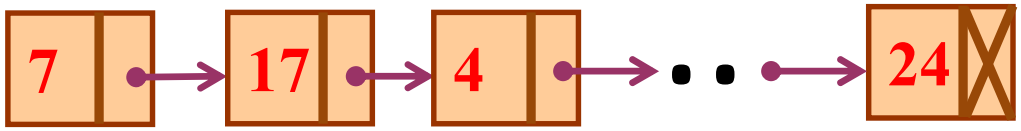


Figure. The concept of copying and storing pointers

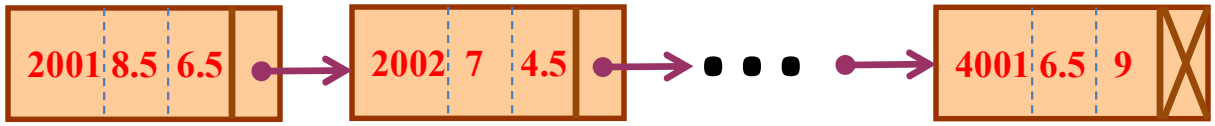


Declare singly linked list in C programming language

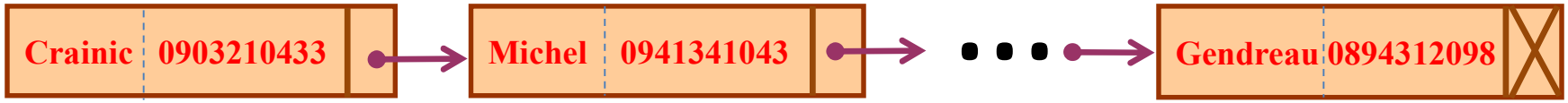
- List of integer numbers:



- List of students with data: student's ID, grade of math and physics



- List of contacts with data: name, phone number



➔ Need to declare:

- the type of data in the node first,
- then the singly linked list consists of (1) data of the node, and (2) the pointer to store the address of the next node in the list

Declare singly linked list

Need to declare:

- the type of data in the node first,
- then the linked list consists of (1) data of the node, and (2) the pointer to store the address of the next node in the list

```
struct NodeType {
    .....
};

struct node {
    NodeType data;
    struct node* next;
}node;
node* head;
```

Define the type of data of the node

Define the singly linked list

This declaration define `node` which is a record consisting of 2 fields:

- `data` : stores data of node, has the type `NodeType` (which was defined in `struct NodeType{…}`, and could consist of several attributes)
- `next` : the pointer which stores the address of the next node in the list

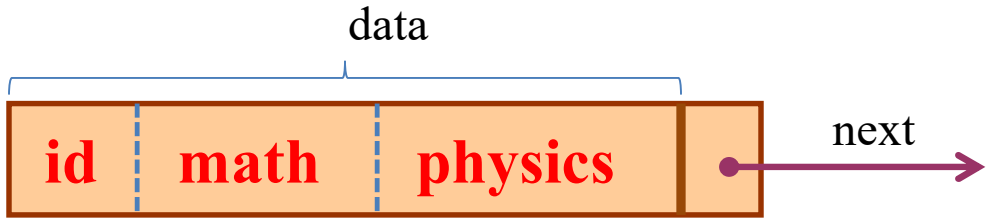
Pointer `head` : store address of the first node in the list

Example1: List of students with data: id of student, marks of 2 subjects: math, physics

```
struct student{
    char id[15];
    float math, physics;
};

struct node {
    student data;
    node* next;
};
node* head;
```

Define the type of data of the node



A node

Declare singly linked list

```
struct NodeType {
    .....
};

struct node {
    NodeType data;
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};
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```

Define the type of data of the node

Define the singly linked list

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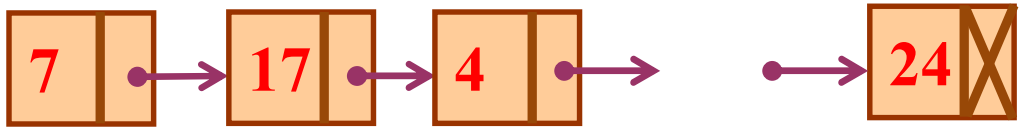
- `next` : the pointer which stores the address of the next node in the list

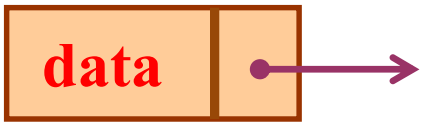
Pointer `head` : store address of the first node in the list

Example2: List of integer numbers




```
struct node {
    int data;
    node* next;
};
node* head;
```

"`int`" is the type of node, so do not need to use "`typedef….NodeType`" to define the type



A node

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```
struct NodeType{
    .....
};

struct node {
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};
node* head;
```

Define the type of data of the node

Define the singly linked list

This declaration define `node` which is a record consisting of 2 fields:

- `data` : stores data of node, has the type `NodeType` (which was defined in `struct NodeType`, and could consist of several attributes)
- `next` : the pointer which stores the address of the next node in the list

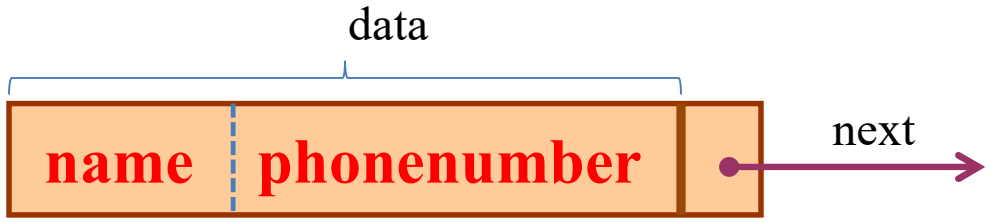
Pointer `head` : store address of the first node in the list

Example 3: List of contacts with data: name and phone number

```
struct contact{
    char name[15];
    char phone[20];
};

struct node{
    contact data;
    node* next;
};
node* head;
```

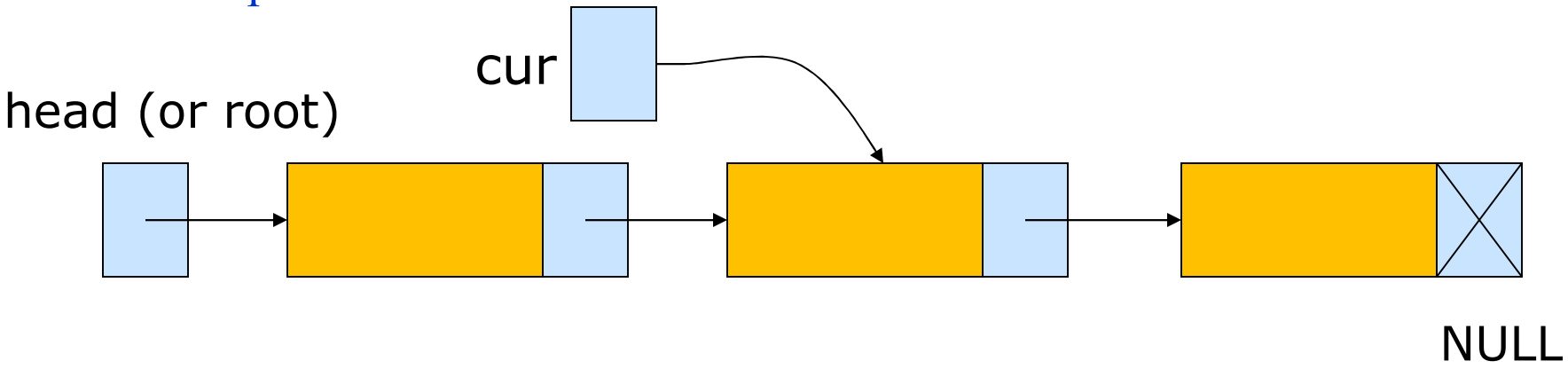
Define the type of data of the node



A node

Important elements of singly linked list

- head: store the address of the first node in the linked list
- NULL: value of the pointer of the last node in the linked list
- cur: the pointer stored the address of current node



- Allocate memory for a new node pointed by the pointer `newN` in the list:

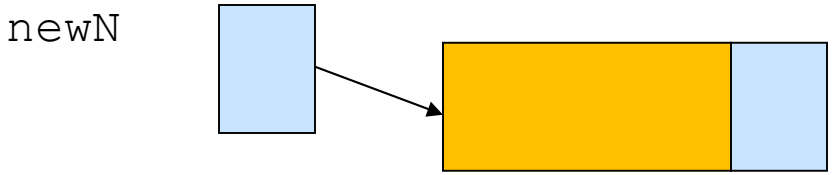
```
newN = new Node;
```

- Access to the data of the node pointed by pointer `newN` :

```
newN->data
```

- Free memory allocated for node pointed by pointer `newN` :

```
delete newN;
```



Operations on singly linked Lists

- Traverse the singly linked list
- Insert a node into the singly linked list
- Delete a node from the singly linked list
- Search data in the singly linked list

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Operations on singly linked Lists

- **Traverse the singly linked list**
- Insert a node into the singly linked list
- Delete a node from the singly linked list
- Search data in the singly linked list

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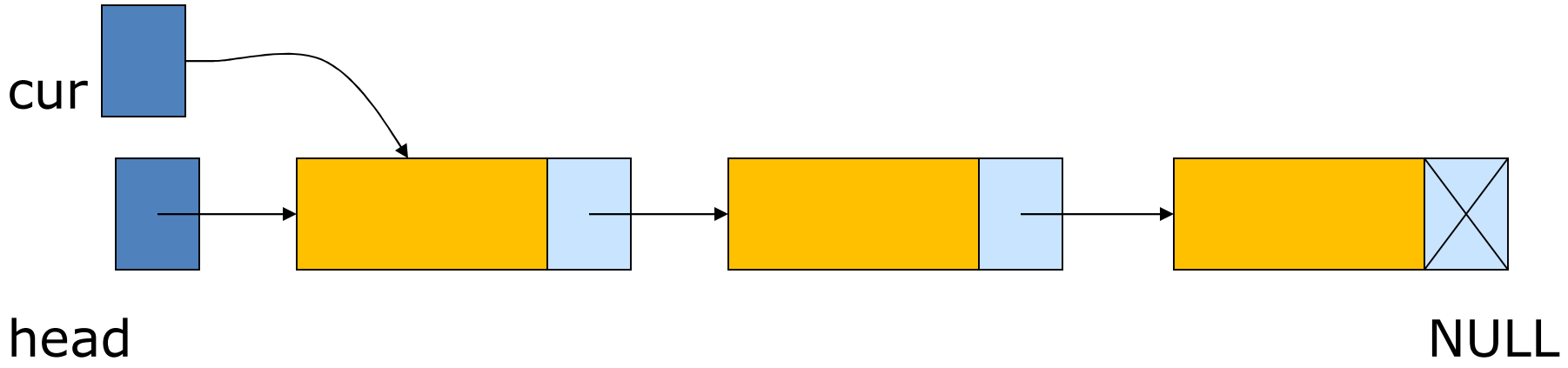
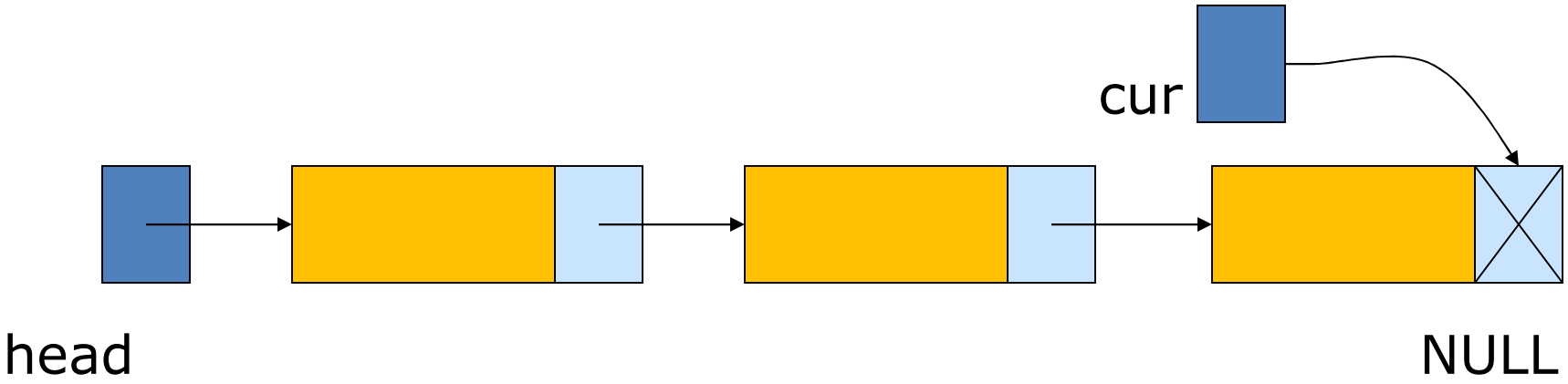
Traversing a singly linked list

```
for ( cur = head; cur != NULL; cur = cur->next )
        showData_Of_Current_Node( cur->data );
```



- Change the value of the pointer cur
- Finish to browse the list when the NULL value is encountered



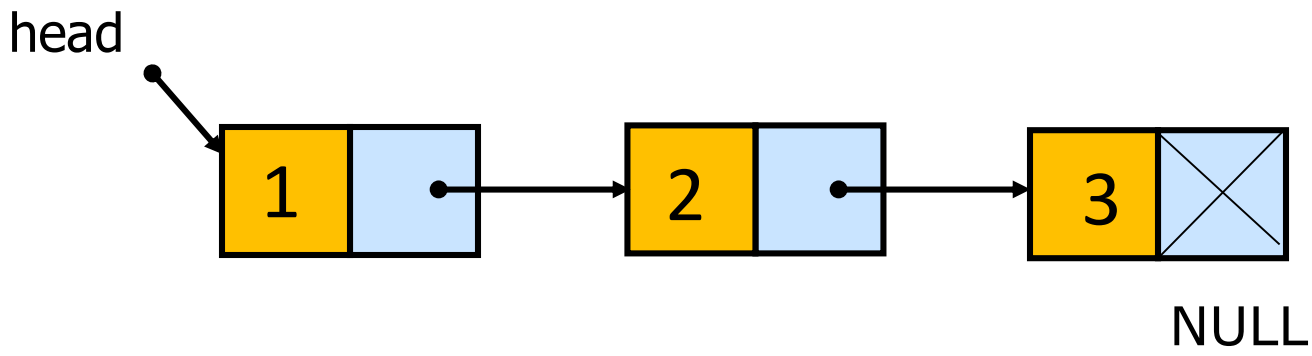
Exercise 1

- A sequence of integers is stored by a singly linked list.

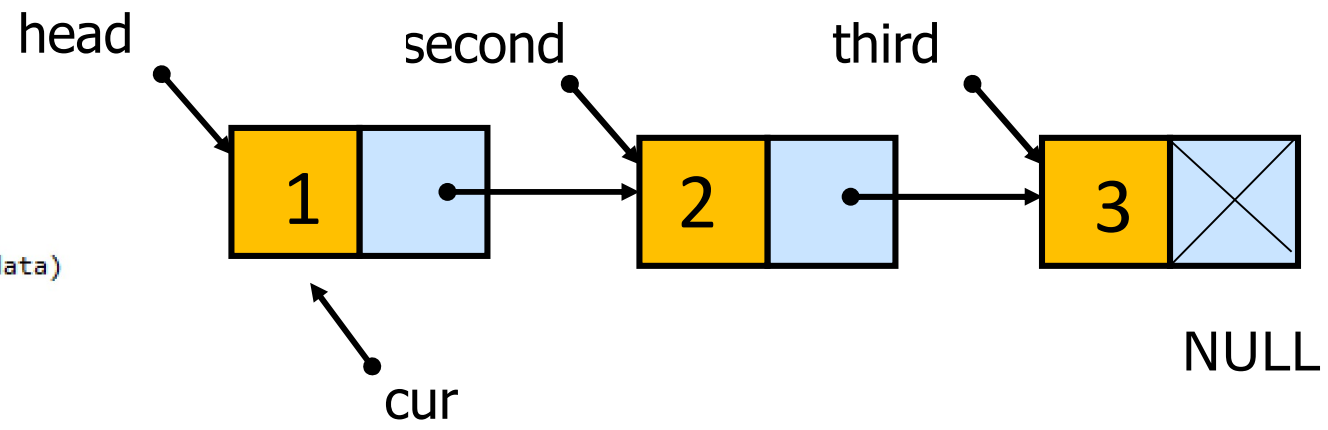
```
struct Node{
    int data;
    Node *next;
};
Node *head;
```

a. Create a list stored 3 integers: 1, 2, 3

b. Print the list of these 3 integers



```cpp
#include <iostream>
using namespace std;

struct Node {
    int data;
    Node *next;
};

void showData_Of_Current_Element(int data)
{
    cout<<"Data = "<<data<<endl;
}
int main()
{
    Node *head, *second =NULL, *third=NULL;

    //allocate memory for 3 nodes in the heap
    head = new Node;
    second = new Node;
    third = new Node;

    head->data = 1; //assign data for the first node
    head->next = second;//connect the first node to second node

    second->data = 2; //assign data for the 2nd node
    second->next = third;//connect the 2nd node to 3rd node

    third->data = 3; //assign data for the 3rd node
    third->next = NULL;

    Node *cur;
    for (cur =head; cur != NULL; cur =cur->next)
      showData_Of_Current_Element(cur->data);

    delete head; delete second; delete third;
    return 0;
}
```



Data = 1

Data = 2

Data = 3

Operations on singly linked lists

- Traverse the singly linked list
- **Insert a node into the singly linked list**
- Delete a node from the singly linked list
- Search data in the singly linked list

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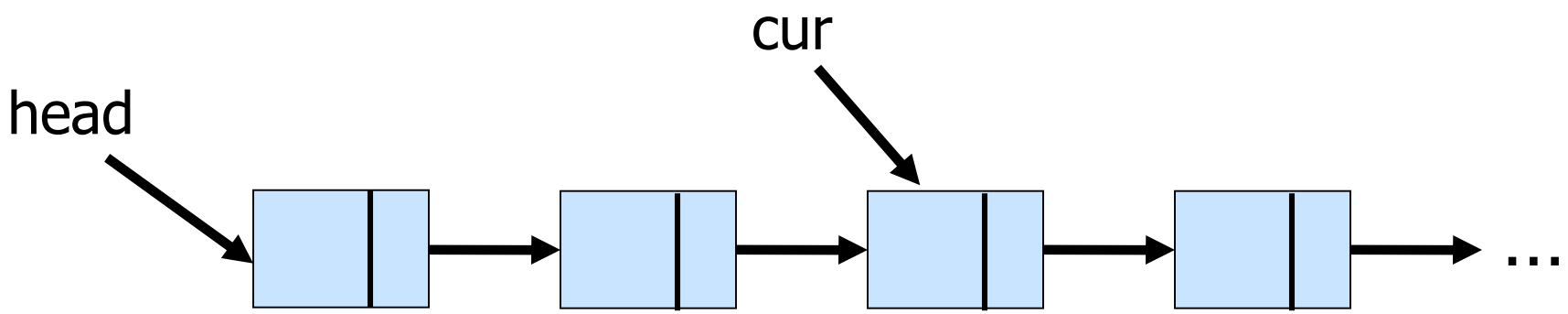
Operations on singly linked list: Insertion

Insert a new node :

- At the beginning of the list
- After the position pointed by the pointer cur
- Before the position pointed by the pointer cur
- At the end of the list

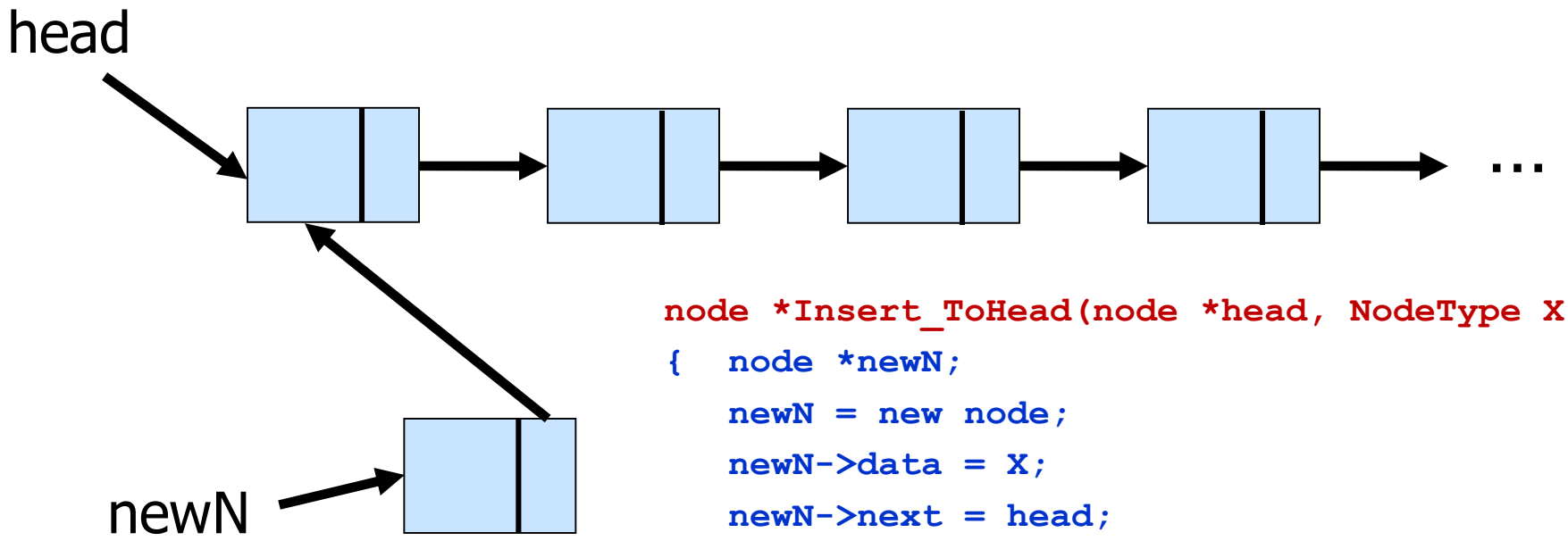


Operations on singly linked list: Insertion

Insert a new node:

- At the beginning of the list

```
<create a new node newN>;
newN->next = head;
head = newN;
```

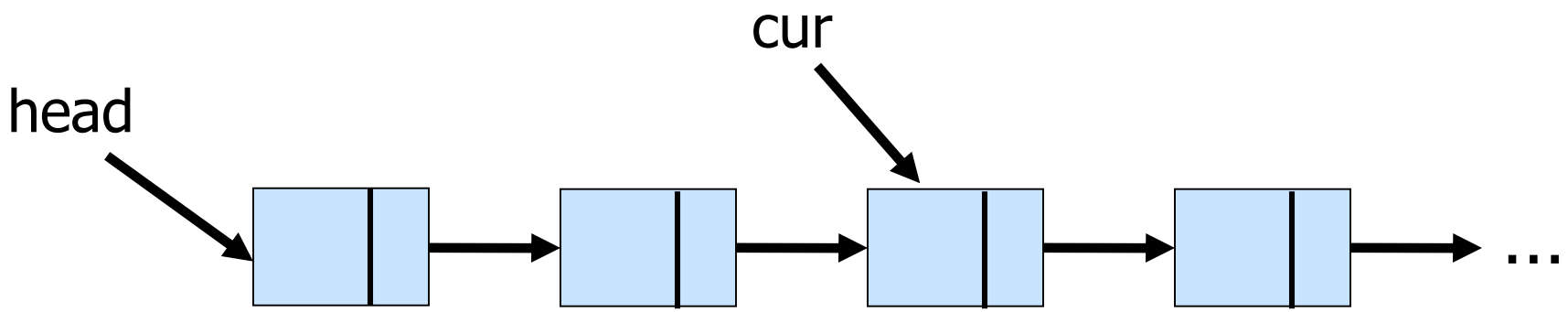



```
node *Insert_ToHead(node *head, NodeType X)
{   node *newN;
    newN = new node;
    newN->data = X;
    newN->next = head;
    head = newN;
    return head;
}
```

Operations on singly linked list: Insertion

Insert a new node :

- At the beginning of the list
- **After the position pointed by the pointer cur**
- Before the position pointed by the pointer cur
- At the end of the list



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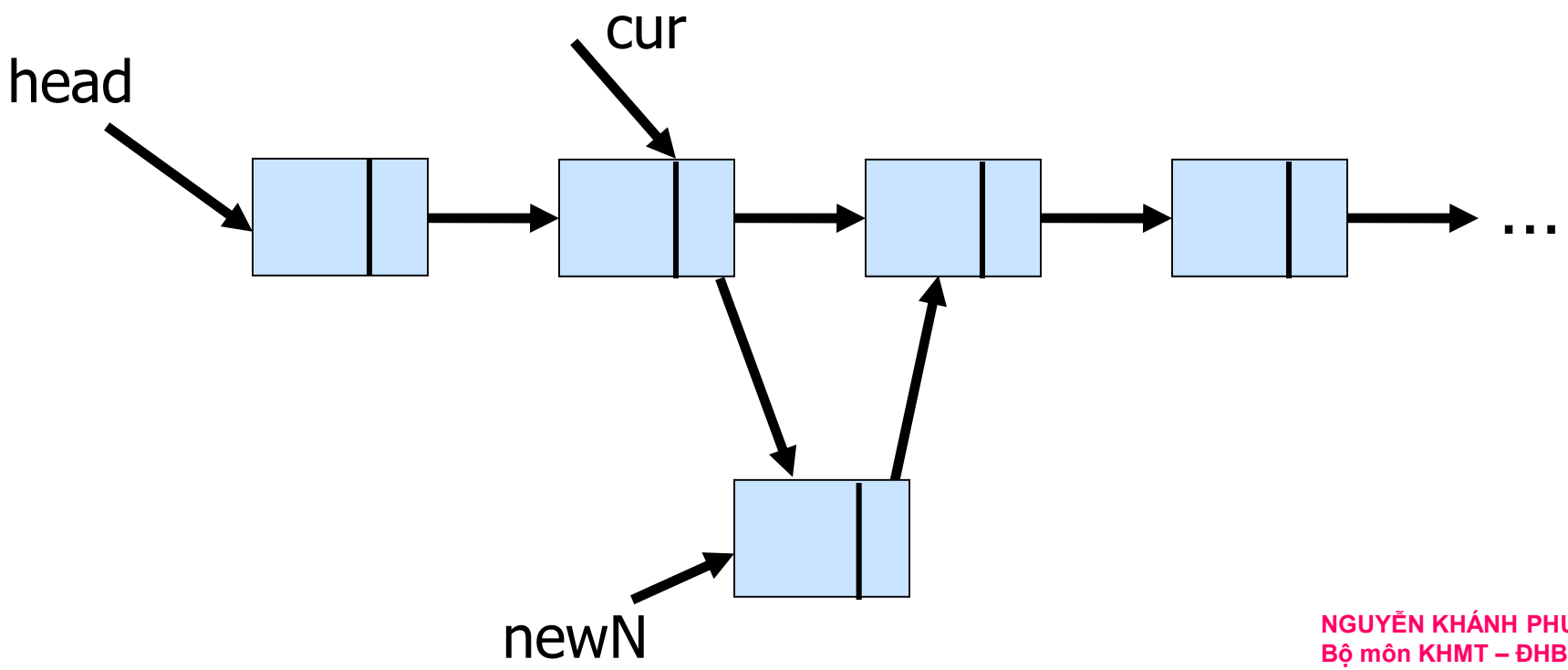
Operations on singly linked list: Insertion

- Insert a new node after the node pointed by the pointer cur:

```
<create a new node newN>;
newN->next = cur->next;
cur->next = newN;
```



Operations on singly linked list: Insertion

- Insert a new node after the node pointed by the pointer cur:

```
<create a new node newN>;
newN->next = cur->next;
cur->next = newN;
```

Write a function to insert a node with data = *X* (having the type «NodeType » after the node pointed by the pointer cur. The function returns the address of the new node:

```
node *Insert_After(node *cur, NodeType X)
{       node *newN;
        newN = new node;                    //(1)
        newN -> data = X;                   //(1)
        newN ->next = cur->next;            //(2)
        cur->next = newN;                   //(3)
        return newN;
}
```

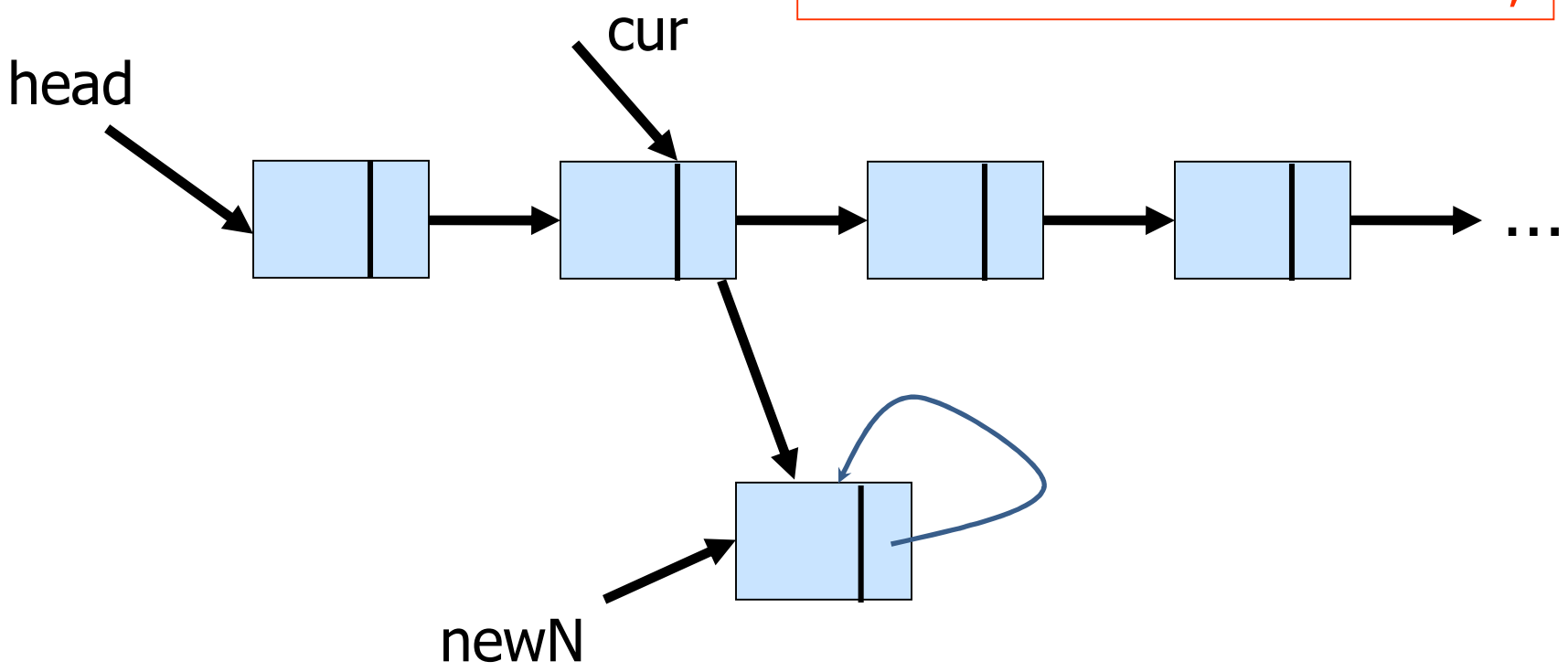
Operations on singly linked list: Insertion

- Insert a new node after the node pointed by the pointer cur:

```
<create a new node newN>;
newN->next = cur->next;
cur->next = newN;
```

?? Empty list

```
// wrong implementation:
cur->next = newN;
newN->next = cur->next;
```



Operations on singly linked list: Insertion

- Insert a new node after the node pointed by the pointer cur:

```
<create a new node newN>;
newN->next = cur->next;
cur->next = newN;
```

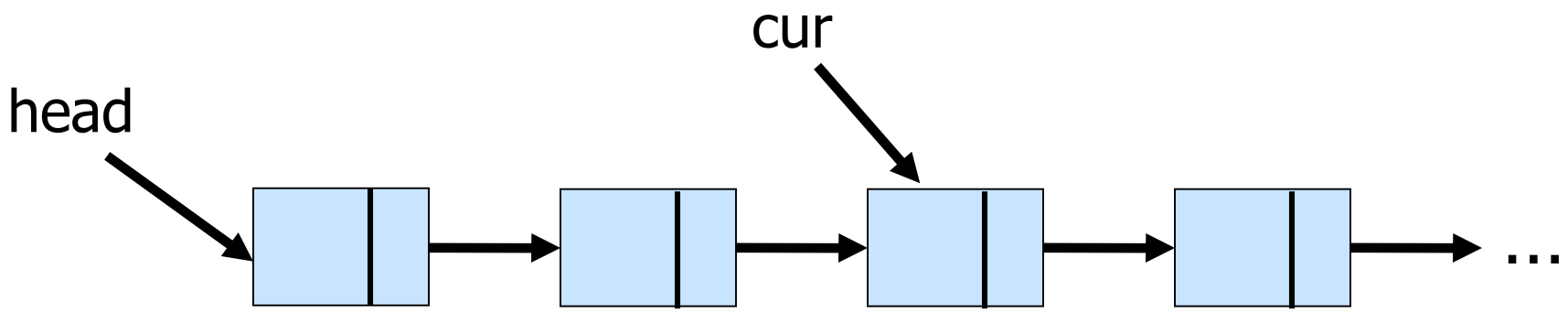
?? Empty list

```
<create a new node newN>;
if (head == NULL) { /* list does not have any node yet */
    head = newN;
    cur = head;
}
else {
   newN->next = cur->next;
   cur->next = newN;
}
```

Operations on singly linked list: Insertion

Insert a new node :

- At the beginning of the list
- After the position pointed by the pointer cur
- **Before the position pointed by the pointer cur**
- At the end of the list



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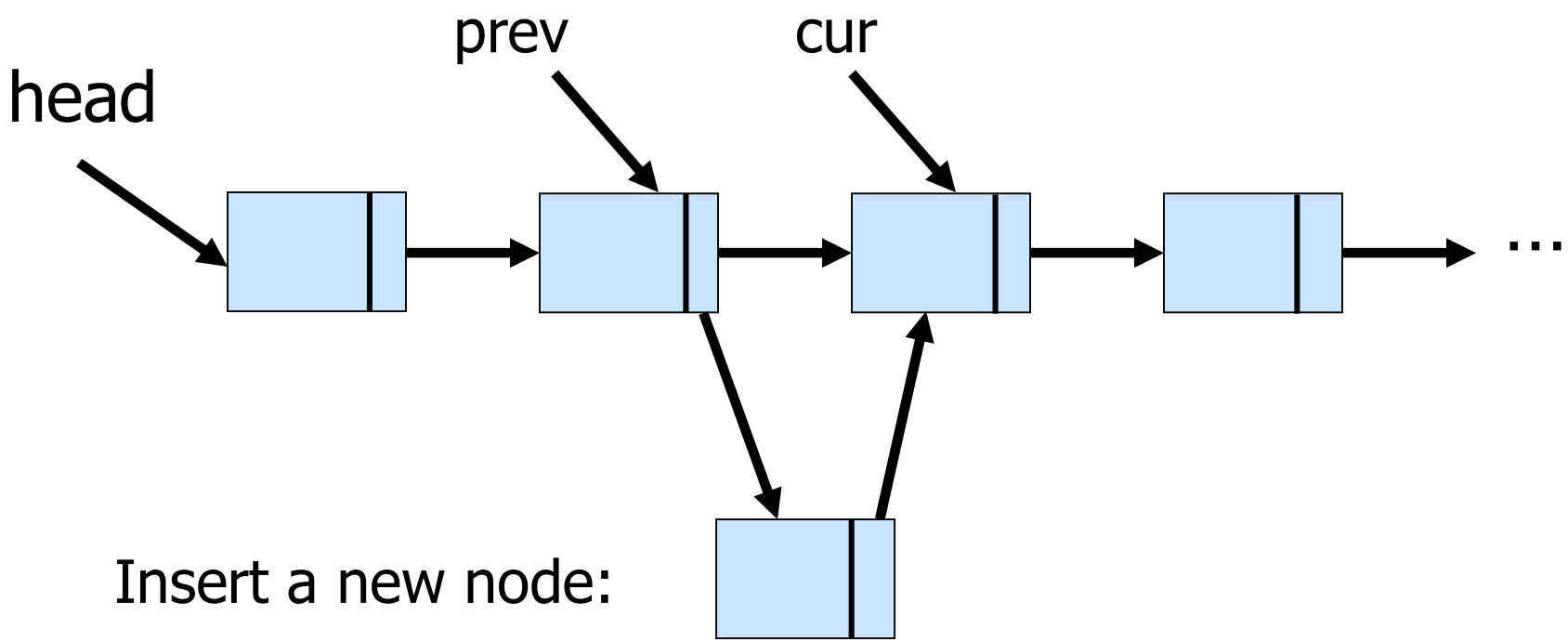
Operations on singly linked list: Insertion

Insert a new node **before the node pointed by the pointer cur**

```
<create a new node newN>;
prev->next = newN;
newN->next = cur;
```

?? List does not have any node yet

?? cur is the first node in the list



Operations on singly linked list: Insertion

Insert a new node **before the node pointed by the pointer cur**

```
<create a new node newN>;
prev->next = newN;
newN->next = cur;
```

?? List does not have any node yet

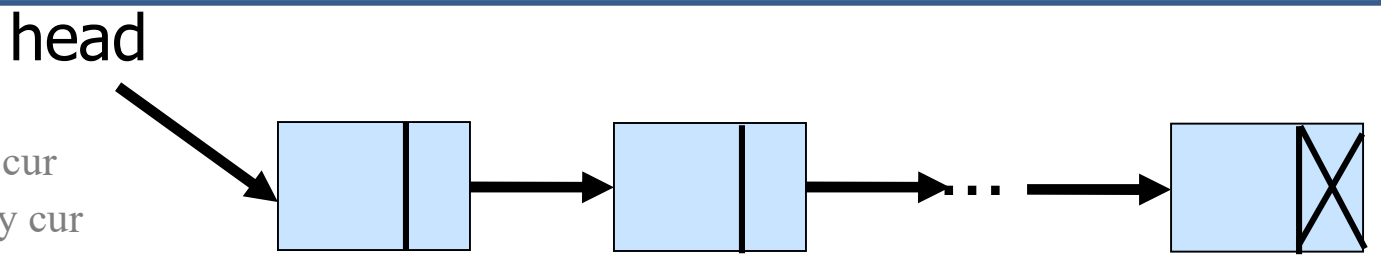
?? cur is the first node in the list

```
<create a new node newN>;
if (head == NULL) { /* list does not have any node yet */
    head = nodeN;
    cur = head;
}
else if (cur == head) { //cur us the first node in the list
    head = newN;
    newN->next = cur;
}
else {
   prev->next = newN;
   newN->next = cur;
}
```

Operations on singly linked list: Insertion

Insert a new node:

- At the beginning
- After the node pointed by cur
- Before the node pointed by cur
- **At the end of the list**




```
<create a new node newN>;
if (head == NULL) { /* list does not have any node yet*/
    head = newN;
}
else {
    //move the pointer to the end of the list:
     node *last = head;
     while (last->next != NULL) last = last->next;
     //Change the pointer next of the last node:
    last->next = newN;
}
```

Complexity is ….

```
node *Insert_ToLast(node *head, NodeType X)
{   node *newN;
    newN = new node;
    newN->data = X;
    if (head == NULL) head = newN;
    else
    {
       node *last;
       last=head;
       while (last->next != NULL)  // move to the last node
          last = last->next;
       last->next = newN;
    }
    return head;
}
```

Operations on singly linked Lists

- Traverse the singly linked list
- Insert a node into the singly linked list
- **Delete a node from the singly linked list**
- Search data in the singly linked list

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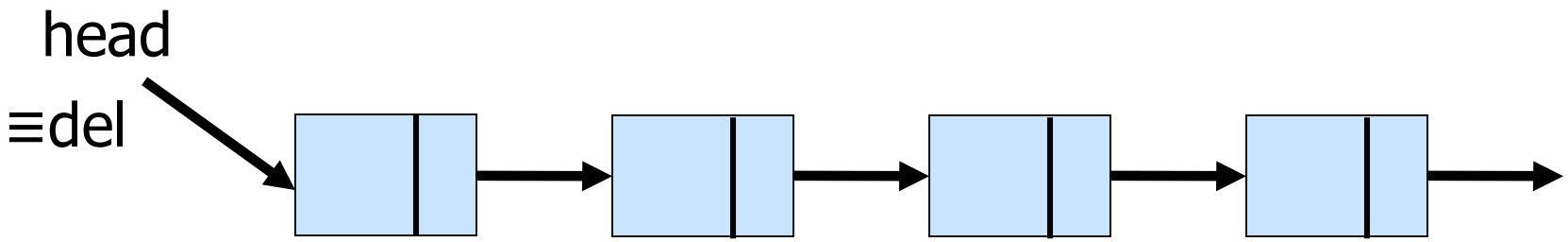
Operations on singly linked lists: Deletion

- **Delete a node**
- Delete all nodes of the list

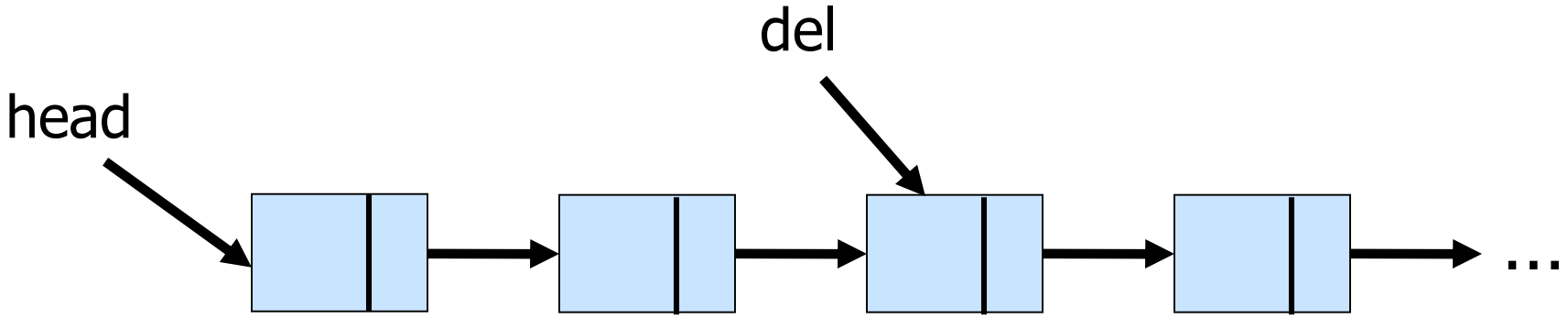
Operations on singly linked lists: Deletion

Delete a node:

- The first node of the list



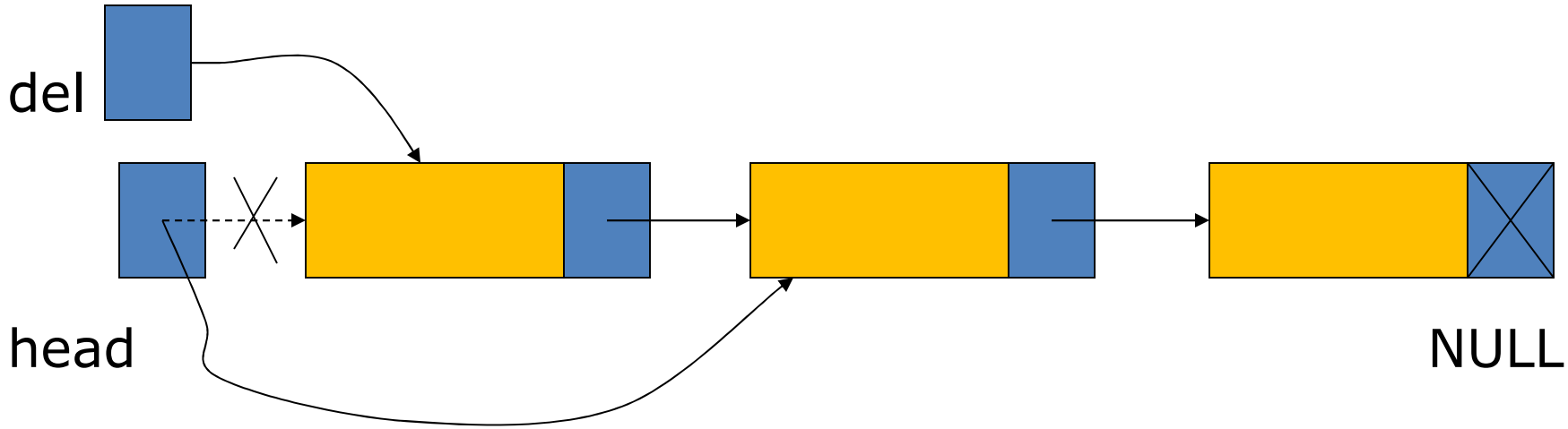
- Middle/last node of the list



Delete the first node of the list

- Delete the node `del` that is currently the first node of the list:

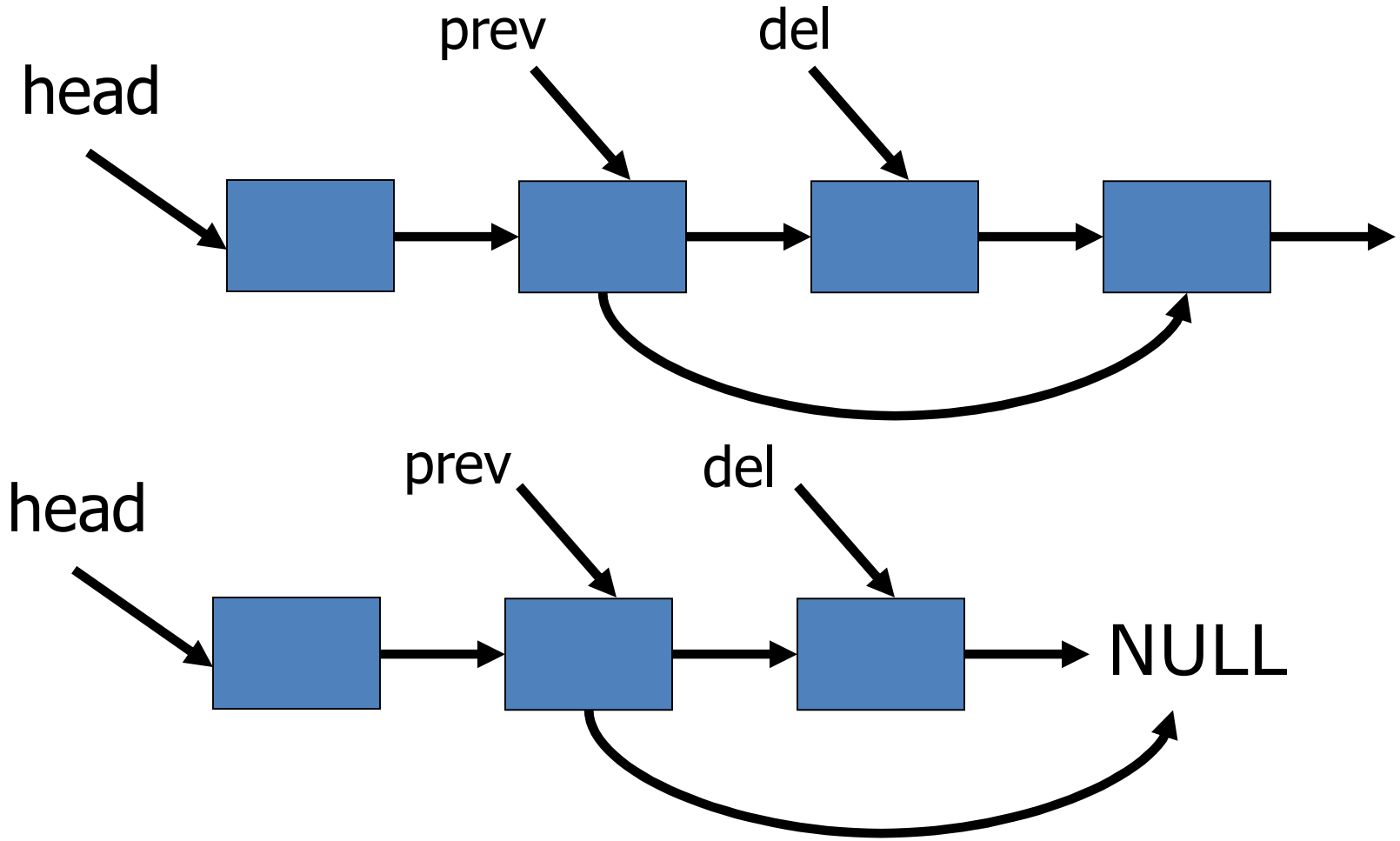
```
head = del->next;
delete del;
```



Delete the node at the middle/end of the list

Delete node `del` that is currently the middle/last node of the list:

```
<Determine the pointer prev pointed to the previous node of del>;
prev->next = del->next; //modify the link
delete del; //delete node del to free memory
```

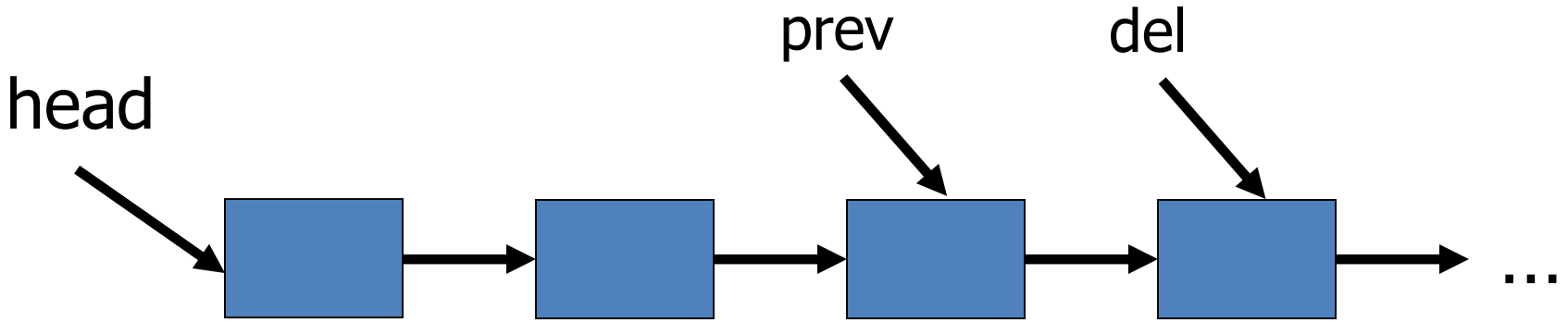


Delete the node at the middle/end of the list

Delete node `del` that is currently the middle/last node of the list:

```
<Determine the pointer prev pointed to the previous node of del>;
prev->next = del->next; //modify the link
delete del; //delete node del to free memory
```

```
node *prev =head;
while (prev->next != del) prev = prev->next;
```



Delete a node pointed by the pointer del

Write the function **`node *Delete_Node(node *head, node *del)`**

to delete a node pointed by the pointer "del" of the list with the first node pointed by the pointer "head".

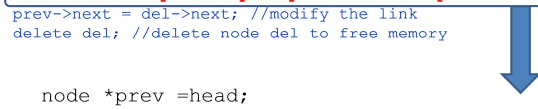
The function returns the address of the first node in the list after deletion:

- Delete the node `del` that is currently the first node of the list:

```
head = del->next;
delete del;
```

Delete node `del` that is currently the middle/last node of the list:

```
<Determine the pointer prev pointed to the previous node of del>;
prev->next = del->next; //modify the link
delete del; //delete node del to free memory
```



```
node *prev =head;
while (prev->next != del) prev = prev->next;
```

```
node *Delete_Node(node *head, node *del)
{
    if (head == del) //del is the first node of the list:
    {
        head = del->next;
        delete del;
    }
    else{
        node *prev = head;
        while (prev->next != NULL) prev = prev->next;
        prev->next =del->next;
        delete del;
    }
    return head;
}
```

Operations on singly linked lists: Deletion

- Delete a node
- **Delete all nodes of the list**

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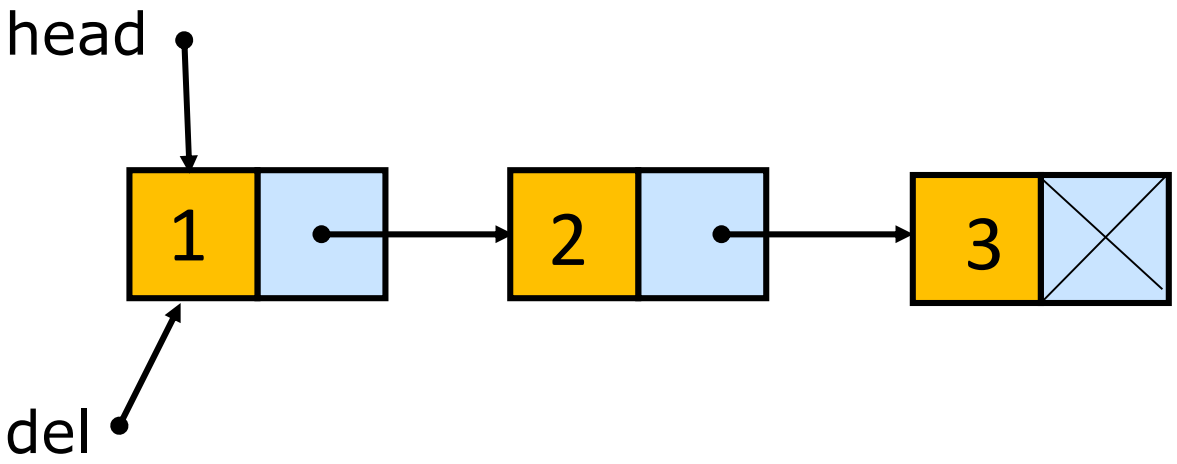
Freeing all nodes of a list

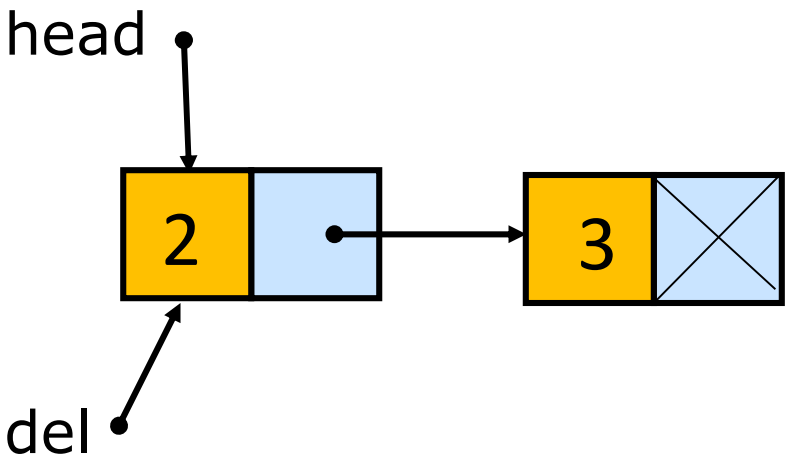
```
del = head ;
while (del != NULL)
{
    head = head->next;
    delete del;
    del = head;
}
```

Traverse elements one by one from the head till the end



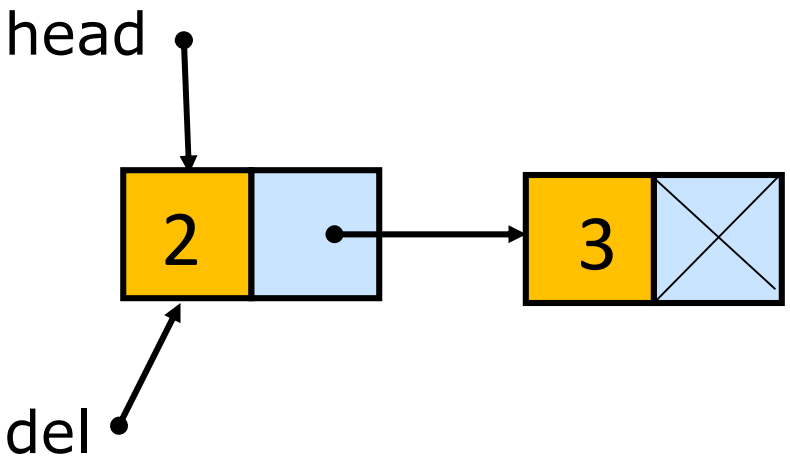
Freeing all nodes of a list

```
del = head ;
while (del != NULL)
{
    head = head->next;
    delete del;
    del = head;
}
```



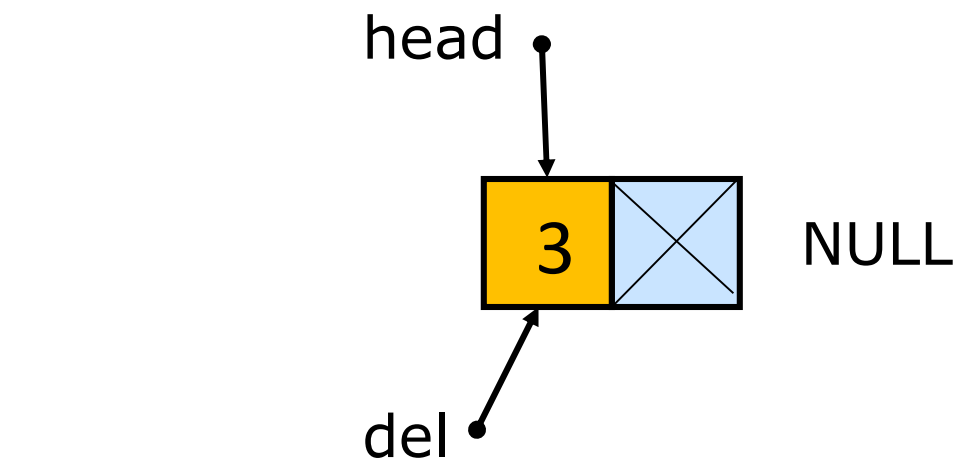
Freeing all nodes of a list

```
del = head ;
while (del != NULL)
{
    head = head->next;
    delete del;
    del = head;
}
```



Freeing all nodes of a list

```
del = head ;
while (del != NULL)
{
    head = head->next;
    delete del;
    del = head;
}
```



Freeing all nodes of a list

Write the function `node* deleteList(node* head)`
to delete all the node in the list having the first node pointed by the pointer `head`
The function returns the pointer `head` after deletion

```
node* deleteList(node* head)
{
   node *del = head ;
   while (del != NULL)
   {
      head = head->next;
      delete del;
      del = head;
   }
   return head;
}
```

Check whether the singly linked list is empty or not

Write the function `int IsEmpty(node *head)`

to check whether the singly linked list is empty or not (the pointer head pointed to the first node of the list).

The function returns 1 if the list is empty; 0 otherwise

```
int IsEmpty(node *head) {
     if (head == NULL)
          return 1;
     else return 0;
}
```

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Operations on singly linked Lists

- Traverse the singly linked list
- Insert a node into the singly linked list
- Delete a node from the singly linked list
- **Search data in the singly linked list**

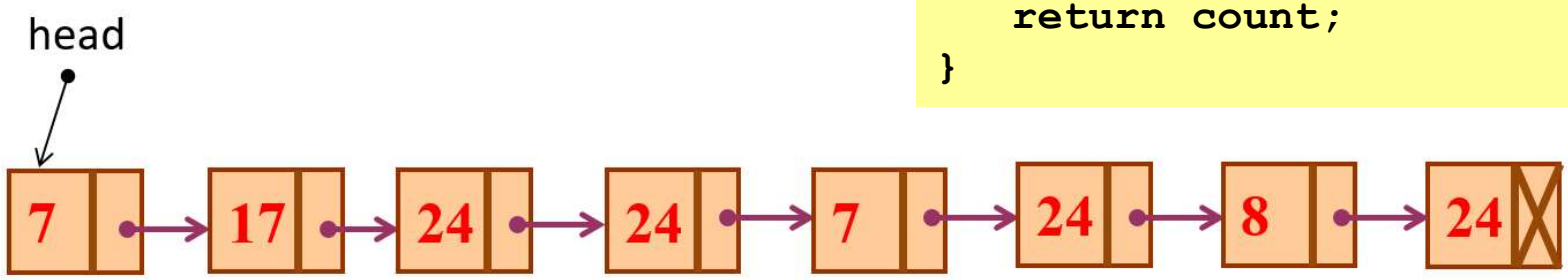
Searching

- To search for an element, we traverse from head until we locate the object or we reach the end of the list.

Example: Given a linked list consisting of integer numbers. Count the number of nodes with data field equal to number x.

```
struct node{
    int data;
    node* next;
};
node* head;
```

```
int countNodes(int x){
    int count = 0;
    node* e = head;
    while (e != NULL){
        if (e->data == x) count++;
        e = e->next;
    }
    return count;
}
```




```
int Result1 = countNodes(24);

int a =7;
int Result2 = countNodes(a);
```

Result1 = ?
Result2 = ?

Time Complexity: Singly-linked lists vs. 1D-arrays

Operation	**ID-Array Complexity**	**Singly-linked list Complexity**
Insert at beginning	O(n)	O(1)
Insert at end	O(1)	O(1) if the list has **tail** reference O(n) if the list has no **tail** reference
Insert at middle*	O(n)	O(n)
Delete at beginning	O(n)	O(1)
Delete at end	O(1)	O(n)
Delete at middle*	O(n): O(1) access followed by O(n) shift	O(n): O(n) search, followed by O(1) delete
Search	O(n) linear search O(log n) Binary search	O(n)
Indexing: What is the element at a given position k?	O(1)	O(n)

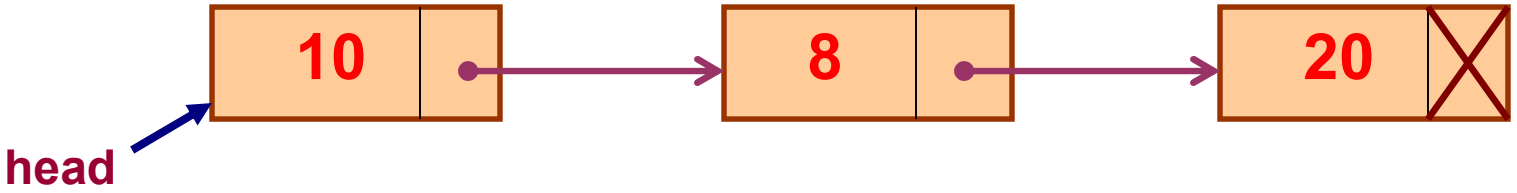
* middle: neither at the beginning nor at the end

Singly-linked lists vs. 1D-arrays

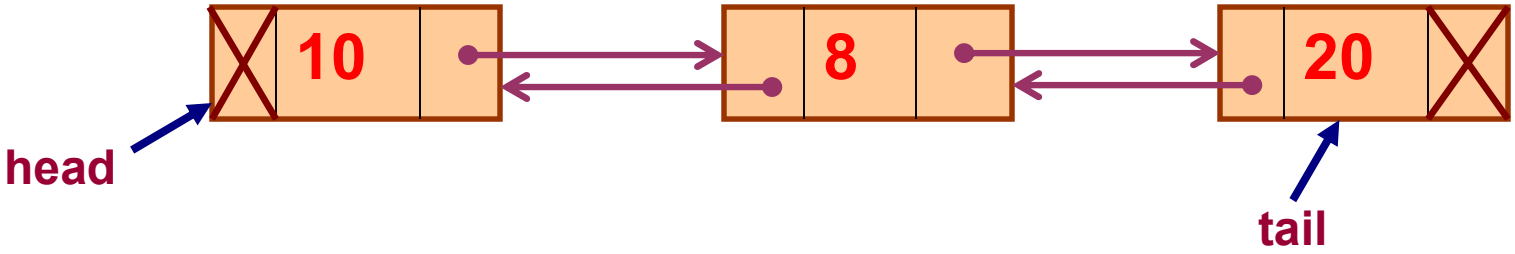
ID-array	Singly-linked list
Fixed size: Resizing is expensive	Dynamic size
Insertions and Deletions are inefficient: Elements are usually shifted	Insertions and Deletions are efficient: No shifting
Random access i.e., efficient indexing	No random access → Not suitable for operations requiring accessing elements by index such as sorting
No memory waste if the array is full or almost full; otherwise may result in much memory waste.	Extra storage needed for references; however uses exactly as much memory as it needs
Sequential access is faster because of greater locality of references [Reason: Elements in contiguous memory locations]	Sequential access is slow because of low locality of references [Reason: Elements not in contiguous memory locations]

2.3. Linked list

- Singly linked list



- **Doubly linked list**



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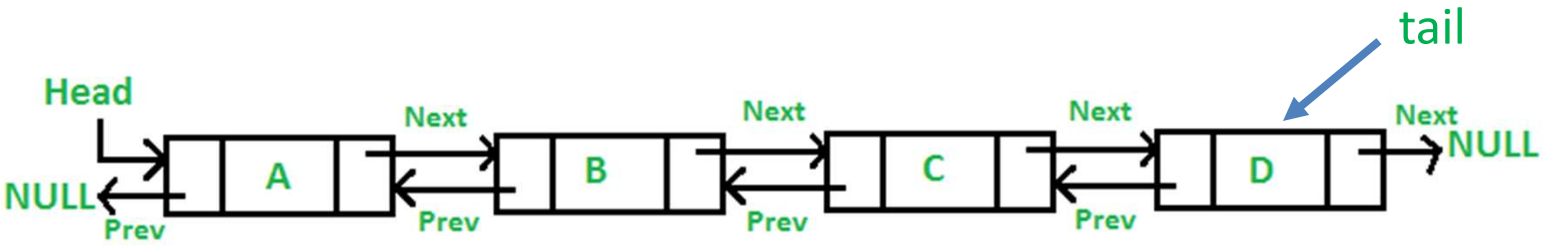
Doubly linked list

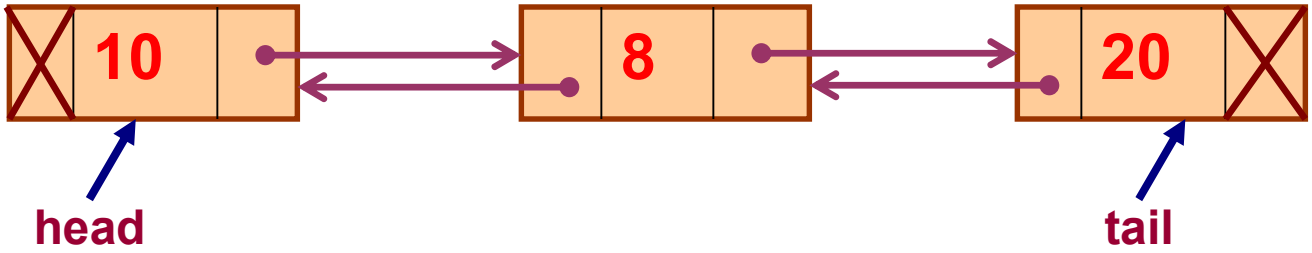
- A **D**oubly **L**inked **L**ist (DLL) contains an extra pointer, typically called *previous pointer*, together with next pointer and data which are there in singly linked list



- 2 special nodes: `tail` and `head`
  - `head` has pointer `prev = null`
  - `tail` has pointer `next = null`
- Basic operations are considered similar as in the singly linked list

Doubly linked list

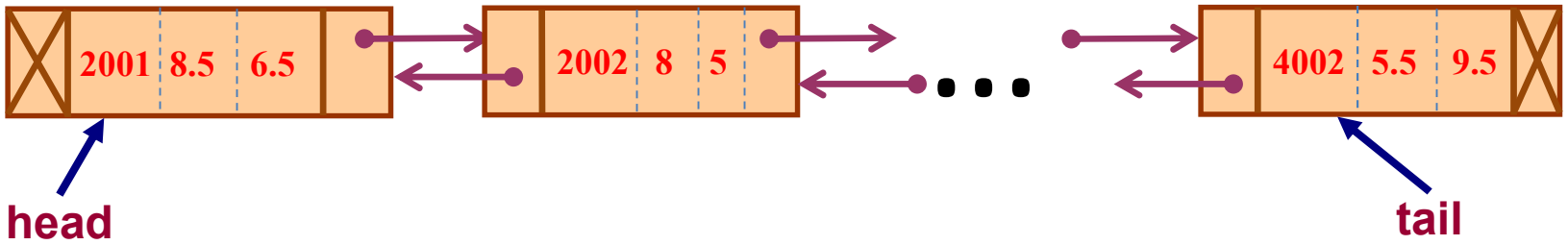
- Declare doubly linked list to store integer numbers:




```
struct dllist {
 int number;
 dllist *next;
 dllist *prev;
};
dllist *head, *tail;
```

Doubly linked list

- Declare doubly linked list store data of students: ID, marks of math and physics




```
struct student{
    char id[15];
    float math, physics;
};

struct dllist {
    student data;
    ddlist* next;
    ddlist* prev;
};
dllist *head, *tail;
```

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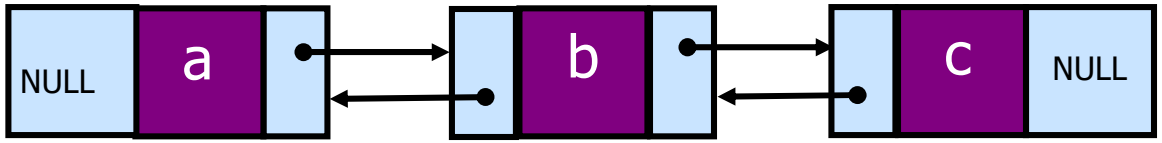
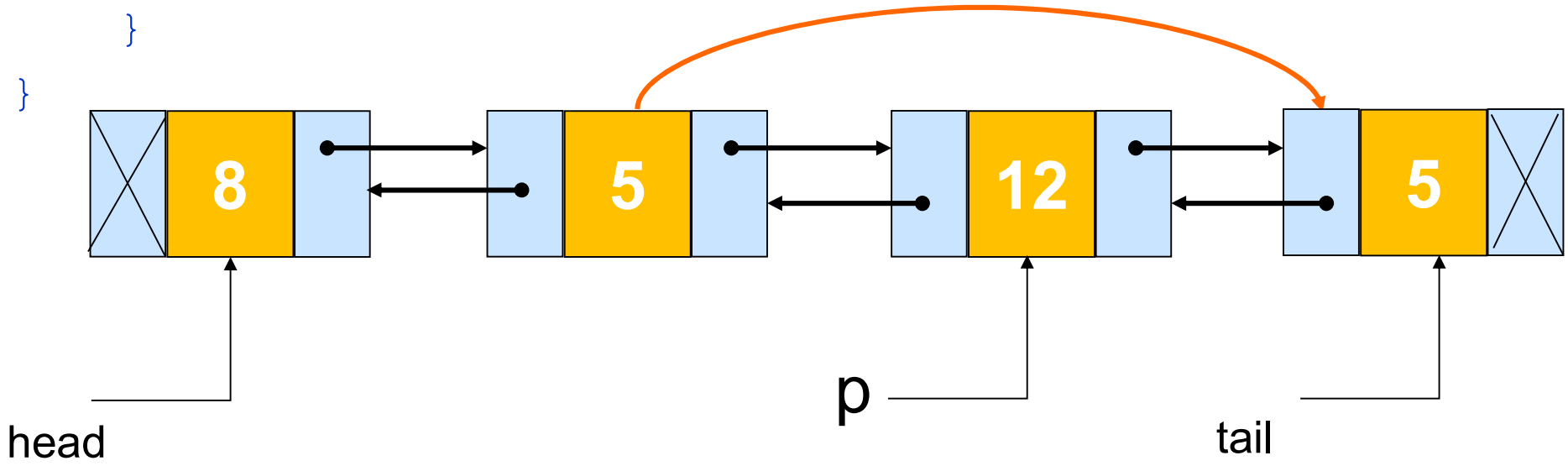
Doubly linked list – Example

```
struct dblist{
    char data;
    dblist *prev;
    dblist *next;
};
dblist *node1, *node2, *node3;
node1 = new dblist; node2 = new dblist; node3 = new dblist;

node1->data='a';
node2->data='b';
node3->data='c';
node1->prev=NULL;
node1->next=node2;
node2->prev=node1;
node2->next=node3;
node3->prev=node2;
node3->next=NULL;
dblist *temp;
for (temp = node1; temp != NULL; temp = temp->next)
    cout<<temp->data<<endl;
```



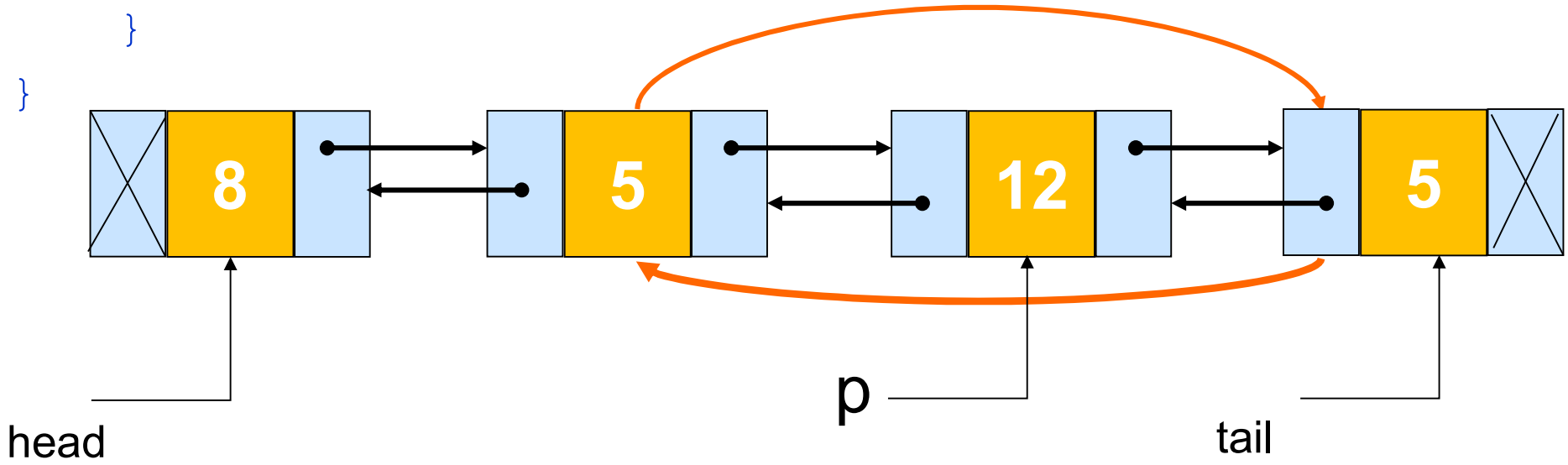
Delete a node pointed by the pointer p

```
void Delete_Node (ddlist *p){
    if (head == NULL) cout<<"The list is empty";
    else {
      if (p==head) head = head->next; //Delete first element
      else p->prev->next = p->next;
      if (p->next!=NULL) p->next->prev = p->prev;
      else tail = p->prev;
      delete p;
    }
}
```



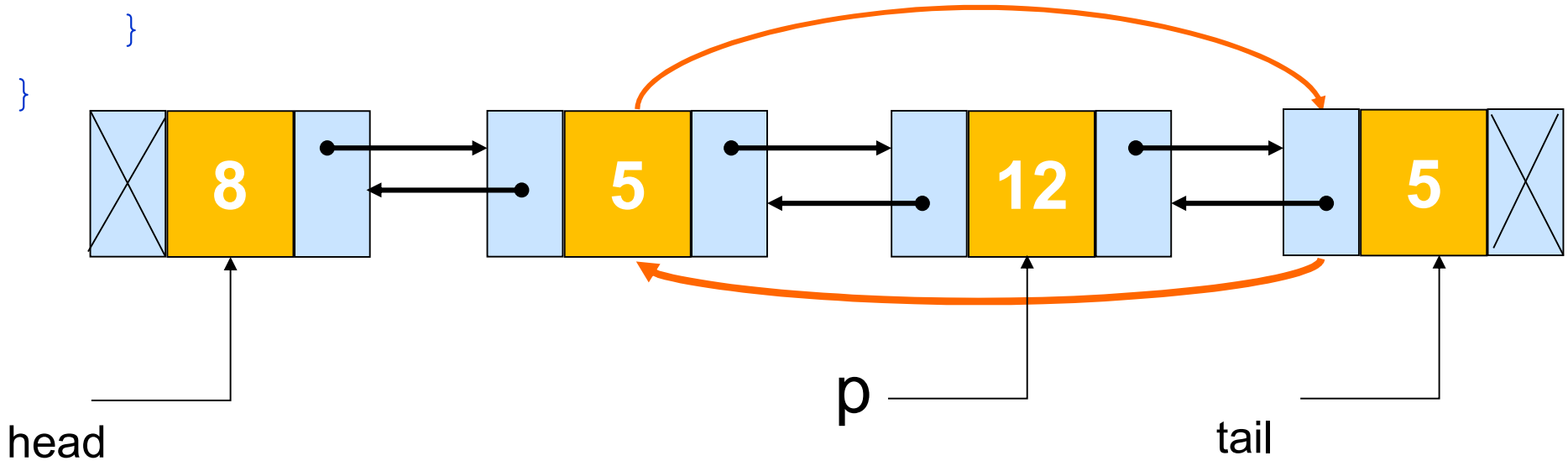
Delete a node pointed by the pointer p

```
void Delete_Node (ddlist *p){
    if (head == NULL) cout<<"The list is empty";
    else {
      if (p==head) head = head->next; //Delete first element
      else p->prev->next = p->next;
      if (p->next!=NULL) p->next->prev = p->prev;
      else tail = p->prev;
      delete p;
    }
}
```



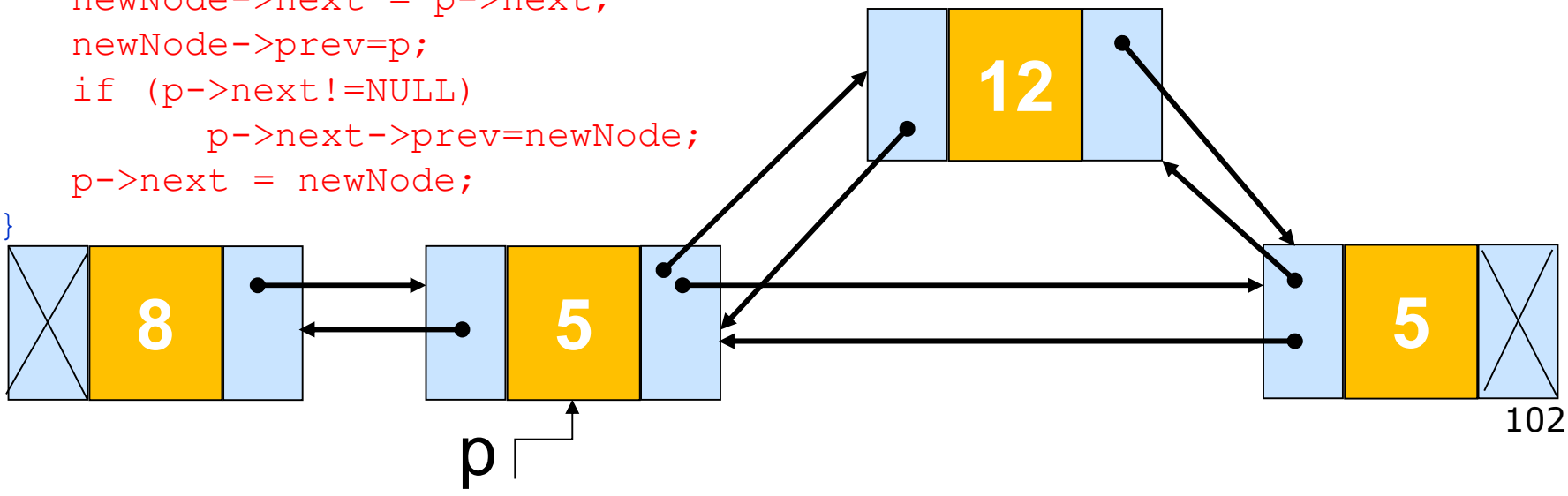
Delete a node pointed by the pointer p

```
void Delete_Node (ddlist *p){
    if (head == NULL) cout<<"The list is empty";
    else {
      if (p==head) head = head->next; //Delete first element
      else p->prev->next = p->next;
      if (p->next!=NULL) p->next->prev = p->prev;
      else tail = p->prev;
      delete p;
    }
}
```



Insert a node after the node pointed by pointer p

```
void Insert_Node (NodeType X, ddlist *p){
   if (head == NULL){  // List is empty
        head = new ddlist;
        head->data = X;
        head->prev =NULL;
        head->next =NULL;
   }
   else{
       ddlist *newNode;
       newNode = new ddlist;
       newNode->data = X;
      newNode->next = NULL;

       newNode->next = p->next;
       newNode->prev=p;
       if (p->next!=NULL)
             p->next->prev=newNode;
       p->next = newNode;
   }
}
```



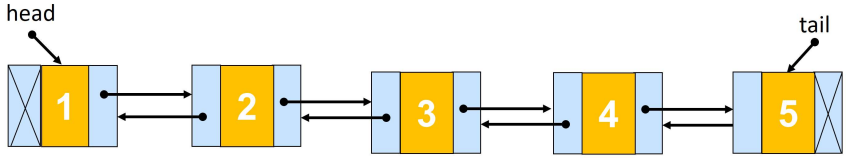
Exercise 2: Create a double linked list store integer numbers

```
#include <iostream>
using namespace std;

struct  dllist {
 int number;
 dllist *next;
 dllist *prev;
};
dllist *head, *tail;

/* Insert a new node p at the end of the list */
void append_node(dllist *p);
/* Insert a new node p after a node pointed by the pointer after */
void insert_node(dllist *p, dllist *after);
/* Delete a node pointed by the pointer p */
void delete_node(dllist *p);
```

```
/* Insert a new node p after a node pointed by the pointer after */
void insert_node(dllist *p, dllist *after) {
 p->next = after->next;
 p->prev = after;
 if (after->next != NULL)
     after->next->prev = p;
 else
     tail = p;
 after->next = p;
}
/* Delete a node pointed by the pointer p */
void delete_node(dllist *p) {
 if(p->prev == NULL)
     head = p->next;
 else  p->prev->next = p->next;
 if(p->next == NULL)
     tail = p->prev;
 else  p->next->prev = p->prev;
 delete p;
}
```

```
/* Insert a new node p at the end of the list */
void append_node(dllist *p) {
   if(head == NULL)
   {
      head = p;
      p->prev = NULL;
   }
   else {
     tail->next = p;
     p->prev = tail;
   }
   tail = p;
   p->next = NULL;
}
```

```
int main( ) {
dllist *tempnode; int i;
/* add some numbers to the double linked list */
for(i = 1; i <= 5; i++) {
    tempnode = new dllist;
    tempnode->number = i;
    append_node(tempnode);
}
```



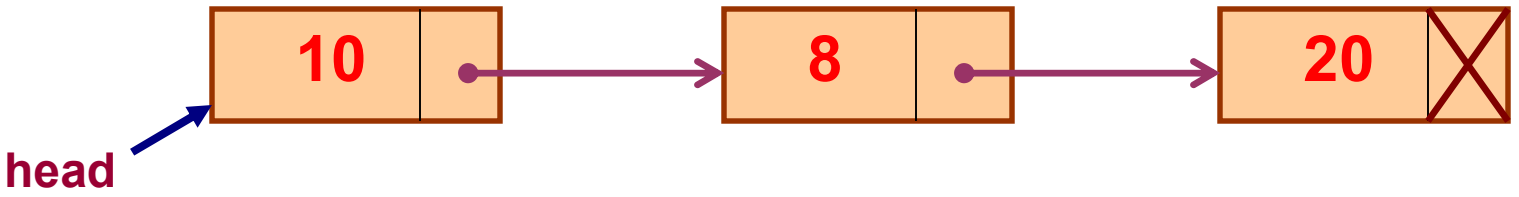

```
/* print the dll list forward */
  printf(" Traverse the dll list forward \n");
  for(tempnode = head; tempnode != NULL; tempnode = tempnode->next)
       printf("%d\n", tempnode->number);

 /* print the dll list backward */
 printf(" Traverse the dll list backward \n");
 for(tempnode = tail; tempnode != NULL; tempnode = tempnode->prev)
       printf("%d\n", tempnode->number);
/* destroy the dll list */
 while(head != NULL) delete_node(head);
 return 0;
}
```

Several variants of linked lists

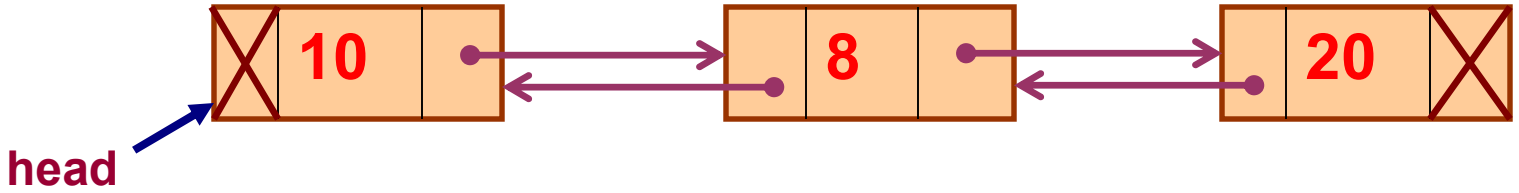
- Some common variants of linked list:
  - Circular Linked Lists
  - Circular Doubly Linked Lists
  - Linked Lists of Lists
- Basic operations on these variants are built similarly to the singly linked list and the doubly linked list that we consider above.

2.3. Linked list

- Singly linked list

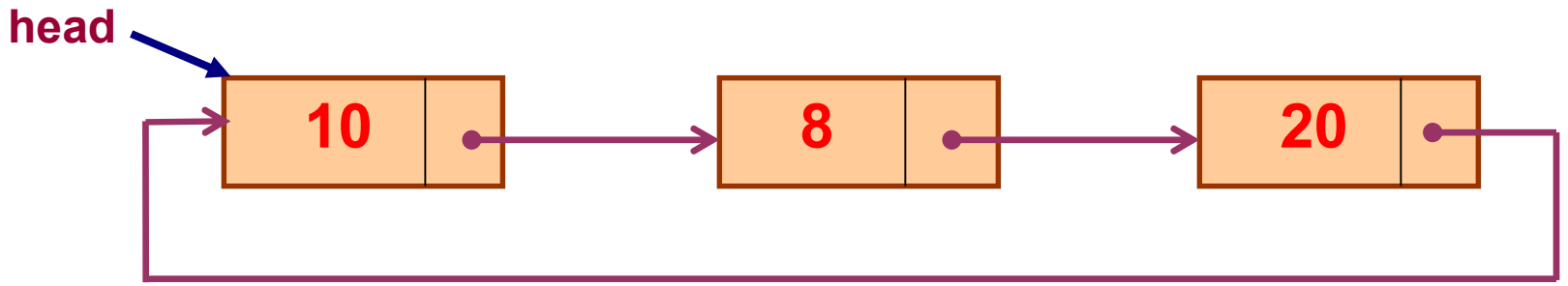


- Doubly linked list



- Circular linked list

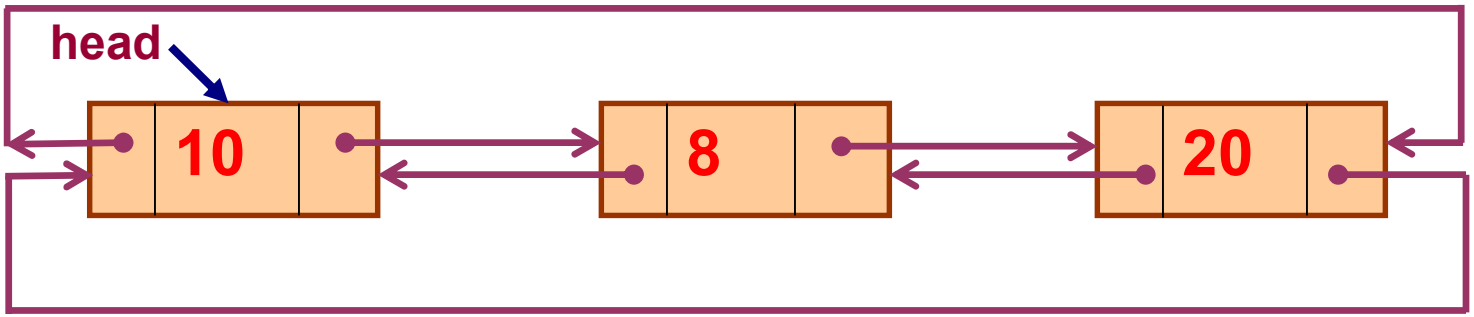
Circular linked list




```
struct node{
 NodeType data;
 struct node * next;
};
```

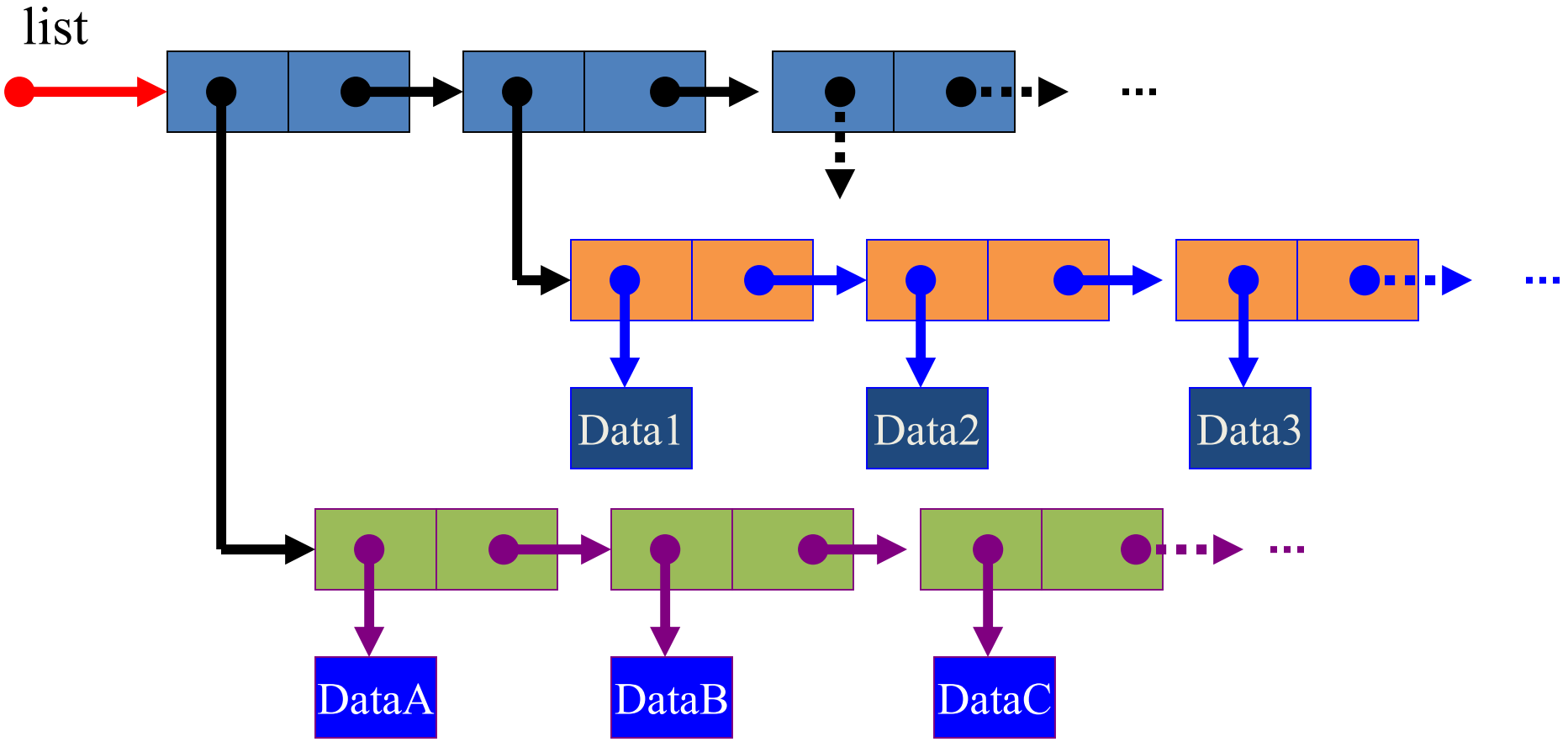
Store data

Circular Doubly Linked Lists




```
struct node {
  NodeType data;
  node * prev;
  node * next;
};
```

Linked Lists of Lists



Exercise: Polynomial Addition

Polynomials: defined by a list of coefficients and exponents

- *degree* of polynomial = the largest exponent in a polynomial

$$p(x) = a_1 x^{e_1} + a_2 x^{e_2} + \cdots + a_n x^{e_n}$$

Example:

Polynomials $A(x) = 3x^{10}+2x^5+6x^4+4$

$B(x) = x^4+10x^3+3x^2+1$

To represent a polynomial, the easiest way is to use array $a[i]$ to store the coefficient of x_i.

Operations with polynomials such as: Add two polynomials, Multiply two polynomials, ..., are thus possible to install simply.

Array a ~A(x)	a[10]	a[9]	a[8]	a[7]	a[6]	a[5]	a[4]	a[3]	a[2]	a[1]	a[0]
	3	0	0	0	0	2	6	0	0	0	4

Array b ~B(x)	b[4]	b[3]	b[2]	b[1]	b[0]
	1	10	3	0	1

$C(x) = A(x) + B(x) = 3x^{10}+2x^5+7x^4+3x^2+5$

Array c ~C(x)	c[10]	c[9]	c[8]	c[7]	c[6]	c[5]	c[4]	c[3]	c[2]	c[1]	c[0]
	=a[10] =3	=a[9] =0	=a[8] =0	=a[7] =0	=a[6] =0	=a[5] =2	=a[4]+b[4] =6+1=7	=a[3]+b[3] =0+10=10	=a[2]+b[2] =0+3=3	=a[1]+b[1] =0+0=0	=a[0]+b[0] =4+1=5

Exercise: Polynomial Addition

$A(x)=2x^{1000}+x^3$

$B(x)=x^4+10x^3+3x^2+1$

Use an array to keep track of the coefficients for all exponents:

…	2	…	0	1	0	0	0	A
…	0	…	1	10	3	0	1	B
	1000	…	4	3	2	1	0	

$A(x) + B(x) =$

advantage: easy implementation

disadvantage: waste space when sparse

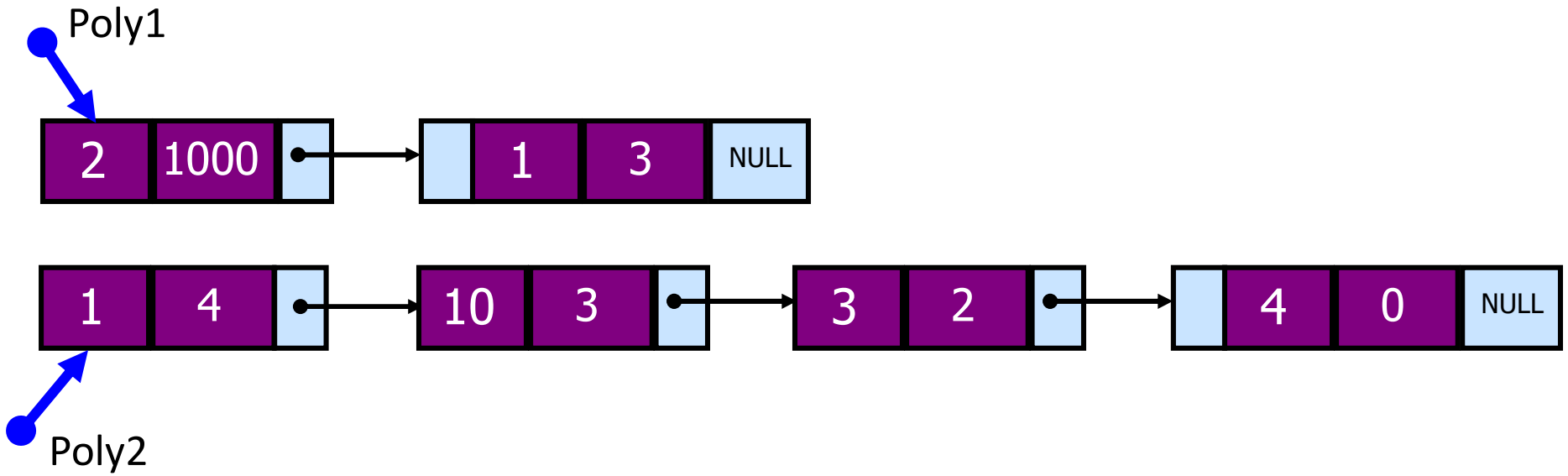
In the case of sparse polynomial (with many coefficients equal to 0), we could represent polynomial by the linked list: We will build a list containing only the coefficients with the value not equal to zero together with the exponent.
However, it is more complicated to implement the operations.

Exercise: Polynomial Addition

$A(x)=2x^{1000}+x^3$

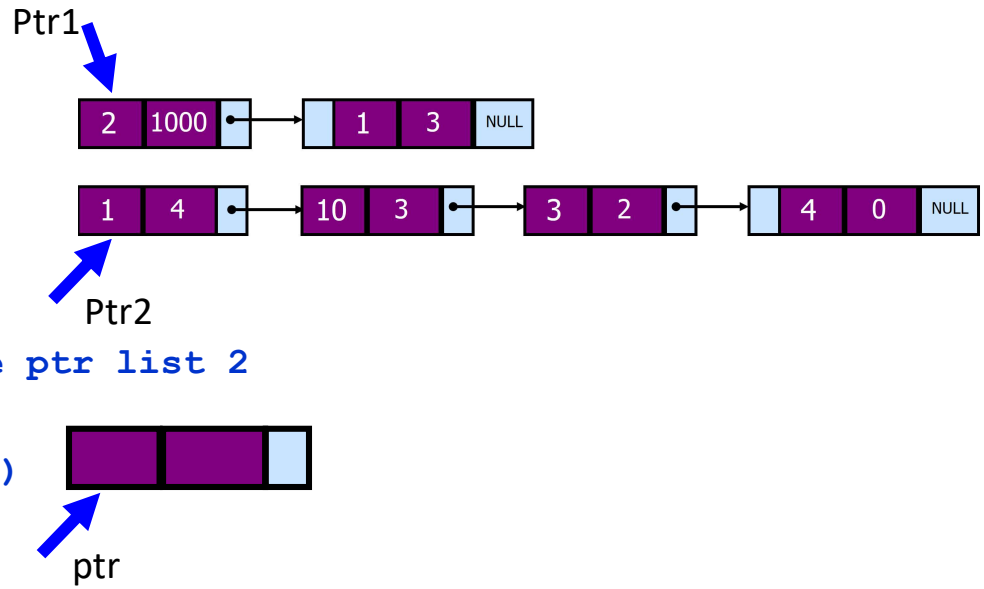
$B(x)=x^4+10x^3+3x^2+4$

Use the linked list to keep track of the coefficients with values ≠ 0 and the exponent:




```
struct Polynom {
    int coef;
    int exp;
    Polynom *next;
};
Polynom *Poly1, *Poly2;
```

```
Polynom *PolySum, *node,*ptr,*ptr1,*ptr2;
node = new Polynom;
PolySum = node;
ptr1 = Poly1; ptr2 = Poly2;
while (ptr1!=NULL && ptr2!=NULL){
    ptr = node;
    if  (ptr1->exp < ptr2->exp ) {
       node->coef = ptr2->coef;
       node->exp = ptr2->exp;
       ptr2 = ptr2->next;   //update ptr list 2
    }
    else if ( ptr1->exp > ptr2->exp )
    {   node->coef = ptr1->coef;
        node->exp = ptr1->exp;
        ptr1 = ptr1->next;   //update ptr list 1
    }
    else
    {  node->coef = ptr2->coef + ptr1->coef;
       node->exp = ptr2->exp;
       ptr1 = ptr1->next;   //update ptr list 1
       ptr2 = ptr2->next;   //update ptr list 2
    }
    node = new Polynom;
    ptr->nextp = node;   //update ptr listResult
} //end of while
```



```
if (ptr1 == NULL)      //end of list  1
{
    while(ptr2!= NULL) //copy the remaining of list2 to listResult
    {   node->coef = ptr2->coef;
        node->exp = ptr2->exp;
        ptr2 = ptr2->next;   //update ptr list 2
        ptr = node;
        node = new Polynom;
        ptr->next = node; //update ptr listResult
     }
 }
 else if (ptr2==NULL)      //end of list  2
 {
    while(ptr1 != NULL) //copy the remaining of list1 to the listResult
    {    node->coef = ptr1->coef;
         node->exp = ptr1->exp;
         ptr1 = ptr1->next;   //update ptr list 2
         ptr = node;
         node = new Polynom;
         ptr->next = node;   //update ptr listResult
   }
 }
 ptr->next = NULL;
}
```

C++ library

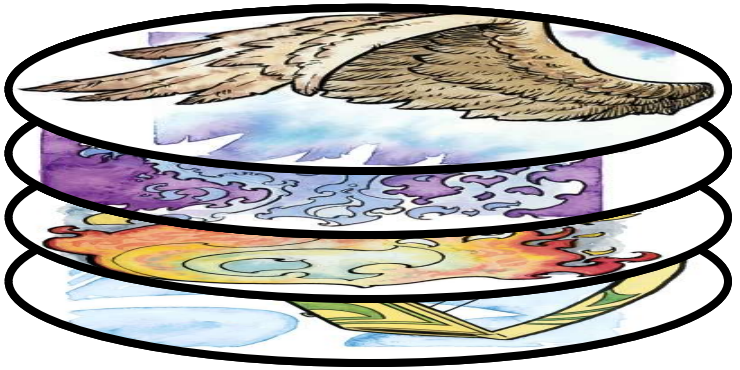
- list: http://www.cplusplus.com/reference/list/list/
- Main operators
  - push_front()
  - push_back()
  - pop_front()
  - pop_back()
  - size()
  - clear()
  - erase()

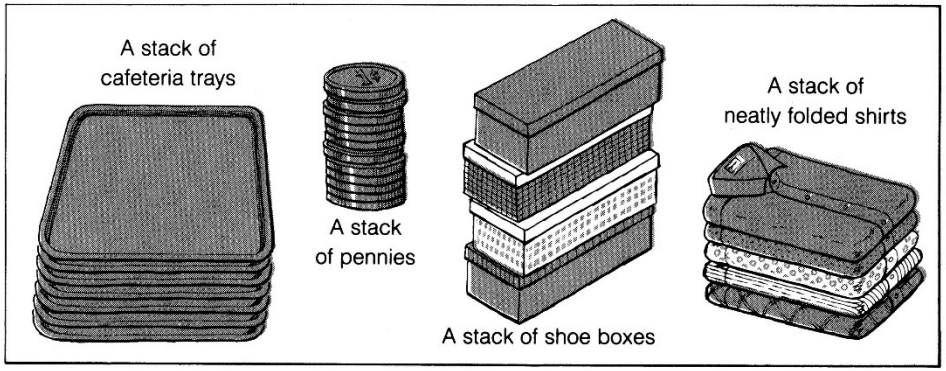
```
#include <iostream>
#include <list>
using namespace std;

int main()
{
  list<int> L;
  for(int i = 1; i <= 10; i++)
    L.push_back(i);
  list<int>::iterator it;
  for(it = L.begin(); it != L.end();it++)
  {
     int x = *it;
     cout<<x;
  }
}
```

Contents

What is a stack?

- A stack is a data structure that only allows items to be inserted and removed at ***one end***
  - We call this end the ***top*** of the stack
  - The other end is called the bottom
- Access to other items in the stack is not allowed
- The last element to be added is the first to be removed (**LIFO**: Last In, First Out)





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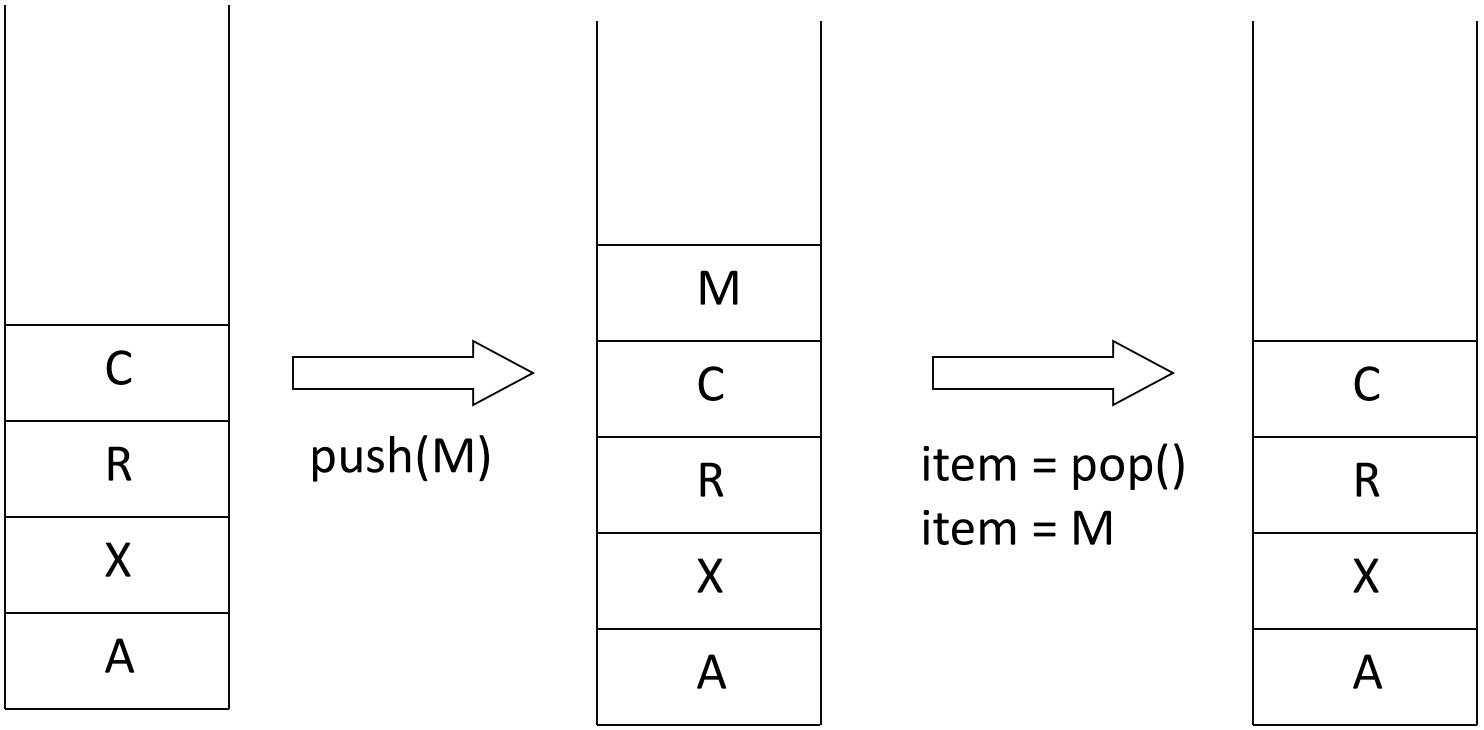
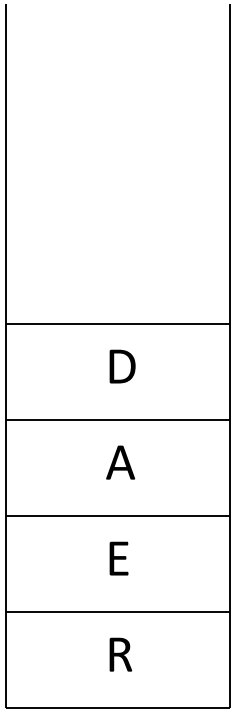
Operation on stack

- *Push*: the operation to place a new item at the top of the stack
- *Pop:* the operation to remove the next item from the top of the stack



Example: Reversing a Word

- We can use a stack to reverse the letters in a word.
- How?
- Example: READ



Push(R)

Push(E)

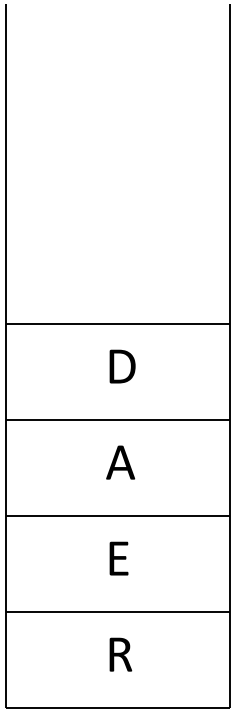
Push(A)

Push(D)

- Read each letter in the word and push it onto the stack

Example: Reversing a Word

- We can use a stack to reverse the letters in a word.
- How?
- Example: READ



Push(R) Pop()

Push(E) Pop()

Push(A) Pop()

Push(D) Pop()

DAER

- Read each letter in the word and push it onto the stack
- When you reach the end of the word, pop the letters off the stack and print them out

Implementing a Stack

- At least two different ways to implement a stack
  - array
  - linked list
- Which method to use depends on the application
  - what advantages and disadvantages does each implementation have?

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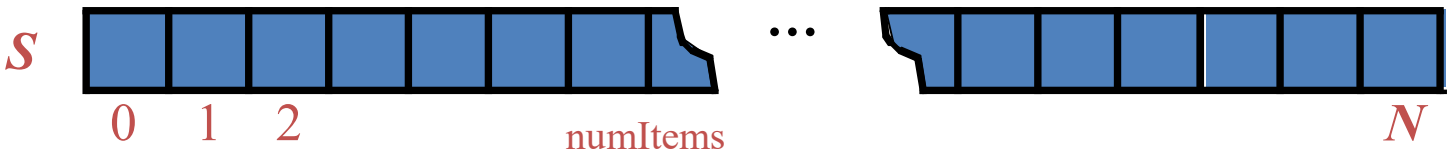
Stack: Array Implementation

- If an array is used to implement a stack what is a good index for the top item?
  - Is it position 0?
  - Is it position numItems-1?
- Note that push and pop must both work in O(1) time as stacks are usually assumed to be extremely fast
- Implementing a stack using an array is fairly easy:
  - The bottom of the stack is at `S[0]`
  - The top of the stack is at `S[numItems-1]`
  - *push* onto the stack at `S[numItems]`
  - *pop* off of the stack at `S[numItems-1]`



NGUYỄN KHÁNH PHƯƠNG
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Array Implementation Summary

- Advantages
  - Easy to implement
  - Best performance: push and pop can be performed in O(1) time
- Disadvantage
  - fixed size: the size of the array must be initially specified because
    - The array size must be known when the array is created and is fixed, so that the right amount of memory can be reserved
    - Once the array is full no new items can be inserted
- If the maximum size of the stack is not known (or is much larger than the expected size) a dynamic array can be used
  - But occasionally push will take O(n) time

maxSize: maximum number of elements in the array

Stack overflow

- The condition resulting from trying to push an element onto a full stack.

```
if(STACKfull())
        STACKpush(item);
```

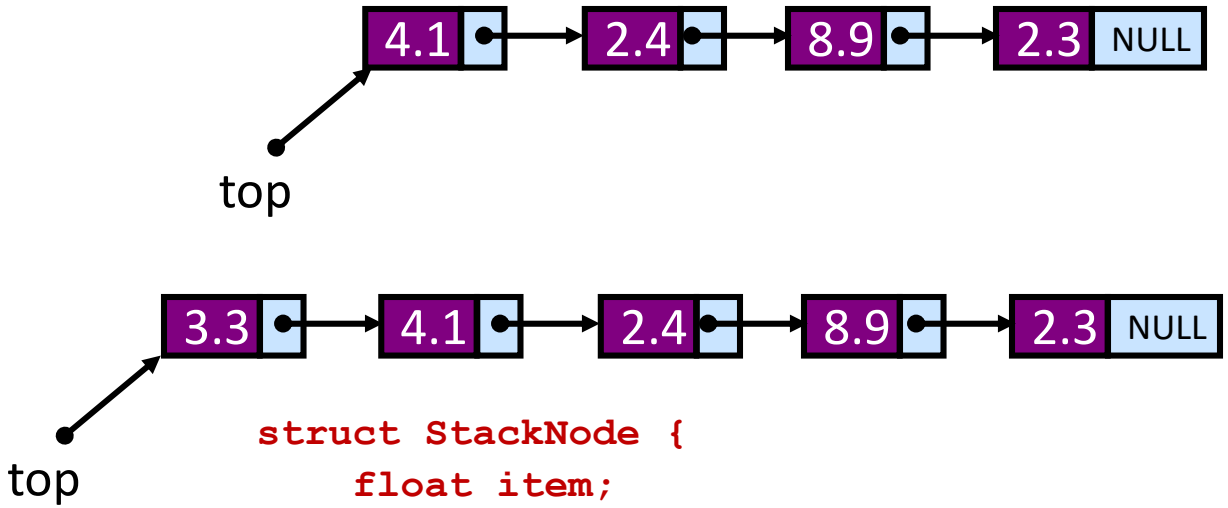
Stack underflow

- The condition resulting from trying to pop an empty stack.

```
if (STACKempty())
      STACKpop(item);
```

Implementing a Stack: using linked list

- Store the items in the stack in a linked list
- The top of the stack is the head node, the bottom of the stack is the end of the list
- *push* by adding to the front of the list
- *pop* by removing from the front of the list




```
struct StackNode {
    float item;
    StackNode *next;
};
struct Stack {
    StackNode *top;
};
```

Operations

1. Init:

```
Stack *StackConstruct();
```

2. Check empty:

```
int StackEmpty(Stack* s);
```

3. Check full:

```
int StackFull(Stack* s);
```

4. Insert a new item into stack (Push): *insert a new item at the top of stack*

```
int StackPush(Stack* s, float* item);
```

5. Remove an item from stack (Pop): *remove and return the item at the top of stack:*

```
float pop(Stack* s);
```

6. Print out all items of stack

```
void Disp(Stack* s);
```

Initialize stack

```
Stack *StackConstruct() {
    Stack *s;
    s = new Stack;
    if (s == NULL) {
      return NULL;  // No memory
    }
    s->top = NULL;
    return s;
}

/****   Destroy stack   *****/
void StackDestroy(Stack *s) {
    while (!StackEmpty(s)) {
      StackPop(s);
    }
    delete s;
}
```

```
/*** Check empty   ***/
int StackEmpty(const Stack *s) {
    return (s->top == NULL);
}

/*** Check full  ***/
int StackFull() {
    cout<<"\n NO MEMORY! STACK IS FULL";
    return 1;
}
```

Display all items in the stack

```
void disp(Stack* s) {
   StackNode* node;
   int ct = 0; float m;
   cout<<"\n\n  List of all items in the stack \n\n";
   if (StackEmpty(s))
       cout<<"\n\n  >>>>>  EMPTY STACK   <<<<<\n";
   else {
      node = s->top;
      do {
         m = node->item;
         cout<<m<<endl;
         node = node->next;
       } while (!(node == NULL));
   }
}
```

Push

Need to do the following steps:

(1) Create new node: allocate memory and assign data for new node

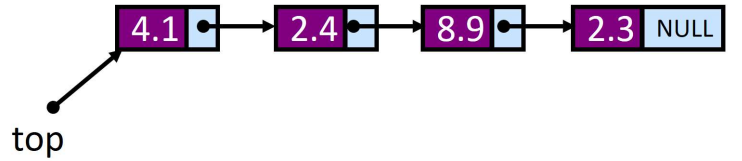
(2) Link this new node to the top (head) node

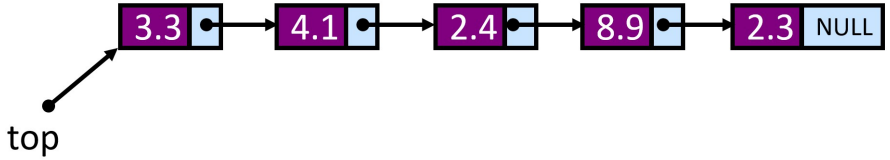
(3) Assign this new node as top (head) node

```
int StackPush(Stack *s, float item) {
  StackNode *node;
  node = new StackNode;
  if (node == NULL) {
    StackFull(); return 1;  // overflow: out of memory
  }
  node->item = item;          //(1)
  node->next = s->top;        //(2)
  s->top = node;              //(3)
  return 0;
}
```

Pop

1. Check whether the stack is empty
2. Memorize address of the current top (head) node
3. Memorize data of the current top (head) node
4. Update the top (head) node: the top (head) node now points to its next node
5. Free the old top (head) node
6. Return data of the old top (head) node






```
float StackPop(Stack *s) {
    float data;
    StackNode *node;
    if (StackEmpty(s))          //(1)
      return NULL;          // Empty Stack, can't pop
    node = s->top;              //(2)
    data = node->item;          //(3)
    s->top = node->next;        //(4)
    delete node;                //(5)
    return item;                //(6)
}
```

Experimental program

```
#include <iostream>
using namespace std;
// all above functions of stacks are put here
int main() {
   int ch,n,i;   float m;
   Stack* stackPtr;
   while(1)
   {  cout<<"\n\n======================\n";
      cout<< " STACK TEST PROGRAM \n";
      cout<<"======================\n";
      cout<<" 1.Create\n 2.Push\n 3.Pop\n 4.Display\n 5.Exit\n");
      cout<<"----------------------\n";
      cout<<"Input number to select the appropriate operation: ";
      cin>>ch; cout<<"\n\n";
```

```
  switch(ch) {
     case 1:     cout<<"INIT STACK";
                 stackPtr = StackConstruct(); break;
     case 2:     cout<<"Input float number to insert into stack: ";
                 cin>>m;
                 StackPush(stackPtr, m); break;
     case 3:     m=StackPop(stackPtr);
                 if (m != NULL)
                   cout<<"\n Data Value of the popped node: %8.3f\n",m;
                 else {
                   cout<<"\n  >>> Empty Stack, can't pop <<<\n";}
                 break;
     case 4:     disp(stackPtr); break;
     case 5:     cout<<"\n  Bye! Bye! \n\n";
                 exit(0);    break;
     default:    cout<<"Wrong choice";
    } //switch
  } // end while
} //end main
```

Implementing a Stack: using linked list

- Advantages:
  - always constant time to push or pop an element
  - can grow to an infinite size
- Disadvantages
  - Difficult to implement

C++ STL(Standard Template Library)

`#include <stack>` http://www.cplusplus.com/reference/stack/stack/

Main operations on stack:

- **`push`**`(object)`: insert the element `'object'` into the stack (on the top of stack)
- **`pop`**`()`: remove the top element from the stack
- `object` **`top`**`()`: return the reference to the top element of the stack
- `int` **`size`**`()`: return the number of elements currently in stack
- `bool` **`empty`**`()`: true if stack is empty

Example:

```
stack <int> s1; //declare stack variable s1 contains data of type int
stack <char> s2; //declare stack variable s2 contains data of type char
```

- We can not declare *`stack<int> myStack (5)`* to get 5 empty elements for stack, because stack does not allow us to declare empty elements.
- Similarly, we can not declare *`stack<int> myStack (5,100)`* to get a stack consisting of 5 elements, each with value of 100.

C++ STL(Standard Template Library)

```cpp
#include <iostream>
#include <stack>
using namespace std;

int main ()
{
    stack <int> S;
    for (int i = 1; i<=5; i++) S.push(i);
    cout<<"Stack size = "<<S.size()<<endl;
    cout<<"The top most element on stack: "<<S.top()<<endl;
    cout<<"Remove each element from the stack in turn: ";
    while (!S.empty())
    {
       cout<<"   "<<S.top();
       S.pop();
    }
    return 0;
}
```

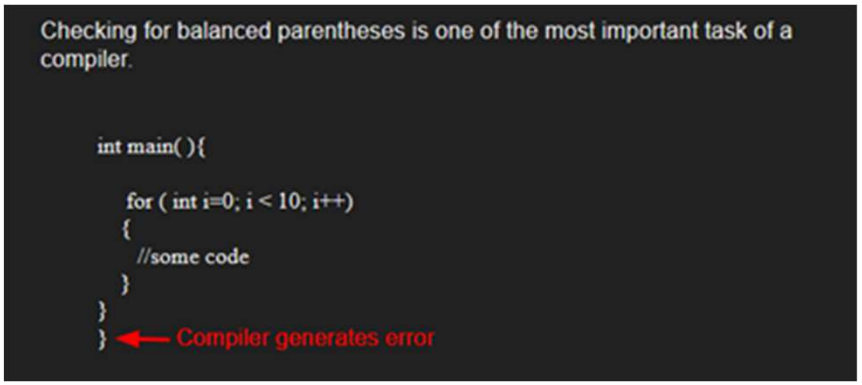
```
Stack size = 5
The top most element on stack: 5
Remove each element from the stack in turn:    5   4   3   2   1
```

Application 1: Parentheses Matching

Check for balanced parentheses in an expression:

Given an expression string expression, write a program to examine whether the pairs and the orders of “{“,”}”,”(“,”)”,”[“,”]” are correct in expression.

For example, the program should print true for expression = “[()]{}{[()()]()}” and false for expression = “[(])”



Algorithm:

1) Declare a character stack S.

2) Now traverse the expression string expression

a) If the current character is a starting bracket (‘(‘ or ‘{‘ or ‘[‘) then push it to stack.

b) If the current character is a closing bracket (‘)’ or ‘}’ or ‘]’) then pop from stack and if the popped character is the matching starting bracket then fine else parenthesis are not balanced.

3) After complete traversal, if there is some starting bracket left in stack then “not balanced”

Application 1: Parentheses Matching

Algorithm ParenMatch(*X,n*):

Input: Array *X* consists of *n* characters, each character could be either parentheses, variable, arithmetic operation, number.

Output: **true if** parentheses in an expression are **balanced**

```
S  = stack empty;
for i=0 to n-1 do {
    if  (X[i] is a starting bracket)
            push(S, X[i]);    // starting bracket ('(' or '{' or '[') then push it to stack
    else
       if (X[i] is a closing bracket)  // compare X[i] with the one currently on the top of stack
       {
           if  isEmpty(S)
                    return false {can not find pair of brackets}
           if  (pop(S)  not pair with bracket stored in X[i])
                    return false {error: type of brackets}
       }
}
if  isEmpty(S)
      return true {parentheses are balanced}
else   return false {there exist a bracket not paired}
```

Application 1: Parentheses Matching

```
#include <stack>
#include <iostream>
using namespace std;

bool isPair(char a, char b){
    if(a == '(' && b == ')') return true;
    if(a == '{' && b == '}') return true;
    if(a == '[' && b == ']') return true;
    return false;
}
```

```
bool solve(char* x, int n){
 stack <char> S;
 for(int i = 0; i <= n-1; i++){
   if(x[i] == '[' || x[i] == '(' || x[i] == '{‘)  S.push(x[i]);
   else
   if(x[i] == ‘]' || x[i] == ‘)' || x[i] == ‘}‘) {
      if(S.empty()) return false;
      else{
        char c = S.top(); S.pop();
        if(!isPair(c,x[i]))  return false;
      }
    }//end if
 }//end for
 return S.empty();
}
int main() {
  bool ok = solve(“[({})]()”,8);
  if (ok) cout<<“Parentheses in the expression are balanced”;
  else cout<<“Not balanced”;
}
```

Application 2: HTML Tag Matching

- In HTML, each <name> has to pair with </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

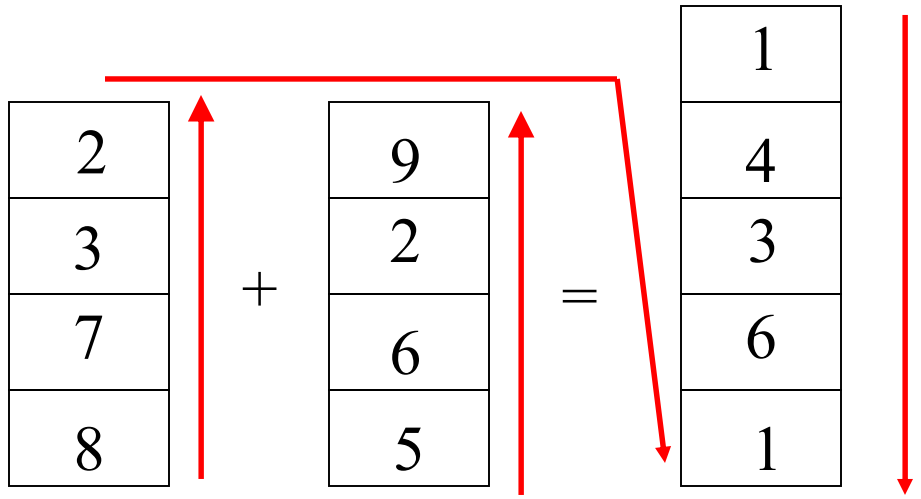
1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Application 3: Adding two large numbers

- Treat these numbers as strings of numerals, store the numbers corresponding to these numerals on two stacks, and then perform addition by popping numbers from the stacks

12345678901234567890123456789012345

78901234567890123456789012345



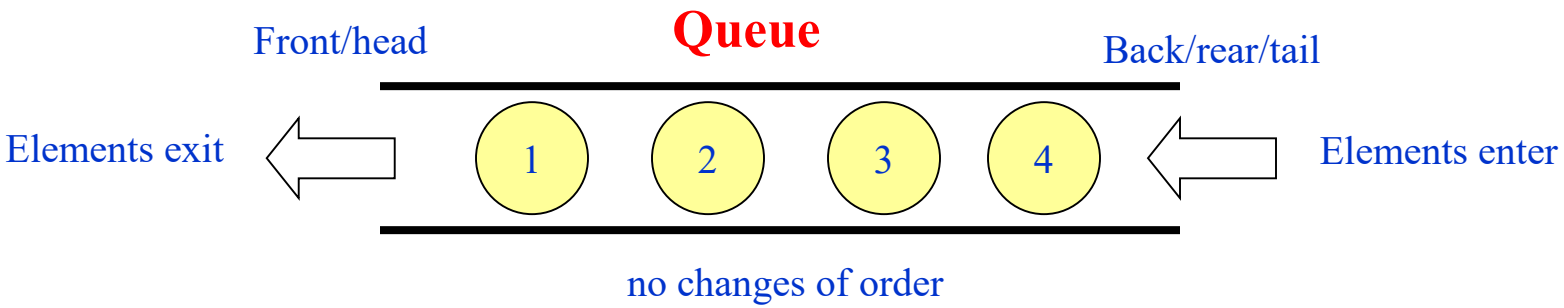
8732 + 5629 = 14361

Contents

What is a Queue?

Queues

- What is a queue?
  - A data structure of ordered items such that items can be inserted only at one end and removed at the other end.



Example: A line at the supermarket

- What can we do with a queue?
  - Enqueue - Add an item to the queue
  - Dequeue - Remove an item from the queue

These operations are also called insert and getFront in order to simplify things.

- A queue is called a FIFO (First in-First out) data structure.

Queue specification

Definitions: (provided by the user)

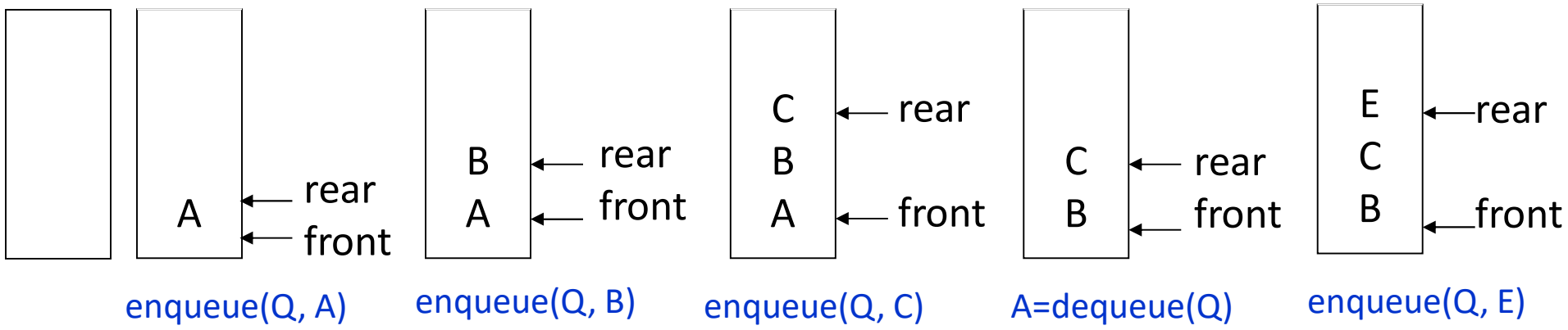
- *maxSize:* Max number of items that might be on the queue
- *ItemType*: Data type of the items on the queue

Operations:

- `Q = init();` initialize empty queue Q
- `isEmpty(Q);` returns "true“ if queue Q is empty
- `isFull(Q);` returns "true“ if Q is full, indicates that we already use the maximum memory for queue; otherwise returns “false”
- `front(Q);` returns the item that is in front (head) of queue Q or returns error if queue Q is empty.
- `enqueue(Q,x);` inserts item x into the back (rear) of queue Q. If before making insertion, the queue Q is full, then give the notification about that.
- `x = dequeue(Q);` deletes the element at the front (head) of the queue Q, then returns x which is the data of this element. If the queue Q is empty before dequeue, then give the error notification.
- `print(Q);` gives the list of all elements in the queue Q in the order from the front to the back.
- `size(Q);` returns the number of elements currently in the queue Q.

FIFO


(Remove/Dequeue)
Add/
Enqueue
Front/Head
Back/Rear


A
rear
front
B
A
rear
front
C
B
A
rear
front
C
B
rear
front
E
C
B
rear
front
enqueue(Q, A)
enqueue(Q, B)
enqueue(Q, C)
A=dequeue(Q)
enqueue(Q, E)

Given the sequence of operations on queue Q as following. Determine the output and the data on the queue Q after each operation:

	Operation	Output	Queue Q
1	`enqueue(5)`	-	(5)
2	`enqueue(Q,3)`	-	(5, 3)
3	`dequeue(Q)`	5	(3)
4	`enqueue(Q,7)`	-	(3, 7)
5	`dequeue(Q)`	3	(7)
6	`front(Q)`	7	(7)
7	`dequeue(Q)`	7	( )
8	`dequeue(Q)`	error	( )
9	`isEmpty(Q)`	true	( )
10	`size(Q)`	0	( )
11	`enqueue(Q,9)`	-	(9)
12	`enqueue(Q,7)`	-	(9, 7)
13	`enqueue(Q,3)`	-	(9, 7, 3)
14	`enqueue(Q,5)`	-	(9, 7, 3, 5)
15	`dequeue(Q)`	9	(7, 3, 5)

Implementing a Queue

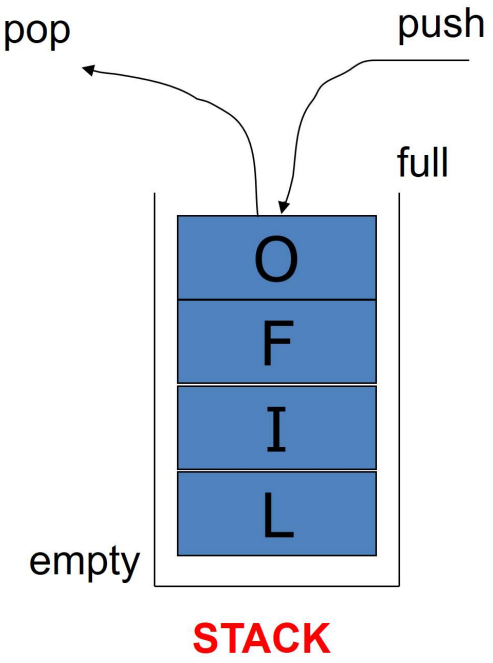
- Just like a stack, we can implementing a queue in two ways:
  - Using an array
  - Using a linked list

Implementing a Queue

- Just like a stack, we can implementing a queue in two ways:
  - **Using an array**
  - Using a linked list

Implementing a Queue: using Array

- Using an array to implement a queue is significantly harder than using an array to implement a stack. Why?
  - A stack: we add and remove at the same end,
  - A queue: we add to one end and remove from the other.

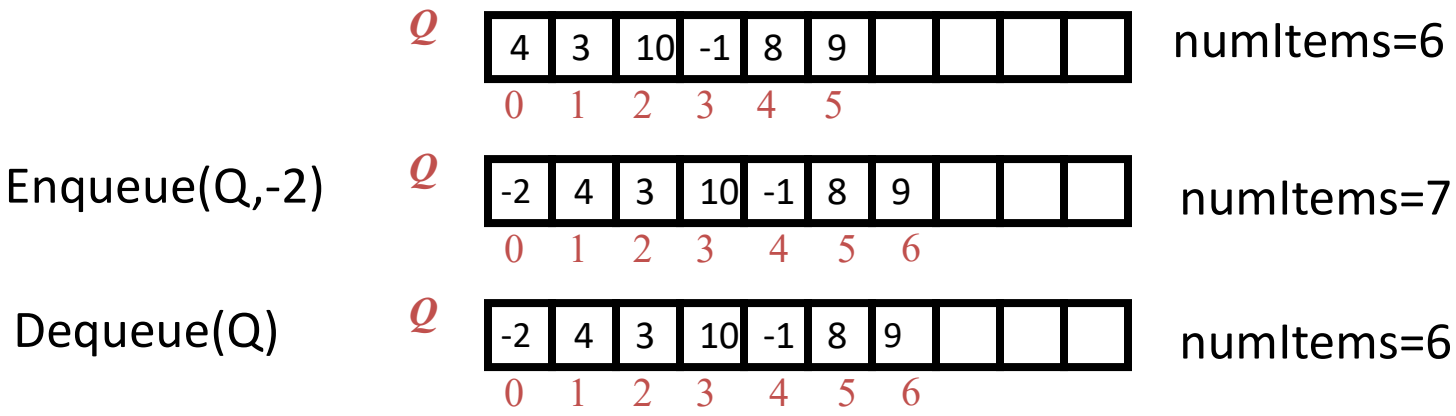


Implementing a Queue: using Array

There are some options for implementing a queue using an array:

- Option 1:
  - *Enqueue* at Q[0] and shift all of the rest of the items in the array down to make room.
  - *Dequeue* from Q[numItems-1]

Example:



- Option 2: circular queue ["wrap around"]

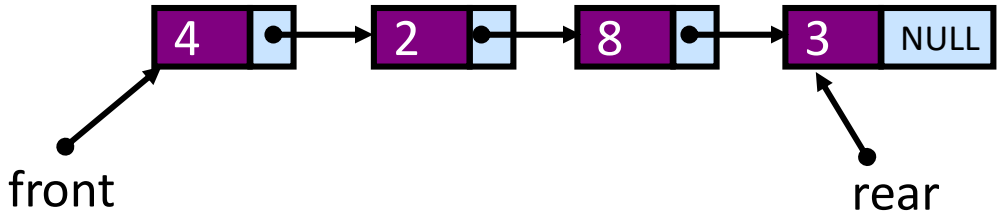
Implementing a Queue

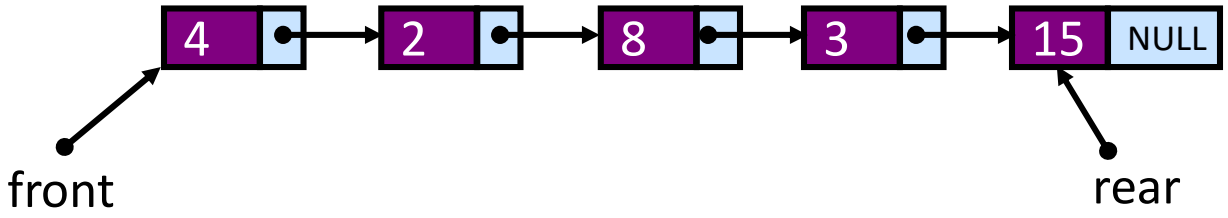
- Just like a stack, we can implementing a queue in two ways:
  - Using an array
  - **Using a linked list**

Implementing a Queue: using linked list

- Store the items in the queue in a linked list
- The front of the queue is stored as the head node of the linked list, the rear of the queue is stored as the tail node (the end of the list)
- *enqueue* by adding to the end of the list
- *dequeue* by removing from the front of the list


```
Struct QueueNode {
    int data;
    QueueNode *next;
};
```





C++ STL(Standard Template Library)

`#include <stack>` http://www.cplusplus.com/reference/stack/stack/

Main operations on stack:

- **push**(object): insert the element 'object' into the queue (at the end of queue)
- **pop**(): remove the first element from the queue
- object **front**(): return the reference to the first element of the queue
- object **back**(): return the reference to the last element of the queue
- int **size**(): return the number of elements currently in queue
- bool **empty**(): true if queue is empty

Example:

```
queue <int> q1;  //declared queue q1 containing elements of int
queue <char> cqueue;  //declared queue cqueue containing elements of char
```

- Similar to stack, we can not declare *queue<int> myQueue (5)* to get 5 elements empty for queue, because queue does not allow us to declare empty elements.
- Similarly, we can not declare *queue<int> myQueue (5,100)* to get a queue consisting of 5 elements, each with value of 100.

Example

```
#include <iostream>
#include <queue>
using namespace std;
int main ()
{
  queue <int> myqueue;
  int myint;
  cout << "Please enter some integers (enter 0 to end):\n";
  do {
    cin >> myint;
    myqueue.push (myint);
  } while (myint);
  cout << "myqueue contains: ";
  while (!myqueue.empty())
  {
       cout << " " << myqueue.front();
       myqueue.pop();
  }
  return 0;
}
```

Application 1: recognizing palindromes

- A *palindrome* is a string that reads the same forward and backward.

Example: NOON, DEED, RADAR, MADAM

Able was I ere I saw Elba

- How to recognize a given string is a palindrome or not:
  - Step 1: We will put all characters of the string into both a stack and a queue.
  - Step 2: Compare the contents of the stack and the queue character-by-character to see if they would produce the same string of characters:
    - If yes: the given string is a palindrome
    - Otherwise: not palindrome

Example 1: Whether "RADAR" is a palindrome or not

Step 1: Put "RADAR" into Queue and Stack:

Current character	Queue (front on the left, rear on the right)	Stack (top on the left)
R	R	R
A	R A	A R
D	R A D	D A R
A	R A D A	A D A R
R	R A D A R	R A D A R

front rear top

Example 1: Whether “RADAR” is a palindrome or not

Step 2: Delete “RADAR” from Queue and Stack:

- Dequeue until the queue is empty
- Pop the stack until the stack is empty

Queue (front on the left)	Front of Queue	Top of Stack	Stack (top on the left)
R A D A R	R	R	R A D A R
A D A R	A	A	A D A R
D A R	D	D	D A R
A R	A	A	A R
R	R	R	R
empty	empty	empty	empty

Conclusion: String "RADAR" is a palindrome

Example 2: recognizing palindromes

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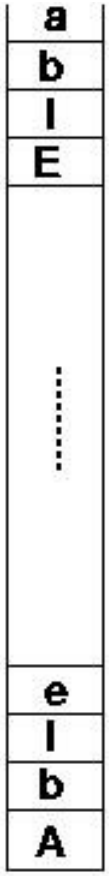

Stack

Queue

Application 2: Convert a string of digits into a decimal number

The algorithm is described as following:

```
// Convert sequence of digits stored in queue Q into decimal number n
// Remove empty space if any
do {      dequeue(Q, ch)
} until ( ch != blank)
//  ch is now the first digit of the given string
// Calculate n from sequence of digit in the queue
n = 0;
done = false;
do {      n = 10 * n + decimal number that ch represents;
          if (! isEmpty(Q) )
                    dequeue(Q,ch)
          else
                    done = true
}  until ( done  || ch != digit)
// Result: n is the decimal number need to be found
```