Learning Material - Experiment in ICT 2

Week 2

Goal of week

Review Boolean Algebra (AND, OR, NOT, XOR… logic functions)

6 axioms

- closure properties
- commutative laws
- existence of identities
- distributive laws
- existence of complement

9 theorems and their proof

Exercises

1. The XOR operation is also called the symmetric difference ($\oplus$) in Boolean algebra and is defined as $x \oplus y = x.\bar{y} + \bar{x}.y$. Prove that $x \oplus (x + y) = \bar{x}.y$
2. Prove the following statements: (x' = not(x))
    a. $\overline{\bar{x} + \bar{y}} = xy$
    b. $x.\bar{y} = 0$ if and only if $xy = x$
    c. x=0 if and only if $y = x.\bar{y} + \bar{x}.y \quad \forall y$ (for all y)
3. Prove the following identities:
    a. $\overline{x \odot y \odot z} = x \odot y \oplus z$
    b. $\overline{x \oplus y \oplus z} = x \oplus y \odot z$
4. Using the rules of Boolean algebra, simplify the following Boolean expressions:
    a. $\overline{\overline{\overline{\overline{x.y}.x}.\overline{\overline{x.y}.y}}}$
    b. $\overline{\overline{\overline{\bar{x}y\bar{w} + xwz} + \overline{\overline{xwz} + \bar{y}\bar{w}\bar{z} + yw\bar{z}}}}$
    c. $\overline{x + y}.\overline{\bar{x} + \bar{y}}$
    d. $y(w\bar{z} + wz) + xy$
    e. $xyz + \bar{x}\bar{y}z + \bar{x}yz + xy\bar{z} + \bar{x}\bar{y}\bar{z}$
5. Find the complements of the following Boolean expressions and reduce them to minimum number if literals.
    a. $(x\bar{y} + \bar{w}z)(w\bar{x} + y\bar{z})$
    b. $(w\bar{x} + \bar{y}\bar{z})(x + y)$
    c. $\bar{x}z + \bar{w}x\bar{y} + wyz + \bar{w}xy$
6. Obtain the truth table of the following functions.
    a. $F_1(w, x, y, z) = xy + \bar{x}z$
    b. $F_2(w, x, y, z) = w\bar{x} + yz + \bar{w}\bar{y}$
7. Obtain the truth table of $F_1 + F_2$ and F_1F_2 where F_1 and F_2 are given in the previous exercise.
8. A self-dual Boolean function is a function whole truth table remains unchanged when all the 0's are interchanged with 1's, and AND are interchanged with OR in the expression. How many self-dual Boolean functions of n variables are there?