Ridesharing Systems with Electric Vehicles

Theodoros Mamalis Subhonmesh Bose Lav R. Varshney

***Abstract*—Ridesharing systems are encouraging drivers in their fleets to adopt electric vehicles and may therefore be able to provide not only transportation services to passengers but also energy services to power grid operators through appropriate contracts. This paper develops a queuing network model of such ridesharing platforms where drivers may decide, at any given time, whether to provide transportation or grid services based on the incentives offered by the ridesharing platform. Then it considers designing driver incentives to maximize revenue for the ridesharing platform, via an analysis of the reward structure and an optimization algorithm. Platform revenue is assessed for various system parameters under optimal incentives.**

***Index Terms*—electric vehicles, queuing networks, sharing economy, transportation, revenue maximization**

I. Introduction

As transportation systems transition to a greater reliance on plug-in electric vehicles (EVs), not only will there be a strong coupling between transit and electrical grid networks [1] but also the possibility of efficiency gains through their joint control. In treating EVs as distributed energy resources (DERs), options for provisioning grid services include both serving as energy sources and sinks, as well as providing functions such as voltage and frequency regulation [2]–[5]. Here we examine the dual role of these *mobile batteries* in electrified transportation within the context of the sharing economy [6]. The sharing economy includes online platforms that mediate the sharing of goods and services by crowd-based agents responding to various incentives.

In particular, we consider ridesharing services, such as Uber and Lyft, where platforms incentivize individual driver/car-owners to provide rides and match them to customers seeking rides.[1] Note that ridesharing fleets are increasingly composed of EVs [9], [10]. If the ridesharing platform were to contract with an electrical utility or a retail aggregator, it could then develop an incentive scheme for drivers to both provide transportation and grid services. At any given time, then, drivers would face the choice of service type—a new kind of *multihoming* where the choice is not between two different platforms such as Uber and Lyft but between two different services on the same platform. Analyzing the driver's choice and optimizing the platform's incentive structure is the focus herein.

In developing mathematical models of two-sided markets that arise in sharing economy platforms, it is of increasing interest to specifically consider dynamics and random arrivals of workers and tasks, modeled as queuing processes [11]–[13]. We adopt a queuing network formalism for performance analysis—as has become standard in a variety of application domains including communication networks, manufacturing systems, and computer systems [14]—largely since equilibrium analysis of queues allows insights into an inherently dynamic process without having to perform full transient analysis as in optimal control approaches. Here, we specifically extend the queuing-theoretic model of ridesharing systems due to Banerjee, Riquelme, and Johari [15] by including a further grid service queue for drivers to choose, yielding a Jackson network and associated equilibrium analysis [16], [17].

The main contributions of the paper are:

- A driver-centric mathematical model of an EV ridesharing platform with both transportation and grid services,
- Characterization of the driver response to incentive schemes, and
- Optimization of platform revenue through design of the incentive scheme (both reward characterization and an optimization algorithm).

The remainder of the paper is organized as follows. We begin by defining the queuing system model for EV drivers providing transportation and energy services in Section II through a ridesharing platform. We pose the problem that the platform seeks to solve for selecting the prices for these two kinds of services in Section III. In Section IV, we analyze the properties of its reward structure, and in Section V, we provide an algorithm to optimize the prices and illustrate how these prices and the platform's reward varies with various parameters of our model. Section VI concludes the paper with a discussion of future work.

II. Ridesharing platform as a queueing system

Consider a peer-to-peer ridesharing company $\mathbf{A}$ which sports an all-electric vehicle fleet. $\mathbf{A}$ helps the drivers to match with two kinds of service requests—providing transportation services to passengers, and utilizing their vehicle as mobile batteries. The second kind of service requires drivers to utilize a portion of their car batteries to transact energy at designated outlets. Consider the scenario where $\mathbf{A}$ may offer this service to a retail aggregator who sells the aggregated capacity from the drivers in the wholesale market. That capacity can be utilized to provide energy, regulation, or reserve services. $\mathbf{A}$ can also sell this aggregated battery capacity to commercial and industrial (C&I) loads that seek to reduce their peak power consumption in order to reduce peak demand charges. In the sequel, call the requests to $\mathbf{A}$ for utilizing battery capacity to provide power as 'grid services'. We now model the driver behavior and $\mathbf{A}$'s business operation as a queuing network as shown in Figure 1.

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[1] Note that by ridesharing, we do not mean the pooling of passengers in the same vehicle [7], [8], though in our abstraction of customers, this is allowed.

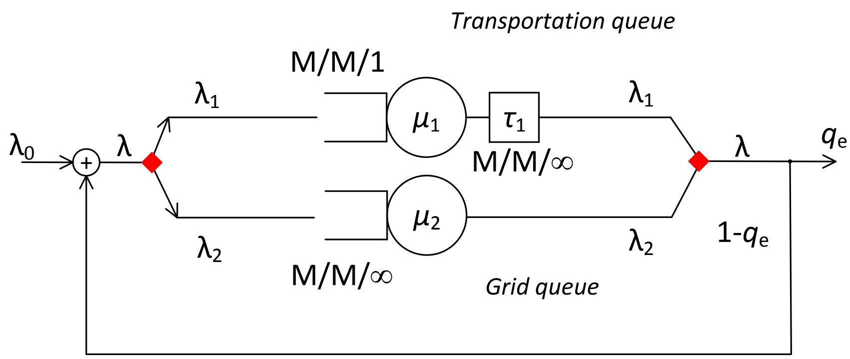

Fig. 1: Queuing network model for $\mathbf{A}$'s business.

A. The drivers and their preferences

Suppose drivers open $\mathbf{A}$'s application to provide services following a Poisson process with rate λ_0. These drivers are joined with others that have already been providing services through $\mathbf{A}$'s app, and decide to re-enter the queue of drivers to serve again. If q_e denotes the probability that a driver leaves the app after providing a service, we have

$$\lambda_0 + (1 - q_e)\lambda = \lambda \implies \lambda = \lambda_0 / q_e.$$

Here, λ denotes the resulting Poisson rate of driver arrivals that are ready to serve. Let drivers choose to serve the transportation and grid queues with probabilities proportional to the expected payments from providing each service. More precisely, let $\mathbf{A}$ charge p_1 per unit time to each passenger for providing a ride. Then, $\mathbf{A}$ shares a $\gamma < 1/2$ fraction of it, i.e., γp_1 with the driver, and keeps $(1-\gamma)p_1$ for itself. Further, it pays a driver at the rate p_2 per unit time for providing grid services. The payment scheme then results in drivers arriving at the transportation and the grid queues according to a Poisson process with rates λ_1 and λ_2, where $\lambda_1/\lambda_2 = \gamma p_1 / p_2$, implying

$$\lambda_1 = \frac{\gamma p_1}{\gamma p_1 + p_2}\lambda, \quad \lambda_2 = \frac{p_2}{\gamma p_1 + p_2}\lambda.$$

The equations can equivalently be viewed as the result of the drivers' rewards from the transportation and grid services being independent exponentially distributed random variables with means $\gamma_1 p_1$ and p_2, respectively. If drivers choose the service with the largest reward, then the probability of choosing each service becomes proportional to the mean reward from that service as we have assumed.

In our model, drivers who commit to providing transportation services will remain in the transportation queue, even if no passengers seek a ride immediately, but do not switch to the grid queue. We remark that the so-called sunk cost fallacy (loss aversion) in queuing systems explains such a modeling choice. People often decide to persist with a chosen course of action, even when alternate and economically better actions are available, e.g., see [18], [19].

B. Modeling transportation service provision

Assume passengers seeking a ride open $\mathbf{A}$'s app according to a Poisson process with rate μ_1. They choose to use $\mathbf{A}$'s services if the price of a ride p_1 does not exceed their reservation price. Further, assume the ride lasts for an exponentially distributed length of time with mean τ_1 (see [20], [21] for details). Let $\overline{F}$ denote the complementary cumulative distribution (tail distribution) function of the reservation prices of consumers. Therefore, passengers who ultimately seek a ride arrive according to a Poisson process with rate

$$\mu_1' := \mu_1 \overline{F}(p_1).$$

Assume that the reservation wages take values in all of $\mathbb{R}_+$ with a finite expectation, denoted by $\mathbb{E}[p_{\text{res}}]$. The passenger arrival can therefore be modeled as a queuing process. Additionally assume that, absent a driver, an arriving passenger immediately leaves. Akin to that in [15], this assumption prevents the queue of passengers from growing unbounded. The queue of available drivers sees exponentially distributed job sizes with average $1/\mu_1'$. In Kendall's notation, the available drivers form an $M/M/1$ queue to provide transport.

C. Modeling grid service provision

Next, we model the grid services. Consider a service request from $\mathbf{A}$ to the drivers that asks them to park their car at a designated location for an exponentially distributed amount of time with mean $1/\mu_2$. When grid connected, the car relinquishes control over a portion of the battery to $\mathbf{A}$ (or a third-party with whom $\mathbf{A}$ has a contract). We model the drivers providing the grid service as an $M/M/\infty$ queuing process. That is, we assume the number of available plug points for the electric vehicles is large. We remark that our modeling choices for the transportation and the grid queues render their departure processes Poisson. As a result, drivers who provide a service and return to serve again follow a Poisson process as well. We have therefore modeled $\mathbf{A}$'s business as an *open Jackson network*, as shown in Figure 1.

Consider a contract between $\mathbf{A}$ and the party receiving the grid service that remunerates $\mathbf{A}$ based on the total battery capacity made available for grid services. For simplicity, assume that the battery capacity of each car, and the portion allotted to provide grid services, are homogeneous across all drivers. Then, the capacity available to provide grid services at any time is proportional to the number of cars plugged in. Let the contract be given by two components—one a forward contract that pays for making a certain number of cars available, and then a reward or penalty for abiding by or violating that contract. Let $\mathbf{A}$ earn at a rate of $f(\theta)$ from the forward contract to make θ cars available, and a reward of $R(k)$ when k cars are connected in realtime, where

$$R(k) := \begin{cases} -c, & \text{if } k < \theta, \\ +c, & \text{otherwise.} \end{cases} \tag{1}$$

The reward structure is such that $\mathbf{A}$ pays a penalty when too few cars ($< \theta$) are available to provide energy services, and makes money otherwise.[2]

A note on the thresholded reward structure: Here, we justify the rationale behind our choice of the reward structure R. Consider the example where the battery capacity from the vehicles is utilized by commercial and industrial (C&I) loads to avoid peak demand charges—payments due to increased retail energy price when energy use surpasses a certain threshold. Many have argued in favor of utilizing vehicle-to-grid services to avoid such payments, e.g., see [22]. For such services, the energy capacity from the vehicles provides no benefit if it is below a certain threshold. In addition, excess capacity beyond that threshold adds no value.

Another example service to motivate the reward structure is that of frequency regulation, where assets respond to requests to adjust their power output every 2-4 seconds. These adjustments facilitate the second-by-second balance of demand and supply of power in the grid. In current performance-based regulation markets, assets contract a certain capacity against a forward payment, and are then paid in realtime based on how closely they follow the regulation signal. Capacity in excess of the forward contract does not garner added revenue. The quality of tracking the regulation signal depends on the capacity available in realtime. And, that quality affects procurements from that asset in future market clearings. That is, an asset that regularly falls short of providing the contracted capacity will ultimately find it challenging to be cleared in the market; see [23] for details on regulation markets. A threshold reward structure emulates the payments from such markets.

D. Computing $\mathbf{A}$'s revenue rate

The rich literature on Jackson networks offers effective ways to analyze various properties of the system in Figure 1, when operating at steady-state. The equilibrium properties provide insights into its long-term behavior. Of particular interest to us is the expected revenue rate of $\mathbf{A}$, the rate at which $\mathbf{A}$ earns in equilibrium, which we characterize in the next result.

Proposition 1. *The expected revenue rate of $\mathbf{A}$ is given by*

$$\begin{aligned} r_{\mathbf{A}}(p_1, p_2) := (1-\gamma)\lambda_1 p_1 - \lambda_2 p_2 \\ + f(\theta) + c\left[1 - 2Q\left(\theta, \lambda_2/\mu_2\right)\right], \end{aligned} \tag{2}$$

where

$$Q(z_1, z_2) := e^{-z_2} \sum_{k=0}^{z_1 - 1} \frac{z_2^k}{k!}$$

for $(z_1, z_2) \in \mathbb{N}_+ \times \mathbb{R}_+$ is the regularized Gamma function.

Proof. The first and the second term in the right hand side of (2) follow from multiplying the rate at which drivers arrive at the two queues and the payments to the drivers.

[2]The penalty and the reward from grid service provision have identical magnitudes c; that assumption can be easily relaxed.

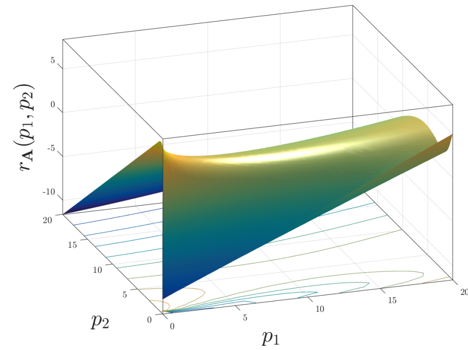

Fig. 2: Variation of expected revenue rate with the prices. Parameters for the experiment are given by $\lambda = 1$, $\mu_1 = \frac{6}{5}$, $\mu_2 = \frac{1}{10}\mu_1$, $\theta = 5$, $c = 10$, $\gamma = \frac{1}{4}$, $f = 0$.

The third term arises from the forward contract for grid service provision. We show that $\mathbb{E}[R]$, the expected reward from grid services, is given by the fourth term. Recall that the grid queue is modeled as an $M/M/\infty$ queue with arrival rate λ_2 and service rate μ_2. Then, the stationary probability distribution of there being k cars at the grid queue is Poisson with mean $\rho_2 := \frac{\lambda_2}{\mu_2}$, implying

$$\begin{aligned} \mathbb{E}[R] &= e^{-\rho_2} \sum_{k=0}^{\theta-1} \frac{\rho_2^k}{k!}(-c) + e^{-\rho_2} \sum_{k=\theta}^{\infty} \frac{\rho_2^k}{k!} c \\ &= -cQ(\theta, \rho_2) + c\left[1 - Q(\theta, \rho_2)\right] \\ &= c\left[1 - 2Q(\theta, \rho_2)\right]. \end{aligned}$$

■

III. The Platform's Price Selection Problem

The expected reward rate in Proposition 1 allows us to formally state $\mathbf{A}$'s objective, given below.

$$\begin{aligned} \underset{p_1 \geq 0, p_2 \geq 0}{\text{maximize}} \quad & r_{\mathbf{A}}(p_1, p_2), \\ \text{subject to} \quad & \lambda_1 < \mu_1 \overline{F}(p_1). \end{aligned} \tag{3}$$

The constraint in (3) arises from stability considerations of the transportation queue. It prevents unbounded increase in the number of drivers queued up to provide transportation services. One of two cases may arise to ensure the stability of the transportation queue. The rate of incoming drivers is low enough that the transportation queue remains stable even if all drivers join it. Then, the price for energy services does not affect the stability considerations. Alternately, when transportation queue cannot handle all interested drivers, then the grid price has to be high enough so as to attract enough drivers away from the transportation queue. The stability constraint in (3) defines an open set. To sidestep possible difficulties in optimizing over an open set, we take a closure with the understanding that μ_1 can be suitably perturbed to keep the closed set feasible. The stability constraint can be succinctly described as

$$p_2 \geq \gamma p_1 \left(\frac{\lambda/\mu_1}{\overline{F}(p_1)} - 1\right)^+ := p_2^{\mathsf{L}}, \tag{4}$$

where we use the notation $z^+ := \max\{z, 0\}$.

Analytical characterization of the optimal prices p_1^*, p_2^* for (3) remains difficult. Naively designing an algorithm to search for these prices can be challenging as well. Problem (3) is nonconvex. Further, the variation of the reward rate as a function of the prices can be quite complex, as Figure 2 illustrates. Motivated to design an algorithm to optimize the reward rate, we analyze the properties of $r_{\mathbf{A}}$ that allow us to systematically narrow the search space for the optimum. Since the forward contract f does not affect the optimization over prices, henceforth, assume $f = 0$.

IV. Properties of the Expected Reward Rate

The design of an algorithm to maximize the expected reward rate $r_{\mathbf{A}}$ relies on understanding how this rate varies with the prices for transportation and grid services. In this section, we provide a sequence of results that capture its essential properties that will allow us to derive such an algorithm in Section V.

Our first result characterizes how $r_{\mathbf{A}}$ varies as a function of the price for grid services p_2, holding the transportation price p_1 constant. The following definition will prove useful in stating the result.

$$p_{\max} := \frac{2c}{\mu_2}\frac{1}{\Gamma(\theta)}(\theta-1)^{\theta-1}e^{1-\theta},$$

where $\Gamma(z) := \int_0^\infty x^{z-1}e^{-x}dx$ is the Gamma function for $z \in \mathbb{R}_+$.

Proposition 2 (Variation with p_2). *The expected reward rate satisfies* $\lim_{p_2\to\infty} r_{\mathbf{A}}(\cdot, p_2) = -\infty$. *Also,* $\frac{\partial r_{\mathbf{A}}}{\partial p_2} \leq 0$, *when*

- $p_1 \geq \frac{1}{1-\gamma}p_{\max}$, $p_2 \geq 0$, *or*
- $p_1 < \frac{1}{1-\gamma}p_{\max}$, $p_2 \geq p_2^{\mathsf{U}}$, *where*

$$p_2^{\mathsf{U}} := -\gamma p_1 + \sqrt{\gamma p_1 p_{\max} - \gamma(1-2\gamma)p_1^2}. \tag{5}$$

Proof. Proposition 1 yields

$$\frac{\partial r_{\mathbf{A}}}{\partial p_2} = \frac{\lambda}{(\gamma p_1 + p_2)^2}(T_2 - T_1), \tag{6}$$

where

$$T_1 := p_2^2 + 2\gamma p_1 p_2 + \gamma(1-\gamma)p_1^2,$$
$$T_2 := \gamma p_1 \frac{2c}{\mu_2}\frac{1}{\Gamma(\theta)}\rho_2^{\theta-1}e^{-\rho_2}.$$

Notice that T_1 is a strictly convex increasing function of p_2, and T_2 is a scaled density function of the Gamma distribution evaluated at ρ_2 with shape and scale parameters θ and unity, respectively. The scaling factor is $2c\gamma p_1/\mu_2$. The mode of the Gamma distribution occurs at $\theta - 1$, implying

$$T_2 \leq \gamma p_1 p_{\max}.$$

However, $T_1 \sim p_2^2$ for large p_2, and hence, we have

$$\lim_{p_2\to\infty}\frac{\partial r_{\mathbf{A}}}{\partial p_2} = -\lambda,$$

i.e., $r_{\mathbf{A}}$ decreases at a rate of $-\lambda$ towards $-\infty$ for large p_2.

A sufficient condition for $\frac{\partial r_{\mathbf{A}}}{\partial p_2} \leq 0$ is given by

$$T_1 \geq \gamma p_1 p_{\max}.$$

The minimum of T_1 occurs at zero, taking the value $\gamma(1-\gamma)p_1^2$. Enforcing it to be greater than the right-hand side of the above inequality, we get the first condition in the proposition. Otherwise the crossing point of T_1 with the right hand side occurs at p_2^{U}, and $r_{\mathbf{A}}$ always decreases for $p_2 > p_2^{\mathsf{U}}$. ■

Owing to the above proposition, the search space for the optimizer of (3) reduces to

$$\begin{aligned} p_1 &\geq \frac{1}{1-\gamma}p_{\max}, \quad p_2 = p_2^{\mathsf{L}}, \\ p_1 &< \frac{1}{1-\gamma}p_{\max}, \quad p_2 \in [p_2^{\mathsf{L}}, p_2^{\mathsf{U}}], \end{aligned} \tag{7}$$

where p_2^{L} is defined in (4) from stability considerations of the transportation queue. Next, we present a result similar to Proposition 2, where we characterize the variation of the expected reward rate with the transportation price alone, keeping the grid price constant.

Proposition 3 (Variation with p_1). *The expected reward rate satisfies* $\lim_{p_1\to\infty} r_{\mathbf{A}}(p_1, \cdot) = \infty$. *Also,* $\frac{\partial r_{\mathbf{A}}}{\partial p_1} \geq 0$, *when*

- $p_2 \geq p_{\max}$, $p_1 \geq 0$ *or,*
- $p_2 < p_{\max}$, $p_1 \geq p_1^{\mathsf{L}}$, *where*

$$p_1^{\mathsf{L}} := -\frac{p_2}{\gamma} + \frac{1}{\gamma\sqrt{1-\gamma}}\sqrt{(1-2\gamma)p_2^2 + \gamma p_2 p_{\max}}. \tag{8}$$

Proof. Using Proposition 1, we have

$$\frac{\partial r_{\mathbf{A}}}{\partial p_1} = \frac{\lambda}{(\gamma p_1 + p_2)^2}(T_3 - T_4), \tag{9}$$

where

$$T_3 := p_2^2 + 2(1-\gamma)p_1 p_2 + \gamma(1-\gamma)p_1^2,$$
$$T_4 := p_2 \frac{2c}{\mu_2}\frac{1}{\Gamma(\theta)}e^{-\rho_2}\rho_2^{\theta-1}.$$

Similar to the proof of Proposition 2, $T_4 \leq p_2 p_{\max}$. Then, a sufficient condition for $\frac{\partial r_{\mathbf{A}}}{\partial p_1}$ to be nonnegative is $T_3 \geq p_2 p_{\max}$. Here, T_3 is a strictly convex increasing function of p_1. The possible crossing point for T_3 with $p_2 p_{\max}$ is given by p_1^{L} in (8). For $p_2 \geq p_{\max}$, one can verify that $p_1^{\mathsf{L}} \leq 0$, and hence, the required derivative is nonnegative for all p_1. ■

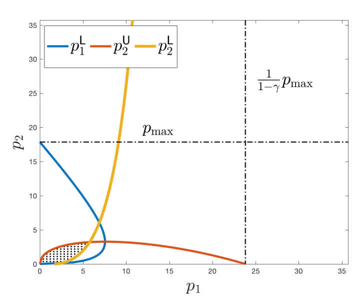

Fig. 3: The search space for the optimal prices includes p_2^{L} and the shaded area. The reservation prices were assumed to be exponentially distributed with mean 20. Other parameters for the experiment are given by $\lambda = 1$, $\mu_1 = \frac{7}{4}$, $\mu_2 = \frac{1}{5}\mu_1$, $\theta = 5$, $c = \frac{2}{3}$, $\gamma = \frac{1}{4}$.

Equipped with Proposition 3, the search space for the optimal prices narrows down to the following two regions: (i) the area below p_2^{U} that lies to the left of p_1^{L} and p_2^{L}, and (ii) along p_2^{L}. See Figure 3 for a graphical illustration. The next result argues that the reward rate ultimately decreases in an unbounded fashion as the prices are increased along the trajectory marking the boundary of the queue stability limit p_2^{L}. Further, it provides a bound on the maximum transportation price beyond which the reward rate reduces below any given threshold, effectively reducing our search space to a bounded region. The following notation is introduced:

$$\psi := \min\{\lambda/\mu_1, 1\}.$$

Proposition 4 (Variation along the queue stability limit). *The expected reward rate satisfies* $\lim_{p_1\to\infty} r_{\mathbf{A}}(p_1, p_2^{\mathsf{L}}) = -\infty$. *Also,* $r_{\mathbf{A}}(p_1, p_2^{\mathsf{L}}) \leq \eta$ *for all* $p_1 \geq \overline{p}_1$, *where*

$$\overline{p}_1 := \begin{cases} \overline{F}^{-1}(\psi), & \text{if } \eta' < 0, \\ \overline{F}^{-1}(\psi/2) \cdot \max\left\{1, \frac{1}{4\eta'}\mu_1^2\psi^2\right\}, & \text{otherwise}, \end{cases}$$

$$\eta' := \frac{\mu_1}{\gamma}\left[(1-\gamma)\mu_1\mathbb{E}[p_{\mathsf{res}}] + c - \eta\right],$$

and $\mathbb{E}[p_{\mathsf{res}}]$ *is the expected reservation price for passengers.*

Proof. For $p_1 \geq \overline{F}^{-1}(\psi)$, we have

$$p_2^{\mathsf{L}} = \gamma p_1\left(\frac{\lambda/\mu_1}{\overline{F}(p_1)} - 1\right) \implies \begin{cases}\lambda_1 = \mu_1\overline{F}(p_1), \\ \lambda_2 = \lambda - \mu_1\overline{F}(p_1),\end{cases}$$

for which Proposition 1 implies

$$\begin{aligned} r_{\mathbf{A}}(p_1, p_2^{\mathsf{L}}) &= (1-\gamma)\mu_1 p_1\overline{F}(p_1) \\ &\quad - \left[\lambda - \mu_1\overline{F}(p_1)\right]\gamma p_1\left(\frac{\lambda/\mu_1}{\overline{F}(p_1)} - 1\right) \\ &\quad + c\left[1 - 2Q\left(\theta, \frac{1}{\mu_2}(\lambda - \mu_1\overline{F}(p_1))\right)\right] \\ &\leq (1-\gamma)\mu_1 p_1\overline{F}(p_1) - \frac{\gamma p_1}{\mu_1\overline{F}(p_1)}\left[\lambda - \mu_1\overline{F}(p_1)\right]^2 + c. \end{aligned}$$

The above follows from the properties of the regularized Gamma function. Further, from Markov's inequality, we get

$$\overline{F}(p_1) \leq \frac{\mathbb{E}[p_{\mathsf{res}}]}{p_1},$$

that in turn allows us to deduce

$$r_{\mathbf{A}}(p_1, p_2^{\mathsf{L}}) \leq (1-\gamma)\mu_1\mathbb{E}[p_{\mathsf{res}}] - \frac{\gamma p_1}{\mu_1\overline{F}(p_1)}\left[\lambda - \mu_1\overline{F}(p_1)\right]^2 + c. \tag{10}$$

The second term decreases to $-\infty$ as p_1 grows unbounded, completing the proof of the first part of the proposition.

To prove the second part, we utilize the upper bound on the reward rate in (10) to search for $\overline{p}_1$ such that

$$\frac{p_1}{\overline{F}(p_1)}\left(\left[\lambda - \mu_1\overline{F}(p_1)\right]^+\right)^2 \geq \eta'$$

for all $p_1 \geq \overline{p}_1$. If $\eta' < 0$, it suffices to choose $\overline{p}_1 := \overline{F}^{-1}(\psi)$. Otherwise, for $p_1 \geq \overline{F}^{-1}(\psi/2)$, we have

$$\frac{p_1}{\overline{F}(p_1)}\left(\left[\lambda - \mu_1\overline{F}(p_1)\right]^+\right)^2 \geq \frac{\mu_1^2\psi^2\overline{F}^{-1}(\psi/2)}{4\overline{F}(p_1)}.$$

Requiring the right hand side of the above equation to

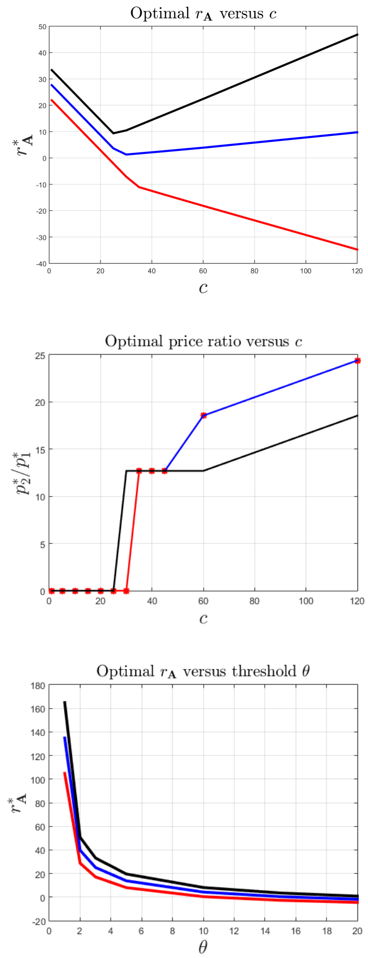

Fig. 4: Plots that illustrate the effect of parameter variations on the optimal prices and expected reward rate for $\lambda = 8$(red), $\lambda = 10$(blue), and $\lambda = 12$(black). In our experiments, we use the parameters $\gamma_1 = \frac{1}{4}, \mu_1 = 10, \mu_2, c = 15, \theta = 5$. The passenger reservation price follows the gamma distribution with shape and scale parameters 2 and 1 respectively.

dominate η' for $p \geq \overline{p}_1$ yields the result. ■

The above result reveals that as $\mathbf{A}$ increases the transportation price p_1, $\mathbf{A}$ loses business from passengers, making the transportation queue prone to becoming unstable. To maintain queue stability, such an increase in transportation price must accompany a corresponding increase in the price for grid service provision. The stability limit is such that for high values of these prices, most drivers essentially end up at the grid queue. $\mathbf{A}$'s reward from energy service provision being bounded above by c, the payout to the drivers in the grid queue ultimately drags $\mathbf{A}$'s revenue down towards $-\infty$.

V. Maximizing the Expected Reward Rate

Having analyzed the variation of $r_{\mathbf{A}}$ with the prices in the last section, we now design a search for the prices that maximize it in the following steps.

- Grid search over (p_1, p_2) in the box $\left[0, \frac{1}{1-\gamma}p_{\max}\right] \times [0, p_{\max}]$ and find the optimizer over all points that satisfy three properties: $p_2 \leq p_2^{\mathsf{U}}$, $p_1 \leq p_1^{\mathsf{L}}$, and $p_2 \geq p_2^{\mathsf{L}}$.

- Call the optimal $r_{\mathbf{A}}$ over the grid search as η.
- Sample multiple $p_1 \in \left[\overline{F}^{-1}(\psi), \overline{p}_1\right]$, where $\overline{p}_1$ is defined as in Proposition 4 with η from the last step.
- With each sample, run a gradient ascent on $r_{\mathbf{A}}$ over the curve (p_1, p_2^{L}) as p_1 varies within $\left[\overline{F}^{-1}(\psi), \overline{p}_1\right]$ with step-sizes varying as $\mathscr{O}(1/\sqrt{T})$ in the T-th iteration.
- Output the maximum $r_{\mathbf{A}}$ encountered.

Our analysis in Section IV narrows the search for optimal prices to a bounded region. One can utilize any nonlinear programming technique or randomized algorithms such as simulated annealing to optimize $r_{\mathbf{A}}$ over that bounded region. To explore that region, we take a two-pronged approach—one over the shaded area in Figure 3, where we do not know how the partial derivatives of $r_{\mathbf{A}}$ behave, and second, on the curve that encodes the stability limit for the transportation queue. We cannot guarantee that the above algorithm produces a global optimal reward rate. Gauging the suboptimality of our search is relegated to future endeavors.

A. Variation of optimal rewards with model parameters

Figure 4 plots the effect of the variation of the parameters λ, c, θ on the optimal prices (p_1^*, p_2^*) and the corresponding expected reward rates $r_{\mathbf{A}}^*$. Increasing c increases the possible penalty from grid service provision. For low driver arrival rates, this penalty leads to a reduction of $r_{\mathbf{A}}^*$. For higher values of λ, the corresponding increase in the possible reward leads to an increase in $r_{\mathbf{A}}^*$ with c beyond a threshold. Again for low values of c, the potential for a small reward from the grid queue dictates that $\mathbf{A}$ chooses to send all its drivers to the transportation queue by setting $p_2^* = 0$. As c increases, the possibility of reward from the grid queue leads $\mathbf{A}$ to choose a nonzero price p_2^*.

As the threshold θ increases, the total expected revenue rate (without accounting for the forward contract $f(\theta)$) drops. We expect that behavior as larger θ binds $\mathbf{A}$ to bring more cars to the grid to overcome this threshold, thereby having to pay the drivers to do so. The variation of $r_{\mathbf{A}}^*$ with θ sheds light on the nature of the forward contract f that $\mathbf{A}$ must sign. Explicitly characterizing the forward contract is left for future work.

VI. Conclusions and future work

We have proposed a novel queuing model for ridesharing systems with electric vehicles that can offer both transportation and grid services, and established an approach for optimizing the platform's revenue. Beyond the basic queuing model for transportation service provision considered here, future work aims to study more detailed geographic factors. Related to such geographic considerations, we also aim to further consider behavioral aspects of drivers' choice behavior, such as *range anxiety*, the phenomenon experienced by EV drivers in settings of insufficient energy replenishment infrastructure [24], [25]. This would lead to a model with battery state dependence, which is mathematically interesting in its own right. Finally, we believe the basic mathematical model we have developed and analyzed may be applicable to a variety of multihoming settings throughout the sharing economy.

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