# Coursera MOOC Econometrics - Test Exercise 5

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**a)** The marginal effect of activity status is defined as

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{active}_i} = \Pr[\text{resp}_i = 1]\Pr[\text{resp}_i = 0]\beta_2$$

We could use this result to construct an activity status elasticity

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{active}_i} \frac{\text{active}_i}{\Pr[\text{resp}_i = 1]} = \Pr[\text{resp}_i = 0]\text{active}_i\beta_2$$

Use these results to compute the elasticity effect of active status for a 50-year-old active male customer. Do the same for a 50-year-old inactive male customer.

**Answer:**
Probability of response 0 of active 50-year male customer:

$$\frac{1}{1 + \exp(\text{-}2{,}49 + 0{,}95 + 0{,}91 + 0{,}07 \times\ 50\ \text{-}0{,}07\ \times 0{,}5^2} \approx 0{,}25$$

Elasticity is probability of above times the active dummy and caution:
$\approx 0{,}25 \times 1 \times 0{,}91 \approx 0{,}23$

For inactive male the dummy is 0 therefore:
$\ldots . \times 0 \times 0{,}91 = 0$

**b)** The activity status variable is only a dummy variable and hence it can take only two values. It is therefore better to define the elasticity as

$$\frac{\Pr[\text{resp}_i = 1;\ \text{active}_i = 1] - \Pr[\text{resp}_i = 1;\ \text{active}_i = 0]}{\Pr[\text{resp}_i = 1;\ \text{active}_i = 0]}$$

Show that you can simplify the expression for the elasticity as

$$(\exp(\beta_2)\text{-}1)\Pr[\text{resp}_i = 0;\ \text{active}_i = 1]$$

**Answer:**

$$\frac{\Pr[\text{resp}_i = 1;\ \text{active}_i = 1] - \Pr[\text{resp}_i = 1;\ \text{active}_i = 0]}{\Pr[\text{resp}_i = 1;\ \text{active}_i = 0]} = \frac{\frac{\exp(\beta_2)2_i}{1 + \exp(\beta_2)2_i} - \frac{2_i}{1 + 2_i}}{\frac{2_i}{1 + 2_i}}$$

$$\frac{\exp(\beta_2)(1 + 2_i)}{1 + \exp(\beta_2)2_i}\text{-}1 = \frac{\exp(\beta_2)(1 + 2_i)\text{-}(1 + \exp(\beta_2)2_i)}{1 + \exp(\beta_2)2_i} = \frac{\exp(\beta_2)\text{-}1}{1 + \exp(\beta_2)2_i}$$

$$(\exp(\beta_2)\text{-}1) \times \frac{1}{1 + \exp(\beta_2)2_i} = (\exp(\beta_2)\text{-}1)\Pr[\text{resp}_i = 0;\ \text{active}_i = 1]$$

**c)** Use the formula in (b) to compute the activity elasticity of 50 years old male active customer

**Answer:**

$$(\exp(0{,}91 = \text{-}1)\frac{1}{1 + \exp(\text{-}2{,}49 + 0{,}95 + 0{,}91 + 0{,}07 + 50\text{-}0{,}07 \times (0{,}5)^2} \approx 1{,}48 \times 0{,}24 \approx 0{,}35$$