Signals and Communications

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1 Signals and systems

Dirac Impluse Relation

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$

Energy of a continuous-time signal

$$E = \int_{-\infty}^{+\infty} x(t)x^*(t)dt$$

Mean power of a continuous-time signal

$$P_x = \lim_{\theta \to +\infty} \frac{1}{2\theta} \int_{-\theta}^{+\theta} x(t)x^*(t)dt$$

This is equal to 0 if $x(t)$ is an energy-type signal.

Correlations

	Cross-correlation	Auto-correlation
energy-type	$\gamma_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)y^*(t-\tau)dt$	$\gamma_{xx}(\tau) = \int_{-\infty}^{+\infty} x(t)x^*(t-\tau)dt$
power-type	$\gamma_{xy}(\tau) = \frac{1}{2\theta}\int_{-\theta}^{+\theta} x(t)y^*(t-\tau)dt$	$\gamma_{xx}(\tau) = \frac{1}{2\theta}\int_{-\theta}^{+\theta} x(t)x^*(t-\tau)dt$

Linearity and time-invariance

A system is linear iff:

$$\forall x_i(t), \forall a_i \in \mathbb{R} \quad f(\sum_i a_i x_i) = \sum_i a_i f(x_i)$$

Convolution

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-u)h(u)du$$

Time shift

$$x(t)\delta(t-\tau) = x(t-\tau)$$

2 The Fourier Transform

Fourier transform of a continuous-time signal

$$X(f) = FT\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt$$

Inverse Fourier transform of a continuous-time signal

$$x(t) = FT^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{+j2\pi ft}df$$

Properties of the Fourier Transform

time shifting

$$FT\{x(t-\tau)\} = e^{+j2\pi ft}X(f)$$

time reversal

$$FT\{x(-t)\} = X(-f)$$

conjugation

$$FT\{x^*(t)\} = X^*(-f)$$

Fourier transform of a covolution product

$$FT\{x(t) * y(t)\} = X(f)Y(f)$$