The self-testing scenario

$\mathscr{L}(\mathscr{H})$ denotes the set of linear operators acting on Hilbert space $\mathscr{H}$.

We know there exist measurement operators $M_{a|x} \in \mathscr{L}(\mathscr{H})$ acting on Alice's Hilbert space and satisfying

$$M_{a|x} \succcurlyeq 0; \forall a, x \sum_a M_{a|x} = Id_A$$

Similarly there exist measurement operators $N_{b|y} \in \mathscr{L}(\mathscr{H})$ acting on Bob's Hilbert space. The measurement operators are therefore projective :

$$\forall a, a' : \quad M_{a|x} M_{a'|x} = \delta_{a,a'} M_{a|x}$$

$$\forall b, b' : \quad N_{b|y} N_{b'|y} = \delta_{b,b'} N_{b|y}$$

Now, from the Born rule, there must exist some quantum state $\rho_{AB} \in \mathscr{L}(\mathscr{H}_\mathrm{A} \otimes \mathscr{H}_\mathrm{B}) \succcurlyeq 0$, and $\mathrm{tr}\rho_{AB} = 1$ such that

$$p(a, b|x, y) = \mathrm{tr}\left[\rho_{AB} M_{a|x} \otimes N_{b|y}\right]$$

In self-testing, one aims to infer the form of the state and the measurement in the trace from knowledge of the correlation $p(a, b|x, y)$ alone, i.e. in device-independent scenario.

Born rule : A key postulate of quantum mechanics which gives the probability that a measurement of a quantum system will yield a given result. More formally, for a state $|\psi\rangle$ and a F_i POVM element (associated with the measurement outcome i), then the probability of obtaining i when measuring $|\psi\rangle$ is given by

$$p(i) = \langle\psi|F_i|\psi\rangle$$

Physical assumptions

1. The experiment admits a quantum description (state and measurement)
2. The laboratories of Alice and Bob are located in separate location in space and there is no communication between the two laboratories.
3. The setting x and y are chosen freely and independently of all other systems in the experiment.
4. Each round of the experiment is independent of all other rounds a physically equivalent to all others (i.e. there exists a single density matrix and measurement operators that are valid in every round).

Impossibility to infer exactly the references

1. *Unitary invariance of the trace* : one can reproduce the statistics of any state $|\psi\rangle$ and measurement $\{M_{a|x}\}, \{N_{b|y}\}$ by instead using the rotated state $U \otimes V |\psi\rangle$ and measurement $\{UM_{a|x}U^\dagger\}, \{VN_{b|y}V^\dagger\}$, where U, V are unitary transformations. Hence, one can never conclude that the state was $|\psi\rangle$ or $U \otimes V |\psi\rangle$.

2. *Additional degrees of freedom* : a state $|\psi\rangle \otimes |\xi\rangle$ and measurements $\{M_{a|x} \otimes Id_\xi\}, \{N_{b|y} \otimes Id_\xi\}$ gives the same correlation as $|\psi\rangle$ and $\{M_{a|x}\}, \{N_{b|y}\}$.