Take-away Impartial Combinatorial Game on Different Geometric and Discrete Structures

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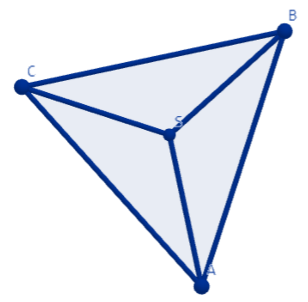
Rules in a Game of Nim on Hypergraphs

Two players take turns to remove the vertices and the hyperedges of a hypergraph.
In each turn, a player must remove either only 1 vertex or only 1 hyperedge.

- When 1 vertex is chosen to be removed, all hyperedges that contain the chosen vertex are also removed.
- When 1 hyperedge is chosen to be removed, only the chosen hyperedge is removed.

Whoever removes the last vertex wins the game.



Hypergraph T:
Set of vertices: { S, A, B, C }
Set of hyperedges: { { S, A, B }, { S, A, C }, { S, B, C }, { A, B, C } }

Figure 1 Hypergraph T

Nim-value of all positions in a Game of Nim on Oddly Uniform Hypergraphs

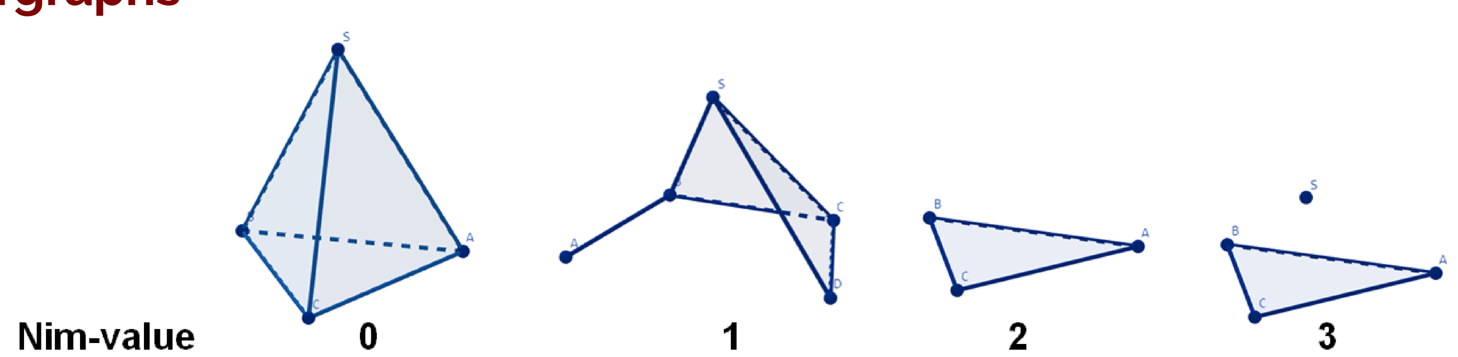


Figure 2 Positions in a Game of Nim on Oddly Uniform Hypergraphs

VERTICES / HYPEREDGES	EVEN	ODD
EVEN	0	1
ODD	3	2

Figure 3 Nim-value of all positions in a Game of Nim on Oddly Uniform Hypergraphs

Nim-value of all positions in a Game of Nim on Evenly Uniform Hypergraphs with a marked coloring

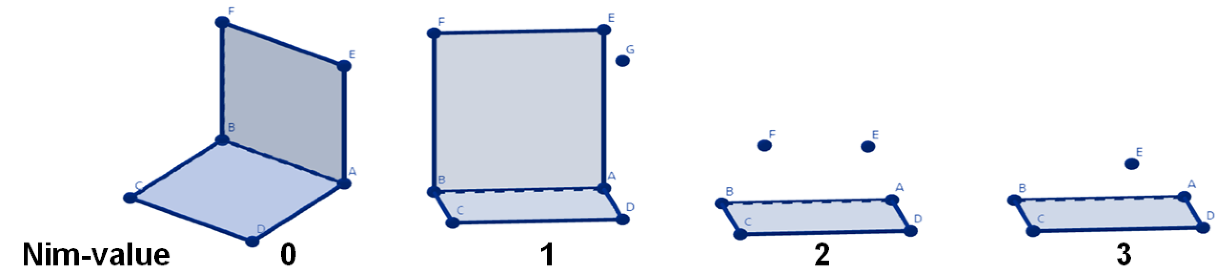


Figure 4 Positions in a Game of Nim on Evenly Uniform Hypergraphs with a marked coloring

VERTICES / HYPEREDGES	EVEN	ODD
EVEN	0	1
ODD	2	3

Figure 5 Nim-value of all positions in a Game of Nim on Evenly Uniform Hypergraphs with a marked coloring

Special Neither Oddly nor Evenly Uniform Hypergraphs

New definition:

- A base: a hyperedge that contains an even number of vertices
- A triangular hyperedge: a hyperedge that contains exactly 3 vertices, exactly 2 out of those 3 vertices are the vertices of a base.

Neither Oddly nor Evenly Uniform Hypergraphs whose vertices and hyperedges must satisfy the special conditions: **1 base, any number of triangular hyperedges, and regarding the only one vertex in each triangular hyperedge that is not a vertex of a base, either that one vertex is shared between all triangular hyperedges or that one vertex is shared between 0 triangular hyperedge**

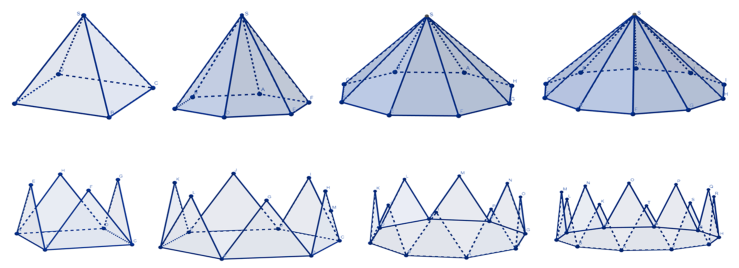


Figure 6 Special Neither Oddly nor Evenly Uniform Hypergraphs

Types of vertices: 0_A, B_1, B_2, B_0

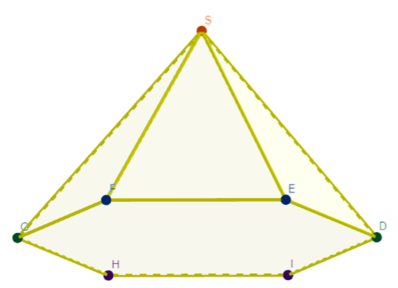


Figure 7 Special Neither Oddly nor Evenly Uniform Hypergraphs

Hypergraph:
Set of vertices: {S, D, E, F, G, H, I}
Set of hyperedges: { { S, D, E }, { S, E, F }, { S, F, G }, { D, E, F, G, H, I } }
Type 0_A: Only vertex S
Type B_1: Vertex D, Vertex G
Type B_2: Vertex E, Vertex F
Type B_0: Vertex H, Vertex I

Theorem 01: For any position T that has 1 base and an odd number of triangular hyperedges, the nim-value of position T is 1.

Theorem 02: For any position T that has 1 base, an even number of triangular hyperedges, and all of the vertices of a base are of type B_2, the nim-value of position T is 3.

Theorem 03: For any position T that has 1 base, and an even number of triangular hyperedges, and all of the vertices of a base are of type B_1, the nim-value of position T is 0.

Theorem 04: For any position T that has 1 base, an even number of triangular hyperedges, and position T does not satisfy Theorem 02 nor Theorem 03, the nim-value of position T is 4.

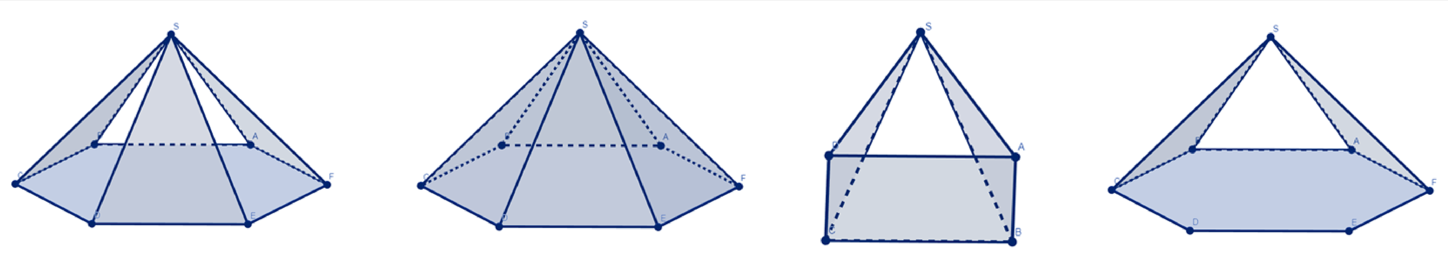


Figure 8 Hypergraphs (Positions) in Theorem 01, Theorem 02, Theorem 03, and Theorem 04,

Special Neither Oddly nor Evenly Uniform Hypergraphs

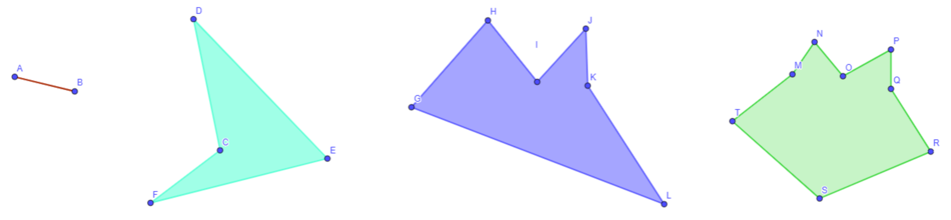


Figure 9 Bases of Special neither Oddly nor Evenly Uniform Hypergraphs

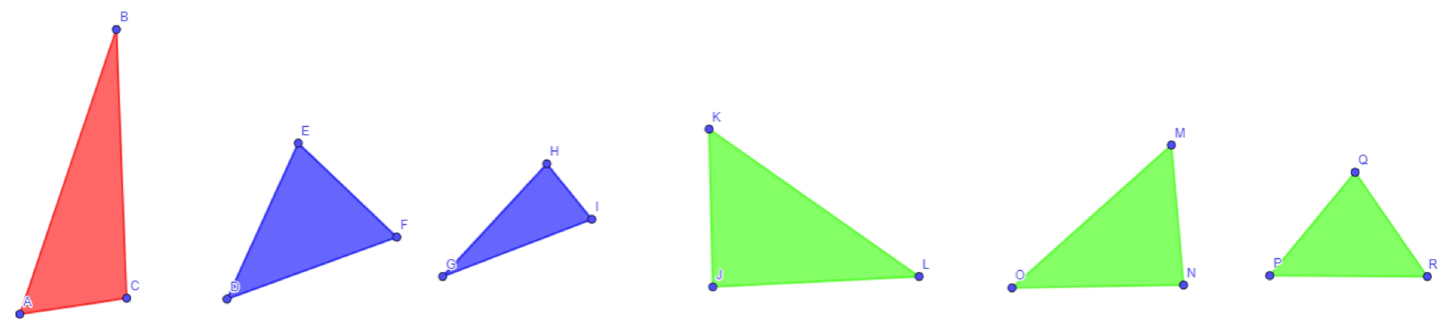


Figure 10 Triangular hyperedges of Special neither Oddly nor Evenly Uniform Hypergraphs

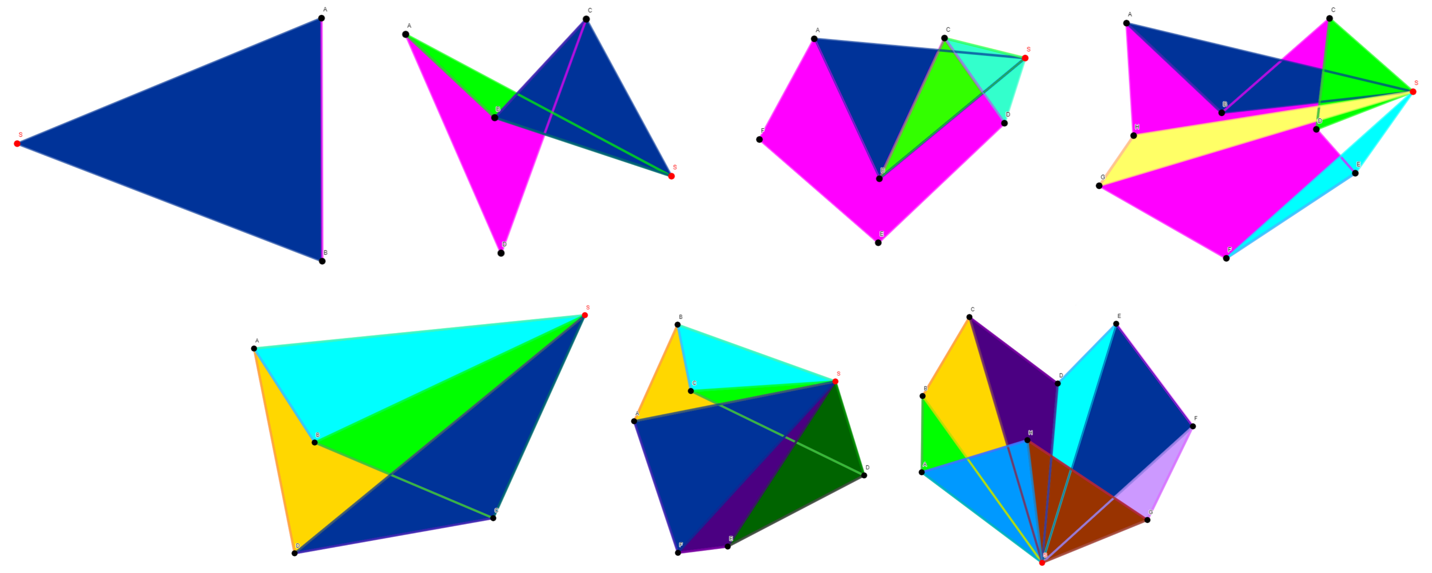


Figure 11 More versions of neither Oddly nor Evenly Uniform Hypergraphs

Theorem 05: For any position T that has 1 base and an odd number of triangular hyperedges, the nim-value of position T is 1.

Theorem 06: For any position T that has 1 base, and an even number of triangular hyperedges and all of the vertices of a base are of type B_2, the nim-value of position T is 2.

Theorem 07: For any position T that has 1 base, and an even number of triangular hyperedges and position T does not satisfy Theorem 06, the nim-value of position T is 3.

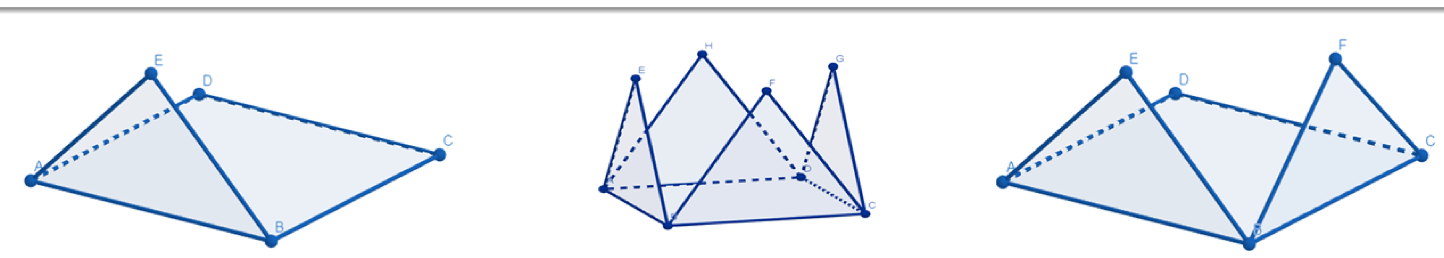


Figure 12 Hypergraphs (Positions) in Theorem 5, Theorem 6, and Theorem 7, respectively

References

[1] Kristen Barnard. Some take-away games on discrete structures. *University of Kentucky, PhD Dissertation.*, 2017.

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