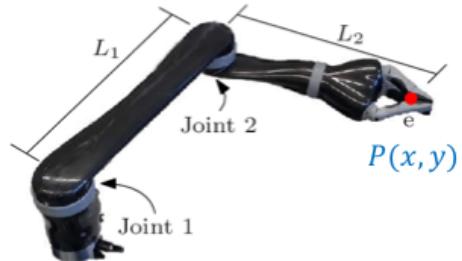


Analytic Inverse Kinematics Approach

- **Goal:** Use geometric properties of a mechanism to find a closed-form analytic solution to the inverse kinematics problem
 - Simple and closed form → super fast to compute
 - Only applicable to simple and specific type of mechanism
 - Difficult to generalize to complex, redundant robot mechanism
 - Requires additional steps to verify multiple solutions

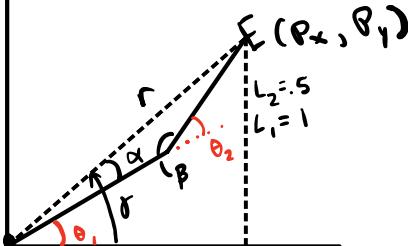
Example: 2R Planar Robot Arm

given a desired position of the end effector $P(x, y)$, determine the joint angles θ_1 and θ_2 .



The Ohio State University

$$\text{Workspace: } \left\{ \left| x^2 + y^2 - \frac{1}{4} \right|, x^2 + y^2 - 1.5^2 \right\}$$



$$r = \sqrt{P_x^2 + P_y^2}$$

$$\alpha = \arccos\left(\frac{r^2 + L_1^2 - L_2^2}{2 \cdot r \cdot L_1}\right)$$

$$\beta = \arccos\left(\frac{L_1^2 + L_2^2 - r^2}{2 \cdot L_1 \cdot L_2}\right)$$

$$\theta_2 = \pi - \beta$$

$$\gamma = \arctan\left(\frac{P_y}{P_x}\right) = \alpha + \theta_1$$

$$\theta_1 = \arctan\left(\frac{P_y}{P_x}\right) - \alpha$$

$$\theta_1 = \arctan\left(\frac{P_y}{P_x}\right) - \arccos\left(\frac{r^2 + L_1^2 - L_2^2}{2 \cdot r \cdot L_1}\right)$$

$$r = \sqrt{P_x^2 + P_y^2}$$

$$\theta_2 = \pi - \arccos\left(\frac{L_1^2 + L_2^2 - r^2}{2 \cdot L_1 \cdot L_2}\right)$$