PRACTICE PROBLEMS

The following information relates to Questions 1–7

Donald Troubadour is a derivatives trader for Southern Shores Investments. The firm seeks arbitrage opportunities in the forward and futures markets using the carry arbitrage model.

Troubadour identifies an arbitrage opportunity relating to a fixed-income futures contract and its underlying bond. Current data on the futures contract and underlying bond are presented in Exhibit 1. The current annual compounded risk-free rate is 0.30%.

Exhibit 1 Current Data for Futures and Underlying Bond

Futures Contract		Underlying Bond	
Quoted futures price	125.00	Quoted bond price	112.00
Conversion factor	0.90	Accrued interest since last coupon payment	0.08
Time remaining to contract expiration	Three months	Accrued interest at futures contract expiration	0.20
Accrued interest over life of futures contract	0.00		

Troubadour next gathers information on three existing positions.

Position 1 (Nikkei 225 Futures Contract):

Troubadour holds a long position in a Nikkei 225 futures contract that has a remaining maturity of three months. The continuously compounded dividend yield on the Nikkei 225 Stock Index is 1.1%, and the current stock index level is 16,080. The continuously compounded annual interest rate is 0.2996%.

Position 2 (Euro/Yen Forward Contract):

One month ago, Troubadour purchased euro/yen forward contracts with three months to expiration at a quoted price of 100.20 (quoted as a percentage of par). The contract notional amount is ¥100,000,000. The current forward price is 100.05, and the current annualized risk-free rate is 0.30%.

Position 3 (JPY/USD Currency Forward Contract):

Troubadour holds a short position in a yen/US dollar forward contract with a notional value of $1,000,000. At contract initiation, the forward rate was ¥112.10 per $1. The forward contract expires in three months. The current spot exchange rate is ¥112.00 per $1, and the annually compounded risk-free rates are –0.20% for the yen and 0.30% for the US dollar. The current quoted price of the forward contract is equal to the no-arbitrage price.

Troubadour next considers an equity forward contract for Texas Steel, Inc. (TSI). Information regarding TSI common shares and a TSI equity forward contract is presented in Exhibit 2.

Exhibit 2 Selected Information for TSI

- The price per share of TSI's common shares is $250.
- The forward price per share for a nine-month TSI equity forward contract is $250.562289.
- Assume annual compounding.

Troubadour takes a short position in the TSI equity forward contract. His supervisor asks, "Under which scenario would our position experience a loss?"

Three months after contract initiation, Troubadour gathers information on TSI and the risk-free rate, which is presented in Exhibit 3.

Exhibit 3 Selected Data on TSI and the Risk-Free Rate

- The price per share of TSI's common shares is $245.
- The risk-free rate is 0.325% (quoted on an annual compounding basis).
- TSI recently announced its regular semiannual dividend of $1.50 per share that will be paid exactly three months before contract expiration.
- The market price of the TSI equity forward contract is equal to the no-arbitrage forward price.

1 Based on Exhibit 1 and assuming annual compounding, the arbitrage profit on the bond futures contract is *closest* to:

A 0.4158.

B 0.5356.

C 0.6195.

2 The current no-arbitrage futures price of the Nikkei 225 futures contract (Position 1) is *closest* to:

A 15,951.81.

B 16,047.86.

C 16,112.21.

3 The value of Position 2 is *closest* to:

A –¥149,925.

B –¥150,000.

C –¥150,075.

4 The value of Position 3 is *closest* to:

A –¥40,020.

- **B** ¥139,913.
- **C** ¥239,963.

5 Based on Exhibit 2, Troubadour should find that an arbitrage opportunity relating to TSI shares is

- **A** not available.
- **B** available based on carry arbitrage.
- **C** available based on reverse carry arbitrage.

6 The *most appropriate* response to Troubadour's supervisor's question regarding the TSI forward contract is:

- **A** a decrease in TSI's share price, all else equal.
- **B** an increase in the risk-free rate, all else equal
- **C** a decrease in the market price of the forward contract, all else equal.

7 Based on Exhibits 2 and 3, and assuming annual compounding, the per share value of Troubadour's short position in the TSI forward contract three months after contract initiation is *closest* to:

- **A** $1.6549.
- **B** $5.1561.
- **C** $6.6549.

The following information relates to Questions 8–16

Sonal Johnson is a risk manager for a bank. She manages the bank's risks using a combination of swaps and forward rate agreements (FRAs).

Johnson prices a three-year Libor-based interest rate swap with annual resets using the present value factors presented in Exhibit 1.

Exhibit 1 Present Value Factors

Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136

Johnson also uses the present value factors in Exhibit 1 to value an interest rate swap that the bank entered into one year ago as the receive-floating party. Selected data for the swap are presented in Exhibit 2. Johnson notes that the current equilibrium two-year fixed swap rate is 1.12%.

Exhibit 2 Selected Data on Fixed for Floating Interest Rate Swap

Swap notional amount	$50,000,000
Original swap term	Three years, with annual resets
Fixed swap rate (since initiation)	3.00%

One of the bank's investments is exposed to movements in the Japanese yen, and Johnson desires to hedge the currency exposure. She prices a one-year fixed-for-fixed currency swap involving yen and US dollars, with a quarterly reset. Johnson uses the interest rate data presented in Exhibit 3 to price the currency swap.

Exhibit 3 Selected Japanese and US Interest Rate Data

Days to Maturity	Yen Spot Interest Rates	US Dollar Spot Interest Rates
90	0.05%	0.20%
180	0.10%	0.40%
270	0.15%	0.55%
360	0.25%	0.70%

Johnson next reviews an equity swap with an annual reset that the bank entered into six months ago as the receive-fixed, pay-equity party. Selected data regarding the equity swap, which is linked to an equity index, are presented in Exhibit 4. At the time of initiation, the underlying equity index was trading at 100.00.

Exhibit 4 Selected Data on Equity Swap

Swap notional amount	$20,000,000
Original swap term	Five years, with annual resets
Fixed swap rate	2.00%

The equity index is currently trading at 103.00, and relevant US spot rates, along with their associated present value factors, are presented in Exhibit 5.

Exhibit 5 Selected US Spot Rates and Present Value Factors

Maturity (years)	Spot Rate	Present Value Factors
0.5	0.40%	0.998004
1.5	1.00%	0.985222
2.5	1.20%	0.970874
3.5	2.00%	0.934579
4.5	2.60%	0.895255

Johnson reviews a 6 × 9 FRA that the bank entered into 90 days ago as the pay-fixed/receive-floating party. Selected data for the FRA are presented in Exhibit 6, and current Libor data are presented in Exhibit 7. Based on her interest rate forecast, Johnson also considers whether the bank should enter into new positions in 1 × 4 and 2 × 5 FRAs.

Exhibit 6 6 × 9 FRA Data

FRA term	6 × 9
FRA rate	0.70%
FRA notional amount	US$20,000,000
FRA settlement terms	Advanced set, advanced settle

Exhibit 7 Current Libor

30-day Libor	0.75%
60-day Libor	0.82%
90-day Libor	0.90%
120-day Libor	0.92%
150-day Libor	0.94%
180-day Libor	0.95%
210-day Libor	0.97%
270-day Libor	1.00%

Three months later, the 6 × 9 FRA in Exhibit 6 reaches expiration, at which time the three-month US dollar Libor is 1.10% and the six-month US dollar Libor is 1.20%. Johnson determines that the appropriate discount rate for the FRA settlement cash flows is 1.10%.

8 Based on Exhibit 1, Johnson should price the three-year Libor-based interest rate swap at a fixed rate *closest* to:

A 0.34%.

B 1.16%.

C 1.19%.

9 From the bank's perspective, using data from Exhibit 1, the current value of the swap described in Exhibit 2 is *closest* to:

A –$2,951,963.

B –$1,849,897.

C –$1,943,000.

10 Based on Exhibit 3, Johnson should determine that the annualized equilibrium fixed swap rate for Japanese yen is *closest* to:

A 0.0624%.

B 0.1375%.

C 0.2496%.

11 From the bank's perspective, using data from Exhibits 4 and 5, the fair value of the equity swap is *closest* to:

A –$1,139,425.

B –$781,323.

C –$181,323.

12 Based on Exhibit 5, the current value of the equity swap described in Exhibit 4 would be zero if the equity index was currently trading the *closest* to:

A 97.30.

B 99.09.

C 100.00.

13 From the bank's perspective, based on Exhibits 6 and 7, the value of the 6 × 9 FRA 90 days after inception is *closest* to:

A $14,817.

B $19,647.

C $29,635.

14 Based on Exhibit 7, the no-arbitrage fixed rate on a new 1 × 4 FRA is *closest* to:

A 0.65%.

B 0.73%.

C 0.98%.

15 Based on Exhibit 7, the fixed rate on a new 2 × 5 FRA is *closest* to:

A 0.61%.

B 1.02%.

C 1.71%.

16 Based on Exhibit 6 and the three-month US dollar Libor at expiration, the payment amount that the bank will receive to settle the 6 × 9 FRA is *closest* to:

A $19,945.

B $24,925.

C $39,781.

SOLUTIONS

1 B is correct.

The no-arbitrage futures price is equal to the following:

$$F_0(T) = \mathrm{FV}_{0,T}(T)[B_0(T + Y) + \mathrm{AI}_0 - \mathrm{PVCI}_{0,T}]$$
$$F_0(T) = (1 + 0.003)^{0.25}(112.00 + 0.08 - 0)$$
$$F_0(T) = (1 + 0.003)^{0.25}(112.08) = 112.1640$$

The adjusted price of the futures contract is equal to the conversion factor multiplied by the quoted futures price:

$$F_0(T) = \mathrm{CF}(T)\mathrm{QF}_0(T)$$
$$F_0(T) = (0.90)(125) = 112.50$$

Adding the accrued interest of 0.20 in three months (futures contract expiration) to the adjusted price of the futures contract gives a total price of 112.70.

This difference means that the futures contract is overpriced by 112.70 – 112.1640 = 0.5360. The available arbitrage profit is the present value of this difference: $0.5360/(1.003)^{0.25} = 0.5356$.

2 B is correct. The no-arbitrage futures price is

$$F_0(T) = S_0 e^{(r_c - \gamma)T}$$
$$F_0(T) = 16{,}080e^{(0.002996 - 0.011)(3/12)} = 16{,}047.68$$

3 A is correct. The value of Troubadour's euro/yen forward position is calculated as

$$V_t(T) = \mathrm{PV}_{t,T}[F_t(T) - F_0(T)]$$
$$V_t(T) = (100.05 - 100.20)/(1 + 0.0030)^{2/12} = -0.149925 \text{ (per ¥100 par value)}$$

Therefore, the value of the Troubadour's forward position is

$$V_t(T) = -\frac{0.149925}{100}(¥100{,}000{,}000) = -¥149{,}925$$

4 C is correct. The current no-arbitrage price of the forward contract is

$$F_t(¥/\$,T) = S_t(¥/\$)\mathrm{FV}_{¥,t,T}(1)/\mathrm{FV}_{\$,t,T}(1)$$
$$F_t(¥/\$,T) = ¥112.00(1 - 0.002)^{0.25}/(1 + 0.003)^{0.25} = ¥111.8602$$

Therefore, the value of Troubadour's position in the ¥/$ forward contract, on a per dollar basis, is

$$V_t(T) = \mathrm{PV}_{¥,t,T}[F_0(¥/\$,T) - F_t(¥/\$,T)]$$
$$= (112.10 - 111.8602)/(1 - 0.002)^{0.25} = ¥0.239963 \text{ per } \$1$$

Troubadour's position is a short position of $1,000,000, so the short position has a positive value of (¥0.239963/$) × $1,000,000 = ¥239,963 because the forward rate has fallen since the contract initiation.

5 A is correct. The carry arbitrage model price of the forward contract is

$$\mathrm{FV}(S_0) = S_0(1 + r)^T = \$250(1 + 0.003)^{0.75} = \$250.562289$$

The market price of the TSI forward contract is \$250.562289. A carry or reverse carry arbitrage opportunity does not exist because the market price of the forward contract is equal to the carry arbitrage model price.

6 B is correct. From the perspective of the long position, the forward value is equal to the present value of the difference in forward prices:

$$V_t(T) = \mathrm{PV}_{t,T}[F_t(T) - F_0(T)],$$

where $F_t(T) = \mathrm{FV}_{t,T}(S_t + \theta_t - \gamma_t)$.

All else equal, an increase in the risk-free rate before contract expiration would cause the forward price, $F_t(T)$, to increase. This increase in the forward price would cause the value of the TSI forward contract, from the perspective of the short, to decrease. Therefore, an increase in the risk-free rate would lead to a loss on the short position in the TSI forward contract.

7 C is correct. The no-arbitrage price of the forward contract, three months after contract initiation, is

$$F_{0.25}(T) = \mathrm{FV}_{0.25,T}(S_{0.25} + \theta_{0.25} - \gamma_{0.25})$$
$$F_{0.25(T)} = [\$245 + 0 - \$1.50/(1 + 0.00325)^{(0.5 - 0.25)}](1 + 0.00325)^{(0.75 - 0.25)} = \$243.8966$$

Therefore, from the perspective of the long, the value of the TSI forward contract is

$$V_{0.25}(T) = \mathrm{PV}_{0.25,T}[F_{0.25}(T) - F_0(T)]$$
$$V_{0.25}(T) = (\$243.8966 - \$250.562289)/(1 + 0.00325)^{0.75 - 0.25} = -\$6.6549$$

Because Troubadour is short the TSI forward contract, the value of his position is a gain of \$6.6549.

8 C is correct. The swap pricing equation is

$$r_{FIX} = \frac{1 - \mathrm{PV}_{0,t_n}(1)}{\sum_{i=1}^{n} \mathrm{PV}_{0,t_i}(1)}$$

That is, the fixed swap rate is equal to 1 minus the final present value factor (in this case, Year 3) divided by the sum of the present values (in this case, the sum of Years 1, 2, and 3). The sum of present values for Years 1, 2, and 3 is calculated as

$$\sum_{i=1}^{n} \mathrm{PV}_{0,t_i}(1) = 0.990099 + 0.977876 + 0.965136 = 2.933111$$

Thus, the fixed-swap rate is calculated as

$$r_{FIX} = \frac{1 - 0.965136}{2.933111} = 0.01189 \text{ or } 1.19\%$$

9 B is correct. The value of a swap from the perspective of the receive-fixed party is calculated as

$$V = NA(FS_0 - FS_t)\sum_{i=1}^{n'} \mathrm{PV}_{t,t_i}$$

The swap has two years remaining until expiration. The sum of the present values for Years 1 and 2 is

$$\sum_{i=1}^{n'} \text{PV}_{t,t_i} = 0.990099 + 0.977876 = 1.967975$$

Given the current equilibrium two-year swap rate of 1.00% and the fixed swap rate at initiation of 3.00%, the swap value per dollar notional is calculated as

$$V = (0.03 - 0.0112)1.967975 = 0.036998$$

The current value of the swap, from the perspective of the receive-fixed party, is \$50,000,000 × 0.036998 = \$1,849,897.

From the perspective of the bank, as the receive-floating party, the value of the swap is –\$1,849,897.

10 C is correct. The equilibrium swap fixed rate for yen is calculated as

$$\hat{r}_{FIX,JPY} = \frac{1 - \text{PV}_{0,t_4,JPY}(1)}{\sum_{i=1}^{4} PV_{0,t_4,JPY}(1)}$$

The yen present value factors are calculated as

$$\text{PV}_{0,t_i}(1) = \frac{1}{1 + r_{Spot_i}\left(\frac{NAD_i}{NTD}\right)}$$

90-day PV factor = 1/[1 + 0.0005(90/360)] = 0.999875.
180-day PV factor = 1/[1 + 0.0010(180/360)] = 0.999500.
270-day PV factor = 1/[1 + 0.0015(270/360)] = 0.998876.
360-day PV factor = 1/[1 + 0.0025(360/360)] = 0.997506.

Sum of present value factors = 3.995757.

Therefore, the yen periodic rate is calculated as

$$\hat{r}_{FIX,JPY} = \frac{1 - 0.997506}{3.995757} = 0.000624 \text{ or } 0.0624\%$$

The annualized rate is (360/90) times the periodic rate of 0.0624%, or 0.2496%.

11 B is correct. The value of an equity swap is calculated as

$$V_t = \text{FB}_t(C_0) - \left(\frac{S_t}{S_{t-}}\right)\text{NA}_E$$

The swap was initiated six months ago, so the first reset has not yet passed; thus, there are five remaining cash flows for this equity swap. The fair value of the swap is determined by comparing the present value of the implied fixed-rate bond with the return on the equity index. The fixed swap rate of 2.00%, the swap notional amount of \$20,000,000, and the present value factors in Exhibit 5 result in a present value of the implied fixed-rate bond's cash flows of \$19,818,677:

Date (in years)	PV Factors	Fixed Cash Flow	PV (fixed cash flow)
0.5	0.998004 or 1/[1 + 0.0040(180/360)]	$400,000	$399,202
1.5	0.985222 or 1/[1 + 0.0100(540/360)]	$400,000	$394,089
2.5	0.970874 or 1/[1 + 0.0120(900/360)]	$400,000	$388,350
3.5	0.934579 or 1/[1 + 0.0200(1,260/360)]	$400,000	$373,832
4.5	0.895255 or 1/[1 + 0.0260(1,620/360)]	$20,400,000	$18,263,205
Total			$19,818,677

The value of the equity leg of the swap is calculated as (103/100)($20,000,000) = $20,600,000.

Therefore, the fair value of the equity swap, from the perspective of the bank (receive-fixed, pay-equity party) is calculated as

V_t = $19,818,677 - $20,600,000 = –781,323

12 B is correct. The equity index level at which the swap's fair value would be zero can be calculated by setting the swap valuation formula equal to zero and solving for S_t:

$$0 = \mathrm{FB}_t(C_0) - \left(\frac{S_t}{S_{t-}}\right)\mathrm{NA}_E$$

The value of the fixed leg of the swap has a present value of $19,818,677, or 99.0934% of par value:

Date (years)	PV Factors	Fixed Cash Flow	PV (fixed cash flow)
0.5	0.998004	$400,000	$399,202
1.5	0.985222	$400,000	$394,089
2.5	0.970874	$400,000	$388,350
3.5	0.934579	$400,000	$373,832
4.5	0.895255	$20,400,000	$18,263,205
Total			$19,818,677

Treating the swap notional value as par value and substituting the present value of the fixed leg and S_0 into the equation yields

$$0 = 99.0934 - \left(\frac{S_t}{100}\right)100$$

Solving for S_t yields

$$S_t = 99.0934$$

13 A is correct. The current value of the 6 × 9 FRA is calculated as

$$V_g(0,h,m) = \{[\mathrm{FRA}(g,h-g,m) - \mathrm{FRA}(0,h,m)]t_m\}/[1 + D_g(h+m-g)t_{h+m-g}]$$

The 6 × 9 FRA expires six months after initiation. The bank entered into the FRA 90 days ago; thus, the FRA will expire in 90 days. To value the FRA, the first step is to compute the new FRA rate, which is the rate on Day 90 of an FRA that expires in 90 days in which the underlying is the 90-day Libor, or FRA(90,90,90):

$$\text{FRA}(g,h-g,m) = \{[1 + L_g(h-g+m)t_{h-g+m}]/[1 + L_0(h-g)t_{h-g}] - 1\}/t_m$$

$$\text{FRA}(90,90,90) = \{[1 + L_{90}(180-90+90)(180/360)]/[1 + L_{90}(180-90)(90/360)] - 1\}/(90/360)$$

$$\text{FRA}(90,90,90) = \{[1 + L_{90}(180)(180/360)]/[1 + L_{90}(90)(90/360)] - 1\}/(90/360)$$

Exhibit 7 indicates that $L_{90}(180) = 0.95\%$ and $L_{90}(90) = 0.90\%$, so

$$\text{FRA}(90,90,90) = \{[1 + 0.0095(180/360)]/[1 + 0.0090(90/360)] - 1\}/(90/360)$$

$$\text{FRA}(90,90,90) = [(1.00475/1.00225) - 1](4) = 0.009978, \text{ or } 0.9978\%$$

Therefore, given the FRA rate at initiation of 0.70% and notional principal of \$20 million from Exhibit 1, the current value of the forward contract is calculated as

$$V_g(0,h,m) = V_{90}(0,180,90)$$

$$V_{90}(0,180,90) = \$20{,}000{,}000[(0.009978 - 0.0070)(90/360)]/[1 + 0.0095(180/360)].$$

$$V_{90}(0,180,90) = \$14{,}887.75/1.00475 = \$14{,}817.37.$$

14 C is correct. The no-arbitrage fixed rate on the 1 × 4 FRA is calculated as

$$\text{FRA}(0,h,m) = \{[1 + L_0(h+m)t_{h+m}]/[1 + L_0(h)t_h] - 1\}/t_m$$

For a 1 × 4 FRA, the two rates needed to compute the no-arbitrage FRA fixed rate are $L(30) = 0.75\%$ and $L(120) = 0.92\%$. Therefore, the no-arbitrage fixed rate on the 1 × 4 FRA rate is calculated as

$$\text{FRA}(0,30,90) = \{[1 + 0.0092(120/360)]/[1 + 0.0075(30/360)] - 1\}/(90/360).$$

$$\text{FRA}(0,30,90) = [(1.003066/1.000625) - 1]4 = 0.009761, \text{ or } 0.98\% \text{ rounded}$$

15 B is correct. The fixed rate on the 2 × 5 FRA is calculated as

$$\text{FRA}(0,h,m) = \{[1 + L_0(h+m)t_{h+m}]/[1 + L_0(h)t_h] - 1\}/t_m$$

For a 2 × 5 FRA, the two rates needed to compute the no-arbitrage FRA fixed rate are $L(60) = 0.82\%$ and $L(150) = 0.94\%$. Therefore, the no-arbitrage fixed rate on the 2 × 5 FRA rate is calculated as

$$\text{FRA}(0,60,90) = \{[1 + 0.0094(150/360)]/[1 + 0.0082(60/360)] - 1\}/(90/360)$$

$$\text{FRA}(0,60,90) = [(1.003917/1.001367) - 1]4 = 0.010186, \text{ or } 1.02\% \text{ rounded}$$

16 A is correct. Given a three-month US dollar Libor of 1.10% at expiration, the settlement amount for the bank as the receive-floating party is calculated as

$$\text{Settlement amount (receive floating)} = \text{NA}\{[L_h(m) - \text{FRA}(0,h,m)]t_m\}/[1 + D_h(m)t_m]$$

$$\text{Settlement amount (receive floating)} = \$20{,}000{,}000[(0.011 - 0.0070)(90/360)]/[1 + 0.011(90/360)]$$

$$\text{Settlement amount (receive floating)} = \$20{,}000/1.00275 = \$19{,}945.15$$

Therefore, the bank will receive \$19,945 (rounded) as the receive-floating party.

PRACTICE PROBLEMS

The following information relates to Questions 1–9

Bruno Sousa has been hired recently to work with senior analyst Camila Rocha. Rocha gives him three option valuation tasks.

Alpha Company

Sousa's first task is to illustrate how to value a call option on Alpha Company with a one-period binomial option pricing model. It is a non-dividend-paying stock, and the inputs are as follows.

- The current stock price is 50, and the call option exercise price is 50.
- In one period, the stock price will either rise to 56 or decline to 46.
- The risk-free rate of return is 5% per period.

Based on the model, Rocha asks Sousa to estimate the hedge ratio, the risk-neutral probability of an up move, and the price of the call option. In the illustration, Sousa is also asked to describe related arbitrage positions to use if the call option is overpriced relative to the model.

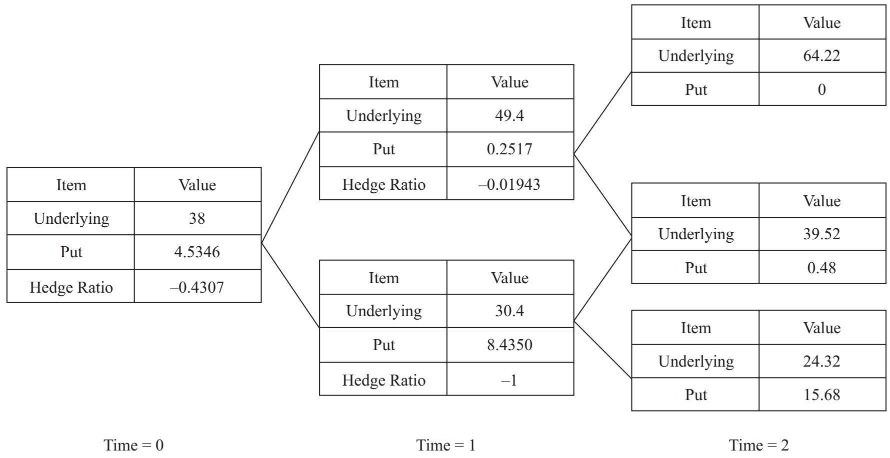
Beta Company

Next, Sousa uses the two-period binomial model to estimate the value of a European-style call option on Beta Company's common shares. The inputs are as follows.

- The current stock price is 38, and the call option exercise price is 40.
- The up factor (u) is 1.300, and the down factor (d) is 0.800.
- The risk-free rate of return is 3% per period.

Sousa then analyzes a put option on the same stock. All of the inputs, including the exercise price, are the same as for the call option. He estimates that the value of a European-style put option is 4.53. Exhibit 1 summarizes his analysis. Sousa next must determine whether an American-style put option would have the same value.

Exhibit 1 Two-Period Binomial European-Style Put Option on Beta Company



Sousa makes two statements with regard to the valuation of a European-style option under the expectations approach.

Statement 1 The calculation involves discounting at the risk-free rate.

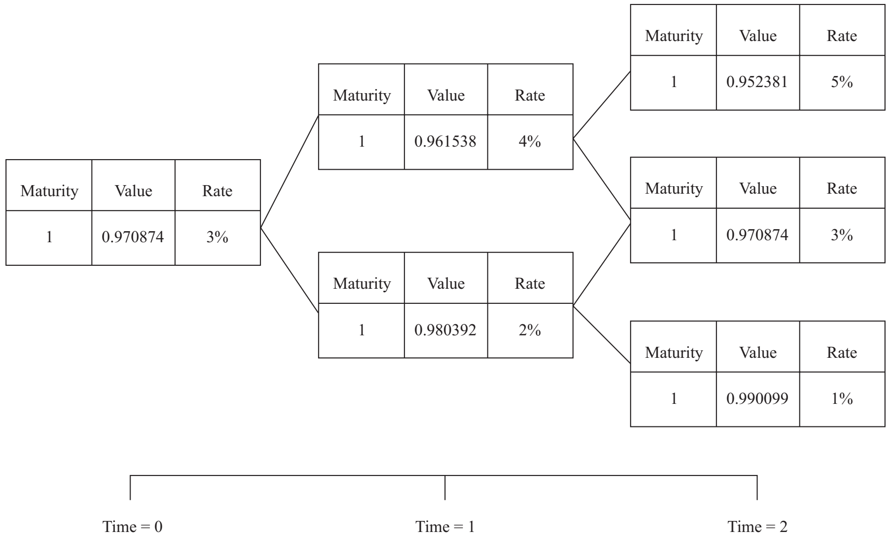
Statement 2 The calculation uses risk-neutral probabilities instead of true probabilities.

Rocha asks Sousa whether it is ever profitable to exercise American options prior to maturity. Sousa answers, "I can think of two possible cases. The first case is the early exercise of an American call option on a dividend-paying stock. The second case is the early exercise of an American put option."

Interest Rate Option

The final option valuation task involves an interest rate option. Sousa must value a two-year, European-style call option on a one-year spot rate. The notional value of the option is 1 million, and the exercise rate is 2.75%. The risk-neutral probability of an up move is 0.50. The current and expected one-year interest rates are shown in Exhibit 2, along with the values of a one-year zero-coupon bond of 1 notional value for each interest rate.

Exhibit 2 Two-Year Interest Rate Lattice for an Interest Rate Option



Rocha asks Sousa why the value of a similar in-the-money interest rate call option decreases if the exercise price is higher. Sousa provides two reasons.

Reason 1 The exercise value of the call option is lower.

Reason 2 The risk-neutral probabilities are changed.

1 The optimal hedge ratio for the Alpha Company call option using the one-period binomial model is *closest* to:

A 0.60.

B 0.67.

C 1.67.

2 The risk-neutral probability of the up move for the Alpha Company stock is *closest* to:

A 0.06.

B 0.40.

C 0.65.

3 The value of the Alpha Company call option is *closest* to:

A 3.71.

B 5.71.

C 6.19.

4 For the Alpha Company option, the positions to take advantage of the arbitrage opportunity are to write the call and:

A short shares of Alpha stock and lend.

B buy shares of Alpha stock and borrow.

C short shares of Alpha stock and borrow.

5 The value of the European-style call option on Beta Company shares is *closest* to:

A 4.83.

B 5.12.

C 7.61.

6 The value of the American-style put option on Beta Company shares is *closest* to:

A 4.53.

B 5.15.

C 9.32.

7 Which of Sousa's statements about binomial models is correct?

A Statement 1 only

B Statement 2 only

C Both Statement 1 and Statement 2

8 Based on Exhibit 2 and the parameters used by Sousa, the value of the interest rate option is *closest* to:

A 5,251.

B 6,236.

C 6,429.

9 Which of Sousa's reasons for the decrease in the value of the interest rate option is correct?

A Reason 1 only

B Reason 2 only

C Both Reason 1 and Reason 2

The following information relates to Questions 10–17

Trident Advisory Group manages assets for high-net-worth individuals and family trusts.

Alice Lee, chief investment officer, is meeting with a client, Noah Solomon, to discuss risk management strategies for his portfolio. Solomon is concerned about recent volatility and has asked Lee to explain options valuation and the use of options in risk management.

Options on Stock

Lee uses the BSM model to price TCB, which is one of Solomon's holdings. Exhibit 1 provides the current stock price (S), exercise price (X), risk-free interest rate (r), volatility (σ), and time to expiration (T) in years as well as selected outputs from the BSM model. TCB does not pay a dividend.

Exhibit 1 BSM Model for European Options on TCB

BSM Inputs

S	*X*	*r*	Σ	*T*
$57.03	55	0.22%	32%	0.25

(continued)

Exhibit 1 (Continued)

BSM Outputs

d_1	$N(d_1)$	d_2	$N(d_2)$	BSM Call Price	BSM Put Price
0.3100	0.6217	0.1500	0.5596	\$4.695	\$2.634

Options on Futures

The Black model valuation and selected outputs for options on another of Solomon's holdings, the GPX 500 Index (GPX), are shown in Exhibit 2. The spot index level for the GPX is 187.95, and the index is assumed to pay a continuous dividend at a rate of 2.2% (δ) over the life of the options being valued, which expire in 0.36 years. A futures contract on the GPX also expiring in 0.36 years is currently priced at 186.73.

Exhibit 2 Black Model for European Options on the GPX Index

Black Model Inputs

GPX Index	X	r	σ	T	δ Yield
187.95	180	0.39%	24%	0.36	2.2%

Black Model Call Value	Black Model Put Value	Market Call Price	Market Put Price
\$14.2089	\$7.4890	\$14.26	\$7.20

Option Greeks

Delta (call)	Delta (put)	Gamma (call or put)	Theta (call) daily	Rho (call) per %	Vega per % (call or put)
0.6232	–0.3689	0.0139	–0.0327	0.3705	0.4231

After reviewing Exhibit 2, Solomon asks Lee which option Greek letter best describes the changes in an option's value as time to expiration declines.

Solomon observes that the market price of the put option in Exhibit 2 is \$7.20. Lee responds that she used the historical volatility of the GPX of 24% as an input to the BSM model, and she explains the implications for the implied volatility for the GPX.

Options on Interest Rates

Solomon forecasts the three-month Libor will exceed 0.85% in six months and is considering using options to reduce the risk of rising rates. He asks Lee to value an interest rate call with a strike price of 0.85%. The current three-month Libor is 0.60%, and an FRA for a three-month Libor loan beginning in six months is currently 0.75%.

Hedging Strategy for the Equity Index

Solomon's portfolio currently holds 10,000 shares of an exchange-traded fund (ETF) that tracks the GPX. He is worried the index will decline. He remarks to Lee, "You have told me how the BSM model can provide useful information for reducing the risk of my GPX position." Lee suggests a delta hedge as a strategy to protect against small moves in the GPX Index.

Lee also indicates that a long position in puts could be used to hedge larger moves in the GPX. She notes that although hedging with either puts or calls can result in a delta-neutral position, they would need to consider the resulting gamma.

10 Based on Exhibit 1 and the BSM valuation approach, the initial portfolio required to replicate the long call option payoff is:

- **A** long 0.3100 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.
- **B** long 0.6217 shares of TCB stock and short 0.1500 shares of a zero-coupon bond.
- **C** long 0.6217 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.

11 To determine the long put option value on TCB stock in Exhibit 1, the correct BSM valuation approach is to compute:

- **A** 0.4404 times the present value of the exercise price minus 0.6217 times the price of TCB stock.
- **B** 0.4404 times the present value of the exercise price minus 0.3783 times the price of TCB stock.
- **C** 0.5596 times the present value of the exercise price minus 0.6217 times the price of TCB stock.

12 What are the correct spot value (S) and the risk-free rate (r) that Lee should use as inputs for the Black model?

- **A** 186.73 and 0.39%, respectively
- **B** 186.73 and 2.20%, respectively
- **C** 187.95 and 2.20%, respectively

13 Which of the following is the correct answer to Solomon's question regarding the option Greek letter?

- **A** Vega
- **B** Theta
- **C** Gamma

14 Based on Solomon's observation about the model price and market price for the put option in Exhibit 2, the implied volatility for the GPX is *most likely*:

- **A** less than the historical volatility.
- **B** equal to the historical volatility.
- **C** greater than the historical volatility.

15 The valuation inputs used by Lee to price a call reflecting Solomon's interest rate views should include an underlying FRA rate of:

- **A** 0.60% with six months to expiration.
- **B** 0.75% with nine months to expiration.
- **C** 0.75% with six months to expiration.

16 The strategy suggested by Lee for hedging small moves in Solomon's ETF position would *most likely* involve:

A selling put options.

B selling call options.

C buying call options.

17 Lee's put-based hedge strategy for Solomon's ETF position would *most likely* result in a portfolio gamma that is:

A negative.

B neutral.

C positive.

SOLUTIONS

1 A is correct. The hedge ratio requires the underlying stock and call option values for the up move and down move. $S^+ = 56$, and $S^- = 46$. $c^+ = \text{Max}(0,S^+ - X) = \text{Max}(0,56 - 50) = 6$, and $c^- = \text{Max}(0,S^- - X) = \text{Max}(0,46 - 50) = 0$. The hedge ratio is

$$h = \frac{c^+ - c^-}{S^+ - S^-} = \frac{6 - 0}{56 - 46} = \frac{6}{10} = 0.60$$

2 C is correct. For this approach, the risk-free rate is $r = 0.05$, the up factor is $u = S^+/S = 56/50 = 1.12$, and the down factor is $d = S^-/S = 46/50 = 0.92$. The risk-neutral probability of an up move is

$$\pi = [\text{FV}(1) - d]/(u - d) = (1 + r - d]/(u - d)$$
$$\pi = (1 + 0.05 - 0.92)/(1.12 - 0.92) = 0.13/0.20 = 0.65$$

3 A is correct. The call option can be estimated using the no-arbitrage approach or the expectations approach. With the no-arbitrage approach, the value of the call option is

$$c = hS + \text{PV}(-hS^- + c^-).$$
$$h = (c^+ - c^-)/(S^+ - S^-) = (6 - 0)/(56 - 46) = 0.60.$$
$$c = (0.60 \times 50) + (1/1.05) \times [(-0.60 \times 46) + 0].$$
$$c = 30 - [(1/1.05) \times 27.6] = 30 - 26.286 = 3.714.$$

Using the expectations approach, the risk-free rate is $r = 0.05$, the up factor is $u = S^+/S = 56/50 = 1.12$, and the down factor is $d = S^-/S = 46/50 = 0.92$. The value of the call option is

$$c = \text{PV} \times [\pi c^+ + (1 - \pi)c^-].$$
$$\pi = [\text{FV}(1) - d]/(u - d) = (1.05 - 0.92)/(1.12 - 0.92) = 0.65.$$
$$c = (1/1.05) \times [0.65(6) + (1 - 0.65)(0)] = (1/1.05)(3.9) = 3.714.$$

Both approaches are logically consistent and yield identical values.

4 B is correct. You should sell (write) the overpriced call option and then go long (buy) the replicating portfolio for a call option. The replicating portfolio for a call option is to buy h shares of the stock and borrow the present value of $(hS^- - c^-)$.

$$c = hS + \text{PV}(-hS^- + c^-).$$
$$h = (c^+ - c^-)/(S^+ - S^-) = (6 - 0)/(56 - 46) = 0.60.$$

For the example in this case, the value of the call option is 3.714. If the option is overpriced at, say, 4.50, you short the option and have a cash flow at Time 0 of +4.50. You buy the replicating portfolio of 0.60 shares at 50 per share (giving you a cash flow of –30) and borrow (1/1.05) × [(0.60 × 46) – 0] = (1/1.05) × 27.6 = 26.287. Your cash flow for buying the replicating portfolio is –30 + 26.287 = –3.713. Your net cash flow at Time 0 is + 4.50 – 3.713 = 0.787. Your net cash flow at Time 1 for either the up move or down move is zero. You have made an arbitrage profit of 0.787.

In tabular form, the cash flows are as follows:

Transaction	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
Sell the call option	4.50	0	–6.00
Buy h shares	–0.6 × 50 = –30	0.6 × 46 = 27.6	0.6 × 56 = 33.6
Borrow $-\text{PV}(-hS^- + c^-)$	–(1/1.05) × [(–0.6 × 46) + 0] = 26.287	–0.6 × 46 = –27.6	–0.6 × 46 = –27.6
Net cash flow	0.787	0	0

5 A is correct. Using the expectations approach, the risk-neutral probability of an up move is

$$\pi = [\text{FV}(1) - d]/(u - d) = (1.03 - 0.800)/(1.300 - 0.800) = 0.46.$$

The terminal value calculations for the exercise values at Time Step 2 are

$$c^{++} = \text{Max}(0,u^2S - X) = \text{Max}[0,1.30^2(38) - 40] = \text{Max}(0,24.22) = 24.22.$$
$$c^{-+} = \text{Max}(0,udS - X) = \text{Max}[0,1.30(0.80)(38) - 40] = \text{Max}(0,-0.48) = 0.$$
$$c^{--} = \text{Max}(0,d^2S - X) = \text{Max}[0,0.80^2(38) - 40] = \text{Max}(0,-15.68) = 0.$$

Discounting back for two years, the value of the call option at Time Step 0 is

$$c = \text{PV}[\pi^2c^{++} + 2\pi(1 - \pi)c^{-+} + (1 - \pi)^2c^{--}].$$
$$c = [1/(1.03)]^2[0.46^2(24.22) + 2(0.46)(0.54)(0) + 0.54^2(0)].$$
$$c = [1/(1.03)]^2[5.1250] = 4.8308.$$

6 B is correct. Using the expectations approach, the risk-neutral probability of an up move is

$$\pi = [\text{FV}(1) - d]/(u - d) = (1.03 - 0.800)/(1.300 - 0.800) = 0.46.$$

An American-style put can be exercised early. At Time Step 1, for the up move, p^+ is 0.2517 and the put is out of the money and should not be exercised early ($X < S$, 40 < 49.4). However, at Time Step 1, p^- is 8.4350 and the put is in the money by 9.60 ($X - S$ = 40 – 30.40). So, the put is exercised early, and the value of early exercise (9.60) replaces the value of not exercising early (8.4350) in the binomial tree. The value of the put at Time Step 0 is now

$$p = \text{PV}[\pi p^+ + (1 - \pi)p^-] = [1/(1.03)][0.46(0.2517) + 0.54(9.60)] = 5.1454.$$

Following is a supplementary note regarding Exhibit 1.

The values in Exhibit 1 are calculated as follows.

At Time Step 2:

$$p^{++} = \text{Max}(0,X - u^2S) = \text{Max}[0,40 - 1.300^2(38)] = \text{Max}(0,40 - 64.22) = 0.$$
$$p^{-+} = \text{Max}(0,X - udS) = \text{Max}[0,40 - 1.300(0.800)(38)] = \text{Max}(0,40 - 39.52) = 0.48.$$
$$\text{p}^{--} = \text{Max}(0,X - d^2S) = \text{Max}[0,40 - 0.800^2(38)] = \text{Max}(0,40 - 24.32) = 15.68.$$

At Time Step 1:

$$p^+ = \text{PV}[\pi p^{++} + (1 - \pi)p^{-+}] = [1/(1.03)][0.46(0) + 0.54(0.48)] = 0.2517.$$
$$p^- = \text{PV}[\pi p^{-+} + (1 - \pi)p^{--}] = [1/(1.03)][0.46(0.48) + 0.54(15.68)] = 8.4350.$$

At Time Step 0:

$$p = \text{PV}[\pi p^+ + (1 - \pi)p^-] = [1/(1.03)][0.46(0.2517) + 0.54(8.4350)] = 4.5346.$$

7 C is correct. Both statements are correct. The expected future payoff is calculated using risk-neutral probabilities, and the expected payoff is discounted at the risk-free rate.

8 C is correct. Using the expectations approach, per 1 of notional value, the values of the call option at Time Step 2 are

$$c^{++} = \text{Max}(0,S^{++} - X) = \text{Max}(0,0.050 - 0.0275) = 0.0225.$$
$$c^{+-} = \text{Max}(0,S^{+-} - X) = \text{Max}(0,0.030 - 0.0275) = 0.0025.$$
$$c^{--} = \text{Max}(0,S^{--} - X) = \text{Max}(0,0.010 - 0.0275) = 0.$$

At Time Step 1, the call values are

$$c^{+} = \text{PV}[\pi c^{++} + (1 - \pi)c^{+-}] .$$
$$c^{+} = 0.961538[0.50(0.0225) + (1 - 0.50)(0.0025)] = 0.012019.$$
$$c^{-} = \text{PV}[\pi c^{+-} + (1 - \pi)c^{--}].$$
$$c^{-} = 0.980392[0.50(0.0025) + (1 - 0.50)(0)] = 0.001225.$$

At Time Step 0, the call option value is

$$c = \text{PV}[\pi c^{+} + (1 - \pi)c^{-}].$$
$$c = 0.970874[0.50(0.012019) + (1 - 0.50)(0.001225)] = 0.006429.$$

The value of the call option is this amount multiplied by the notional value, or 0.006429 × 1,000,000 = 6,429.

9 A is correct. Reason 1 is correct: A higher exercise price does lower the exercise value (payoff) at Time 2. Reason 2 is not correct because the risk-neutral probabilities are based on the paths that interest rates take, which are determined by the market and not the details of a particular option contract.

10 C is correct. The no-arbitrage approach to creating a call option involves buying Delta = $N(d_1)$ = 0.6217 shares of the underlying stock and financing with $-N(d_2)$ = −0.5596 shares of a risk-free bond priced at $\exp(-rt)(X) = \exp(-0.0022 \times 0.25)(55)$ = \$54.97 per bond. Note that the value of this replicating portfolio is $n_S S + n_B B = 0.6217(57.03) - 0.5596(54.97)$ = \$4.6943 (the value of the call option with slight rounding error).

11 B is correct. The formula for the BSM price of a put option is $p = e^{-rt}XN(-d_2) - SN(-d_1)$. $N(-d_1) = 1 - N(d_1) = 1 - 0.6217 = 0.3783$, and $N(-d_2) = 1 - N(d_2) = 1 - 0.5596 = 0.4404$.

Note that the BSM model can be represented as a portfolio of the stock ($n_S S$) and zero-coupon bonds ($n_B B$). For a put, the number of shares is $n_S = -\text{N}(-d_1) < 0$ and the number of bonds is $n_B = -N(d_2) > 0$. The value of the replicating portfolio is $n_S S + n_B B = -0.3783(57.03) + 0.4404(54.97)$ = \$2.6343 (the value of the put option with slight rounding error). B is a risk-free bond priced at $\exp(-rt)(X) = \exp(-0.0022 \times 0.25)(55)$ = \$54.97.

12 A is correct. Black's model to value a call option on a futures contract is $c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$. The underlying F_0 is the futures price (186.73). The correct discount rate is the risk-free rate, r = 0.39%.

13 B is correct. Lee is pointing out the option price's sensitivity to small changes in time. In the BSM approach, option price sensitivity to changes in time is given by the option Greek theta.

14 A is correct. The put is priced at \$7.4890 by the BSM model when using the historical volatility input of 24%. The market price is \$7.20. The BSM model overpricing suggests the implied volatility of the put must be lower than 24%.

15 C is correct. Solomon's forecast is for the three-month Libor to exceed 0.85% in six months. The correct option valuation inputs use the six-month FRA rate as the underlying, which currently has a rate of 0.75%.

16 B is correct because selling call options creates a short position in the ETF that would hedge his current long position in the ETF.

Exhibit 2 could also be used to answer the question. Solomon owns 10,000 shares of the GPX, each with a delta of +1; by definition, his portfolio delta is +10,000. A delta hedge could be implemented by selling enough calls to make the portfolio delta neutral:

$$N_H = -\frac{\text{Portfolio delta}}{\text{Delta}_H} = -\frac{+10{,}000}{+0.6232} = -16{,}046 \text{ calls.}$$

17 C is correct. Because the gamma of the stock position is 0 and the put gamma is always non-negative, adding a long position in put options would most likely result in a positive portfolio gamma.

Gamma is the change in delta from a small change in the stock's value. A stock position always has a delta of +1. Because the delta does not change, gamma equals 0.

The gamma of a call equals the gamma of a similar put, which can be proven using put–call parity.