Case study Hop Phat

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Class Group Number:	**Group 3**
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1. Developing the linear programing

Step 1: Identify the problems and understanding the optimisation

Information given by the case study				
Hours	**Holding Cost/Bag**	**Raw Material Cost**	**Beginning Inventory**	**Beginning worker**
4	3	$ 15.00	500	20

Standard Working (hours/month)	**Standard Wages ($/month)**
160	$ 1,500.00
Maximum OT	**Over Time Wage ($/hour/worker)**
20.00	$ 13.00

Cost of Hired Worker ($)	**Cost of Fired worker ($)**
$ 1,600.00	$ 2,000.00

Through the case study, Hop Phat's initial **inventory of 500**, with **20 workers** in the Company. The wage for each employee is **$1500 per month** for **160 hours of work**, with a maximum overtime per worker **is 20 hours**, and overtime prices are **$13 per hour**. However, to meet the demand, the Company must change the number of workers; hence, the hiring price is **$1600 per worker**, and firing is **$2000 per worker**. This report will analyse and calculate the optimal solution for the forecast demand.

The Company wants to determine the labour schedule to produce a unit of bag that meets the demand forecast while not exceeding the required work hours per worker. This will be based on finding the optimal solution for minimising the cost of production scheduling.

Step 2: Define the decision variable

W_t: Number of available workers in month t

X_t: Number of bag product in month t

H_t: Number of workers hiring in month t

F_t: Number of worker firing in month t

I_t: Ending inventory in month t

With *t:* The month

Step 3: Objective function

$$\boxed{\text{Minimise cost } Z = \sum_{t=1}^{4} 1500 \times w_t + 15 \times X_t + 3 \times I_t + 1600 \times H_t + 2000 \times F_t + 13 \times O_t}$$

Step 4: Constrains

- Production capacity

$$\boxed{X_t \leq [(160 \times W_t) + OT_t]/4}$$

This constraint defines the maximum number of bags produced in a month. Taking into account the standard working hours (160 hours per worker) and the overtime hours (OT_t) for all the workers (W_t). The total available hours are then divided by 4, assuming that it takes 4 hours to produce one bag. This ensures the production plan does not exceed the available working hours.

- Inventory balance

$$\boxed{I_{t-1} + X_t - \mathit{Forcast\ demand} = I_t}$$

This equation balances the inventory from month to month. It states that the ending inventory for the previous month (I_{t-1}), plus the production in the current month (X_t), minus the forecast demand, should equal the ending inventory for the current month (I_t) This ensures continuity in inventory levels and helps in planning for sufficient stock to meet demand.

- Workforce balance

$$\boxed{W_{t-1} + H_t - F_t = W_t}$$

This constraint ensures the number of workers available each month is accurately accounted for. It states that the total workforce in the current month (W_t) is equal to the previous month's workforce (W_{t-1}) plus hires worker (H_t) minus firings worker (F_t). This helps in planning and adjusting the workforce according to the production neeeds.

- Overtime limit

$$\boxed{OT_t \leq 20 \times W_t}$$

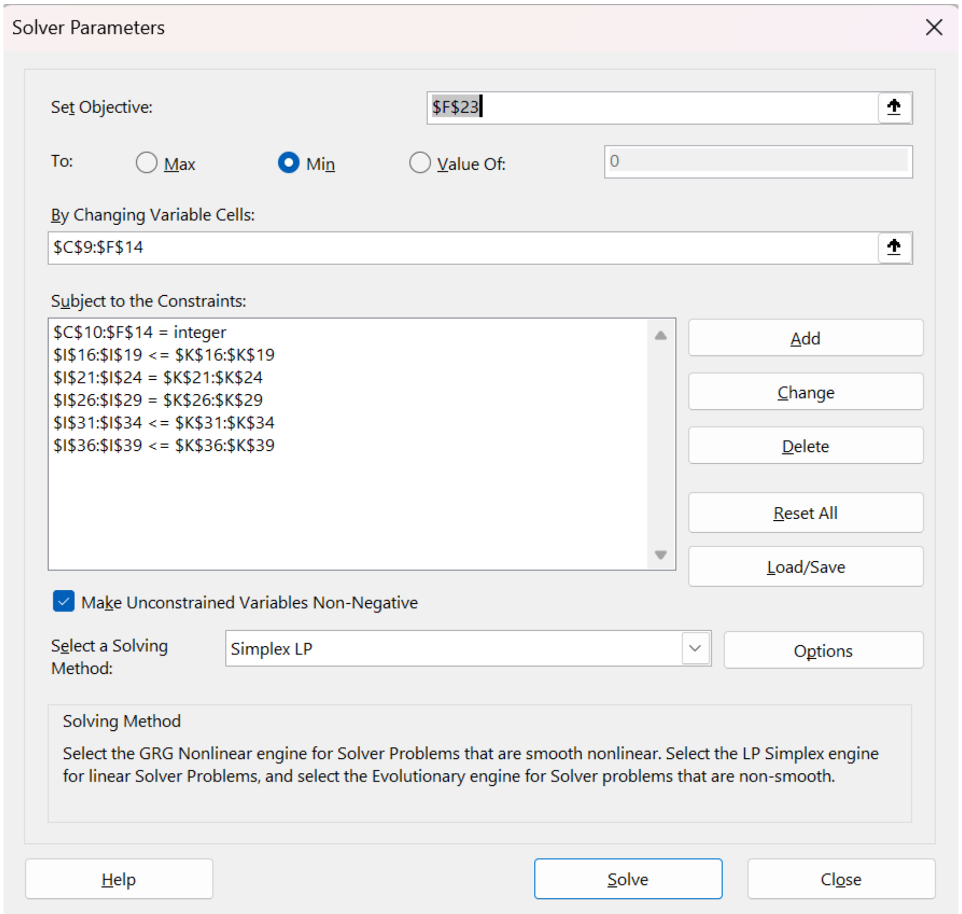
This function is to limit the total overtime hours in any given month. It ensures that the overtime (OT_t) does not exceed 20 hours per worker (W_t). This constrains is require due to the company policy does not allow excessive overtime and balance the workload for each employee.

- Alternative production capacity

$$\boxed{X_t \leq (160 + 20)/4 \times (W_{t-1} + H_t - F_t)}$$

This is another way to express the production capacity, considering both standard and overtime hours (a total of 180 hours per worker). It states that the production (X_t) is limited by the total work hours available from the workforce (including new hires and firings) divided by 4 (hours to produce one bag).

Step 5: Entering the solve with the constrains.



Input the objective function as mentioned above, furthermore since the case study requires to calculate the optimal cost which could reduce the expenses of all the changing variables. In the constrains box, input the integer for all variables to eliminate value that contains fractions.

2. Interpret the result

Production scheduling cost (initial 20 workers)					
Period	**Month 1**	**Month 2**	**Month 3**	**Month 4**	**Total**
Forcast	3000	5000	2000	1000	11000
Output					
No of bags produces	3720	3780	2000	1000	10500
Overtime	360	560	0	0	920
Output 2 forcast	720	-1220	0	0	-500
Inventory					
Beginning	500	1220	0	0	
Ending	1220	0	0	0	
Average	860	610	0	0	
Cost					
Output					
Raw material	$ 55,800.00	$ 56,700.00	$ 30,000.00	$ 15,000.00	
Overtime	$ 18,720.00	$ 21,840.00	$ -	$ -	
Worker	$ 126,000.00	$ 126,000.00	$ 75,000.00	$ 75,000.00	
Hire	$ 102,400.00	$ -	$ -	$ -	
Fire			$ 68,000.00		
Inventory	$ 3,660.00	$ -	$ -	$ -	
Total	$ 306,580.00	$ 204,540.00	$ 173,000.00	$ 90,000.00	**$ 774,120.00**

Table 1: Aggregate planning scheduling

Completing the outcome in the production schedule planning

Comparing the forecast and the productions

Over the course of four months, the firm anticipates a cumulative demand of 11,000 bags. The production plan details the manufacturing of 10,500 bags, with a surplus of 720 bags in Month 1 and a deficit of 1,220 bags in Month 2. This suggests a possible overestimation of capacity in Month 1 and an underestimating in Month 2, resulting in an imbalance in inventory.

In order to address the imbalance, the firm has scheduled overtime for the first two months, namely 360 and 560 hours, respectively. This measure is intended to possibly fulfil the projected increase in demand or to make up for any shortfall in normal working hours. There will be no overtime scheduled for Months 3 and 4, in accordance with the decreased production needs.

Inventory Management

The initial inventory stands at 500 bags, then surges to 1,220 bags at the end of Month 1 due to overproduction, and after that declines to 0 for the rest of the time. Indications imply that a modification was implemented after the first month to synchronise output with the projected demand more effectively.

The mean inventory peaks in Month 1 and then declines in Month 2, with no inventory carried over into Months 3 and 4. This may suggest a deliberate attempt to minimise carrying expenses or a reaction to a decrease in demand.

Cost Analysis

The costs of raw materials and labour are directly proportional to production volumes, meaning that when the number of bags produced reduces, the expenses also decrease. The surge in recruitment expenses in Month 1 signifies a preliminary upsurge in the labour pool. In contrast, termination expenses in Month 3 imply a downsizing of the workforce, most likely due to the declining demand. The overtime expenses are substantial during the first two months but are absent during the last two months. This indicates the initial concentration of manufacturing resources to meet the early surge in demand.

Although the inventory cost was only spent in Month 1, it may have been caused by excessive production compared to demand, suggesting an opportunity for cost reduction by improving demand alignment. Therefore, according to the provided data, the outcome indicates a cumulative expenditure of $774,120 to fulfil two primary objectives: meeting the anticipated labour hours and satisfying the projected production demand.

3. The change of initial worker

Looking deeper into the data through the sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C10	X_t (Bags Produced) Month 1	3750	0	15	51.22222222	57.22222222
D10	X_t (Bags Produced) Month 2	3750	0	15	57.22222222	51.22222222
E10	X_t (Bags Produced) Month 3	2000	0	15	58.61111111	28
F10	X_t (Bags Produced) Month 4	1000	0	15	28	18
C11	W_t (Workers) Month 1	83.33333333	0	1500	3465	2305
D11	W_t (Workers) Month 2	83.33333333	0	1500	3735	2305
E11	W_t (Workers) Month 3	50	0	1500	1080	1000
F11	W_t (Workers) Month 4	50	0	1500	500	1000
C12	H_t (Workers Hired) Month 1	63.33333333	0	1600	3465	2305
D12	H_t (Workers Hired) Month 2	0	1732.5	1600	1E+30	1732.5
E12	H_t (Workers Hired) Month 3	0	3600	1600	1E+30	3600
F12	H_t (Workers Hired) Month 4	0	3100	1600	1E+30	3100
C13	F_t (Workers Fired) Month 1	0	3600	2000	1E+30	3600
D13	F_t (Workers Fired) Month 2	0	1867.5	2000	1E+30	1867.5
E13	F_t (Workers Fired) Month 3	33.33333333	0	2000	1000	1080
F13	F_t (Workers Fired) Month 4	0	500	2000	1E+30	500
C14	I_t (Inventory) Month 1	1250	0	3	51.22222222	57.22222222
D14	I_t (Inventory) Month 2	0	58.61111111	3	1E+30	58.61111111
E14	I_t (Inventory) Month 3	0	28	3	1E+30	28
F14	I_t (Inventory) Month 4	0	18	3	1E+30	18
C15	OT_t (Overtime Hours) Month 1	1666.666667	0	13	6.779411765	13
D15	OT_t (Overtime Hours) Month 2	1666.666667	0	13	7.573529412	13
E15	OT_t (Overtime Hours) Month 3	0	6.75	13	1E+30	6.75
F15	OT_t (Overtime Hours) Month 4	0	13	13	1E+30	13

Table 2: sensitivity report

In order to generate the sensitivity report in the Excel spreadsheet, it is necessary to eliminate the integer constraints. This is because sensitivity analysis depends on the linearity and continuity of the solution space of the issue (Pulch et al., 2019). In addition, it should be noted that the reliability of a sensitivity report prepared for an integer programme may be compromised due to the rounding off of the best solution to the closest integer (Picelli et al., 2020). The sensitivity ranges may not accurately reflect the real integer answer.

Considering all factors, the production variables The final data displays the monthly manufacturing quantities of bags, indicating a greater output over the first two months. The goal coefficient for making one bag is $15. The permissible increase/decrease represents the extent to which this cost may vary without compromising the optimality of the existing production plan.

The workforce variables, denoted as (Wt), indicate the permissible fluctuation in labour costs, with an increment or decrement of $1500 per worker. This implies that the model allows for some adaptability in labour expenses, particularly during the first two months.

- Number of Workers Hired (Ht): There was a substantial increase in hiring during the first month, with no permissible reduction. This suggests that recruiting was crucial to the strategy, and any reduction could impact the solution. No employment occurs in the following months.
- Terminated Employees (Ft): Only employment is terminated in the third month. The permissible increment is substantial, indicating that the termination rate may have been greater without impacting the ideal solution.

Furthermore, the ending inventory (It) in month 1 is high, with a substantial permissible reduction, suggesting that inventory levels might have been lower. The inventory is consistently zero in the following months, albeit with fluctuating sensitivity.

Finally, The overtime variables (Ot) exhibit a significant increase over the first two months, displaying a certain level of sensitivity. Months 3 and 4 exhibit no instances of overtime, while month 3 allows for a permissible rise, indicating that the inclusion of overtime may be possible without altering the current solution.

Therefore, the sensitivity analysis demonstrates that Hop Phat's production plan is resilient to fluctuations in labour prices but not as much to variations in production costs and inventory holding costs. The organisation should explore solutions that enhance production flexibility to respond to future cost fluctuations while maintaining operational optimality effectively.

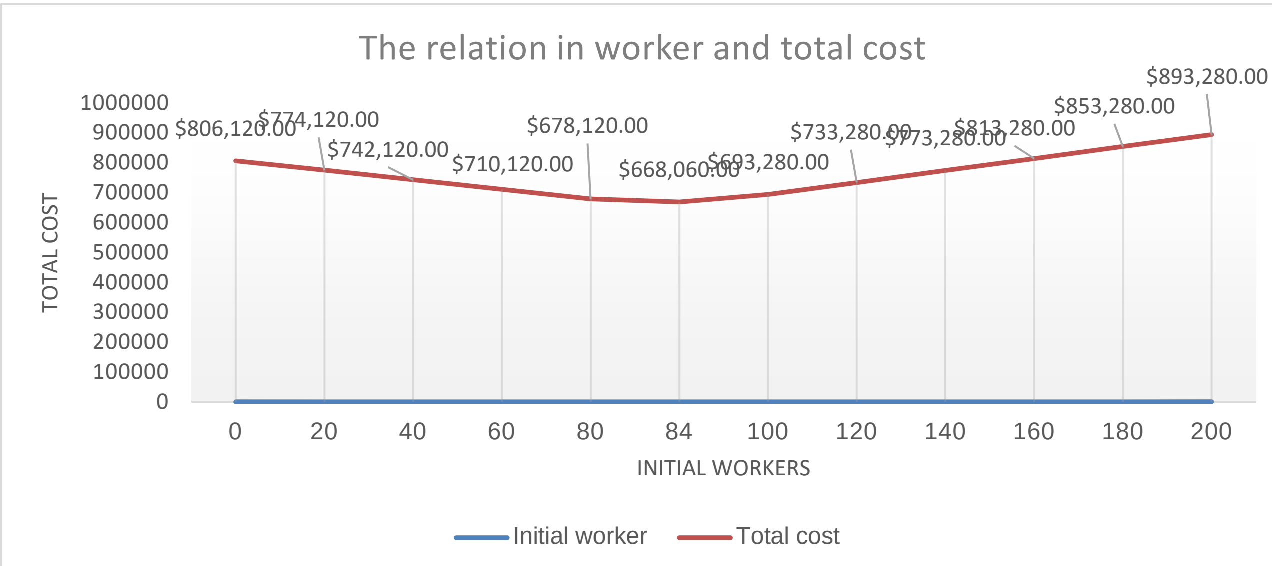


Figure 1: The change in initial worker affect on the total cost

Through the sensitive report, the variables that have a major impact on the total cost is available worker in the firm. With 0 workers, the cost is highest at $806,120. This would be due to the high costs of not meeting production demands, leading to either lost sales or the need for expensive last-minute production measures. As the number of workers increases to 20, the cost decreases to $774,120, suggesting that having some workers is more cost-effective than starting with none. Continuing this trend, the total cost decreases as the number of workers increases to 40 ($742,120), 60 ($710,120), and reaches the lowest at 84 workers ($668,060). Beyond this point, the total cost begins to increase with more workers: 100 workers result in a cost of $693,280, and the cost continues to rise steadily until it reaches $893,280 with 200 workers.

Following the trendline and the sensitive report, there is an optimal number of workers where the cost is minimized, which, in this case, is at 84 workers. Starting with too few workers likely incurs extra costs due to overtime, inability to meet demand, or the need for expensive rapid hiring. Conversely, starting with too many workers leads to excessive labor costs that are not offset by the benefits of additional production.

Reccomendation

As mentioned above, the optimal solution would be having 84 workers to meet the forcast demand. Creating a new production schedule to have more insight on the cost of production

Production scheduling cost (for initial 84 workers)					
Period	**Month 1**	**Month 2**	**Month 3**	**Month 4**	**Total**
Forcast	3000	5000	2000	1000	11000
Output					
No of bags produces	3720	3780	2000	1000	10500
Overtime	360	420	0	0	780
Output 2 forcast	720	-1220	0	0	-500
Inventory					
Beginning	500	1220	0	0	
Ending	1220	0	0	0	
Average	860	610	0	0	
Cost					
Output					
Raw material	$ 55,800.00	$ 56,700.00	$ 30,000.00	$ 15,000.00	
Overtime	$ 18,720.00	$ 21,840.00	$ -	$ -	
Worker	$ 126,000.00	$ 126,000.00	$ 75,000.00	$ 75,000.00	
Hire	$ -	$ -	$ -	$ -	
Fire	$ -	$ -	$ 68,000.00	$ -	
Inventory					
Total	$ 200,520.00	$ 204,540.00	$ 173,000.00	$ 90,000.00	**$ 668,060.00**

Table 3: aggregate schedule planning

Comparing to the aggregate schedule planning for initial 20 workers, one of the changes in the production is the hiring cost. To balance the work hours of the workers the optimal worker for the first month is 4 since the company already has 80 workers in hand, which only hire extra 4 people this reduce from $774,120 to $668,1060 (a $106,060 reduction).

Reference

Picelli R, Sivapuram R. and Xie Y.M. A (2021) ‘A 101-line MATLAB code for topology optimization using binary variables and integer programming’, *Struct Multidisc Optim,* 63: 935–954. doi:10.1007/s00158-020-02719-9.

Pulch R, Narayan A and Stykel T (2021) ‘Sensitivity analysis of random linear differential–algebraic equations using system norms’, *Journal of Computational and Applied Mathematics,* 397: 113666. doi:10.1016/j.cam.2021.113666.