A Fourier-series-based Virtual Fields Method for the identification of modulus distributions

Nguyen TT[1], Huntley JM[1], Ashcroft IA[2], Ruiz PD[1] and Pierron F[3]

[1] Loughborough University, Wolfson School of Mech. and Manuf. Engineering, Loughborough LE11 3TU, UK
[2] University of Nottingham, Faculty of Engineering, Nottingham NG7 2RD, UK
[3] University of Southampton, Faculty of Engineering and the Environment, Highfield, Southampton SO17 1BJ, UK

Introduction

- ✓ Requirement of a fast and accurate inverse technique to determine spatially-varying properties of materials has arisen in many research fields.
- ✓ Non-contact full-field measurement techniques have become more attractive to research community.
- ✓ A modest number of inverse techniques using full-field measurement data available in the literature.

From the classical Virtual Fields Method...

Equation (general form) of the principle of virtual work

$$-\int_V \boldsymbol{\sigma}:\boldsymbol{\epsilon}^* dV + \int_{S_f} \mathbf{T}\cdot\mathbf{u}^* dS_f + \int_V \mathbf{f}\cdot\mathbf{u}^* dV = \int_V \rho\mathbf{a}\cdot\mathbf{u}^* dV$$

written for a 2-D linear elastic isotropic case as

$$\int_S \left((\epsilon_{xx}+\nu\epsilon_{yy})\epsilon^*_{xx} + (\epsilon_{yy}+\nu\epsilon_{xx})\epsilon^*_{yy} + \frac{1-\nu}{2}\epsilon_{ss}\epsilon^*_{ss}\right) Q_{xx}\, dS = \int_\ell (T_x u^*_x + T_y u^*_y)\, d\ell$$

...to the Fourier Virtual Fields Method

- ✓ Parameterisation of spatially-varying modulus/stiffness by a Fourier series expansion:

$$Q_{xx}(x,y) = \sum_{m=0}^{M}\sum_{n=-N}^{N} a_{m,n}\cos 2\pi\left(\frac{mx}{L_x}+\frac{ny}{L_y}\right) + \sum_{m=0^*}^{M}\sum_{n=-N^*}^{N} b_{m,n}\sin 2\pi\left(\frac{mx}{L_x}+\frac{ny}{L_y}\right)$$

- ✓ Selection of virtual deformation fields as cosine/sine functions of spatial variables.

Modulus reconstruction with unspecified boundary conditions

- ✓ Application of an appropriate window function $W(x,y)$ to zero unknown traction components on the boundary:

$$\int_\ell (T_x \hat{u}^*_x + T_y \hat{u}^*_y)\, d\ell = \int_\ell (T_x W u^*_x + T_y W u^*_y)\, d\ell = 0$$

Application to experimental data

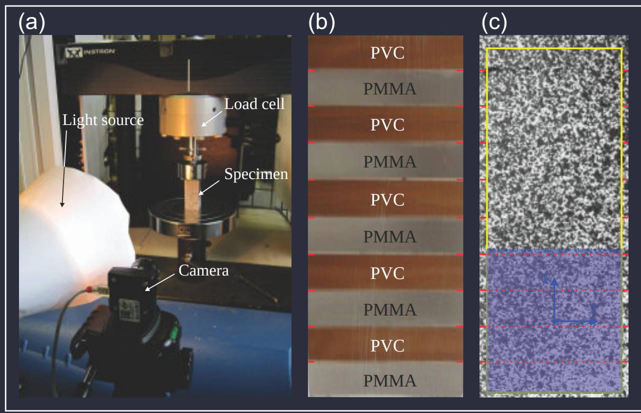

Figure 1: (a) Experimental setup of a multi-layered prismatic plastic specimen under uniaxial compression. (b) Specimen's layer-up. (c) ROI highlighted.

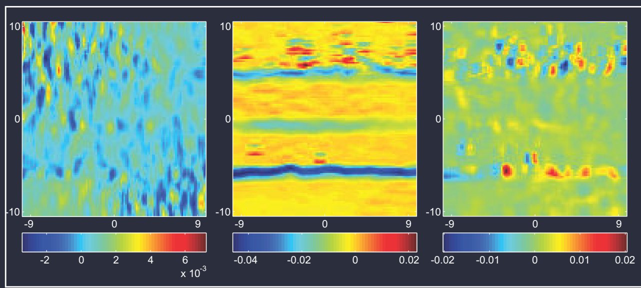

Figure 2: Experimental strain fields measured within the ROI by 2-D DIC. Left to right: ϵ_{xx}, ϵ_{yy} and ϵ_{ss}.

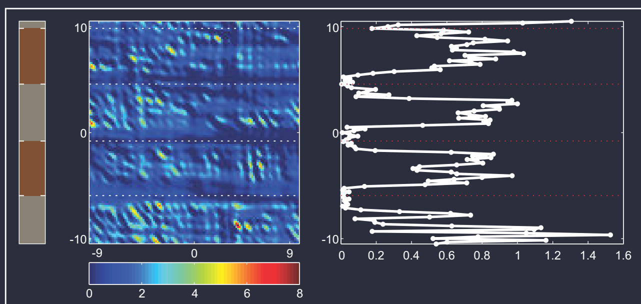

Figure 3: Modulus map reconstructed by the F-VFM. The graph on the right shows mean values of every row of the modulus map. The modulus ratio of the two materials is determined ~1.05 ± 0.1 and compared with the real ratio of 1.2.

Conclusions

- ✓ Development of a Fourier-series-based method able to reconstruct spatially-varying modulus distributions.
- ✓ Adaptation of the method to challenging situation of limited knowledge of the boundary conditions.
- ✓ Computational efficiency achieved by using the fast algorithm of the proposed technique, which returns nearly a thousand of variables in ~3 seconds.