Fast Fourier Virtual Fields Method for Determination of Modulus Distributions from Full-field Optical Strain Data

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1 Introduction

Inspection of parts for manufacturing defects or in-service damage is often carried out by full-field optical techniques (e.g., digital speckle pattern interferometry, digital holography) where the high sensitivity allows small anomalies in a load-induced deformation field to be measured. Standard phase shifting and phase unwrapping algorithms provide full-field displacement and hence strain data over the surface of the sample. The problem remains however of how to quantify the spatial variations in modulus due, for example, to porosity or damage-induced micro-cracking. Finite element model updating (FEMU) is one method to solve problems of this type, by adjusting an approximate finite element model until the responses it produces are as close to those acquired from experiments as possible.

An alternative approach is the Virtual Fields Method (VFM) [1]. The advantage of the VFM over other methods is its ability to solve inverse problems of this type without iterative computation. Several approaches based on polynomial virtual fields with the material properties considered as to be single valued within the domain have been developed [2-5]. The first attempt to parameterize the material properties as a function of spatial variables was proposed in [6].

In this paper, we retain the basic concepts of the VFM but approach the parameterization of the material properties in the spatial frequency, rather than spatial, domain by performing a 2-D Fourier series expansion of the stiffness distribution over the region of interest. Furthermore, the virtual fields are not selected as polynomials of spatial variables as in the previous VFM literature, but from a set of simple cosine or sine functions of different spatial frequencies. This Fourier version of the VFM will be denoted the F-VFM. An example of its successful application to the identification of an unknown stiffness distribution under known boundary conditions is summarized here; further details are given in [7].

2 Theoretical

2.1 Virtual Fields Method formulation

2.2 Fourier series expansion of stiffness distribution

2.3 Selection of virtual fields in the F-VFM

2.4 Fast Fourier VFM implementation

3 Example application

4 Conclusions

5 References

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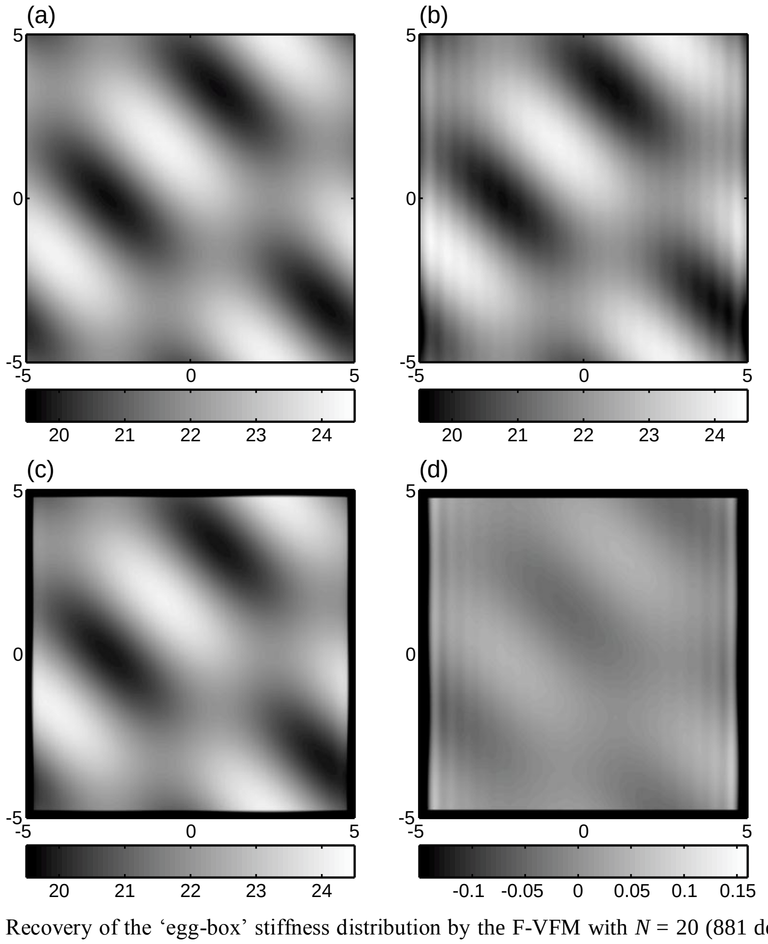

Fig. 1. Recovery of the 'egg-box' stiffness distribution by the F-VFM with $N = 20$ (881 degrees of freedom, units: MPa). (a) Reference stiffness map (1000×1000 pixels); (b) Recovered stiffness map by F-VFM; (c) as (b) after smoothing by a 50×50 pixel kernel; (d) Error map