Research review

Game Tree Searching by Min/Max Approximation

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Summary of the research's goals or techniques

- The paper proposes a new algorithm to select the next node to explore in a large Min/Max tree.
- The algorithm is a case of penalty-based search.
    - A penalty/weight is assigned to each edge of the game board, bad moves are penalized more than good moves.
    - The proposed algorithm expands the node with the least penalty within the time allowed
- The relative penalty is calculated via the value of D(x,y) in formula 11, 12, 13 and 14 of the paper (https://people.csail.mit.edu/rivest/pubs/Riv87c.pdf). To implement the algorithm, apply the formulas on page 11 to 14 on page 86 to all nodes on the tree, then backtrack and add all weight on each edges of the tree to get the penalty score from root to current node.
- Expand the node with the least penalty/weight, then update the game tree and repeat the process within allocated time.
- The paper also proposes an alternative implementation without calculating the generalized mean (an expensive operation) and use the approximated min/max values, wherein the weight is calculated with formula 15 (in the paper). However, the paper does not provide a clear method to select the optimal p value. This should be left to the problem-specific implementation to decide.

Summary of the paper's results

- The proposed solution was implemented on a game of Connect-Four on a 6 columns x 7 cells board.
- The algorithm competed against the classical Alpha-beta pruning algorithm, using 2 different criteria for benchmarking: CPU time (1 to 5 seconds in 1s increment) and number of moves considered (1000 to 5000 in 1000-move increments)
- In time-bound benchmark, Alpha-beta pruning wins most the matches.
- In move-bound benchmark, the new Min/Max Approximation wins most matches
- Number of distinct positions considered by Alpha-beta was ~3 times for the same time period.
- Conclusion:
    - Min/Max approximation is superior to Alpha-beta pruning when the “move” operation is expensive to run.

Notes: Formulas from the paper are referenced here as their numbers from the original paper instead of copying them over. This review should be read side-by-side with the original paper.