Assignment 5-1: Euler Angle Rotations (2 Points)

(a) (1 Point) Given is a coordinate frame $\{A\}$. The coordinate frame $\{B\}$ is created from $\{A\}$ via rotation around the z-axis by $\frac{\pi}{2}$, then intrinsic rotation around the y-axis by $\frac{\pi}{2}$ and intrinsic rotation around the x-axis by $\frac{\pi}{2}$. Write down the whole formula (with all rotation matrices in the right order) to calculate ${}^{A}_{B}R$. Calculate ${}^{A}_{B}R$.

(b) (1 Point) Given is a coordinate frame $\{A\}$. The coordinate frame $\{C\}$ is created from $\{A\}$ via rotation around the z-axis by $\frac{\pi}{2}$, then extrinsic rotation around the y-axis by $\frac{\pi}{2}$ and extrinsic rotation around the x-axis by $\frac{\pi}{2}$. Write down the whole formula (with all rotation matrices in the right order) to calculate ${}^{A}_{C}R$. Calculate ${}^{A}_{C}R$.

a)

$${}^{A}_{B}R_{z'y'x'}\left(\tfrac{\pi}{2},\tfrac{\pi}{2},\tfrac{\pi}{2}\right) = R_z\left(\tfrac{\pi}{2}\right) R_y\left(\tfrac{\pi}{2}\right) R_x\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} \frac{\pi}{2}z_1 & \frac{\pi}{2}y_1 & \frac{\pi}{2}x_1 \\ \frac{\pi}{2}z_2 & \frac{\pi}{2}y_2 & \frac{\pi}{2}x_2 \\ \frac{\pi}{2}z_3 & \frac{\pi}{2}y_3 & \frac{\pi}{2}x_3 \end{pmatrix}$$

b)

$${}^{A}_{C}R_{z'y'x'}\left(\tfrac{\pi}{2},\tfrac{\pi}{2},\tfrac{\pi}{2}\right) = R_x\left(\tfrac{\pi}{2}\right) R_y\left(\tfrac{\pi}{2}\right) R_z\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} \frac{\pi}{2}x_1 & \frac{\pi}{2}y_1 & \frac{\pi}{2}z_1 \\ \frac{\pi}{2}x_2 & \frac{\pi}{2}y_2 & \frac{\pi}{2}z_2 \\ \frac{\pi}{2}x_3 & \frac{\pi}{2}y_3 & \frac{\pi}{2}z_3 \end{pmatrix}$$

Assignment 5-2: Rodrigues Rotations (3 Points)

Given is the axis-angle rotation vector $\Theta = (2, 2, 0)$.

(a) (0.5 Points) Calculate the unit vector of the rotation axis k and the angle θ

(b) (1.5 Points) Derive the rotation matrix R representing the same rotation, using the exponential map, and show, that your matrix is orthogonal.

(c) (1 Point) Given a vector $P_A = (1, 2, 3)$ Rotate Vector P_A by Θ using Rodrigues' formula

Provide calculation steps for each of the above tasks.

a) $k = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\theta = 2$

b) $K = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

$$R = I + \sin(\theta) K + (1 - \cos(\theta)) K^2$$

$$= \begin{pmatrix} 1 & 0 & \sin(\theta) \\ 0 & 1 & -\sin(\theta) \\ -\sin(\theta) & \sin(\theta) & 1 \end{pmatrix}$$

c) $P_{A\Theta} = P_A + \underbrace{(1-\cos(\theta))\, k \times (k \times P_A)}_{0} + (\sin\theta)(k \times P_A)$

$$= \begin{pmatrix}1\\2\\3\end{pmatrix} + 0 + \begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$= \begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}3\\3\\1\end{pmatrix}$$

$$= \begin{pmatrix}4\\5\\4\end{pmatrix}$$

5.3.

a)

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^Cthore@Ubuntu-T:~/uni/robotik/thornavid/src/assignment5/src$ python3 wheelspeedClass.py
speed:0.3, steering:0.0, tick count:611, distance:2 ,tick / distance ratio:305.5
^Cthore@Ubuntu-T:~/uni/robotik/thornavid/src/assignment5/src$ python3 wheelspeedClass.py
speed:0.3, steering:1, tick count:613, distance:2 ,tick / distance ratio:306.5
^Cthore@Ubuntu-T:~/uni/robotik/thornavid/src/assignment5/src$ python3 wheelspeedClass.py
speed:0.3, steering:-1, tick count:611, distance:2 ,tick / distance ratio:305.5
```

b)