Kendall τ

Our initial plan

- for every pair of edges $a, b \in E$ choose random orientations
- indicator variables $X^{\text{con}}_{a,b}$ and $X^{\text{dis}}_{a,b}$ that the resulting points are concordant and discordant, respectively
- count the pairs: $X^{\text{con}} = \sum X^{\text{con}}_{a,b}$ and $X^{\text{dis}} = \sum X^{\text{dis}}_{a,b}$
- $\tau = \frac{\mathbb{E}[X^{\text{con}}] - \mathbb{E}[X^{\text{dis}}]}{\mathbb{E}[X^{\text{con}}] + \mathbb{E}[X^{\text{dis}}]}$

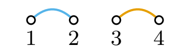
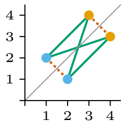
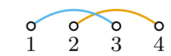
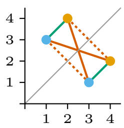
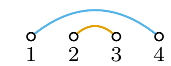
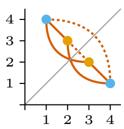
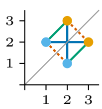
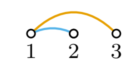
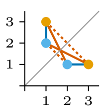
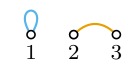
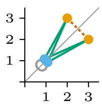
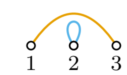
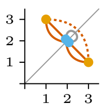
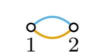
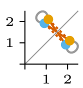
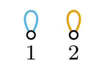
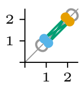
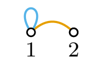
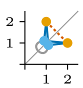
Kandell τ_b for both directions

- for every edge, create two points (one for both directions)
- for these points, count then umber of concordant, discordant and tied pairs (tied in only one direction; pairs of equal points are skipped): #con, #dis, #tie
- $\tau_b = \frac{\#\text{con} - \#\text{dis}}{\#\text{con} + \#\text{dis} + \#\text{tie}/2}$

Comments

- we probably want to include the tied pairs in the denominator as in τ_b
- we probably don't want to count the pairs comparing an edge to itself
- modulo these two points, the two variants are the same (see comparison below)

Possible configurations of a pair of edges

		random direction				both directions			
		Pr[con]	Pr[dis]	Pr[tie]	Pr[tietie]	#con	#dis	#tie	#tietie
		1	0	0	0	4	$0+2$	0	0
		$\frac{1}{2}$	$\frac{1}{2}$	0	0	2	$2+2$	0	0
		0	1	0	0	0	$4+2$	0	0
		$\frac{1}{2}$	0	$\frac{1}{2}$	0	2	$0+2$	2	2
		0	$\frac{1}{2}$	$\frac{1}{2}$	0	2	$0+2$	2	2
		1	0	0	0	4	$0+1$	0	1
		0	1	0	0	0	$4+1$	0	1
		0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$2+2$	0	2
		1	0	0	0	4	$0+0$	0	2
		0	0	1	0	0	$0+1$	4	1
		0	0	0	1	0	$0+0$	0	6