A Fast Wavelet-Based Hybrid Method to Model the RCS of Metallic Targets in Maritime Environment

Corentin Carré, Thomas Bonnafont, Ali Khenchaf

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Corentin Carré*, Thomas Bonnafont*, Ali Khenchaf*
*Lab-STICC, UMR CNRS 6285, ENSTA, Institut Polytechnique de Paris, Brest, France, givenname.surname@ensta.fr

***Abstract*—This article focuses on a hybrid wavelet-based model to compute the radar cross section (RCS) of metallic targets in a maritime environment. The latter is based on the parabolic wave equation, solved in the wavelet domain, for the propagation to the target and the wavelet-based method of moments for the scattered field. Key advantages include its ability to account for terrain relief and refraction effects within the propagation channel, as well as providing accurate RCS calculations. Additionally, wavelet-domain compression enables a flexible trade-off between computational efficiency and precision. Numerical experiments in the VHF-band are performed to validate the method.**

***Index Terms*—radar cross section, split-step method, integral equations, hybridization, wavelets**

I. Introduction

Modeling the radar cross section (RCS) of targets is of major interest for numerous applications, *e.g.*, radar detection, or SAR imaging. In this work, we propose a precise and fast hybrid method to model the RCS of a metallic target in a maritime environment. The main idea is to introduce a fast and accurate method to model the scene from the antenna to the propagation channel and finally the target.

First, to model the long-range propagation from an antenna above the sea, we need to account for various phenomena such as refraction, relief (waves or islands), and ground composition. In this context, the parabolic wave equation (PWE) [1] is widely used since it allows for wide steps in the propagation direction while accounting for all the mentioned phenomena. Indeed, fast algorithms have been proposed to solve the PWE. Split-step Fourier [2], [1] (SSF), which uses the Fourier transform to expedite the computations in the spectral domain, is widely used in this context. Nonetheless, recently, split-step wavelet [3], [4] (SSW) has been proposed to reduce the method's complexity in space and time. It has shown promising results in tropospheric [3], [4] and maritime propagation [5]. Nevertheless, these methods are not well suited to account for a target in the propagation channel since the mesh size used is around 50λ in the propagation direction to be efficient in time.

On the other hand, to compute the RCS of targets, integral equation-based methods [6], [7] are widely used. Indeed, they allow for a precise calculation of the scattered field, requiring only to mesh the surface of the target. Indeed, the method of moments (MoM) [7] allows for precise computation of any object's RCS while accounting for its composition. Nevertheless, a mesh size of at most $\lambda/6$ is required to achieve good accuracy. Therefore, asymptotic methods such as the physical optics (PO) [8] have been developed, and improved [9], to compute the RCS of large targets in a reasonable amount of time at the cost of accuracy. This is why wavelets have emerged as a solution as basis functions for the MoM [10] to reduce its computation time, based on the compression allowed by the wavelets. Nonetheless, these methods would not be well suited to model a complete tropospheric propagation channel.

Therefore, to build on the advantages of both strands, hybrid methods have been developed [11], [12], [13]. Indeed, the PWE is used to model the propagation to the target, and an integral equation-based solver is used to account for the target. In general, to expedite the calculations, the PWE is hybridized with the PO [12], since hybridization with the MoM requires more computation resources.

In this article, we propose a first hybridization between the PWE/SSW method with the MoM in the wavelet domain. Indeed, the idea is to use the advantages of the wavelet for both parts. First, SSW has a lower complexity than SSF allowing for faster computation while less memory resources are needed. Second, the wavelet MoM (wMoM) allows expediting computation using the compression in the wavelet domain. In addition, this gives us a parameter to choose between accuracy and, time and memory efficiency. Finally, it seems appropriate to match the function used in the PWE solver to the one in the MoM for the hybridization. Numerical experiments in the VHF band are provided to highlight the advantages of the method.

The remainder of this article is organized as follows. Section II introduces the wavelet-based hybrid method proposed in this article. In Section III, some numerical experiments are performed to validate the method. Section IV concludes the paper and gives perspectives for future works.

II. The hybrid method

The main idea of our hybrid method is depicted in Fig. 1. Indeed, to account for the environment (refraction, relief, and ground) the PWE model is used to propagate to the target. The latter is solved with the SSW method, which will be briefly recalled in Section II-B. Then, the target RCS is computed at the reception point using the wavelet MoM, recalled in Section II-C. Hybridization is performed between both methods at the vertical of the target x_{t} as explained later in Section II-D.

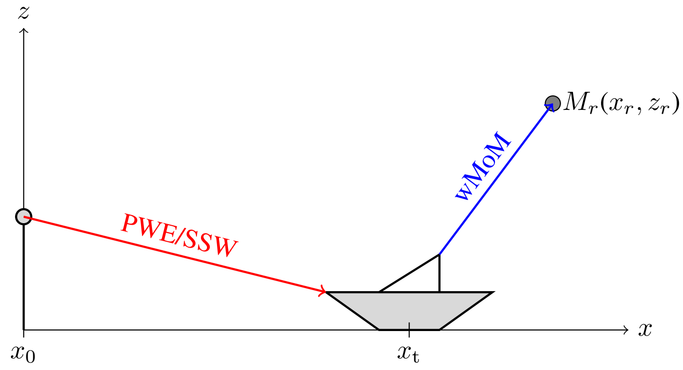


Fig. 1. Schematic representation of the proposed hybrid method.

A. Notations and discretization

First, let us introduce the notations that are used throughout this paper.

An $\exp(j\omega t)$ time dependence is assumed here. For the PWE model, we assume that the refractive index n is slowly varying in the propagation direction. We consider a source placed at $x_s \leq 0$ and the field to be known at $x_0 = 0$. Indeed, otherwise, it only corresponds to translating our coordinate system. The field is computed above the ground, such that $z \geq 0$ and forward from the emitter, $x \geq 0$. We denote by x_{t} the position of the obstacle, its center, and as $M_r = (x_r, z_r)$ the position of the reception antenna.

For obvious numerical reasons, the domain is discretized. Here, since both methods do not share the same mesh, we introduce first, the one used for the PWE, and second the one used for the wMoM. For the PWE, we denote by $\Delta^{\text{PWE}}x$ and $\Delta^{\text{PWE}}z$ the respective steps along the x and z axis. We also denote by $u_x[\cdot]$ a semi-discretized version of u along z at position x. For the wMoM, we denote the step size as $\Delta^{\text{wMoM}}z$.

B. Propagation to the target: PWE/SSW

In this section, we focus on the propagation to the object and thus place ourselves in the domain $(x, z) \in [0, x_o] \times [0, z_{\max}]$. We assume that we are in the transverse electric (TE) case, but all can be extended directly to the TM case. We denote by u the reduced field [1] given by

$$u(x, z) = \sqrt{k_0 x} E_y(x, z) \exp(jk_0 x), \tag{1}$$

where E_y is the only non-zero of the TE field and k_0 is the wavenumber.

The latter is the solution of the wide-angle parabolic equation, which reads as

$$\frac{\partial u}{\partial x} = -j\left(\sqrt{k_0^2 + \frac{\partial^2}{\partial z^2}} - k_0\right) u - jk_0(n-1)u. \tag{2}$$

This equation is widely used in the context of long-range tropospheric propagation since large mesh steps can be performed in the propagation axis and the refraction and the ground can be easily accounted for. Nonetheless, this equation is valid only in the forward direction and in a $90°$ cone along the x-axis here.

Split-step methods [1] are very efficient in solving the PWE. Here, we choose to use the SSW method [3], [4], since it has a lower complexity in both space and time than SSF[1]. In a few words, SSW, as for SSF, solved the PWE iteratively[2]. One step from x to $x + \Delta^{\text{SSW}}x$ is given as follows :

$$u_{x+\Delta^{\text{SSW}}x}[\cdot] = \mathbf{L}\mathbf{R}\mathbf{W}^{-1}\mathbf{P}_{V_p}\mathbf{C}_{V_s}\mathbf{W}u_x[\cdot], \tag{3}$$

where $\mathbf{W}$ is the fast wavelet transform, $\mathbf{C}$ a hard-threshold compression (threshold V_s), $\mathbf{P}_{V_p}$ the compressed wavelet-to-wavelet propagator, see [4] for more information, $\mathbf{R}$ a phase screen operator to account for n and $\mathbf{L}$ an operator to account for the relief. The latter is performed using the staircase model. Finally, the ground is accounted for using the local image theorem [3].

It shall be noted that the compression performed in the wavelet domain allows for improving the computation time and manipulating sparse vectors and operators. Furthermore, the threshold can be chosen to respect a defined accuracy [14].

C. Propagation from the target: wMoM

Now let us consider the field scattered by the target. Indeed, the mesh size used for the PWE/SSW method is not suited for a target. Therefore, we develop here a wavelet-based MoM [10] to efficiently compute its RCS.

In this article, to compute the RCS at a receptor far from the target, we consider the following general scalar integral equation [15]

$$v^{\text{inc}}(x) = \int G(x, x')J(x')dx', \tag{4}$$

where $v^{\text{inc}}(x)$ is a known source term, either the incident electric or magnetic field, G the Green kernel associated with our problem, and $J(x')$ the current density that is unknown here. In this scalar form, it would correspond to looking at one component of the associated vectors.

To solve (4), we use the Galerkin method. Here the test and basis functions considered are the wavelets. Indeed, they have good properties to compress operators and also are widely used for preconditioning [16]. Furthermore, the error from the compression is directly related to the threshold used [17], [18].

In the following, we denote by $\chi_{l,p}$ a wavelet of level $l \in [0, L]$, where L is the chosen maximum level, translated by p [17]. Indeed, the scaling function would correspond to the level 0 here and has the same support as the level L wavelet. It shall be noted that they form an orthonormal basis, leading to

$$J(x) = \sum_{i=0}^{L} \sum_{p \in Supp(\chi_{l,p})} J_{l,p}\chi_{l,p}(x), \tag{5}$$

[1]Furthermore, wMoM is more efficient than a usual MoM, thus wavelets are a good candidates also for solving the PWE to be self-consistent.

[2]Indeed the PWE is an ordinary differential equation in x.

where $Supp(\chi_{l,p})$ is the support of the wavelet [17], [18]. Using also the wavelets as the test functions, we obtained the following linear system

$$\mathbf{A}_{V_a}\mathbf{J}^w = \mathbf{V}_{V_g}, \tag{6}$$

where $\mathbf{J}^w$ corresponds to the vector of $J_{l,p}$, and $\mathbf{V}_{V_f}$ is the source term computed as

$$\mathbf{V} = \int v^{\text{inc}}(x')\chi_{l,p}(x')dx'. \tag{7}$$

This term can be efficiently calculated since it corresponds to the wavelet transform of V. The subscript V_g corresponds to the case when compression is applied. To conclude on this system, the matrix $\mathbf{A}$ is block-constructed from the following general terms

$$A_{p,h}[l,m] = \int\int \chi_{l,p}(x')\chi_{m,h}(x)G(x,x')dx'. \tag{8}$$

The complete matrix is then

$$\mathbf{A} = \begin{pmatrix} A_{0,0} & A_{0,1} & \cdots & A_{n,n} \\ A_{1,0} & A_{1,1} & \vdots & A_{1,n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n,0} & A_{n,1} & \cdots & A_{n,n} \end{pmatrix} \tag{9}$$

that can also be efficiently computed since all the operations can be recast as wavelet transforms. The subscript V_a corresponds to a compression with hard threshold V_a applied.

Then, by solving this linear system, the vector Jw can be easily obtained. Finally, the current is retrieved using an inverse wavelet transform and used to compute the scattered field with the Stratton-Chu formula.

D. Coupling both methods

Now that both methods have been explained, one can wonder how to hybridize them.

As said earlier, the PWE solved by SSW allows computing the reduced field at the target location, at x_t. Therefore, the incident field, either magnetic or electric, is known from the PWE using (1). Nonetheless, we can not directly use it as the source term for the wMoM. Indeed, the field is known on the vertical at x_t, but not on the object's surface. Furthermore, the mesh size between the two methods is very different with $\Delta_{wMoM}z$ that needs to be much lower than $\Delta_{SSW}z$ or $\Delta_{SSW}x$.

From this remark, the problem of hybridization between SSW and wMoM can be stated as follows: *how to compute the field on a thinner mesh from a coarser one?* This can be viewed as an interpolation problem. Here, using the stationary phase theorem, we assume that the phase of the field is constant over our target. Therefore, we interpolate only the amplitude of the field at each new grid center. To do so, we continuously interpolate to a coarser grid by dividing the grid size by 2 at each step until we reach the desired $\Delta_{wMoM}z$.

III. Numerical experiments

In this section, we propose numerical experiments to validate and highlight the proposed method's advantages. All the tests are performed at $f = 300$MHz.

A. Why using the wavelet MoM ?

In this section, we propose a simple test to highlight the advantages of the wMoM over a usual MoM, that uses pulse function as basis and test functions. Here, we consider the matrix A_{V_a} for different thresholds V_a associated with different error ϵ. The idea is to show the advantages of the wMoM over the MoM in terms of computation time and memory usage. Therefore, for N increasing, i.e. the number of segments, we plot the sparseness of the matrix, its memory usage, and the computation time to solve the system, in Fig. 2 (a), (b), and (c), respectively. In each one, the plain line corresponds to the MoM with pulse functions, while the dotted ones correspond to the wMoM for different thresholds.

This shows that the wavelet basis allows obtaining a highly sparse matrix even for reasonable errors, such as $\epsilon = 10^{-5}$, leading to better efficiency in terms of both memory usage and computation time in this case. It shall be noted that wavelet basis also allows for better conditioning of the matrix [10]. To highlight this, we plot the condition number of the obtained matrix with pulsed functions and with the wavelets for different thresholds in Fig. 3.

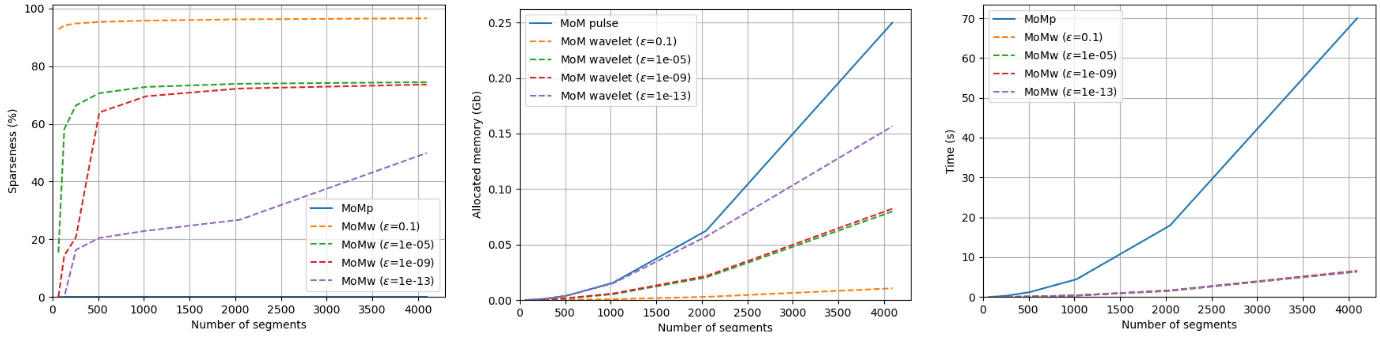
As can be seen in Fig. 3, using the wavelet allows for better conditioning of the system. Finally, the threshold can thus be seen as a cursor that allows choosing between computation efficiency and accuracy.

B. Validation of the method: a strip

First, let us a very simple case to validate the method. Indeed, we study the propagation from a complex source point (CSP) to a strip, our target here, of size $4\lambda = 4$ m. The source is placed at $x_s = -50$ m, $z_s = 256$ m, and has a width $w_0 = 5$ m. The strip is placed at $x_\text{t} = 15$ km and $z_\text{t} = z_s$, where z_t corresponds to the altitude of the center of the target. The parameters are chosen to have the strip directly in the beam, to be able to compare to the MoM to validate the hybridization. Indeed, SSW is used to compute the propagation to the strip, then the RCS of the target is computed with the wMoM. In Fig. 4, we thus plot the normalized RCS obtained with the hybrid SSW/wMoM, dotted blue line, method and with the MoM alone, dotted orange line.

As expected, both curves are closely matching, thus validating the hybridization between SSW and the wMoM. It shall be noted that for z, the step used in SSW was $\lambda/2$ while the mesh size used for the wMoM is $\lambda/258$. Hence, the proposed interpolation is working well and the SSW/wMoM method can now be used to account for more difficult conditions or targets.

To further validate the method, we consider a larger strip of size 8λ. As before, we plot the normalized RCS computed with the MoM and the hybridized SSW/wMoM method in Fig. 5.



(a) Sparseness of the matrix. (b) Memory usage of the matrix. (c) Computation time to solve $\mathbf{A}\mathbf{J}^w = \mathbf{V}$.

Fig. 2. Evolution of the sparseness of the matrix, its memory usage, and the computation time to solve the system with the number of segments.

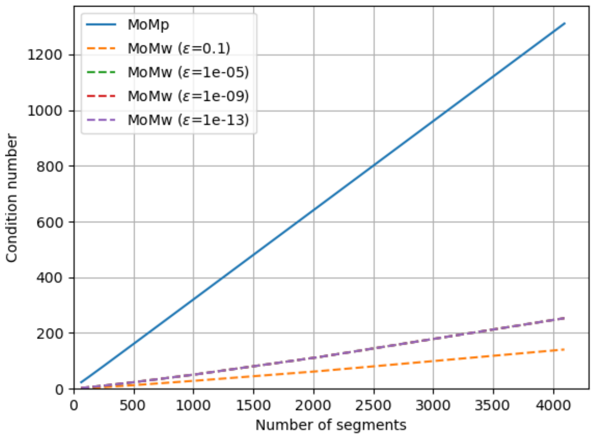


Fig. 3. Condition number of the matrix $\mathbf{A}$ when pulse or wavelet functions are used. For the wavelets, different thresholds are considered.

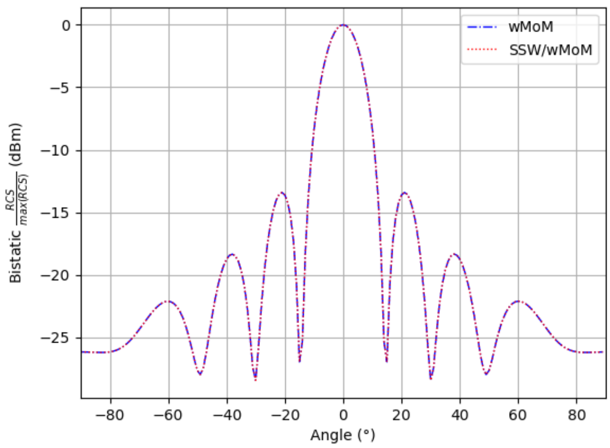


Fig. 4. RCS of the strip computed either with the hybrid SSW/wMoM or with the MoM.

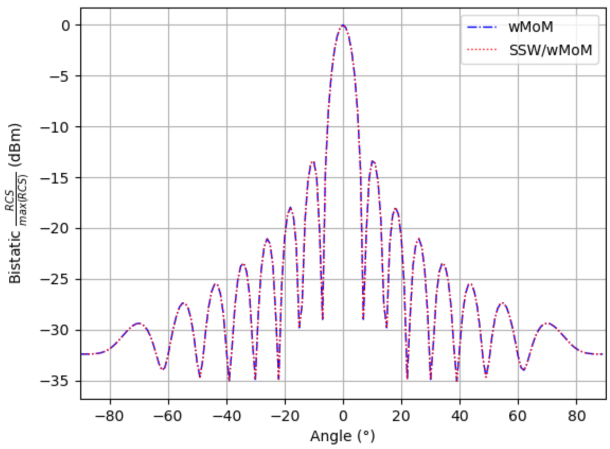


Fig. 5. RCS of the strip computed either with the hybrid SSW/wMoM or with the MoM.

As before, the RCS computed with the MoM and the hybrid SSW/wMoM closely match, validating the interpolation for the mesh size, and thus the hybridization. Here, the mesh size used for the wMoM is $\lambda/128$

IV. Conclusions

This paper presents a wavelet-based hybrid model for computing the RCS of a metallic target in a maritime environment. It combines two methods: the PWE solved with SSW for propagation to the target and the wMoM to compute the target's RCS.

Numerical experiments in the VHF band have been provided to highlight the method's advantages. Indeed, the compression allowed by using the wavelets improves the method's efficiency mainly regarding computation time and memory usage. Furthermore, the compression threshold allows for a cursor between accuracy and computation time. Therefore, this method could be used for larger targets.

Therefore, we are currently working on generalizing the method to account for more complex targets and phenomena. In particular, to account for the waves with the wMoM. Furthermore, we are working on hybridization in the wavelet domain to avoid passing by in the spatial domain to hybridize.

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REFERENCES

[1] M. Levy, *Parabolic equation methods for electromagnetic wave propagation*. No. 45, IET, 2000.

[2] J. R. Kuttler and G. D. Dockery, "Theoretical description of the parabolic approximation/fourier split-step method of representing electromagnetic propagation in the troposphere," *Radio science*, vol. 26, no. 2, pp. 381–393, 1991.

[3] H. Zhou, R. Douvenot, and A. Chabory, "Modeling the long-range wave propagation by a split-step wavelet method," *Journal of Computational Physics*, vol. 402, p. 109042, 2020.

[4] T. Bonnafont, R. Douvenot, and A. Chabory, "A local split-step wavelet method for the long range propagation simulation in 2d," *Radio science*, vol. 56, no. 2, pp. 1–11, 2021.

[5] T. Bonnafont, O. Benhmammouch, and A. Khenchaf, "A two-way split-step wavelet scheme for tropospheric long-range propagation in various environments," *Remote Sensing*, vol. 14, no. 11, p. 2686, 2022.

[6] E. Vasilev, V. Solodukhov, and A. Fedorenko, "The integral equation method in the problem of electromagnetic waves diffraction by complex bodies," *Electromagnetics*, vol. 11, no. 2, pp. 161–182, 1991.

[7] W. C. Gibson, *The method of moments in electromagnetics*. Chapman and Hall/CRC, 2021.

[8] J. S. Asvestas, "The physical optics method in electromagnetic scattering," *Journal of Mathematical Physics*, vol. 21, no. 2, pp. 290–299, 1980.

[9] J. Gutiérrez-Meana, J. Á. Martínez-Lorenzo, and F. Las-Heras, "High frequency techniques: The physical optics approximation and the modified equivalent current approximation (meca)," *Electromagnetic waves propagation in complex matter*, pp. 207–230, 2011.

[10] T. K. Sarkar, M. Salazar-Palma, and M. C. Wicks, *Wavelet applications in engineering electromagnetics*. Artech House, 2002.

[11] V. Fabbro, P. Combes, and N. Guillet, "Apparent radar cross section of a large target illuminated by a surface wave above the sea," *Progress In Electromagnetics Research*, vol. 50, pp. 41–60, 2005.

[12] L. Claudepierre, R. Douvenot, A. Chabory, and C. Morlaas, "A deterministic vor error modeling method—application to wind turbines," *IEEE transactions on Aerospace and Electronic Systems*, vol. 53, no. 1, pp. 247–257, 2017.

[13] H. Yu, C. Liao, J. Feng, Y. Shang, and Q. Ouyang, "A novel 3d hybrid approach for simulating electromagnetic scattering from electrically large targets in ducting maritime environments," *IEEE Antennas and Wireless Propagation Letters*, 2024.

[14] T. Bonnafont, R. Douvenot, and A. Chabory, "Determination of the thresholds in split-step wavelet to assess accuracy for long-range propagation," *Radio Science Letters*, vol. 3, 2021.

[15] A. J. Poggio and E. K. Miller, *Integral equation solutions of three-dimensional scattering problems*. MB Assoc., 1970.

[16] B. Z. Steinberg and Y. Leviatan, "On the use of wavelet expansions in the method of moments (em scattering)," *IEEE Transactions on Antennas and Propagation*, vol. 41, no. 5, pp. 610–619, 1993.

[17] S. Mallat, *A wavelet tour of signal processing*. Academic Press, 1999.

[18] A. Cohen, *Numerical Analysis of Wavelet Methods*. Elsevier, 2003.