Walking Dead

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1 Problem Description

The Walking Dead universe is an m ($2 \leq m \leq 2000$) by m matrix map, the distance from location (x, y) to (a, b) is computed by $max(|x - a|, |y - b|)$. There are multiple supply stations at different locations, and each of them has a supply range k (which means that if a location is covered by this supply station, the distance of this location from the supply station is at most k). Rick Grimes starts at a certain location, and he is trying to get to his destination. To ensure his safety and survival on the road, any cell he steps foot on must be within range of at least n supply stations. He must also actively avoid a growing zombie horde in order to reach his destination. If a cell on the map contains a zombie, it will spread to all four adjacent cells in the next second. There are multiple zombies in the beginning. Each step Rick Grimes takes must be from a position in the matrix to any of the four neighboring cells (top, bottom, left, and right), and will take one second. Neither zombies nor Rick may step into a supply station. Initially, Rick's starting location, destination, supply stations, and zombies are at distinct locations. Can Rick get to the destination before zombies get to him?

The first line of input has three integers m ($2 \leq m \leq 2000$), n ($1 \leq n \leq 10000$), and k ($1 \leq k \leq m$) separated by a single space. The next m lines each has a string of length m consisted of characters r (Rick), d (destination), s (supply station), z (zombie), or x (empty location). Determine if there exists such a path. If not, output "*impossible*". If multiple paths exist, output the shortest path. Each step to the cell above, output 't'; each step to the cell below, output 'b'; each step to the cell on the left, output 'l'; each step to the cell on the right, output 'r'. If there exists multiple shortest paths, output the lexicographical smallest one.

2 Solutions

- First use a $m \times m$ matrix to store a prefix sum array for each row by enumerating supply stations and adding and subtracting $+1$ and -1 at the corresponding four corners. Time complexity for this step is $O(m^2)$.

- Enumerate rows from top to bottom, compute prefix array, and then compute prefix sum for each location. Time complexity for this step is $O(m^2)$
- Note there exists an implicit graph consisting of all valid vertices (locations) with at least n supply stations within range and edges between neighbors.
- Run one BFS shortest path algorithm from the destination to all zombie positions on the graph given by valid positions in whole matrix; run BFS shortest path algorithm from the destination to the starting position on the graph given by the implicit valid (within range of n supply stations) positions graph. Compare Rick's shortest path length with the minimum of all zombies' shortest path lengths. Handle picking lexicographical smallest path during the process. Time complexity is $O(m^2)$.

3 Anti-Solutions

- Enumerate all squares within range $O(m)$ for every supply station. TLE $O(m^4)$
- Run a separate BFS SP for every zombie rather than running one BFS from the destination. TLE $O(m^4)$
- WA where output is not the lexicographical smallest path.

4 Test Case Design

Our test cases will be designed using the following processes:

- Three extremely small test cases that will be displayed to students with correct output provided.
- Three small test cases that are generated with random integers within the given constraints.
- Generate two "*impossible*" test cases. One with no valid path and one with zombies overrunning destination.
- One test case will have the maximum number of supply stations.
- One test case with multiple shortest paths, but only one lexicographical smallest.
- Generate two large test cases where it is possible to find a valid shortest path from source to destination.

A total of 9 secret cases will be generated.