Numerical Model of a Baseball Pitch

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Abstract

A numerical expression was developed to model a pitch thrown by a Major League Baseball pitcher. Given a set of initial values, future positions and velocities were determined using the improved euler midpoint method. These produced values were then plotted to show the effects of all forces on a baseball during flight. The main objective was to determine how the magnus force specifically affects the trajectory of a baseball. A fastball was compared to a curveball to determine the displacement of the baseball with spin to a baseball thrown with no spin. After the model was plotted, it was shown that the magnus effect has a large influence on the trajectory of a baseball as the curveball deviated 0.59 meters in the -$\hat{k}$ direction and 0.5 meters in the -$\hat{\jmath}$ direction.

Introduction

Using the forces present during projectile motion, the flight path of a baseball launched from a pitching machine will be numerically modeled. There are multiple forces that are present on an object during flight through air. These forces will determine the position, velocity, and acceleration values of the object during any moment of time in flight. The forces interacting with the baseball after launch were gravity, air drag, and the magnus force. The magnus force, which is caused due to the spin of the ball, was the main point of interest as we determined the effect that this force has on the end position of the baseball as it passed over home plate. For this model, the standard dimensions of a Major League baseball field were used with a 90 mph pitch as the standard. Using initial values for position, velocity, and angular velocity, a numerical model was produced using Euler's method. All computations were found by developing a program written in FORTRAN. These produced values were then plotted using Matlab[1]. Figure 1 displays the forces present on the ball during flight.

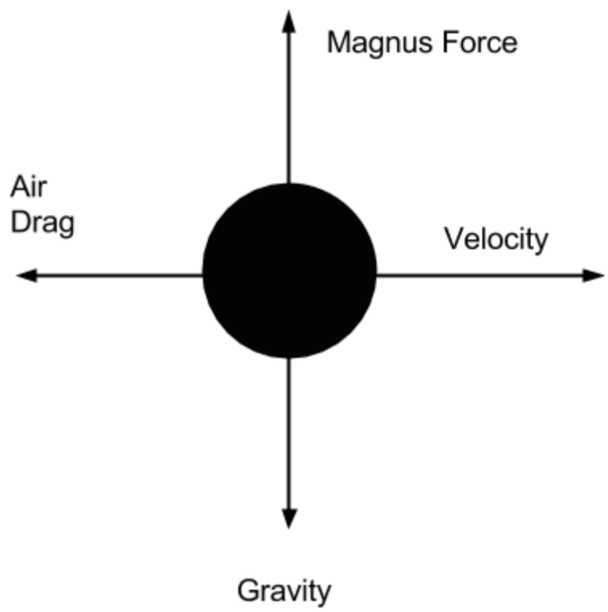

Figure 1: Free body diagram to show the forces present on a baseball as it travels through air. The baseball will be given an initial velocity and while in air the ball will be, pulled down by gravity, slowed down by air drag, and acted on by the magnus force depending on the direction of spin applied to the ball.

Methods

To accurately model the movement of a baseball, we needed to account for all forces acting on the ball through flight. These forces included gravity, air drag, and the magnus force. By summing all of the forces together, we produced a net force on the ball in each direction. These directional forces could then be used to develop future points in time in order to chart the flight path of the baseball. In this model, we used the Euler midpoint method in order to solve for future points in position. This was developed by first beginning with the kinematic equation,

$$R_f = R_0 + v\Delta t \tag{1}$$

Where R is the radial point that can be decomposed into X, Y, and Z components. However, in order to determine the values for velocity, an expression for the acceleration in each direction must be found. These directional acceleration values are found through Newton's law,

$$F = m\ddot{R} \tag{2}$$

Where again, R is defined as the radial distance which can be decomposed. In order to solve this equation for the directional accelerations, each force must be express completely.

Air Drag and Gravity

Air drag is one of the forces acting on the baseball as it travels through the medium. This force acts opposite the direction of motion. The drag on the baseball is expressed as,

$$F_D = \frac{1}{2} C_D \rho \mathrm{A} (\vec{v} \cdot \vec{v}) \tag{3}$$

Where C_D is the drag coefficient, ρ is the density of air, A is the cross sectional area of the baseball, and $(\vec{v} \cdot \vec{v})$ is the v^2 term, however we are treating the velocity as a vector.

Gravity also acts on the baseball and it traverses through the air. The force of gravity is defined as,

$$F_g = mg \tag{4}$$

Magnus Effect

The Magnus effect is a direct result of the spin applied to an object in projectile motion. This effect has a very profound result on the motion of an object in reference to one without spin. The resulting force can be expressed as [2],

$$F_m = \frac{1}{2} C_L \rho \mathrm{A} (\vec{v} \cdot \vec{v}) \ (\vec{\omega} \times \vec{v}) \tag{5}$$

The constants present are nearly the same with respect to air drag, however, the Magnus force also has a associated term that depends on the angular and linear velocity of the object. This cross product was found using,

$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\omega_x}{|\omega_x|} & \frac{\omega_y}{|\omega_y|} & \frac{\omega_z}{|\omega_z|} \\ \frac{v_x}{|v_x|} & \frac{v_y}{|v_y|} & \frac{v_z}{|v_z|} \end{vmatrix} \tag{6}$$

Net Force

After defining each force individually, all forces could be combined to produce a net force on the baseball during flight. This net force would then carry separate directions for each force (drag, Magnus, gravity, etc) throughout flight. This net force was expressed as,

$$\ddot{R} = -[(\frac{C_D \rho A(\vec{v}\cdot\vec{v})^{1/2}}{2m})(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})] - [g]\,\hat{k} +$$
$$\left[\left(\frac{C_L \rho A(\vec{v}\cdot\vec{v})}{2m|w||v|}\right)\left([w_y v_z - w_z v_y]\,\hat{i} - [w_x v_z - w_z v_x]\,\hat{j} + [w_x v_y - w_y v_x]\,\hat{k}\right)\right] \tag{7}$$

This expression can then be decomposed into three expressions for each direction $(\hat{i}, \hat{j}, \hat{k})$,

$$\begin{aligned}
a_x &= \left[\left(\frac{C_D\,\rho\,A\,(\vec{v}\cdot\vec{v})^{1/2}}{2m}\right)(v_x\,\hat{i})\ + \left(\frac{C_L \rho A(\vec{v}\cdot\vec{v})^{1/2}}{2m|w||v|}\right)[w_y v_z - w_z v_y]\,\hat{i}\right] \\
a_y &= -\left[\left(\frac{C_D\,\rho\,A\,(\vec{v}\cdot\vec{v})^{1/2}}{2m}\right)(v_y\,\hat{j})\ + \left(\frac{C_L \rho A(\vec{v}\cdot\vec{v})^{1/2}}{2m|w||v|}\right)[w_x v_z - w_z v_x]\,\hat{j}\right] \\
a_z &= \left[\left(\frac{C_D\,\rho\,A\,(\vec{v}\cdot\vec{v})^{1/2}}{2m}\right)(v_z\,\hat{k})\ - [g]\,\hat{k}\left(\frac{C_L \rho A(\vec{v}\cdot\vec{v})^{1/2}}{2m|w||v|}\right)[w_x v_y - w_y v_x]\,\hat{k}\right]
\end{aligned} \tag{8}$$

After decomposing the acceleration vector into its components, we then can apply the kinematic equation and apply the euler time step to determine the next velocity point (i+1) in each direction as well.

$$\begin{aligned}
v_x(i+1) &= v_x(i) + a_x h \\
v_y(i+1) &= v_y(i) + a_y h \\
v_z(i+1) &= v_z(i) + a_z h
\end{aligned} \tag{9}$$

where i is the index counter and h is the time step through euler's method. After solving for these velocity equations, it is then possible to produce position vectors for the each step of time. Euler midpoint was used to determine the next point, this is described as,

$$R_{i+1} = R_i + \frac{v_{i+1} + v_i}{2}\,t \tag{10}$$

After each position is calculated for each step in time, the values were plotted via Matlab to produce trajectory paths.

Results

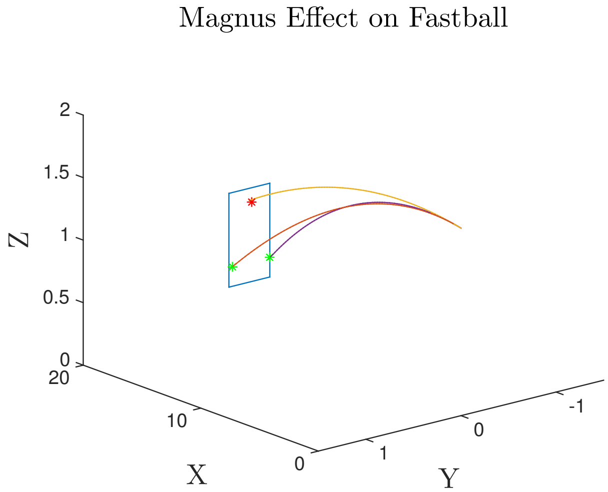

Figure 2: The effect of the magnus force are seen as fastball with no spin is compared to two pitches with different directions of spin. The ball is placed in the lower left hand corner of the zone when a $+\hat{k}$ spin is applied and in the lower right hand corner when a -$\hat{k}$ spin is applied.

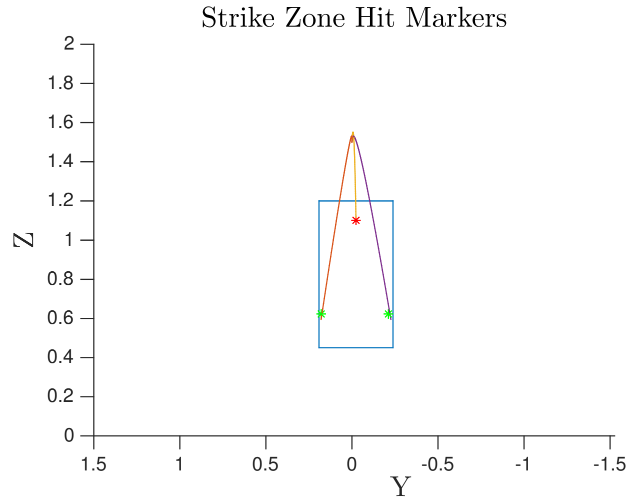

Figure 3: This figure shows where the ending location of the ball is when it crosses home plate. It is shown that all pitches will be called a strike, and depending on which preference of a batter is up (either left or right handed), the bottom two pitches will be more difficult to hit.

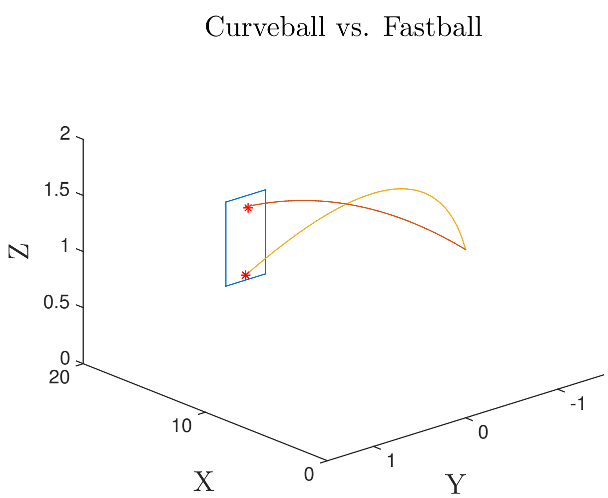

Figure 4: By adjusting the ratio of $\hat{j}$ to $\hat{k}$ spin, a frightening curveball can be thrown into the strike zone.

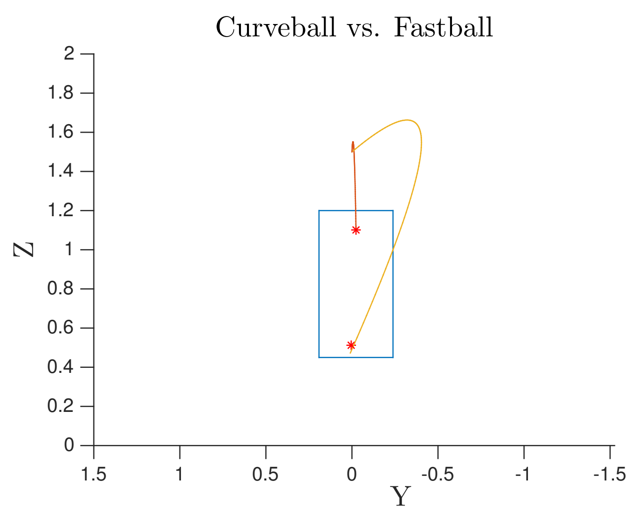

Figure 5: By only adjusting the spin on the baseball, it is possible to obtain a curveball that sinks .59 m down with a maximum y displacement of .5 m, making this curveball nearly impossible to hit.

Table 1: Parameters of Model

Air Density (ρ) $[\frac{kg}{m^3}]$	Drag Coefficient (C_D)	Magnus Coefficient (C_L)	Mass (m) [kg]	Cross Sectional Area (A) $[m^2]$	Number of iterations (n)	Time Step (h)
1.2	0.25	0.12	0.145	1.25×10^{-3}	10000	0.001

Table 2: Initial Values for Numerical Model

	$v_x[\frac{m}{s}]$	$v_y[\frac{m}{s}]$	$v_z[\frac{m}{s}]$	$\omega_x[\frac{rad}{s}]$	$\omega_y[\frac{rad}{s}]$	$\omega_z[\frac{rad}{s}]$	$r_x[m]$	$r_y[m]$	$r_z[m]$
Fastball	45	0	1	0	0	0	0	0	1.5
Left Fastball	45	0	1	0	5	2	0	0	1.5
Right Fastball	45	0	1	0	5	-2	0	0	1.5
Curveball	45	-4	3	14	0	16	0	0	1.5

From the plotted points, it was shown that the Magnus effect is extremely influential on the trajectory of a baseball. Figure 2 shows just how the spin on a baseball can control its final position. In this figure, it is shown that without changing the initial velocity from a regular fastball it is possible to throw a pitch that is nearly impossible to hit. Figure 5 specifically shows how spin can force the ball to displace 0.5 meters in the y direction before bending back into the box for a strike. Figure 3 shows the z displacement of the fastball with only an applied spin. This force is able to displace the baseball 0.5 meters and still deliver a strike across home plate.

Conclusion

The displacements achieved through only the produced magnus force prove conclusively that the spin placed on the ball is extremely important in determining the final position of the pitch. By applying spin to a regular fastball, we see vast differences in resulting pitch. When the magnus effect is applied to a pitch, it is shown just how difficult it would be to hit a 90 mph curveball.

References and Notes

1. MATLAB and Statistics Toolbox Release 2012b, The MathWorks, Inc., Natick, Massachusetts, United States.

2. Nathan, Alan. *The effect of spin on the flight of a baseball*. Department of Physics, University of Illinois, Urbana, Illinois 61801 13 October 2007.