Συστήματα Αναμονής (Queuing Systems)

2ο Εργαστήριο
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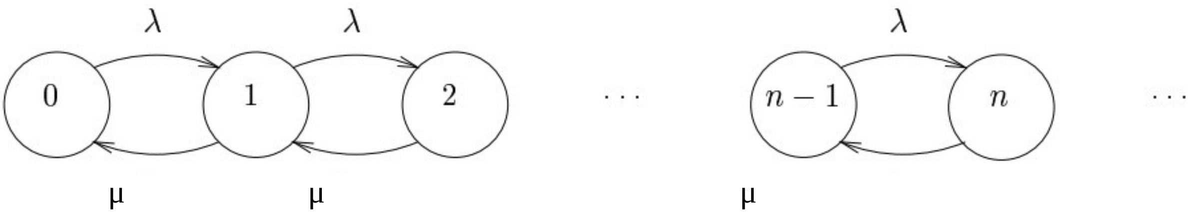
Θεωρητική μελέτη της ουράς M/M/1

α) Προκειμένου να είναι η ουρά M/M/1 εργοδική πρέπει $\rho = \frac{\lambda}{\mu} < 1$



Έτσι, έχουμε Κατανομή Poisson με παράμετρο λ πελάτες ανά δευτερόλεπτο και οι εξυπηρετήσεις εκθετική κατανομή με παράμετρο μ πελάτες το δευτερόλεπτο.

$\lambda_0 = \lambda_1 = ... = \lambda_i = ... = \lambda, i = 1,2,3,\ldots$

και $\mu_0 = \mu_1 = ... = \mu_i = ... = \mu, i = 1,2,3,\ldots$

Χρησιμοποιώντας τις εξισώσεις ισορροπίας (τοπικές και γενικές) είναι $\lambda P_{i-1} = \mu P_i, i = 1,2,3,\ldots$

και $(\lambda_k + \mu_k) P_k = \lambda_{k-1} P_{k-1} + \mu_{k+1} P_{k+1},\ k = 1,2,3,\ldots$.

Επιπλέον έχουμε την κανονικοποίηση της εργοδοτικής πιθανότητας $P_0 + P_1 + ... + P_N = 1$ έχουμε

$$\lambda P_0 = \mu P_1 \Rightarrow P_1 = \frac{\lambda}{\mu} P_0$$

$$(\lambda + \mu) P_1 = \lambda P_0 + \mu P_2 \Rightarrow P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0 = \rho^2 P_0 \Rightarrow \text{επαγωγικά προκύπτει } P_k = \rho^k P_0$$

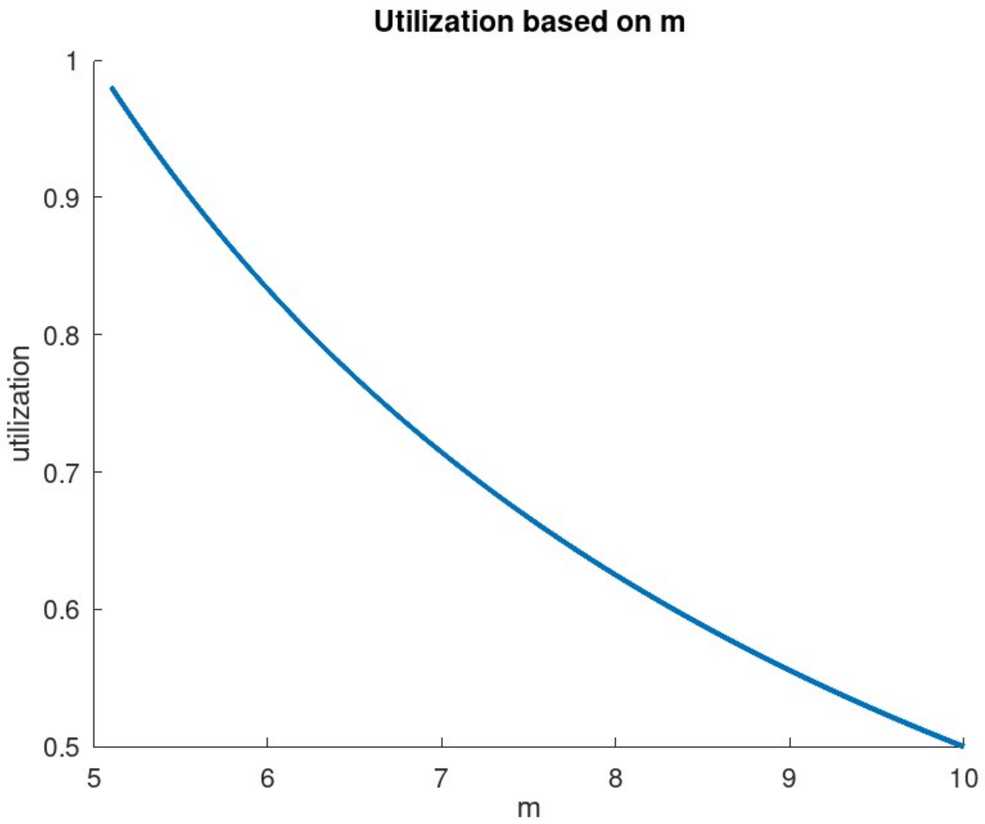
$$\frac{1}{1-\rho} P_0 = 1 \Rightarrow P_0 = 1 - \rho \Rightarrow P_k = (1-\rho)\rho^k, k > 0 \text{ and } P(n(t) > 0) = 1 - P_0 = \rho$$

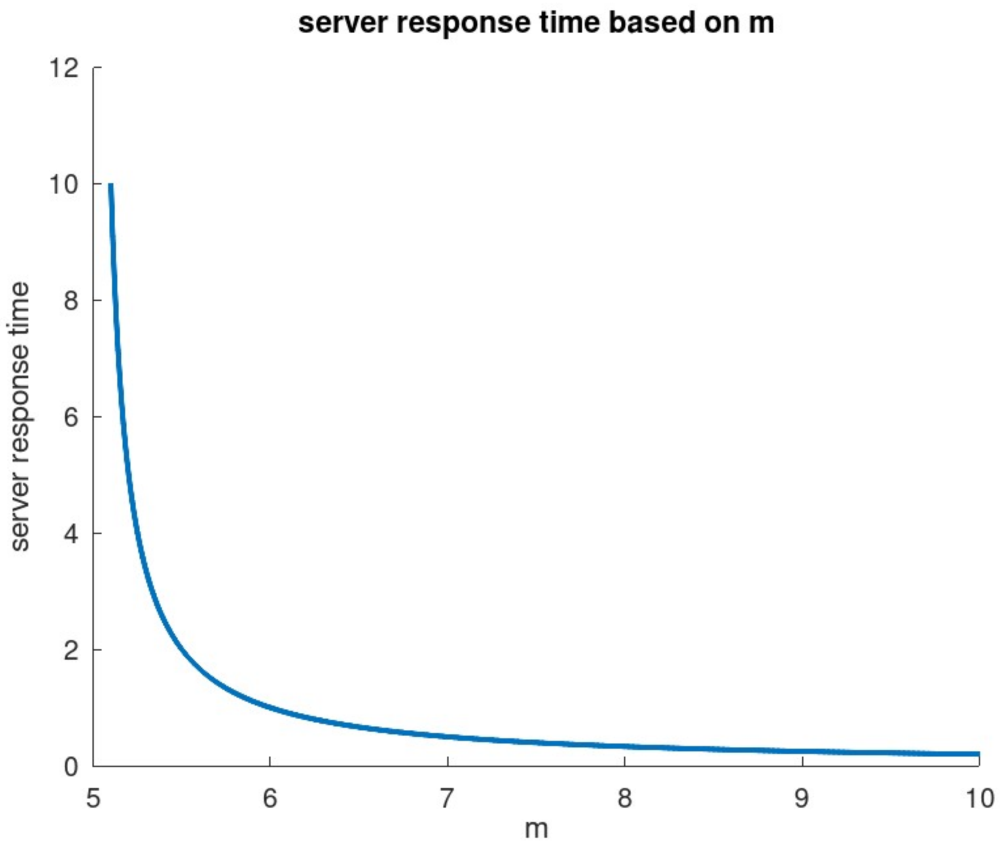
β) Από τον Little’s Law ξέρουμε πως ο μέσος χρόνος αναμονής ενός πελάτη στο σύστημα σε ισορροπία είναι $E(T) = E\frac{[n(t)]}{\gamma} = E\frac{[n(t)]}{\lambda} = \frac{1}{\mu}(1-\rho)$

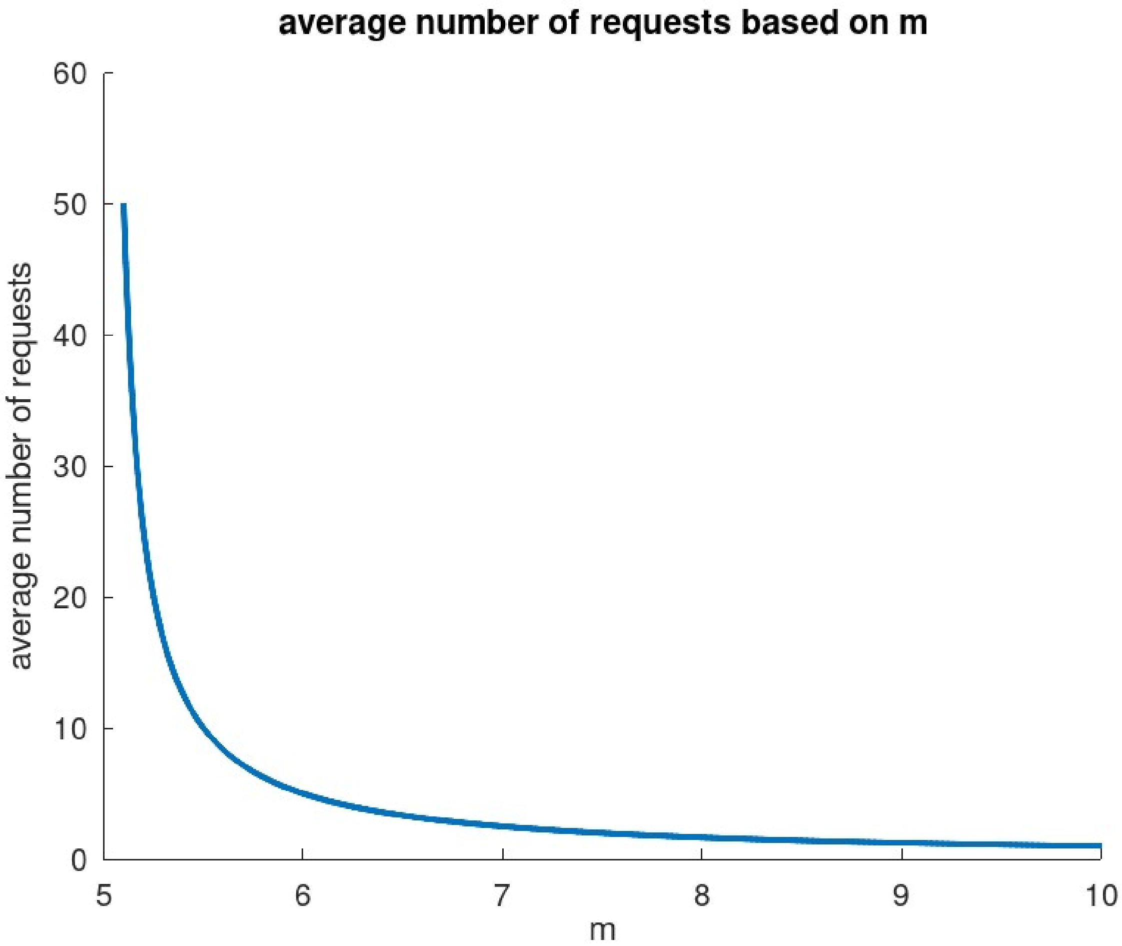
γ) Για k = 57 είναι $P_{57} = (1-\rho)\rho^{57}$ που μας οδηγεί σε μια πιθανότητα θετική που είναι να έχουμε 57 πελάτες.

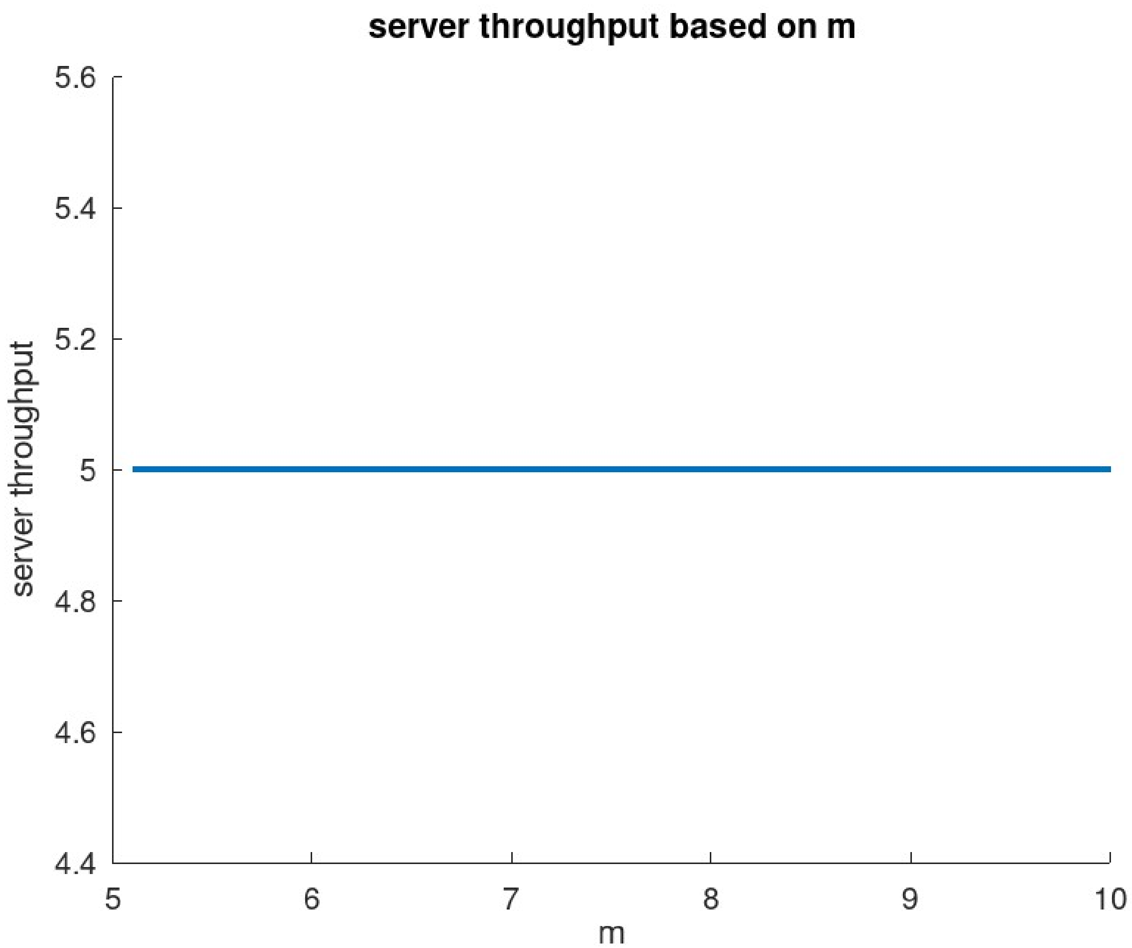
Ανάλυση ουράς M/M/1 με Octave

α) Από πριν έχουμε πως πρέπει μ > λ για να είναι εργοδοτικό. Επιλέγουμε 50 δείγματα που οδηγούν σε 10 πελάτες το λεπτό.
β) (Το m = μ)






average number of requests based on m
average number of requests
60
50
40
30
20
10
0
5
6
7
8
9
10
m


server throughput based on m
server throughput
5.6
5.4
5.2
5
4.8
4.6
4.4
5
6
7
8
9
10
m

γ) Παρατηρούμε πως ο μέσος χρόνος

δ) Γνωρίζουμε πως γ = λ(1 – P(χασουμε πελάτη)). Σε ένα M/M/1 σύστημα, η ουρά έχει άπειρη χωρητικότητα οπότε P(χασουμε πελάτη) = 0 και έτσι το throughput είναι σταθερό και ίσο με λ

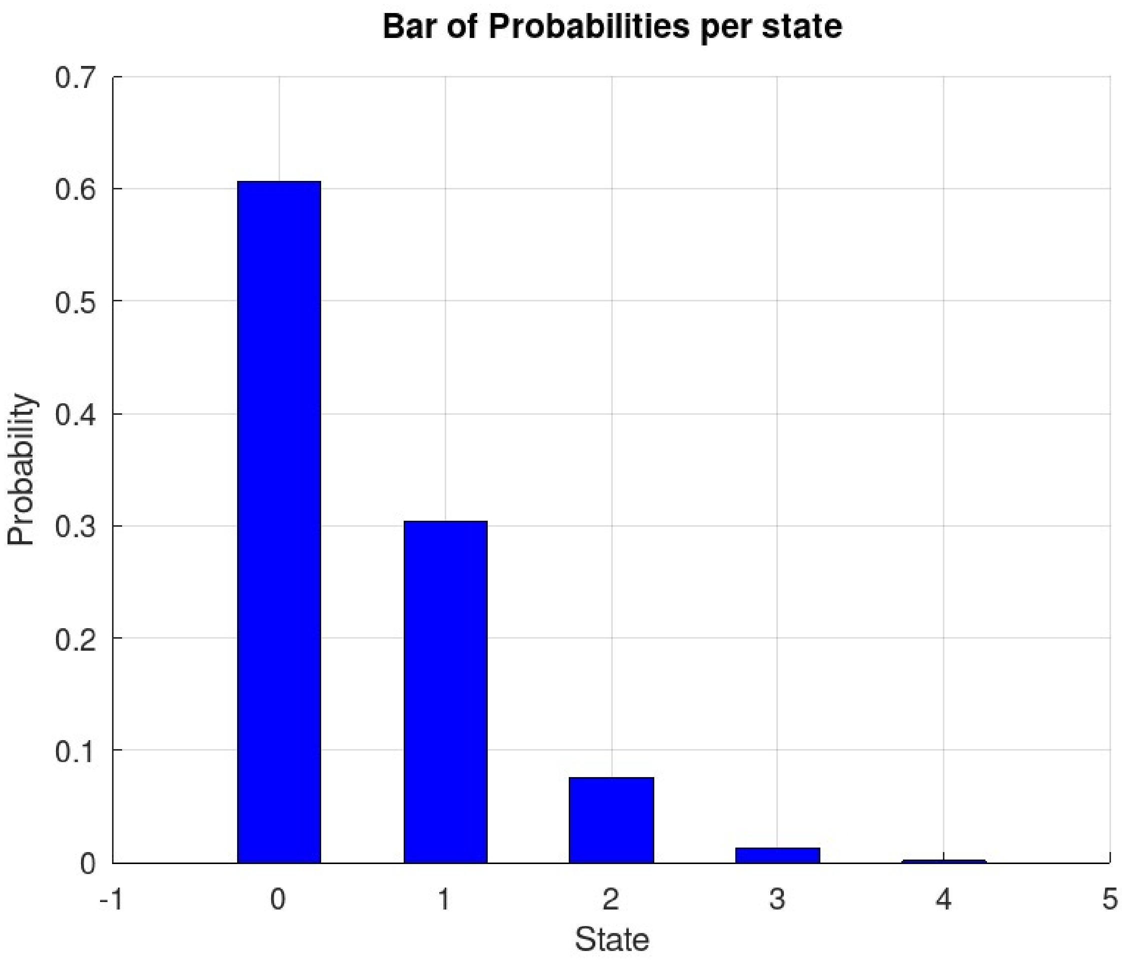
Κώδικας

```
clc;
clear all;
close all;
pkg load statistics
pkg load queueing

lamda = 5;
utilization = [0,500];
server_response_time = [0,500];
average_number_of_requests = [0,500];
server_throughput = [0,500];

m = [5.1:0.01:10];

for i=1:columns(m)
   [utilization(i),server_response_time(i),average_number_of_requests(i),server_throughput(i)] =
qsmm1(lamda, m(i));
endfor

figure(1);
hold on;
title("Utilization based on m");
plot(m,utilization,"linewidth", 2.2);
xlabel("m");
ylabel("utilization");
hold off;

figure(2) ;
hold on;
title("server response time based on m");
plot(m,server_response_time,"linewidth", 2.2);
xlabel("m");
ylabel("server response time");
hold off;

figure(3) ;
hold on;
title("average number of requests based on m");
plot(m,average_number_of_requests,"linewidth", 2.2);
xlabel("m");
ylabel("average number of requests");
hold off;
figure(4);
hold on;
title("server throughput based on m");
plot(m,server_throughput,"linewidth", 2.2);
xlabel("m");
ylabel("server throughput");
hold off;
```

β)
i) To transition array είναι:

-5.0000	5.0000	0	0	0
10.0000	-12.5000	2.5000	0	0
0	10.0000	-11.6667	1.6667	0
0	0	10.0000	-11.2500	1.2500
0	0	0	10.0000	-10.0000

ii)

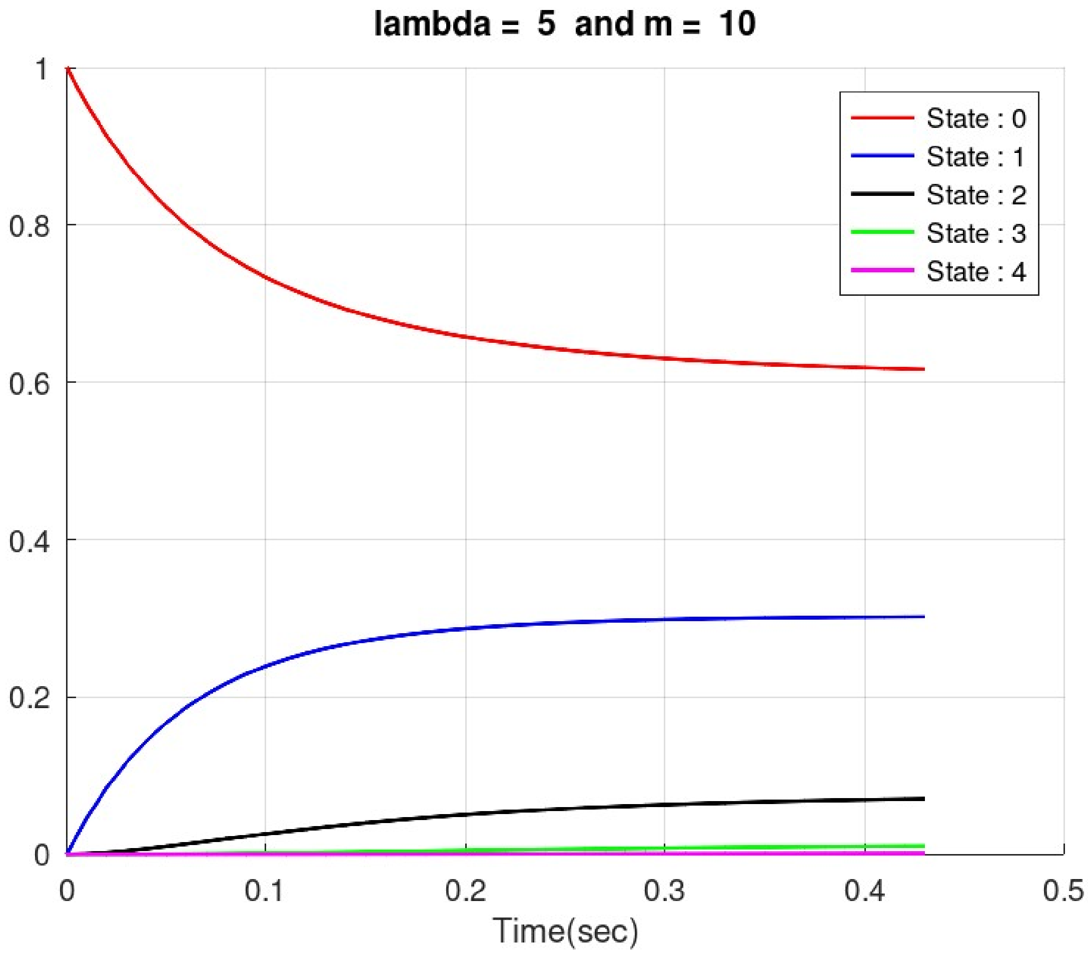


iii)
P = 6.0664e-01   3.0332e-01   7.5829e-02   1.2638e-02   1.

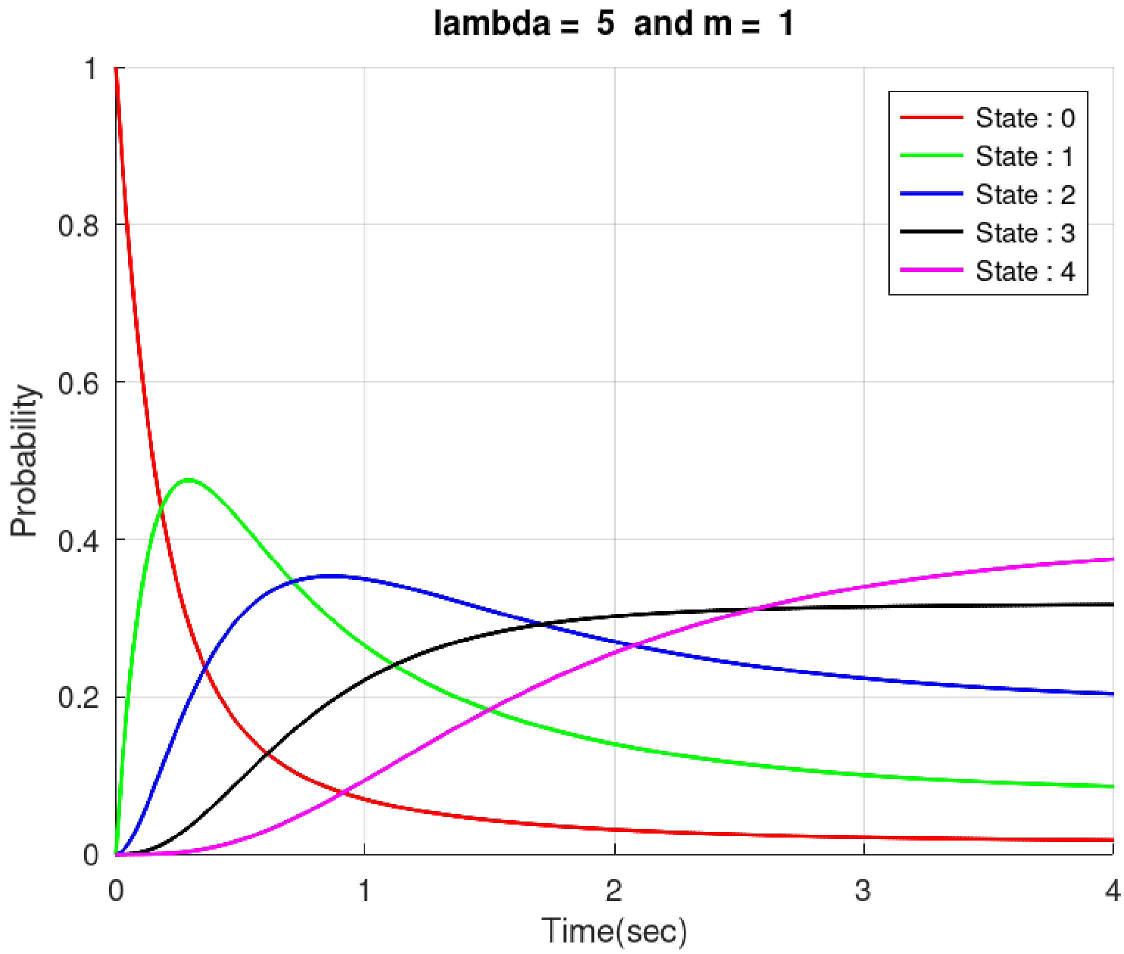
Average Number of customers in the system : Σ k Pk = 0.4992

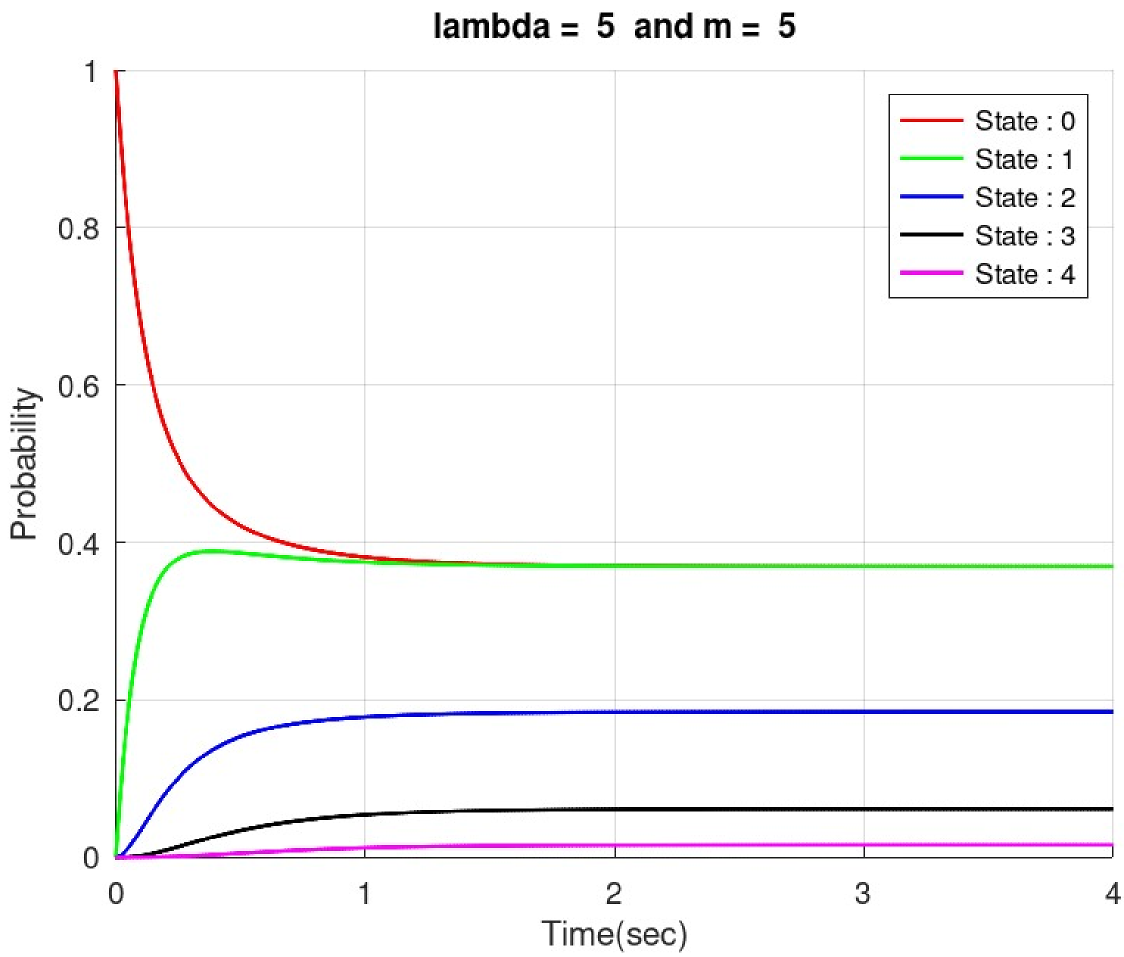
iv) P4 = 0.0012638

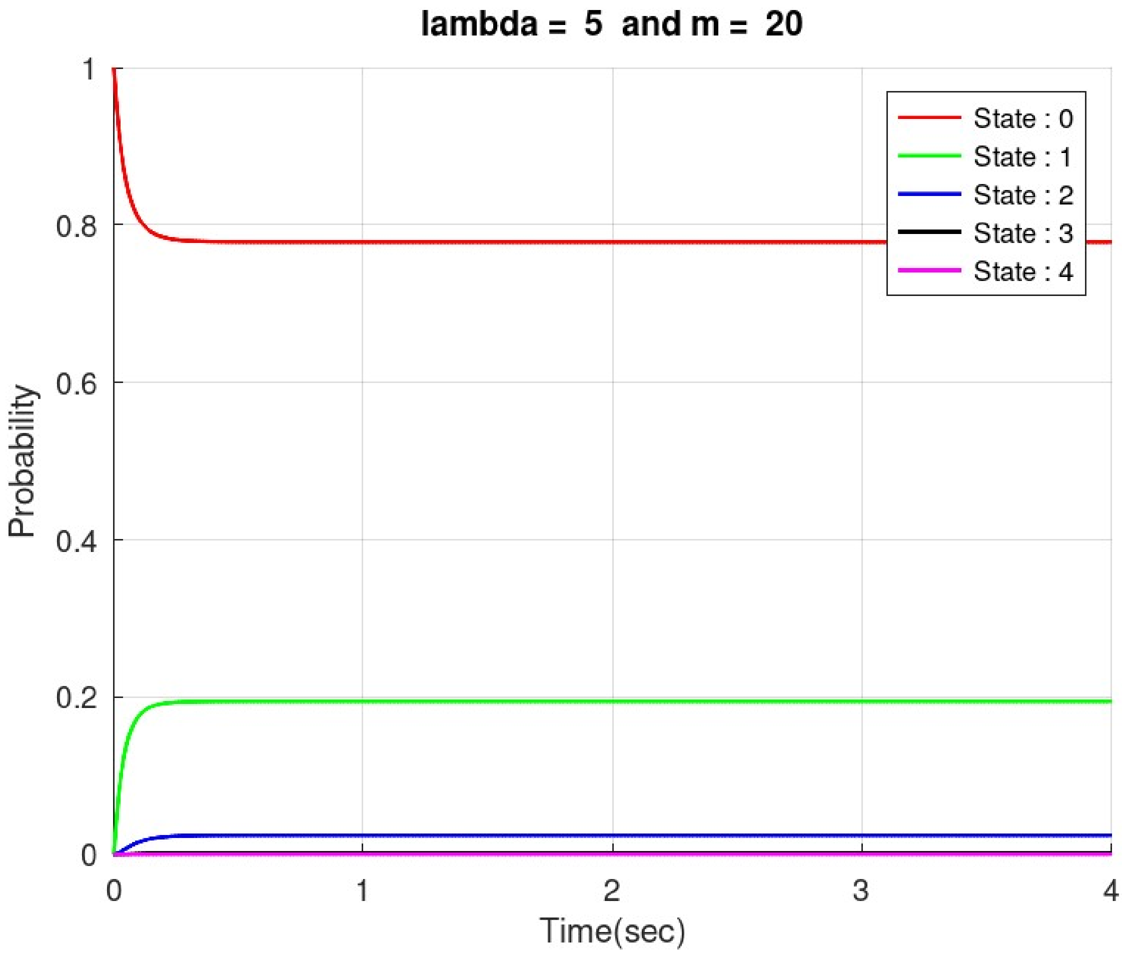
v)



vi)


lambda = 5 and m = 1
State : 0
State : 1
State : 2
State : 3
State : 4
Probability
Time(sec)
0
0.2
0.4
0.6
0.8
1
1
2
3
4


lambda = 5 and m = 5
State : 0
State : 1
State : 2
State : 3
State : 4
Probability
Time(sec)
0
0.2
0.4
0.6
0.8
1
1
2
3
4



Κωδικας

```
clc;
clear all;
close all;

pkg load statistics
pkg load queueing

lambda = 5;
m = 10;
states = [0, 1, 2, 3, 4]; % system with capacity 4 states
% the initial state of the system. The system is initially empty.
initial_state = [1, 0, 0, 0, 0];

% define the birth and death rates between the states of the system.
births_B = [lambda, lambda/2, lambda/3, lambda/4];
deaths_D = [m, m, m, m];

% get the transition matrix of the birth-death process
transition_matrix = ctmcbd(births_B, deaths_D);
display (transition_matrix)

# (ii)
% get the ergodic probabilities of the system
P = ctmc(transition_matrix);
display (P);
```

```
figure(1);
hold on;
title("Bar of Probabilities per state")
xlabel("State")
ylabel("Probability")
bar(states, P, "b", 0.5);
grid on;
hold off;
# (iii)
display( " Average Number of customers in the system : ")
display( sum(P.*[0,1,2,3,4]))

# (iv)
display( " Probability of blocking a customer :")
display( P(5) )

%P[Blocking]

P_Blocking = P(5);
display(P_Blocking)

index = 0;
for T = 0 : 0.01 : 50
  index = index + 1;
  Po = ctmc(transition_matrix, T, initial_state);
  Prob0(index) = Po(1);
  Prob1(index) = Po(2);
  Prob2(index) = Po(3);
  Prob3(index) = Po(4);
  Prob4(index) = Po(5);
  if Po - P < 0.01
    break;
  endif
endfor

T = 0 : 0.01 : T;
figure(2);
title(strjoin({"lambda = ",num2str(lambda)," and m = ",num2str(m)}))
xlabel("Time(sec)")
hold on;
plot(T, Prob0, "r", "linewidth", 1.5);
plot(T, Prob1, "b", "linewidth", 1.5);
plot(T, Prob2, "k", "linewidth", 1.5);
plot(T, Prob3, "g", "linewidth", 1.5);
plot(T, Prob4, "m", "linewidth", 1.5);
legend("State : 0","State : 1","State : 2","State : 3","State : 4");
grid on;
hold on;


m = [1,5,20];
for i=1:columns(m)
  deaths_D = [m(i), m(i), m(i), m(i)];
  transition_matrix = ctmcbd(births_B, deaths_D);
  index = 0;
  for T = 0 : 0.01 : 4
    index = index + 1;
```

```
    P0 = ctmc(transition_matrix, T, initial_state);
    Prob0(index) = P0(1);
    Prob1(index) = P0(2);
    Prob2(index) = P0(3);
    Prob3(index) = P0(4);
    Prob4(index) = P0(5);
    if P0 - P < 0.01
      break;
    endif
  endfor


  T = 0 : 0.01 : T;
  figure(i+2);
  title(strjoin({"lambda = ",num2str(lambda)," and m = ",num2str(m(i))}))
  xlabel("Time(sec)")
  ylabel("Probability")
  hold on;
  plot(T, Prob0, "r", "linewidth", 1.5);
  plot(T, Prob1, "g", "linewidth", 1.5);
  plot(T, Prob2, "b", "linewidth", 1.5);
  plot(T, Prob3, "k", "linewidth", 1.5);
  plot(T, Prob4, "m", "linewidth", 1.5);
  legend("State : 0","State : 1","State : 2","State : 3","State : 4");
  grid on;
  hold off;
endfor
```