The Subminimum Wage and Employment

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Abstract:

To describe the employment effects of the minimum wage, researchers currently rely on the assumption that there is a singular effective minimum wage rate within a state or city. In practice, the federal Fair Labor Standards Act and state level policy suggest a vector of possible minimum wages that may be paid to a class of worker based on special characteristics such as firm headcount, firm revenue, visa status, age, or disability status. I introduce a framework to research the effects of a vector of minimum wage tiers on employment, using the stacked differences estimator used by Cengiz, Dube, Lindner, and Zipperer. I expand on this estimator by adding an identification of workers allowed to be paid at the lower minimum wage rate, or those in between. Results suggest there is still no unemployment effect associated with an increase in the headline minimum wage, and some models suggest no effect from the subminimum, although these results are sensitive to model specification.

JEL Codes: J31 - Wage level and structure, J38 - Public Policy, J71 – Labor Discrimination

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1 INTRODUCTION

In 1994, David Card and Alan Krueger studied the change in unemployment in the food service industries of New Jersey and Pennsylvania in response to an increase in the New Jersey minimum wage (Card & Krueger, 1994). Their study relied on distance and on statewide variation in the minimum wage to identify the change in employment, under the assertion that macroeconomic factors would be comparable between states due to their proximity. Their results, that there was no statistically significant change in unemployment in response to the minimum wage, sparked a debate in the economic community that continues today.

Current researchers expand Card and Krueger's scope to include changes in the minimum wage across the nation. Those who use similar geographic controls trend to find negligible effects of employment from the minimum wage (Dube, Lester, & Reich, 2010) (Allegretto, Dube, Reich, & Zipperer, 2015) (Gittings & Schmutte, 2015). Those opposed to these geographic regression discontinuities prefer synthetic control methods or pooled fixed effects by state and year, and tend to find a significant and negative change in employment in response to the minimum wage, around a 0.2% drop in total employment for each 10% minimum wage increase (Neumark, Salas, & Wascher, 2014) (Meer & West, 2016).

Although the debate over geographic heterogeneity has provided valuable insights into the nature of employment in the United States, each of these studies assume the effective minimum wage within a state is a single number[1]. The reality is that within a state, even within a single firm, the minimum wage that may be paid to an employee may vary. This suggests that regressions that use a single minimum wage rate may inaccurately identify the economic effects of a minimum wage regime. Although workers that are eligible for this subminimum wage are few, their sensitivity to the effects of a minimum wage is worthy of investigation precisely because they represent the difference between the minimum wage as written vs as implemented by policy.

[1] Apart from those who study the tipped minimum wage credit, which I will discuss momentarily.

Does a tiered approach to the minimum wage preserve employment among targeted groups, or is employment unaffected exemptions to the minimum wage? In other words, are minimum wage tiers necessary to preserve employability of chosen policy groups, or does this policy only serve to transfer rents from employees to employers?

Previous literature specifically on this subject consists of studies from the early 1990s, after the federal government established an exemption to the minimum wage for training purposes. These studies include a survey of businesses from the Yellow Pages (Freeman, Gray, & Ichniowski, 1981), and a survey of select restaurant franchise locations (Spriggs, 1993). These studies find the training exemption to the minimum wage was often deemed too cumbersome by employers and was thereby seldom adopted. Finally, the subminimum wage has been studied recently in the context of states that allow a lower minimum wage for tipped workers using a geographic discontinuity model with parameters for the log of minimum wage and for the log of the tipped minimum wage (Allegretto & Nadler, 2015).

I contribute to the literature on the subminimum wage in three ways. First, I estimate the employment effects of minimum wage tiers on a statewide scale, estimating the effects on overall state employment, not just employment within a sector or within a franchise chain. Second, I apply the Stacked Difference in Difference estimator instead of border discontinuity Difference in Difference or Pooled Fixed Effects. Third, although the tip credit allows workers to be paid less than the minimum by their employer, the employer is still responsible for covering wages if tips and wages fall below the minimum. My paper is on a different kind of subminimum wage, in that take home hourly pay is allowed to fall below the headline minimum for select classes of worker.

Using historical measures of the federal and state minimum wages, employment data from the Current Population Survey, and measures of subminimum wage tiers from the Bureau of Labor Statistics, my paper will use the stacked difference in difference estimator to control for the level of employment at different wage levels, and derive a second measure for wage groups that are allowed to be paid the lowest minimum wage according to their eligibility (Cengiz, Dube, Lindner, & Zipperer, 2019). This will provide a measure of the relative aptness of including a vector of measures of the minimum wage within a state.

In section 2, I discuss current theories surrounding the tiered minimum wage, and summarize the economic implications of these theories. Section 3 details the theory behind labor demand. In section 4, I list the empirical methodology deployed to estimate and identify the effect of the tiered minimum wage. Section 5 displays the results of the empirical analysis described in section 4, and section 6 includes results from similar specifications as a robustness test. Section 7 details conclusions that can be drawn from the analysis of this paper.

2 POLICY BACKGROUND

Before detailing the features of my empirical model, it is prudent to distinguish between some of the key concepts in recent minimum wage literature.

2.1 THE EFFECTIVE MINIMUM WAGE

Because there are overlapping jurisdictions that establish their own minimum wage, the statutory minimum wage of a state may not reflect the wage rate that is paid to workers. The effective minimum wage for a state is the greater of the statutory minimum wage defined by a state and that which is defined by the federal government. This is notable for states that do not have any statutory minimum wage, because it ensures that there is a binding effective minimum wage back stop for all states, in the form of the federal minimum wage regime. In cases when the federal minimum wage allows for exemptions and tiers, it may still be possible that states introduce statutory language to cover these exempt groups, however states may also extend those exemptions, meaning for some there is no binding minimum wage.

2.2 TIERED MINIMUM WAGE RATES

Many states design a minimum wage regime to allow tiers, exemptions, or extensions of the minimum wage with the intention of reducing the burden of compliance for small businesses, targeted interests, and select classes of workers. The following examples of tiers to the minimum wage in Minnesota serve as a backstop to ensure some binding minimum wage rate for firms that do not fall under the purview of the Federal Fair Labor Standards Act (Federal FLSA).

Subminimum wage tiers take many forms. In 2016 the state of Minnesota defined the minimum wage as $9.50 an hour for employees of firms that earn more than $500,000 in gross sales receipts in the previous year. Firms earning less than $500,000, however, may pay their employees $7.75 an hour (Minnesota Department of Labor and Industry, 2017). This allows a variation of the effective minimum wage rate between firms.

There may also be variation in the effective minimum wage within a single firm. Employers in Minnesota in 2016 may pay $7.25 to employees hired under a J-1 visa and teenagers receiving training during the first 90 days of their employment. Yet another wage tier may be paid to employees with a disability that limits their productivity. This subminimum wage, sometimes called the commensurate wage, is equal to the wage of a worker's peers scaled proportionally by the observed relative productivity of the worker with a disability.

As a backstop, the commensurate wage could not be less than fifty percent of the wage of the worker's peers, although this backstop was removed three decades ago as a part of federal policy changes. Firms paying the commensurate wage must be certified by the Minnesota Department of Labor and Industry or by the Federal Department of Labor. Because the commensurate wage is defined as a function of the wage rate of a worker's peers, this wage rate may also vary from firm to firm. Current conventional measures of the effective minimum wage would only capture the relative employment effect of the $9.50 rate and would not capture the relative effect of these tiers.

The policy design of this subminimum wage regime is not unique to Minnesota, similar wage regimes are adopted by other states based on employee headcount, industry sector, or health insurance provision (Vaghul & Zipperer, 2016) , (United States Department of Labor, 2017) .

2.3 CHANGES IN ELIGIBILITY FOR THE SMALL FIRM TIER

In 1990, the Federal Fair Labor Standards Act (hereafter FLSA) was amended to include a subminimum wage rate for teenage workers for their first six months with an employer to serve as a training period. This represented the introduction of a subminimum wage rate that relied on worker characteristics, whereas previous subminimum wage tiers were defined by industry factors, like pre-tipped income for the restaurant industry, or work study income for universities. The introduction of a tier based on firm and worker characteristics was a novelty, and similar subminimum wage tiers throughout the 1990s were soon to follow.

Before 1990, the federal FSLA did not stipulate a minimum wage for firms earning below $500,000 in annual revenues (Linder, 1998). By 1991, Minnesota established a minimum wage tier for firms earning less than $362,500 a year in revenues, Ohio established a similar tier for firms earning $500,000 in revenues, and Oklahoma established a similar tier for firms with $100,000 in revenues but also employed over 10 workers. By 1992, Montana established a similar minimum wage for firms with $110,000 in revenues. By 1994, Washington D.C. established its own minimum wage rate, and introduced tiers based on Fair Labor Standards Act coverage (United States Department of Labor, 2017).

A wave of subminimum wages were introduced in 1997. The federal minimum wage defined a tipped worker credit as half the minimum wage for a non-tipped worker, this credit now was set to $2.13. The overall wage rate also increased from $4.75 to $5.15. In Indiana, Michigan, and Vermont, a subminimum wage tier was established for firms with two or fewer employees. In Arkansas, Illinois, Nebraska, and Virginia, a subminimum wage tier was established for firms with four or less employees. In Georgia and West Virginia, a similar subminimum wage tier was established for firms with six or fewer employees (United States Department of Labor, 2017). The District of Colombia recently removed a grandfathered minimum wage tier for laundromats (District of Comombia, 2017).

2.4 THE STATE OF THE TIERED WAGE DISCUSSION

In the 1980s, educational organizations were allowed to pay student workers below the minimum wage to compensate for the availability of other benefits offered to the student. Despite the existence of this wage tier, youth employment remained relatively unchanged (Brown, Gilroy, & Kohen, 1983), (Freeman, Gray, & Ichniowski, 1981).

In April 1990, the Fair Labor Standards Act was revised to include a provision that defined a training wage for all teenagers, similar to the training wage described in Section 2.2 of this paper. Studies of this subminimum wage measured the relative change in employment in states that did not have a supplemental minimum wage ordinance that would supersede the binding subminimum wage rate. These studies measure the change in employment among teenage and young male workers in response to this policy using first differences analysis of Current Population Survey (hereafter CPS) employment data, and some use a primary instrument based on franchise geography (Neumark & Wascher, 1994), (Card, Katz, & Krueger, 1994), (Katz & Krueger, 1991), (Neumark & Wascher, 1991), (Spriggs, 1993), (Smith & Vavrichek, 1992).

These studies found conflicting measures of the employment effects of the minimum wage, but they all only track the prevalence of the subminimum wage with respect to teenage workers in the United States. Discussions of the employment effects of the subminimum wage defined by firm size, worker disability status, or visa status remain largely underdiscussed by researchers.

2.5 A BRIEF NOTE ON TIPPING

In many states, and at the federal level, workers that receive gratuities may be paid less than the minimum wage by their employer if tips cover the difference. Although this reflects a statutory subminimum wage rate, the design of the tipping credit stipulates that if tips do not sum to exceed the minimum wage rate for untipped workers, the employer is responsible for making up the difference. This is a key distinction from treating a tipped wage regime as a subminimum.

From the perspective of the firm, so long as the employees are sufficiently tipped, there is no change in the cost of employing the marginal worker. From the perspective of the employee, however, there is an effective earnings backstop against possible poor tips. There may be an argument for a change in behavior leading to an increase in earnings (Even & Macpherson, 2014), but there remains an effective backstop against poor tips in the form of this employer responsibility. For the purposes of this paper, tipped workers will be categorized according to their post-tip earnings, as is reported in the CPS.

2.6 OTHER USES OF THE TERM SUBMINIMUM

The term "subminimum wage" does not have a concrete sense in the economics of employment literature as of 2023. In this paper I only intend to use the word "subminimum wage" to refer to those who belong to a policy class that allows them to legally be paid less than the headline minimum wage, except for incarcerated persons. I will briefly discuss three other meanings for context.

When discussing low wages some use the term "subminimum" to refer to workers who misreport their wages on surveys, and thereby only appear to earn less than the headline minimum wage. For the sake of this paper, I will refer to such workers as "misreported workers".

Some use the term "subminimum" to draw attention to the problem of wage theft. Wage theft occurs when an employee is illegally coerced to work hours off the books, lowering their effective hourly wage. Wage theft may also occur if an employee is informally paid in cash with no official records, paid at a rate lower than their entitled minimum. I do not engage with this kind of breach in my paper.

Finally, incarcerated persons may be paid less than the minimum wage if they take on a job with their facility. The survey data I use for this paper only includes the non-incarcerated civilian population, so I cannot investigate the employment of incarcerated persons with this data in this study.

3 A THEORY OF LABOR SUBSTITUTION

The stacked difference in difference estimator is a semi-parametric technique that provides a framework to identify inelasticities in the labor market subject to a change in the price of labor. This stacked difference in difference estimator controls for the distribution of wages in a state by assuming a similar distribution of marginal labor productivity. To understand the implications of the empirical results of the stacked difference in difference model, I must first clarify the current theory of semi-inelastic labor that explains the distribution of wages[2].

3.1 PRODUCTION IN A SINGLE MINIMUM WAGE REGIME

Assume firms produce a single output according to the following constant elasticity of substitution production function:

Equation 1: Production

$$Y = \left(\int_{j \in J} \phi_j \, l_j^{\frac{\sigma-1}{\sigma}} \; \partial j \right)^{\frac{\sigma}{\sigma-1}}$$

With single output *Y*, differentiated worker types *j*, each with marginal productivity ϕ_j, quantity of labor demanded l_j, and elasticity of labor type substitution σ. Firms treat capital as fixed, and output will vary according to the level of labor consumed of each type. Without loss of generality, let *j* be ordered

[2] This discussion is framed in terms of a perfectly competitive labor market, equations for a monopsonistic market are included in Appendix 4.

from lowest paid worker type $\underline{w_j}$ to highest paid worker type $\overline{w_j}$. To prevent intractable solutions to firms' optimization problem, assume $\underline{w_j} > 0$ & $\overline{w_j} < \infty$.

Note that in this model work type need not be uniform. Type *j* suggests labor differentiation, which allows this model to characterize market wide distributional outcomes, and the ripple effects of the minimum wage on higher paid workers. Each type of work incorporates its own labor supply and demand based on their marginal productivity, their elasticity of substitution with other workers, and workers' willingness to accept a given wage, although these rates are related. The household is willing to supply labor is characterized by the following:

Equation 2 : Household Labor Supply

$$\int_{\underline{w}}^{\overline{w}} l_j \, \partial j = \int_{\underline{w}}^{\overline{w}} \kappa_j * w_j{}^{\gamma} \, \partial j$$

Where κ_j is noted by the same *j* as ϕ_j and γ is an elasticity of scale parameter. Assume firms have fixed capital, and face the following labor cost function:

Equation 3 : Conditional Cost

$$C = \int_{\underline{w}}^{\overline{w}} w_j \, l_j \, \partial j$$

Once a minimum wage *m* is introduced, firms face a segmented cost function based on the share of workers who are bound by the minimum wage:

Equation 4 : Conditional Cost with a Minimum Wage

$$\check{C} = \int_{\underline{w}}^{m} m \, \check{l_j} \;\; \partial j \; + \; \int_{m}^{\overline{w}} w_j \, \check{l_j} \;\; \partial j$$

Where terms which have been re-calculated to account for the introduction of a single minimum wage are denoted with a check character ($\check{x}$). One derives the Hicksian demand for labor by solving for the first order conditions of the firm's implied Lagrangian. This produces a cost function that is segmented by workers who are bound by the minimum wage, and those who are not. [3]

This leads to the following change of employment at the minimum wage:

[3] A proof of this is given in available in appendix files A1, A2, and A3, with monopsony in A4.

Equation 5 : Tail of Employment

$$b = \int_{\underline{w}}^{m} l_j \,\partial j - a \;;\; a = \int_{\underline{w}}^{m} \breve{l}_j \,\partial j$$

Where *a* is the number of workers bunched at the minimum wage, and *m* is the new binding minimum wage. The density of workers at or below the minimum wage during the pretreatment period denoted as *b*.

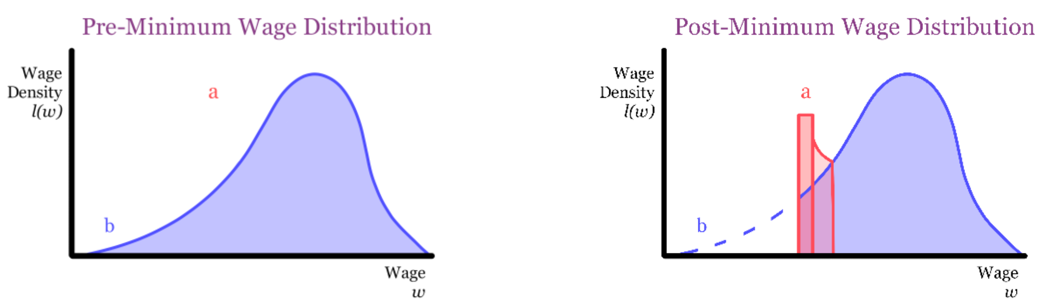
The change in employment at the spike of the minimum wage is as follows:

Equation 6 : Elasticity of The Employment Spike at the Minimum Wage

$$\frac{\partial a}{\partial m}\frac{m}{a} = -\sigma\ m^{-1}\int_{\underline{w}}^{m} \left(1 - s_j\right) Y\ \frac{\partial \check{C}}{\partial Y}^{\sigma}\ \phi_j^{\sigma}\ m^{-\sigma}\ \partial j\ \frac{m}{a}$$

Where *s* is the share of marginal costs from those employed at wage level *j*, in this case the minimum wage. If labor supply is elastic, and labor demand is inelastic ($\sigma = 0$), the density of workers below that wage in the pre-treatment period will equal the density of workers at the minimum wage in the post-treatment period (this occurs when *b* = *a*). Then there will be no loss of employment in response to a change in the minimum wage, but workers will shift from one pay range to another. If labor supply is not perfectly elastic ($\sigma \neq 0$), or labor demand is not perfectly inelastic, the minimum wage will increase costs, causing an increase in marginal cost $\frac{\partial \check{c}}{\partial Y}$, which decreases hires at *a*, employment at the minimum wage. This occurs when the area of *a* is less than the lost area of *b*.

Graphic 1: Compression at the Minimum Wage [4]



[4] Image inspired by Cengiz et al.

3.2 PRODUCTION IN A SUBMINIMUM WAGE REGIME

In the last section, terms which account for the introduction of a single minimum wage were denoted with a check character ($\check{x}$). Let terms with a hat of the form ($\hat{x}$) denote a change in terms to account for two tiers of minimum wage.

To distinguish between two wage rates specified in a wage regime vector, I adopt the following language: the higher tier minimum wage binding to most workers is the *headline minimum wage* denoted with bar ($\overline{m}$), and the lower tier minimum wage for only a special class of worker is the *subminimum wage* denoted with underbar ($\underline{m}$).

There are restrictions to the kinds of workers that are allowed to be paid the subminimum wage (further discussed in Section 3.3). Let α_j^1 denote the share of employees eligible to be paid an effective subminimum wage within a given wage bin *j*. Let α_j^0 denote the share of baseline employees ineligible to be paid the subminimum wage, and therefore bound by the headline minimum wage. If a worker's clearing wage is less than their binding minimum wage, worker bin *j* denotes their counterfactual competition wage. This class type is independent of the wage bin distribution, and no one type is expected to be more prevalent in any wage bin due to their type before the enactment of the subminimum wage[5].

The production function expands from Equation 1 into the following new production function, which accounts for both classes of worker:

Equation 7 : Output After Subminimum Wage

$$Y = \left(\int_{\underline{w}}^{\underline{m}} \phi_j \left(\left(\alpha_j^0 + \alpha_j^1 \right) * l_j \right)^{\frac{\sigma-1}{\sigma}} \partial j \quad + \int_{\underline{m}}^{\overline{m}} \phi_j \left(\left(\alpha_j^0 + \alpha_j^1 \right) * l_j \right)^{\frac{\sigma-1}{\sigma}} \partial j \right.$$
$$\left. + \int_{\overline{m}}^{\overline{w}} \phi_j \left(\left(\alpha_j^0 + \alpha_j^1 \right) * l_j \right)^{\frac{\sigma-1}{\sigma}} \partial j \quad \right)^{\frac{\sigma}{\sigma-1}}$$

[5] This may be relaxed if one has data to support this distributional claim. I will remain neutral in this paper.

Where the first range of integration represents workers of type *j* who would be paid below the subminimum wage pretreatment, the second range represents workers who would be paid between the subminimum and the headline minimum wage, and the third region represents workers who would be paid above the headline minimum wage.

Expanding the production function will alter the cost function from Equation 4 into the following form:

Equation 8 : Cost After Subminimum Wage

$$\check{C}(w,\underline{m},\overline{m},Y) = \int_{\underline{w}}^{\underline{m}} (\alpha_j^0) * \overline{m} * \check{l}_j \, \partial j + \int_{\underline{w}}^{\underline{m}} (\alpha_j^1) * \underline{m} * \check{l}_j \, \partial j + \int_{\underline{m}}^{\overline{m}} (\alpha_j^0) * \overline{m} * \check{l}_j \, \partial j + \int_{\underline{m}}^{\overline{m}} (\alpha_j^1) * w_j * \check{l}_j \, \partial j + \int_{\overline{m}}^{\overline{w}} (\alpha_j^0) * w_j * \check{l}_j \, \partial j + \int_{\overline{m}}^{\overline{w}} (\alpha_j^1) * w_j * \check{l}_j \, \partial j$$

Solving the implied Lagrangian for these production and cost functions produces the new Hicksian demand curves for labor if households supply is elastic. This replacement and subsequent derivation are included in the appendix. This suggests the following employment elasticities at each minimum wage tier[6]:

Equation 9

$$\frac{\partial \underline{a}}{\partial \underline{m}} * \frac{\underline{m}}{\underline{a}} = (-\sigma) * \left(1 - s^{\alpha^1}_{\underline{w}\ \underline{m}}\right)$$

Equation 10

$$\frac{\partial \underline{a}}{\partial \overline{m}} * \frac{\underline{a}}{\overline{m}} = (\sigma) * \left(s^{\alpha^0}_{\underline{w},\ \underline{m}} + s^{\alpha^0}_{\underline{m},\ \overline{m}}\right)$$

Equation 11

$$\frac{\partial \bar{a}}{\partial \underline{m}} \frac{\underline{m}}{\bar{a}} = (\sigma) * \left(s^{\alpha^1}_{\underline{w},\ \underline{m}}\right)$$

[6] Derivation in the elastic case is available in Appendix 3. Further proof with a monopsony based model available upon request in appendices 4 and 5.

Equation 12

$$\frac{\partial \bar{a}}{\partial \bar{m}} \frac{\bar{m}}{\bar{a}} = (-\sigma) * \left(1 - \left(s^{\alpha^0}_{\underline{w},\ \underline{m}} + s^{\alpha^0}_{\underline{m},\ \bar{m}} \right)\right)$$

For equations Equation 11 and Equation 12, the right most term represents the share of subminimum wage eligible workers who happen to have market clearing wage exactly equal to $\underline{m}$ or $\bar{m}$ respectively. Note, since σ is positive, this implies that own price elasticities are negative, and cross price elasticities are positive, suggesting workers provide normal factors of production and substitutes for each other.

Since the cross-price elasticities are not zero, changes in one minimum wage will affect the workers that are bound by the other, and the researcher must consider both changes when quantifying any employment effects. The scale of this may however be minimal if $s^{\alpha^1}_{\underline{w},\underline{m}} + s^{\alpha^1}_{\underline{m},\bar{m}}$ is small.

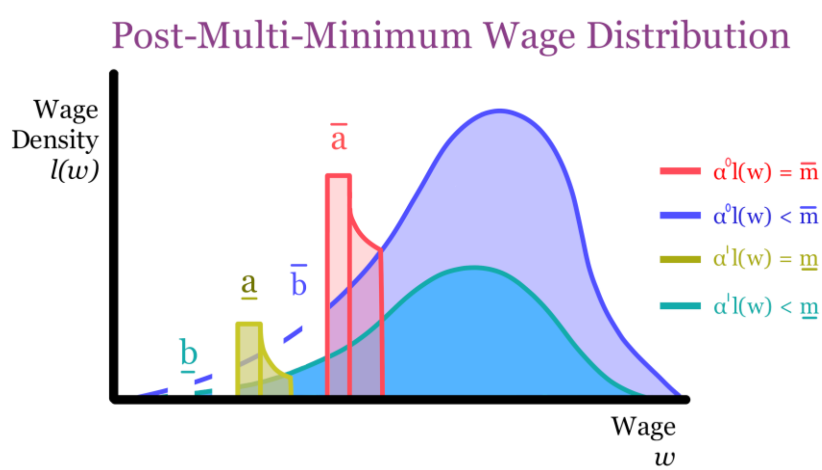
Furthermore, $\underline{a}$ represents the density of employment at the subminimum wage, $\bar{a}$ represents the density of employment at the headline minimum wage, and $s^{\alpha^x}_{w_1,w_2}$ represents the share of the change in marginal cost due to changes in the workforce of subminimum wage eligibility α, with pre-minimum-wage productivity-clearing-wages within the range of w_1 and w_2.

3.3 The Differences Between a Single Minimum Wage and a Minimum Wage Vector

In the Cengiz et al model of the employment effects of the minimum wage the change in employment is simply the change in the number of persons at the minimum wage (Δa) less the number of people who would have worked at the portion of the wage distribution below the minimum wage (Δb). Their model accounts for the employment effects of a change in the headline minimum wage but does not account for the change in employment caused by a change in the subminimum wage. To test if the subminimum wage exhibits a bespoke employment effect, I specify how the employment effects of the headline minimum wage may differ or be independent of the subminimum wage.

Assume workers of types α^1 & α^0 follow two distributions with respect to wage j before a wage policy change:

Graphic 2 : Compression with Multiple Minimum Wages



The upper blue line represents the distribution of non-exempt workers or type α^0. Once the headline minimum wage is established, the lower left tail of this distribution disappears as workers are no longer allowed to be paid below the headline (represented as the dotted region of the blue line). Workers who once earned below $\bar{m}$ either become unemployed or are given a raise to the new headline minimum (represented by the red spike). Workers earning slightly above the minimum wage may see a slight increase as well as relative wages are renegotiated (represented by the tapering of the red spike).

The lower teal like still represents type α^1 workers, and establishing the subminimum wage produces the same dynamics as the non-exempt workers, but in relation of the binding subminimum wage $\underline{m}$ (represented by the dotted region of the teal line and the yellow spike with taper).

Adding the height of these two distributions generates the aggregate employment curve (not pictured). One cannot differentiate between the two classes of worker when aggregated. After policy dictates a two minimum wage regime, workers are either unemployed or they are raised to their appropriate higher wage bin. Adding these two distributions produces the aggregate employment curve (not pictured), but unlike before the minimum wage policy, one can now identify exempt type workers, but only in the region below the headline minimum wage.

Those earning the subminimum wage should only include the exempt class of worker. Those earning above the subminimum wage but below the headline minimum wage should also only include this exempt class. These in-between workers earn their market clearing wage, suggesting a share of workers that earn below the headline minimum wage, but are still unaffected by small changes in the wage regime.

Those earning at or above the headline minimum wage will include both classes of worker, non-exempt workers and exempt workers with high enough productivity who achieve their market clearing wages. When the headline minimum wage increases, workers who were paid below it are moved to the new wage level, creating a spike at that point in the distribution. This spike includes new workers whose wages rose after the policy, but it also stacks on top of the count of workers who were previously already earning that wage, both exempt and non-exempt worker types.

Including two classes of worker and giving each a respective minimum wage produces six possible regions of employment: exempt workers with perfect competition wages that would clear below the subminimum, exempt between the subminimum and headline minimum, exempt above the headline minimum, not exempt below the subminimum, not exempt between the subminimum and headline minimum, and not exempt above the headline minimum. This distribution captures a counterfactual in the absence of any kind of minimum wage and is used to construct employment levels in the absence of any wage regime. In the sake of brevity, I do not share the elastic case in the body of this paper but share the results of the monopsonistic model below. Using similar identifying assumptions as the single minimum wage case, a change in the minimum wage will produce the following elasticities[7]:

Equation 13

$$\frac{\%\,\Delta(\,\underline{a}-\underline{b}\,)}{\%\,\Delta\,\underline{m}} = \sigma * \frac{\left(s^{\alpha^1}_{\underline{w},\underline{m}}\right)}{1-\ \left(\ \frac{\sigma}{\gamma+\sigma}\ \right) * \left(s^{\alpha^1}_{\underline{m}\,,\overline{m}} + s^{\alpha^0}_{\overline{m},\,\overline{w}} + s^{\alpha^1}_{\overline{m},\,\overline{w}}\right)}$$

Equation 14

$$\frac{\%\,\Delta(\,\bar{a}-\bar{b})}{\%\,\Delta\,\overline{m}} = -\sigma * \left(1-\left(1+\left(\frac{\gamma}{\gamma+\sigma}\right)\right) * \left(\frac{\left(s^{\alpha^0}_{\underline{w},\ \underline{m}}\right)+\left(s^{\alpha^0}_{\underline{m}\,,\overline{m}}\right)}{1-\left(\ \frac{\sigma}{\gamma+\sigma}\ \right) * \left(s^{\alpha^1}_{\underline{m}\,,\overline{m}} + s^{\alpha^0}_{\overline{m},\,\overline{w}} + s^{\alpha^1}_{\overline{m},\,\overline{w}}\right)}\right)\right)$$

Equation 15

$$\frac{\%\,\Delta(\,\underline{a}-\underline{b}\,)}{\%\,\Delta\,\overline{m}} = \sigma * \left(\frac{\left(s^{\alpha^0}_{\underline{w},\ \underline{m}}\right)+\left(s^{\alpha^0}_{\underline{m}\,,\overline{m}}\right)}{1-\left(\ \frac{\sigma}{\gamma+\sigma}\ \right) * \left(s^{\alpha^1}_{\underline{m}\,,\overline{m}} + s^{\alpha^0}_{\overline{m},\,\overline{w}} + s^{\alpha^1}_{\overline{m},\,\overline{w}}\right)}\right)$$

Equation 16

$$\frac{\%\,\Delta(\,\bar{a}-\bar{b}\,)}{\%\,\Delta\,\underline{m}} = \sigma * \left(\frac{\left(s^{\alpha^1}_{\underline{w},\underline{m}}\right)}{1-\ \left(\ \frac{\sigma}{\gamma+\sigma}\ \right) * \left(s^{\alpha^1}_{\underline{m}\,,\overline{m}} + s^{\alpha^0}_{\overline{m},\,\overline{w}} + s^{\alpha^1}_{\overline{m},\,\overline{w}}\right)}\right)$$

Note that each term includes the term $s^{\alpha^1}_{\underline{m}\,,\overline{m}}$, suggesting that the measure of any change in employment ought to account for those in between the two rates. This term is a novel addition to the CDLZ approach and suggests the need for an additional control or an additional identifying assumption.

[7] Derivations of these terms are available upon request in Appendix 3 and 4.

The employment dynamics of a multi-minimum wage regime is similar to the dynamics of a single wage regime. The existence of this multi-minimum wage regime suggests reported earnings below the headline minimum wage are not necessarily a result of measurement error, recall bias, or the violation of labor laws, but instead reflect a legitimate and accurate reporting of their circumstance.

Deriving the employment effects of the subminimum wage yields two useful policy conclusions; first to discover any effect among low wage workers of the exempt class, and second to discover if such a policy influences the dynamics of employment among the non-exempt class. The introduction of the subminimum wage to the stacked difference in difference model allows me to consider six hypotheses:

1. If $\frac{\partial(\underline{a}+\bar{a}+\underline{b}+\bar{b})}{\partial \underline{m}} \neq 0$ then the subminimum wage has a bespoke effect on overall employment.
2. If $\frac{\partial(\underline{a}+\underline{b})}{\partial \underline{m}} \neq 0$ then the subminimum wage has an employment effect on those who are subject to its eligibility (where α^1) .
3. If $\frac{\partial(\bar{a}+\bar{b})}{\partial \underline{m}} < 0$, and $\frac{\partial(\underline{a}+\underline{b})}{\partial \underline{m}} < 0$, then the loss of one type of worker causes the loss of the other type of worker, suggesting workers are considered complements of the production process.
4. If $\frac{\partial(\bar{a}+\bar{b})}{\partial \underline{m}} > 0$, and $\frac{\partial(\underline{a}+\underline{b})}{\partial \underline{m}} < 0$, then the loss of one type of worker causes their replacement with the other type of worker, and worker types are considered substitutes of the production process.
5. If $\frac{\partial(\bar{a}+\bar{b})}{\partial \underline{m}} = 0$ then workers of type α^1 are cross price inelastic, suggesting either that the production markets that employ workers according to their type α are independent of each other, or that monopsony power in the labor market

determines labor demand, and the increase in minimum wages represents only a transfer of rents but not a change in quantity[8].

6. Empirical studies that link pay slips with recall survey data find that recall bias tends to be even worse as wages increase (Prati, 2017). It is possible that observed employment effects do not reflect actual changes in employment, but the change in the subminimum wage reminds workers of their actual wage, reducing misreported wages. My paper does not access administrative data to verify its results, so my results may demonstrate if there is a change in reported wages. This is a necessary condition but not a sufficient one to test the recall bias hypothesis.

The next section of this paper discusses the empirical tools needed to identify the bespoke effects of the subminimum wage, and connects their identification to the stacked difference in difference method used by Cengiz et al.

4 DATA

4.1 THE QUARTERLY CENSUS OF EMPLOYMENT AND WAGES

The Quarterly Census of Employment and Wages (QCEW) is conducted by the Bureau of Economic Analysis. This data series reflects employer based information while firms file their necessary records for unemployment insurance compliance. This data reflects payroll distributional data, and the results are then aggregated to the County level so no one firm is identifiable. Since collection of QCEW data is tied to the administrative obligation of payroll compliance, the QCEW exhibits a higher response rate than the Current Population Survey, at the cost of less detail. The QCEW will capture the overall level of employment in a given state at a given quarter, but to differentiate between different wage levels, I will need to merge this with the Current Population Survey.

[8] 14-c certified Community Rehabilitation Programs that apply the commensurate wage for persons with a disability are a notable exemption, since they are regulated to serve the best interests of their clients. Most are nonprofit firms with grants from the State, and would not be monopsonists, although the firms they contract with may still be so.

4.2 The Current Population Survey – MORG

The Current Population Survey (CPS) is conducted by the United States Census Bureau and the Bureau of Economic Analysis. The CPS surveys 60,000 households every month and collects data on basic demographic data as well as economic data and government program use. The CPS follows a semi-panel structure, where respondents are randomly selected according to their residential geography. Respondents are asked to complete a survey of their household and economic outcomes for months one through four, withheld from the survey for months five through twelve, and re-surveyed for months thirteen through sixteen. In months four and sixteen, respondents are given a more comprehensive survey. This subset of respondents is called the Monthly Outgoing Rotation Group (MORG) because in any given month, they represent the respondents who are either about to go on a survey hiatus for eight months, or to exit the sample entirely. This study employs the Minnesota Population Center's version of the CPS (Flood, King, Ruggles, & Warren, 2017). The Bureau of Labor statistics temporarily changed their protocols for imputing missing wages, making it inappropriate to compare observations from 1994 through 1995 to the rest of the sample. I will exclude these years from my regression unless otherwise stated.

4.3 A Note on Cengiz, Dube, Linder and Zipperer's Data

To estimate the prevalence of misreported wages, Cengiz et al received aggregated, anonymized, administrative data from the Unemployment Insurance programs of the states of Minnesota, Oregon, and Washington. This data is the underlying source of the QCEW, but the detailed records supplied allow the researcher to test the distribution of wages as well as overall employment.

Although the replication data from their paper's replication files do include the aggregated data from these three states, I do not use it here. They use this administrative data to apply a deconvolution algorithm (similar to a repeated Fourier Transformation filter) to find the underlying distribution of wages, under the assumption that true wages and misreported wages are independently distributed. This assumption is at odds with a key assumption of my paper – that the class of exempt workers may have a counterfactual wage that is linked to the rate of their non-exempt peers. Also, I would like to obtain my own copy of this data to verify its accuracy before I feel comfortable using it in my own work.

Furthermore, the CDLZ team uses the public use data from the National Bureau of Economic Research archive of the CPS, whereas I use the Minnesota Population Center version. This suggests that my employment history can only go back to 1989, instead of 1979. In the 1980's there are many changes in employment survey protocols and changes in the commensurate wage for workers with disabilities. This decade will require separate analysis in a future study.

4.4 VARIABLE CONSTRUCTION

4.4.1 Earning the Subminimum Wage

The sample is limited to those with wage or salary income in the month of the survey. The primary measure of income in this section will reflect the HOURWAGE variable, a measure of the amount earned per hour at the current job for workers paid an hourly wage. The secondary measure of income is the reported EARNWEEK and UHRSWORKORG, which capture the reported weekly earnings and usual hours worked of the respondent. Dividing weekly earnings by usual weekly hours worked captures the approximate hourly wage of workers who may be paid under salary or under some other non-hourly contract.

The distribution of wages is constructed by placing individuals in the CPS into 25 cent bins, from a wage rate of $0.01, through $30.00. Individuals with wages above $30.00 are placed in the same bin as those earning $30.00. To account for inflation, wage bins are determined by their 2016 real value according to the Consumer Price Index[9].

Hourly wage in the CPS is designed to be inclusive of income from commissions and tips, so if someone's combined income is above the minimum wage, they will be classified above the minimum wage.

4.5 AGGREGATING TO THE UNIT OF ANALYSIS

4.5.1 The Observation Unit of Analysis

The CPS is collected at the household per month level, but the unit of analysis in Cengiz et al is the statewide wage bin per quarter. The statewide bins allow the researcher to observe the aggregate employment level, and the quarterly aggregation accounts for the fact that many of these wage bins will have sparse counts in any one particular month[10].

I aggregate employment within each wage bin to the state-month-bin level, applying the CPS weights for earnings. I then average each three month bin to estimate quarterly employment to generate state-quarter-bin units of analysis. This reflects the same unit of analysis as Cengiz et al. I then divide the employment in each bin by the state's population average in that quarter (based on household weights) to generate state-quarter-bin employment to population shares. This accounts for changes in population of each state over time, and again is necessary to reflect the same outcome of interest as Cengiz et al.

After aggregating the unit of analysis up to the 25 cent wage bin in a given state during a given quarter, initial exposure to the treatment is defined with a two-way fixed effect binary based on the number of 25 cent bins and the number of quarter

[9] Stacked difference in difference estimators use time fixed effects to absorb macroeconomic price effects of employment, but indexing ensures the width of the 25 cent bins are consistent over time.
[10] Furthermore, this aggregation is necessary to merge QCEW employment counts with CPS wage distributions.

periods needed to shift to the proximate minimum wage change. I will describe this more in the next section.

4.5.2 The Unit of Treatment

4.5.2.1 Wage Increments

Multiple treatment effects are necessary to capture the difference in treatment intensity across firms. Individual firms may see a more drastic change in their demand for labor based on the distribution of wages of their employees before the policy. If the minimum wage increases to a new level, then workers who were already earning that level will be affected by the policy less than workers who were previously earning well below this new minimum.

Even if workers are not directly bound by the minimum wage, the increase in workers at the new minimum wage may trigger a renegotiation of relative wages at the firm to account for the distribution of occupations and tasks included. This suggests that both workers who earned well below the minimum wage before the policy and those who earned above the minimum wage after the policy may still exhibit some sensitivity due to these equilibrium ripple effects.

Cengiz et al propose a stacked difference in difference estimator with a series of treatment effects for each group of affected workers based on their relative wage bin distance from the new minimum wage to account for this heterogeneity of the treatment. Their preferred model will also include several fixed effects as controls, which will limit the ability to identify the treatment effect due to autocorrelation. They propose a unit of treatment that is wider than the unit of analysis in terms of time and deeper in terms of wage bin distance.

Since there are several wage bins at a given time in a given state with no reported CPS respondents, this relative bin distance is grouped together and grouped into relative 100 cent increments. Although the depth of this increment is $1, the boundaries of this group may begin at any 25 cent bin. This suggests that for every $1 treatment increment there are four wage bins included.

For example, the $1 treatment increment assignment for the $8.25 wage bin in a given state in a given time will include the $8.25 wage bin, $8.50 bin, the $8.75 then, and the $9.00 bin. If the minimum wage effective in that state at that time belongs to any four of these bins, then the $8.25 bin is classified as treated.

Cengiz et al refer to both the 25 cent increments of the unit of analysis and the 100 cent increments of the unit of treatment as “bins.” To avoid confusion, I will adopt a different terminology, and instead refer to 100 cent treatment increments as wage "shelves". Each $1 treatment wage shelf will contain information from up to four 25 cent wage bins.

4.5.2.2 Time Increments

The timing of the minimum wage may also lead to a bias in the estimation of its effects. If firms anticipate a drastic change in the minimum wage, then they may change their hiring practices before the policy takes place in anticipation. In such a case the researcher will observe disemployment effects during the pre-treatment period but is actually observing an anticipation of the treatment soon to come. It is also possible that employment shocks generate initial changes in employment, but over time firms readjust to the new labor market and exhibit a diminished effect.

Cengiz et al include a series of treatment effects for each wave of affected workers based on the relative time until a change in the minimum wage. Even after aggregating the 25 cent wage bins into 100 cent wage shelves, there still are several wage bins in a given state at a given time with no CPS respondents. The authors choose to additionally aggregate these treatment increments into year-long distances.

Although these steps are in one year increments, they still may begin at any given quarter. This suggests that if a given wage bin will include a new minimum wage in the next 6 months, it's treatment assignments will include information from its own quarter, the next quarter, the first quarter after the minimum wage change, and the following quarter.

Again, Cengiz et al use the terms lags and leads to both the one quarter increment of time between each observation in the unit of analysis and the four quarter increment of time in each treatment. To avoid confusion, I will instead call each observation's quarter of time it’s "period", and each treatment’s four quarter distance it’s treatment "wave."

4.5.2.3 Combining Time Waves and Wage Shelves

The overall treatment effect is defined by the combination of wage shelves and timing waves. The CDLZ authors propose treatment shelves from $4 below through $16 above the relative minimum wage and including times 3 years before and four years after a new minimum wage change. Each of these units are interacted to create hundreds of possible treatment shelf-waves.

For example, suppose we observe 1,000 employed people in a state at the $8.25 wage bin in the year 2010 during quarter three. Suppose that in this state in 2012 during quarter one the minimum wage will increase from $7 to $10. The new minimum wage is greater than $1 away but less than $2 away from this observation, and it is more than one year but less than 2 years away from changing. This 1,000 person unit would be considered treated in the indicator reserved for plus $1 shelf and plus one year wave. Similarly, suppose we also observe 2,000 people in the same state employed in the $9.00 bin in 2014 during quarter one. These 2,000 workers are observed two years after the same minimum wage event and are within $1 of that minimum wage. These 2,000 workers would be considered in the treatment group of the plus $0 shelf and minus 2 years wave.

4.5.2.4 Bin, Shelf, Period, and Wave Size Considerations

Since the unit of treatment and the unit of analysis are not the same, it is possible that this approach will overcount or undercount some of the workers at the minimum wage. This may introduce bias when estimating treatment effects with a binary, because there is no empirical way to know if the outcome of the non-treated members of your assigned treatment cohort would have in absence of the policy (often referred to as the Moulton Effect).

For example, in 2017 in the state of Minnesota the headline minimum wage was $9.65, which is only 15 cents above the boundary of its relative wage bin, therefore everyone earning $9.51 up through $9.75 will be assigned to the treatment group even if $9.74 was technically above the minimum wage.

There are two reasons the severity of this issue is limited in the case of the minimum wage stacked difference in difference estimator. Since the stacked estimator includes several possible treatment wave-shelves, overcounting in one bin suggests undercounting in an adjacent bin. The stacked estimator will aggregate an entire set of calculated effects, and the composition of each individual term is not as important as it would be in the typical regression setting to identify a single treatment effect. Second, since the minimum wage is a parameter, and since there is already evidence that there are ripple effects of the policy on employment up to 15% above the minimum wage (Dube, Lester, & Reich, 2010), and since the stacked estimator is designed to capture those ripple effects, the Moulton Effect is noteworthy, but minimal. Future studies may seek to address this issue using a bin size with finer resolution. The best semi-parametric bin and shelf size is a matter of debate, but since this paper relies so heavily on the design of Cengiz et al., I choose similar 25 cent bins and 100 cent shelves, so my regression results are comparable.

4.6 HISTORY OF THE MINIMUM WAGE REGIME

In 2016, Kavya Vaghul and Ben Zipperer published such a history of state and federal minimum wage rates, as well as subminimum wage rates, such as the small firm wage rate in Minnesota. Their history of the effective minimum wage cites statutory language that outlines the regional minimum wage. This history represents the federal, state, and when applicable, municipal minimum wage rate by year, month, and day the given minimum wage ordinance took effect (Vaghul

& Zipperer, 2016). Because the unit of observation for the CPS is person-per-month, the daily minimum wage is averaged to the monthly level (so if the state minimum wage was $9.00 on July 1st, and $10.00 on July 16th, the state minimum wage for all of July is reported as 9.00*(15/31) + 10.00*(16/31) = $9.51). When the CPS quarterly pseudo panel is constructed, each state's minimum wage is averaged by the monthly minimum wage within each quarter [11]. The subminimum wage is reported for California, Minnesota, Montana, Nevada, and Pennsylvania based on subminimum wages for firm size and health insurance provision.

4.7 UNLISTED AND OTHER EXAMPLES OF THE SUBMINIMUM WAGE

Other states have wage triggers that influence worker eligibility for exemptions from the headline minimum wage rate. For example, Oklahoma adopts the federal headline minimum wage but requires firms to pay that same level even if they are not covered by the Federal FLSA if that firm has a high enough employee headcount. Such wage regimes are reported by the Department of Labor, separate from Vaghul and Zipperer, but are only reported at the annual level. I include a separate set of estimates including such changes under the assumption they initiate on January 1st of each year.

To account for the subminimum wage in states with no reported subminimum in Vaghul and Zipperer or in my own repository of the Department of Labor Records, I include four different possible measures of the subminimum. First, if a state does not have a listed subminimum wage, I set it equal to the headline, I set it to zero, I set it to missing, and I set it to one half the headline minimum. The half-rate is based on the backstop of the disability commensurate wage from the 80s, which only allows eligible workers to be paid half the rate of their peers, although updates to the FLSA removed this backstop over time.

[11] This supposes that each of the three months has the same amount of work days. It is possible to use a different series of the minimum wage history that is weighted by the distribution of hours over a quarter, but the difference in results from this approach will be negligible compared to the aggregation already deployed in the definition of the treatment shelves and waves.

4.8 ESTIMATING THE SHARE OF EXEMPT WORKERS

The first method of identifying subminimum wage workers is to assume those eligible to earn wages below the minimum are uniformly distributed at 10%. If the headline minimum wage changes, and only some of the workers in a given wage bin shift to a higher wage, then those who remain may be of the exempt class. This is admittedly a strong assumption, although no stronger than the implicit assumption in any model that assumes the entire bin reflects wage mismeasurement[12]. I test the 1% and 5% level as well[13].

5 EMPIRICAL METHODOLOGY

5.1 THE STACKED DIFFERENCES ESTIMATOR FOR A SINGLE WAGE

Cengiz et al propose a stacked difference in difference estimator to identify the employment effects of the minimum wage. They first define their dependent term as the employment to population ratio for a given bin:

$$Y_{jst} = \frac{Employment_{jst}}{Population_{st}}$$

CDLZ addresses the issue of a multi-phase change in the minimum wage regime by defining minimum wage increases that take place incrementally by adding a

[12] These are two extremes; In future drafts, I will test with many shares and compare the differences in results. For now, assume this share is 10%. In the future I will assume that the share of eligible workers is uniformly 1%, 5%, 10%, 90%, and 100% across wage bins.

[13] Future drafts will include the count of possibly eligible workers based on characteristics reported in the CPS. Identifying the share of workers at small headcount firms is difficult in the CPS, since that data is only available during the March survey supplement, when survey responses are matched with other administrative records. Counts are reported in size bins, but the count thresholds change over time. I assume the count of workers in a state in a wage bin at a firm with under 10 employees in March remains constant for the whole year, until the next ASEC.
If a worker in the CPS is in school, if they are a recent immigrant, if they report a work limiting disability, or if they earn tips, I count their number in the same manner as detailed in section 4.5.1, aggregating their mass to the bin-state-quarter level. In one test, I include this employment mass as its own dependent variable outcome, and in another, I discuss it as a possible future robustness check using the controlled direct effect estimator (Acharya, Blackwell, & Sen, 2016), discussed in section 11.

binary control if a minimum wage increase is within three years of another change within the same state. They use the placeholder Ω to represent this control.

The following semiparametric model to captures the employment effects of the singular minimum wage regime:

Equation 17

$$Y_{jst} = \alpha + \int_{\tau=-3}^{+4} \int_{k=-4}^{+17} \beta^{\tau k}\, \mathbb{I}_{j'st'}^{k\tau}\, \partial k\, \partial \tau + \mu_{js} + \psi_{jt} + \Omega_{jst} + \left(\epsilon_{jst} + v_s\right)$$

Where *j* is the wage bin discussed in Section 3.1, *s* is the state, *t* is the quarter, τ is a treatment relative time wave, and *k* is the treatment relative wage shelf.

For example, if *k* = 0 and $\tau = 1$, $\beta^{\tau k}$ captures the average change in the employment rate among sixteen observations. This includes the 25 cent wage bin *j* at the minimum, and the three bins just above it and that are in the same treatment shelf. These four bins are included at the time during the minimum wage increase and during the three times after that would be in the same treatment wave. If k = 5 and $\tau = 4$, $\beta^{\tau k}$ captures the ripple effects of a binding change in the minimum wage for wage bin *j* on the employment rate of those earning $5.00 per hour more than *j*, during the time that is four years ahead of the time of that observation.

The stacked difference in difference estimator includes two sets of fixed effects as general controls. Macroeconomic shocks that influence the entire country are captured by a wage bin × time fixed effect ψ_{jt}, and regional factors that influence the prevalence of certain occupations, and therefore income distributions, are captured by state × wage bin fixed effect μ_{js}. Terms $\left(\epsilon_{jst} + v_s\right)$ reflect any residuals to the estimation of the treatment, with v_s clusters at the state level to account for state specific factors that may influence the adoption of the treatment.

This stacked differences estimator acts like a triple difference estimator and relies on three levels of variation. The first variation is the change in employment in a given wage bin and in a given state after it changes its minimum wage. The second variation for a given bin is the difference in employment between states that changed the minimum wage and those that did not. The third variation is the difference in employment between wage bins close to their state's minimum wage and those that are distant from it.

If there are underlying economic circumstances that are unrelated to the minimum wage, but would influence nationwide employment, then both the treated states and the untreated states will be affected, but the net difference will be cancelled out. If there are underlying labor market factors that may confound the distribution of wages between states, then the stacked estimator will still account for this by comparing the change in employment in that wage bin to the changes in employment in the same state but relative to those that earned well above the minimum and are thereby not directly affected [14].

By construction of $\mathbb{I}$, $\beta^{\tau k}$ can be summed across wage shelves and time waves to derive a series of useful estimators. Since there are a number of these relative bins, the results of any single bin are not informative, but the summation of these bins may reflect some useful effects for the sake of policy analysis.

The change of employment for those in a given wage shelf *k* with respect to minimum wage *m*, is calculated by averaging the effects in that shelf the first five waves of the treatment, $\tau \in [0,4]$, subtracting the effect of that same shelf one wave before the treatment as follows:

Equation 18

$$\hat{\beta}_m^k = \left(\frac{1}{5} * \sum_{\tau=0}^{+4} \hat{\beta}_m^{k\,\tau}\right) - \hat{\beta}_m^{k\,\tau=-1}$$

The benefit of this method is that it implicitly defines the change in employment relative to the period just before the treatment, while the regression itself still

[14] Cengiz et al also include a special control Ω for small federal changes in the minimum wage that may confound estimation of each of these treatment effects. I choose to keep small and federal changes in my estimation. Changes in the federal minimum wage are important to determine effects in Minnesota in the mid 2000's, when the federal wage was above the state headline wage, but then many state subminimums subsequently surpassed the federal headline wage.

controls for variation that may take place over a longer horizon. Furthermore, this creates a separate estimate for each treatment shelf, allowing the researcher to differentiate between the employment effects for shelves below the new minimum wage, those at or slightly above the new minimum wage, and those well above the minimum wage.

The change in employment for those in a given wage wave with respect to a minimum wage is calculated by averaging the effect for that wave in the first five treatment shelves at or above the minimum wage and subtracting the average effect of those same shelves one period before the change:

Equation 19

$$\hat{\beta}_m^{\tau} = \frac{1}{5} * \sum_{k=0}^{+4} \hat{\beta}_m^{k\,\tau} - \frac{1}{5} * \sum_{k=0}^{+4} \hat{\beta}_m^{k\,\tau=-1}$$

One feature of this model is that it allows the researcher to incorporate attenuation effects from changes in the treatment over time after the labor market accommodates for the change in price structure. One limitation of this model is that since the difference in employment is measured with respect to the time period just before the minimum wage change, τ = -1 becomes the baseline and no inference can be made based on that period since it is used as the standard.

5.2 THE STACKED ESTIMATOR FOR MULTIPLE MINIMUM WAGES

Defining the treatment effects of the subminimum wage is a difficult task. The subminimum wage is defined in relation to the headline minimum wage; it is meaningless to consider the subminimum wage alone. There are empirical approaches to consider the treatment effects of multiple simultaneous treatments, but such approaches usually assume the exogeneity of the assignment to each individual treatment arm. This is not the case with the subminimum wage. Even if one could empirically separate the effects of the headline minimum wage from the subminimum wage, such estimates would not be economically useful since one is always defined in relation to the other.

The stacked estimator is particularly well suited for this situation, however, because its semiparametric form is designed to account for the distribution of wages, and its partial effects are designed to be added together later. Likewise, the relative treatment effects for the subminimum wage may be identified as partial effects and can be added back together to reflect aggregate labor equilibrium in a way that is useful for policy discussion purposes.

Each 25 cent wage bin within a state during a quarter defines an observation. The headline minimum wage treatment $\mathbb{I}_{jst}^{\bar{k}\tau}$ is set relative to each observation and the nearest headline minimum wage change. The treatment is based on number of time waves and bin shelves from the observation to the next minimum. A separate vector of shelves and waves $\mathbb{J}_{jst}^{\underline{k}\tau}$ are defined for the proximate distance of the subminimum wage change. I note these effects with $\bar{\beta}^{\bar{k}\tau}$ and $\underline{\beta}^{\underline{k}\tau}$ respectively.

In Section 3.3, I established the relationship between changes in employment from the headline minimum wage and its relationship to the subminimum wage. This theory demonstrates that the elasticity of employment with respect to either minimum will not only include the share of marginal costs attributable to workers of its own given type, but that there is a scaling $s_{\underline{m},\bar{m}}^{\alpha^1}$ That represents the share of exempt workers with wages that would clear between the minimums in their absence. This represents a region of workers who do not distort employment in a perfect competition regime. Since the CPS and the QCEW do not have information about the entire cost structure of the firm, I instead rely on a proxy indicator variable that represents the relative marginal cost change from a given wage bin, $\mathbb{K}$.

It is difficult to distinguish misreported wages from those eligible to be paid below the headline minimum wage. This may include those earning above a state's subminimum wage, like those at low revenue firms in Minnesota, but may even include workers who are exempt from both minimums, like workers with a disability at a section 14-c certified firm. Among those who report earning less than the headline minimum wage, some are eligible for the subminimum, and are accurately reporting their wages. The rest are ineligible and misreported their wage. But even if their reported wages are wrong, their employment status is still a function of their true wage. Such observations will be more sensitive to the headline minimum than the subminimum because the headline minimum is binding for them.

I construct a weighted share of eligibility as a control to test the sensitivity of this factor. Let $\mathbb{K}$ reflect a weighted share of the ratio of the marginal cost of labor between exempt workers and total workers in a given wage bin. If a wage bin is above the headline minimum wage, then both exempt and non-exempt workers are paid *w* and are above the minimum wage, and therefore this share will just be based on the share that is eligible for the subminimum. If the wage bin is below the subminimum, this represents a fixed share of the subminimum to all workers in a given state in a given time. This effect will just scale the wage bin fixed effects in the stacked estimator. Wage bins in between the headline and sub-minimum are weighted according to the share of the wage bin presumed to be eligible and the relative distance of that wage bin to each minimum.

Equation 20 : K

$$\mathbb{K}^{\tau} = \frac{\alpha_j^1\, \tilde{w}_j}{\alpha_j^0\, \tilde{w}_j + \alpha_j^1\, \tilde{w}_j} = \begin{cases} \dfrac{\alpha_j^1 \underline{m}}{\alpha_j^0 \overline{m} + \alpha_j^1 \underline{m}} \; if\ b+k \in [\underline{w}_{s\,t+\tau}, \underline{m}_{s\,t+\tau}] \\ \dfrac{\alpha_j^1 w_j}{\alpha_j^0 \overline{m} + \alpha_j^1 w_j} \; if\ b+k \in [\underline{m}_{s\,t+\tau}, \overline{m}_{s\,t+\tau}] \\ \dfrac{\alpha_j^1 w_j}{\alpha_j^0 w_j + \alpha_j^1 w_j} \; if\ b+k \in [\overline{m}_{s\,t+\tau}, \overline{w}_{s\,t+\tau}] \end{cases}$$

Since wage bins are defined semi parametrically, this proxy is flexible to any distribution of productivity ϕ_j across wage bins. Also, one cannot identify $\mathbb{K}$ with respect to treatment shelves since becomes colinear with the bin-quarter fixed effects and the treatment shelf and wave vectors. One can still identify $\mathbb{K}$ by treatment waves.

To control for the difference in treatment effects from incremental vs lump sum wage changes, CDLZ define a moving window to identify units with multiple treatments three years before or after each observation. I use a window of one year before and two years after either minimum wage change to avoid issues of collinearity, and I define two versions of Ω_{jst}, one for the headline minimum wage and another for the subminimum. This control is useful because there are rare instances when the effective headline minimum wage will change without any change in the subminimum wage or vice versa[15].

This all together produces the following reduced form:

Equation 21

$$Y_{jst} = \alpha + \int_{\tau=-3}^{+4} \left(\int_{k=-4}^{+17} \beta_{\overline{m}}^{\overline{k}\,\tau}\, \mathbb{I}_{j+\overline{k}\,s\,t+\tau}^{k\tau}\, \partial\overline{k} + \int_{\underline{k}=-4}^{+17} \beta_{\underline{m}}^{\underline{k}\,\tau}\, \mathbb{J}_{j+\underline{k}\,s\,t+\tau}^{k\tau}\, \partial\underline{k} \right) + \left(\gamma^{\tau}\, \mathbb{K}_{j\,s\,t+\tau}^{\tau}\right) \partial\tau + \Omega_{jst}^{\mathbb{I}} + \Omega_{jst}^{\mathbb{J}} + \mu_{js} + \psi_{jt} + \left(\epsilon_{jst} + v_s\right)$$

5.3 Estimation Strategies

The stacked estimator in CDLZ relies on three levels of variation to identify its effect. First is the difference in employment in each wage bin and in each state by comparing before and after it increased the minimum wage. Second is the difference in employment in each wage bin and between states that increased their minimum and those that did not. Third is the difference in employment in wage bins that are close to their state's minimum and those distant from it. By adding a distance to the subminimum, I add more variation based on the difference in employment between wage bins that are close to its state's subminimum rate.

The stacked difference in difference estimator from CDLZ relies on two identifying assumptions. First, the researcher can only observe employment in the post treatment period based on the actual behavior adopted by the states. The researcher cannot observe the counterfactual post minimum wage increase levels of employment level among states that didn't actually increase the minimum

[15] Unlike the CDLZ approach which includes all federal minimum wage changes as a control Ω_{jst}, I treat federal changes the same as state changes, relying on degree for variation.

wage, and visa-versa. The researcher must use their judgment to construct an appropriate counterfactual to evaluate the policy. Since I use the same methods as CDLZ, I adopt the same identifying assumptions.

First, I assume that trends in employment in each state and wage bin combination are consistent over time unless there are increases in either minimum wage. This "parallel trends" assumption allows me to construct a counterfactual estimate of employment during the time after a state's minimum wage increase. Even if there are different average levels of employment between each state's wage bins, assuming their trends are otherwise unrelated to the minimum wage allows the researcher to construct a counterfactual. I presume that had the states who increased the minimum wage chosen instead to do nothing, their employment pattern in the pre-treatment period can be extended into post treatment period to construct this counterfactual. This suggests that if there is a trend in employment in a given state × wage bin combination, that this trend would continue unchanged until there is an increase in the minimum wage, in which case the change in employment is considered to be due to the policy.

$$\mathbb{E}\left[\left(Y_{jst}^{1}-Y_{jst}^{0}\right)-\left(Y_{jst-1}^{1}-Y_{jst-1}^{0}\right)\middle|\,\mathbb{I}_{jst}^{\bar{k}\tau}=1\ or\ \mathbb{J}_{jst}^{k\tau}=1\right]=0$$

The second identifying assumption is that among firms in the states who choose to increase their minimum wage, they do not change their behavior before a new minimum wage goes into effect. This is the "no anticipation" assumption, made famous by Orly Ashenfelter and David Card and is often called the "Ashenfelter Dip" (1985); this is also a common feature in many difference in difference models. Although the researcher can observe employment in a state × wage bin combination before the adoption of a new wage regime, the researcher cannot appropriately ascribe the cause of employment variations during pretreatment.

One hopes such pretreatment variations are uncorrelated with the change of the minimum wage because such confounding would effectively confuse the classification of those who are considered treated versus untreated by the change.

The researcher does not make this assumption lightly, and although this empirical method does not model the true cause of pretreatment variations, we can still observe employment of both treated and untreated state × wage bins before a minimum wage change to establish if such an assumption is at least reasonable. This assumption allows me to treat the pre-wage change time period as untreated, to construct the basis of comparison.

$$\mathbb{E}\left[Y_{js\,t-1}^{1} - Y_{js\,t-2}^{1} \,\middle|\, \mathbb{I}_{jst}^{\bar{k}\tau} = 1 \vee \mathbb{J}_{jst}^{k\tau} = 1\right] = 0$$

Together, these assumptions allow the researcher to interpret the changes in employment in a given state × wage bin combination over a period of time as the change that is due to the minimum wage uniquely and not due to other macroeconomic shocks, policy, or sudden changes in technology. This constructs the conditional average treatment effect on the treated, the change in employment due to the minimum wage in the states that increased their minimum wages conditional on μ_{js} , ψ_{jt}, $\mathbb{K}$, and Ω.

Since the states who do not increase their minimum wages are presumed to have consistent employment trends while other states change their minimums, I cannot estimate the effects of the untreated states which may have been due to ripple effects from employment in adjacent states. To gain the ability to interpret the results of the stacked estimator as an average treatment effect on the treated units, I lose the ability to identify patterns in the untreated units, and therefore cannot claim insight into a conditional average treatment effect on the untreated or their counterfactual. This suggests the estimates from the stacked estimator are descriptive of changes in employment among the states who increased their minimums, and not prescriptive for the states who did not. Such prescriptive measures would require stronger identifying assumptions than I make here.

Since this estimator includes many treatment waves and shelves, and since the goal of the stacked estimator is to aggregate these results into useful categories, the estimation results will be aggregated according to the methods used by CDLZ, as specified in Equation 18 for the shelf effects and Equation 19 for the wave effects[16]. I include the coefficients from $\mathbb{K}$ alongside the wave effects when included, although they will exhibit a different scale.

6 RESULTS <AS OF 10/2023>

Since the stacked difference in difference estimator from CDLZ is a cumulative estimator, the results below depict the results graphically to display the distribution of the change in the employment to population ratio from the regressions listed above. The numerical tables are included after the body of this paper in section 10. The shelf estimates report the average change in employment in a given treatment shelf during the treatment waves after the change in the minimum relative to the level of employment in the wave just before it as described in Equation 18. The wave estimates report the average change in employment in a given treatment wave among the shelves within $5 above the new minimum relative to the shelf just below it as described in Equation 19.

Each shelf and wave combination receives its own treatment effect, making 177 treatments. In many cases there are more fixed effects and treatment measures than there are observations to contribute to its estimation, producing some models with no estimable effect due to collinearity. In these cases, I switched from a model with binary fixed effects to a model where each state wage bin combination receives its own quadratic trend. Even in these cases many treatment groups are unable to be identified. I report the results of my regressions in this section for discussion but include more detail in the appendix and will elaborate on the inferences that can still be made even with noisy estimations.

In states that have no statutory subminimum, I set it equal to zero since they have no policy. Some may argue that such states should be treated with a subminimum equal to their headline since another interpretation could be that state policy

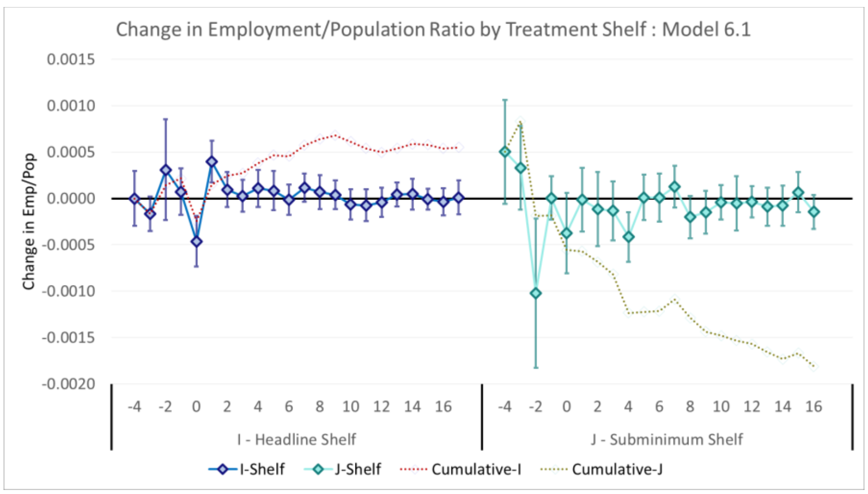
[16] I run variations of the stacked estimator to account for the conditional average treatment effect of the treated on different subgroups, like persons with a disability or those in schooling. These results will be available in a future draft.

defines no difference, or defines its differences relative to the headline, suggesting the headline is the appropriate comparison. I will include this as a later test.

6.1 The Two Treatment Arm Case

First is a regression that includes 𝕀 headline effects and 𝕁 subminimum effects, but not 𝕂 or Ω controls. This reflects the results suggested from simply adding 𝕁 and 𝕀 with no control for treatment interactions [17].

Figure 1: Model 6.1 Shelf Effects

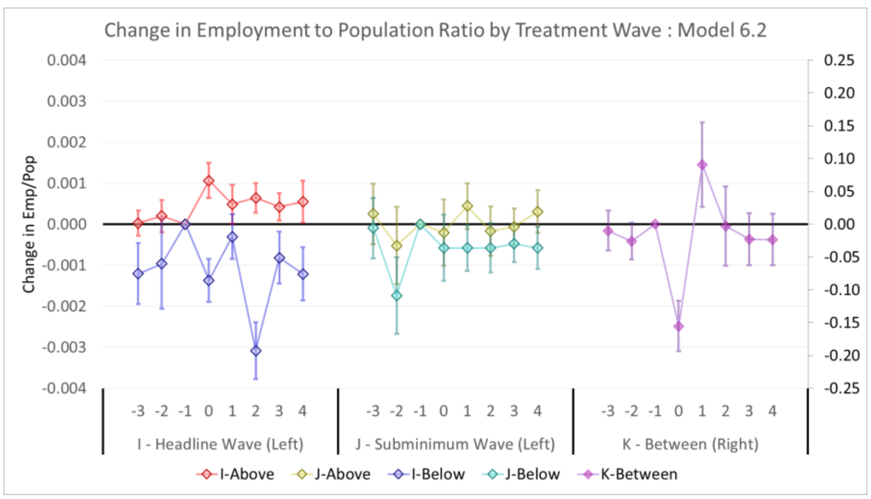


The results of this estimator in Figure 1 demonstrate a decrease in employment below the headline minimum wage, an increase in employment at the headline minimum wage, and null effects above the headline minimum wage. Furthermore, this estimator demonstrates a negative effect of employment below the sub-minimum wage, a null effect of employment at the subminimum wage, and a negative effect $4 above the subminimum but a null effect elsewhere above the subminimum wage. This result is similar to CDLZ's results on the effect on the headline minimum wage, although recall that CDLZ's sample includes more observations from the 1980's because they use a different archive for the CPS.

[17] I include the graphic results in this paper to demonstrate general patterns of results, since the complete results of specific treatment measures are not useful in isolation. Tables of the raw regression results are available on request, and table versions of these graphs are in section 10.

With respect to the subminimum wage there is a drop in employment of those $2 below the subminimum wage, and a drop among those $4 above the subminimum wage. Although the drop in employment below the subminimum wage is to be expected after the subminimum wage increases, the additional drop higher among the distribution suggests that employment is not directly substituted into headline minimum wage jobs. Furthermore, unlike the results for the headline wage, there is not a characteristic drop just below the subminimum followed by a spike at and above it. This could suggest after an increase in the subminimum wage that jobs are lost to attrition and not made up later.

Figure 2: Model 6.1 Wave Effects



With respect to time waves, the effects of an increase in the subminimum wage have an immediate shock the first two years of the new regime. Employment recovers in wages below the subminimum two years after the minimum wage change. Bins with wages just above the subminimum recover three years later. The headline minimum wage demonstrates a null effect across most of the periods close to the minimum wage change, although there is a small increase in employment following the headline minimum wage increase.

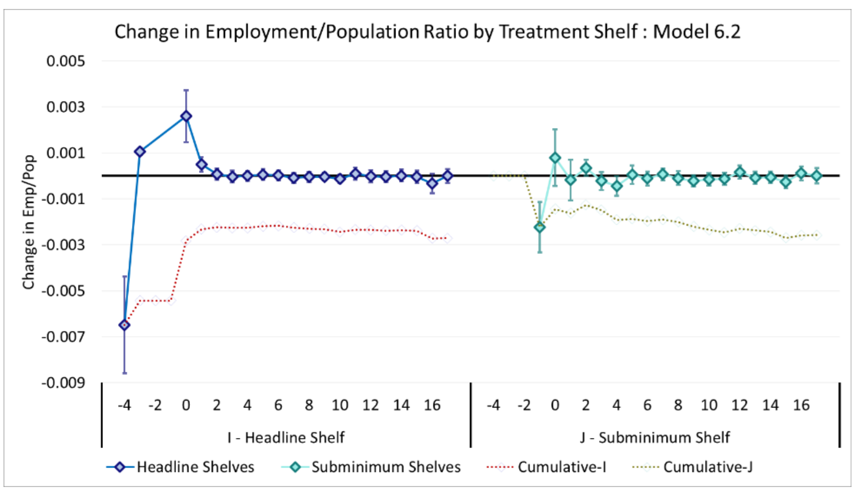
This result is unlike CDLZ’s. Their treatment waves for the those just below the headline (in blue) fall after a change in the minimum and stay down, whereas my results are more unstable. This suggests that including the subminimum wage introduces a treatment arm that changes the basis for comparison for the headline effects. These results would still be consistent with a subminimum wage change leading to an increase in employment just below the headline minimum wage.

Among shelves that are below the headline minimum wage, the fact that the treatment wave effect one year after an increase in the minimum wage exhibits a slightly positive effect is at first puzzling. So is the finding that bins with wages below the subminimum recover two years after the wage change. However, results from this specification should be read with caution. These results depend on the strong assumption that the headline and subminimum wage treatment arms are mostly independent of each other because there is no treatment interaction or weighted share included. I will explore this assumption further in the next model.

6.2 The Two Treatment Arm Case with Controls and time Binaries

Model 6.2 includes $\mathbb{I}$ and $\mathbb{J}$, but adds $\mathbb{K}$ and Ω. The results of this model vary from model 6.1**,** suggesting new negative effects from the headline wage but also dropping many treatment measures due to multicollinearity.

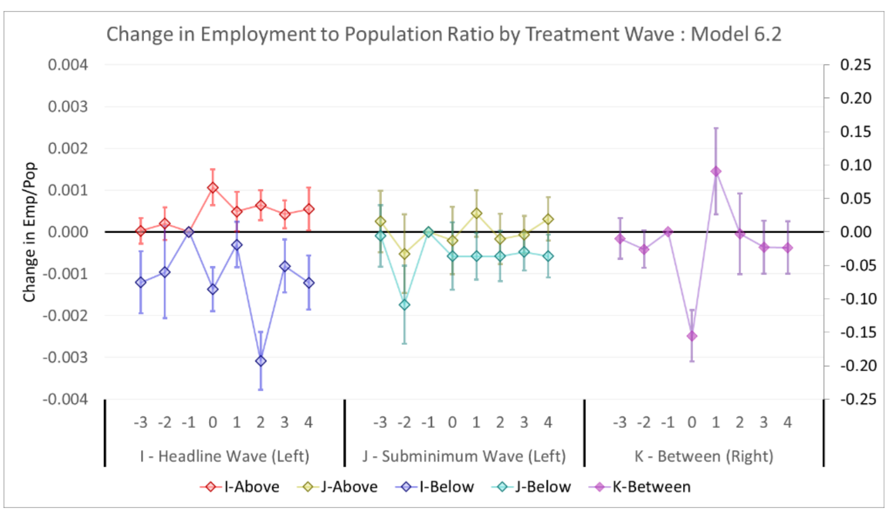
Figure 3 : Model 6.2 Shelf Effects



Similar to CDLZ, there is a decrease in employment below the minimum wage and a spike above it. There is an increase in employment at the new headline minimum wage and $1 above it, with ambiguous results in higher wage bins. Estimating the cumulative employment effects below the headline minimum is difficult with this model. There is a large negative effect of those $4 below the headline minimum on an order of magnitude larger than the rest of these results. However, due to multicollinearity issues after introducing Ω, there are no results available $2 and $1 below the headline minimum. This makes the cumulative effect difficult to interpret because it is unclear if these lower bins would absorb some of the variation from the $4 below shelf and thereby mitigate some of its effect, or if they would have demonstrated a similar effect if identified.

With respect to the subminimum wage, $4 through $2 below the subminimum are multicollinear and cannot be interpreted, however employment decreases $1 below the subminimum wage and increases at the new sub-minimum wage, although this is not statistically significant. There is a small significant effect $2 above the new subminimum wage, which may be consistent with the concurrent increase in the headline minimum wage. In short, including Ω and $\mathbb{K}$ yield results that are qualitatively different from model 6.1, but must still be taken with skepticism due to the inability to identify the effects in noteworthy ways.

Figure 4 : Model 6.2 Wave Effects

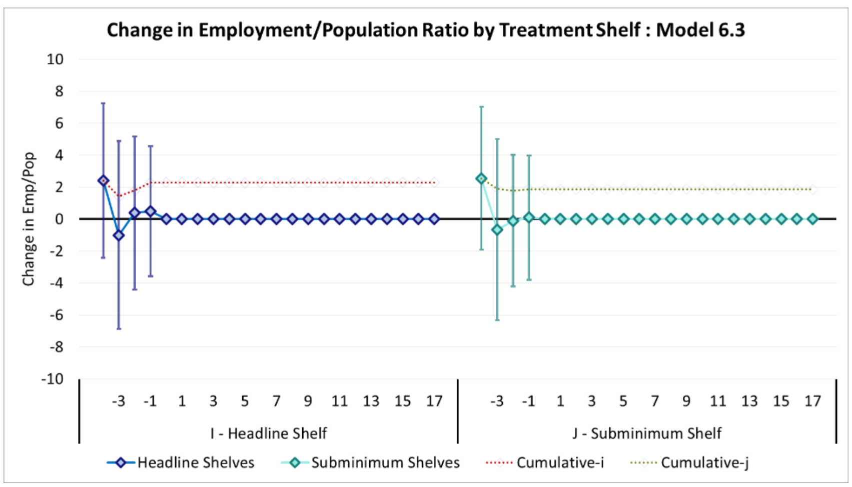


With respect to time, the headline minimum wage exhibits a pattern similar to that of CDLZ, where employment among wage bins just below the minimum wage decrease over time, while there is a positive effect among those above the headline minimum wage. With respect to the subminimum wage, there is a decrease two years before the subminimum among those below the subminimum.

6.3 The Two Treatment Arm Case with Controls and time Quadratics

Model 6.3 includes the same terms as model 6.2 but replaces semi parametric quarter time bins with a quadratic polynomial for each bin-state combination, effectively replacing $\mu_{js} + \psi_{jt}$ with $\mu_{js} * \left(1 + \eta_{js} * t + \rho_{js} * t^2\right)$[18].

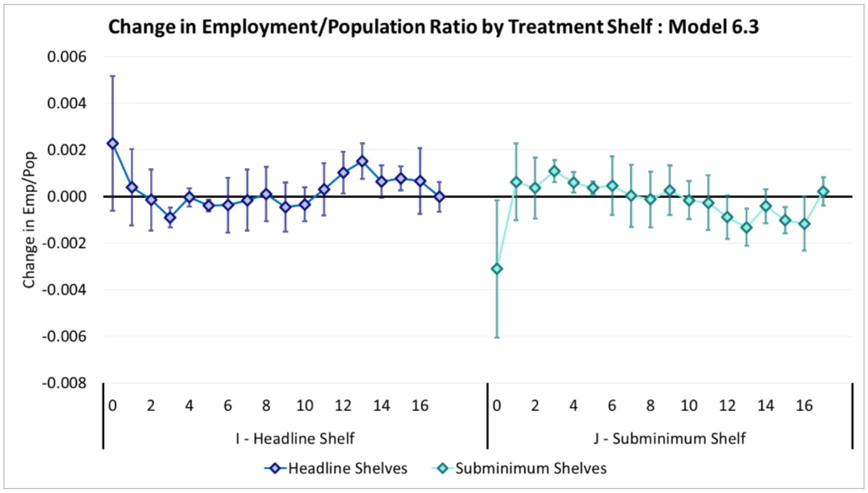
Figure 5 : Model 6.3 Shelf Effects (Un-Truncated)



<continued on the next page for space>

[18] Time t=0 is indexed according to Stata 14's default, Quarter 1 of 1960.

Figure 6 : Model 6.3 Shelf Effects (Truncated)



Compared to the previous regression these results include more degrees of freedom and remove fewer bins due to collinearity, although the restrictive form of time yields results that span orders of magnitude. The effects of the headline minimum wage are slightly positive just above the new headline and mostly ambiguous everywhere else. There is a massive negative effect $3 below the headline minimum wage and is orders of magnitude larger than the other effects combined, however this then also exhibits much higher standard errors and is overall statistically insignificant. There is also a negative effect $4 below the headline minimum, it remains statistically significant and is still notably larger than the effect in other wage shelves.

With respect to the subminimum wage, many of the shelves below the sub-minimum are omitted due to collinearity. There is a drop and employment at the new subminimum wage but an increase $7 above and a decrease $11 above. The net effect is that overall employment among these subminimum wage bins is lower after an increase in the subminimum wage. It is surprising that there is no spike in employment at the new subminimum wage after a change. This may be consistent with a labor substitution model where workers of the policy class are substituted for those who are not bound by the subminimum wage, and employment effects are captured by changes in the headline accordingly.

Figure 7: Model 6.3 Wave Effects (Un-Truncated)

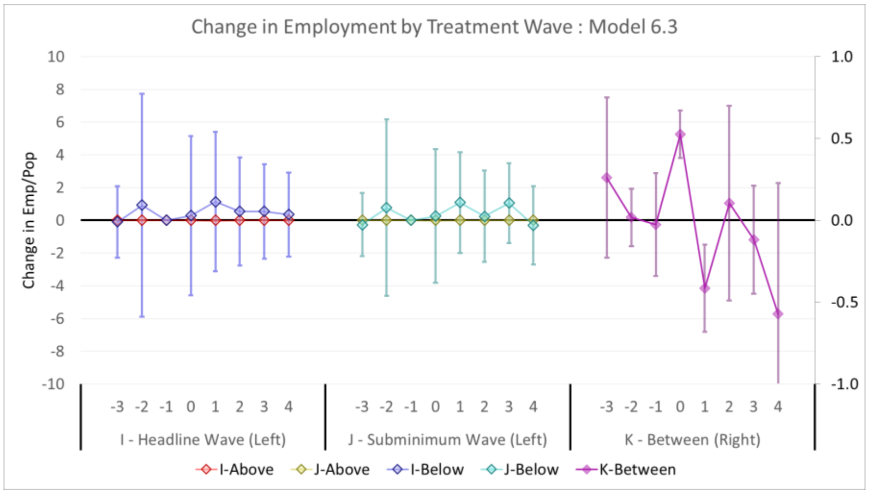
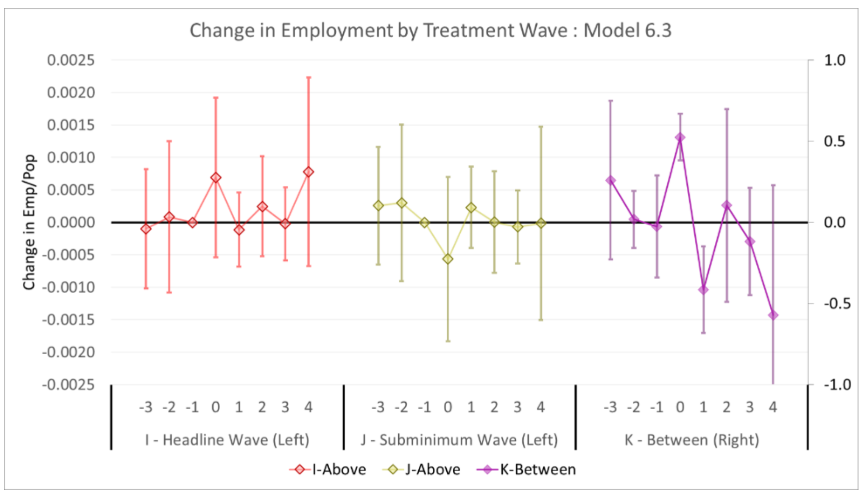


Figure 8: Model 6.3 Wave Effects (Truncated)



In the case of the treatment wave effects, none of the waves demonstrate statistically significant differences in employment, although again their slopes vary by orders of magnitude.

6.4 The Two Treatment Arm Case with Broad Fixed Effects

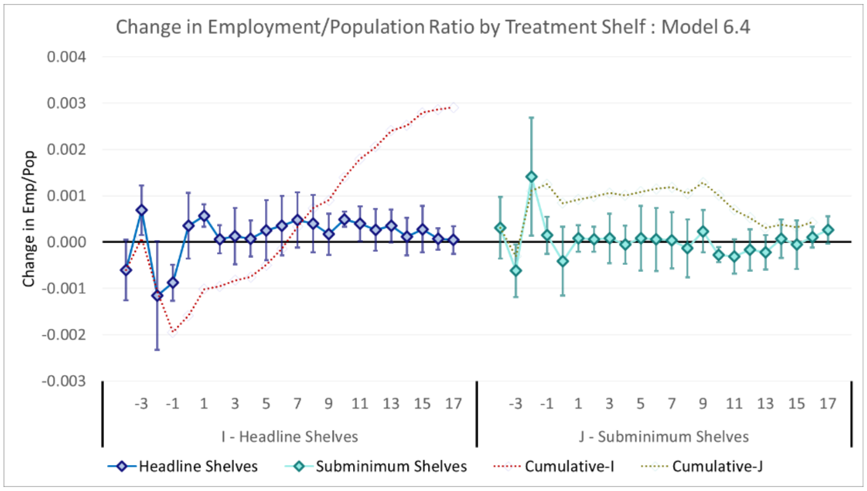
In model 6.1, many of the observations that began below the sub minimum wage are dropped due to autocorrelation. Of the 415,000 observations used to estimate the entire model, only 1,300 of them inform the effects of wage bins that are below the new subminimum rate. Only a few observations are from California since its subminimum wage policy occurred late in the sample, about 250 of these observations are from Minnesota, 150 are from Montana, 140 are from Nevada, and over 750 are from Pennsylvania. This suggests using the negative treatment shelves as a basis for measuring the change in employment disproportionately will represent changes from Pennsylvania and does not accurately reflect all of the states that have a subminimum wage policy.

Model 6.4 includes the same factors as model 6.1, but broadens the fixed effects included in the regression. Instead of granting each 25 cent wage bin in each state its own fixed effect, I pool together every four bins to generate 100 cent fixed effects. I repeat this for the wage bins over time fixed effect as well. This effectively replaces $\mu_{js} + \psi_{jt}$ with $\mu_{\lfloor 0.25*j \rfloor s} + \psi_{\lfloor 0.25*j \rfloor t}$. The unit of analysis as a 25 cent bin per state per quarter is unchanged, as is the treatment wage shelf, only the control is pooled by bin groups of four.

This is more flexible than the quadratic function although not as flexible as including the full set of fixed effects. This reduces the number of wage bin × state fixed effects from 6,077 to 1,581, and reduces the number of wage bin × time effects from 11,757 down to 3,161. This allows over 200 new observations to contribute to the regression that were previously dropped due to autocorrelation, 170 of which are untreated units.

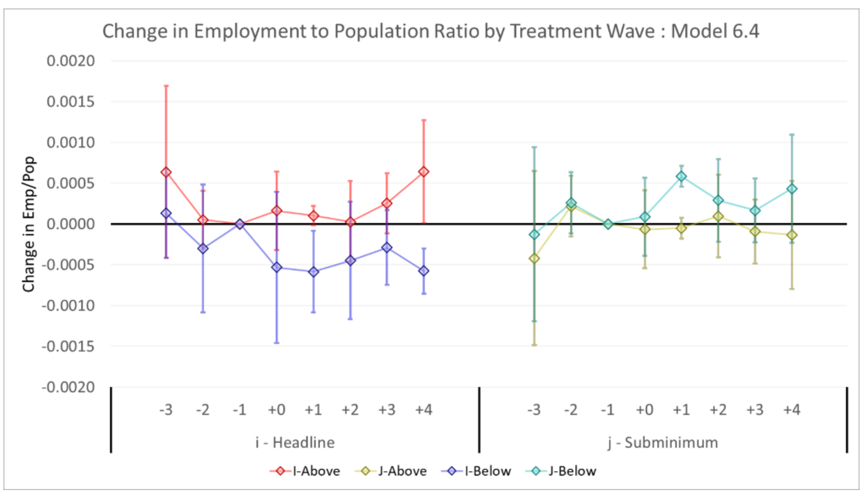
In terms of treatment shelves, there remains no statistically significant unemployment effect from an increase in the headline minimum wage. Similar to the original model there is a negative effect in wage shelves that are below the new headline minimum, and a positive or null effect on shelves above it. There is a slight shift from this tipping point being located at shelf zero or $1.00 above it. The more notable difference is that the effect of the sub minimum is now largely positive. This demonstrates even without the weighted control of $\mathbb{K}$, even if the researcher treats each treatment arm as independent there still is an ambiguous effect of the sub minimum on employment due to the sensitivity of the fixed effect size.

Figure 9: Model 6.4 Shelf Effects



It is noteworthy that the spike in employment with respect to the subminimum wage does not occur at the baseline treatment shelf, but instead occurs $2 below the subminimum[19].

Figure 10: Model 6.4 Wave Effects



[19] I find two possible explanations. Even though the CPS asks the respondent to include tips in their wage estimations, it is possible that some respondents do not do so. Separate estimates for tipped workers are reserved for future discussion in section 11.4. The second cause might be in Pennsylvania in 1997, when it raised the subminimum incrementally. The first wage step was an increase of less than $3, starting from $0. I will explore Pennsylvania in section 11 in a later draft.

Regarding the effect of the headline minimum wage in terms of treatment waves, this model suggests no statistically significant change in employment until after four years. This is notably slower than the results suggested by CDLZ. The wave effects of the subminimum are largely not significant over time except in the wave one year after a change in the subminimum, where the share of workers below the subminimum grow faster than the share just above it.

7 ROBUSTNESS

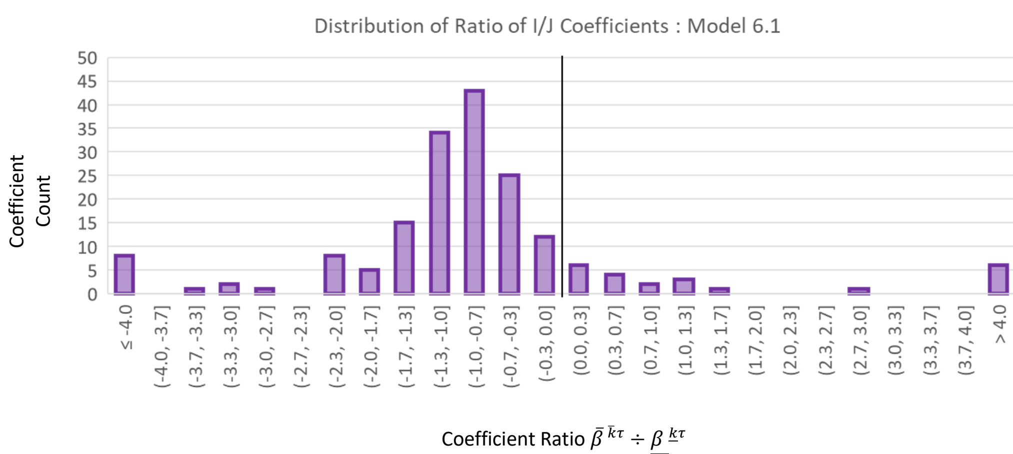
Each of these initial results are difficult to interpret in different ways. In models that include the $\mathbb{K}$ weighted shares wages between the two wage rates, many treatment units are unable to be defined. In cases where I replaced the quarter fixed effects with quadratic trends, I identify slopes for each of these treatments, however their scale often differs by several orders of magnitude, suggesting sensitivity to outliers.

Upon closer inspection, all treatment arms in 6.3 with negative shelf values $\bar{k}, \underline{k} < 0$ do not convey a representative sample of employment across the United States. There are only 12 observations with which to identify the effect of the sub-minimum, and 35 observations to identify the effect of the headline minimum. All of these cases are from Pennsylvania in the late 1990s and early 2000s. Most of them are also from one year before the Great Recession's financial crisis. The difference in the scale of effects reflects sample selection collinearity issues rather than the true average treatment effect on the treated.

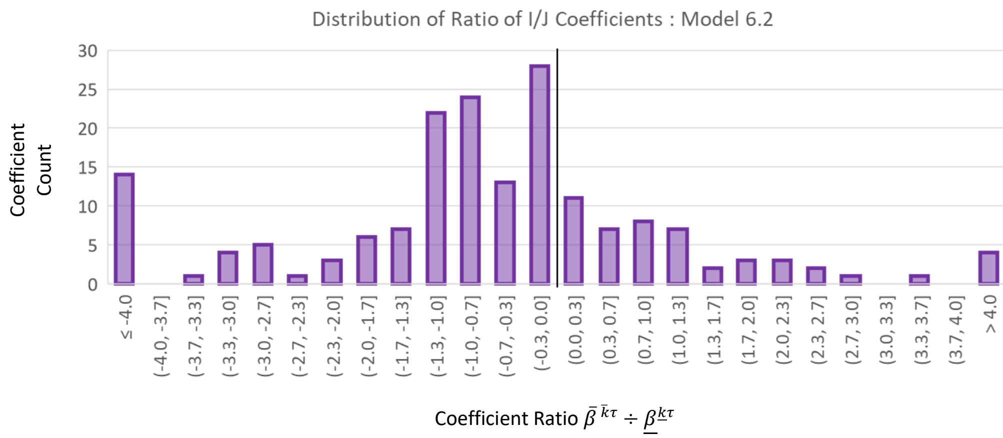
That said, some useful patterns may still be inferred based on relative comparisons between the two treatment distances. Both treatment arms exhibit the same pattern of coefficients on the scale of 0.001 for all treatment arms with positive shelves, but coefficients on the order of 1.000 among negative shelf arms. I demonstrate the relative difference between each arm in a series of correlations below. Admittedly this does not separately identify the causal effect of the subminimum wage, but it may demonstrate if there are consistent patterns in these effects to form testable hypotheses for future study.

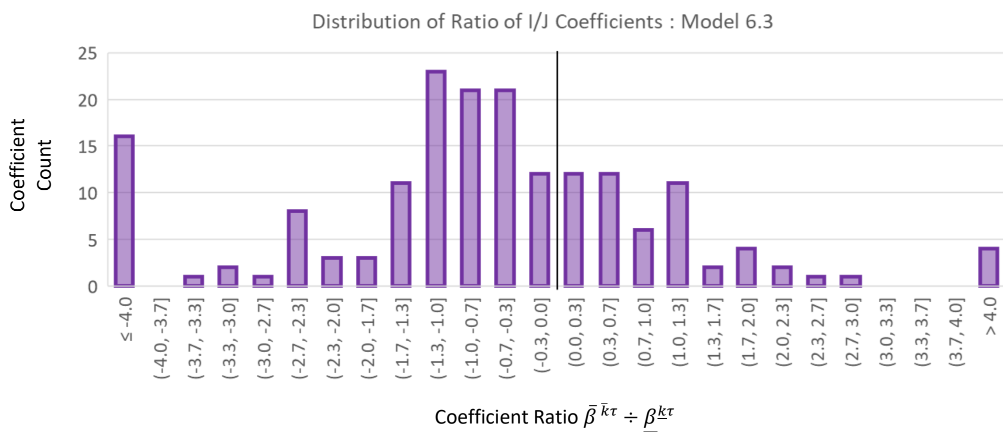
I construct the ratio of the coefficient for a given treatment shelf and wave with respect to the headline minimum divided by the coefficient for the same treatment shelf and wave with respect to the sub minimum. If this ratio is positive and greater than one, then the headline minimum exhibits a larger effect and has the same sign as the subminimum effect. If the ratio is between zero and one, then the headline minimum exhibits a smaller effect but still has the same sign as the subminimum effect. If the ratio is between zero and negative one, then the headline minimum exhibits a smaller effect and has the opposite sign as the sub-minimum effect. If the ratio is less than negative one, then the headline minimum exhibits a larger effect and has an opposite sign of the sub minimum effect.

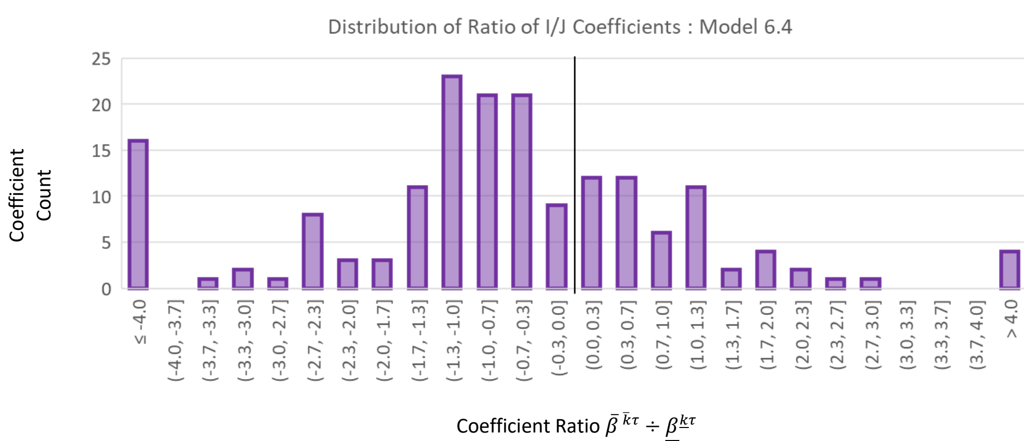
The coefficients of these regressions in some cases varied by orders of magnitude, making them difficult to directly compare to other coefficients of other shelves and waves within the same minimum type, but this magnitude was comparable within shelves and waves but across minimum type. This suggests that making one a ratio of the other allows me to compare the relative size of each effect even though these effects scale orders of magnitude[20].



[20] A previous draft of this paper included a set of scatter plots of coefficient results but to address the issue of scale I utilize the inverse hyperbolic sine transformation. I use the ratio of coefficients because its pattern is easier to interpret.







In all four cases, the most common result was that coefficients were close to a ratio of -1:1, suggesting that in many cases the coefficient results had similar magnitudes but opposite signs. The majority of results fell between -1 and 1, suggesting that the headline effects were most often smaller than the subminimum effects.

Negative ratios would be consistent with the hypothesis that workers are substitutes because an increase in the price of either is correlated with the inverse effect on the employment of the other. This suggests that the effect of the headline minimum is more muted on average, although may still reflect a larger share of the population overall. For other discussions of the robustness tests considered for this research, see section 11.

8 CONCLUSIONS

8.1 DISCUSSION OF RESULTS

Does the subminimum wage achieve its policy goals? In terms of employment, most of the models presented in this paper suggest that the headline minimum wage remains largely unchanged after accounting for a state's subminimum wage, but the results of the subminimum wage itself are ambiguous and sensitive to model specification. If one believes that the controls $\mathbb{K}$ & Ω_{jst} provide necessary control for mediating employment outcomes in response to a minimum wage change, then increasing the subminimum wage has likely yielded no significant unemployment effects.

If one believes that the controls $\mathbb{K}$ & Ω remove too much information from the estimation of the treatment effect, and that the headline minimum and sub-minimum should be entered into the regression as independent terms with no weighted share control, it yields results that there is a statistically significant negative effect of increasing the subminimum wage conditional on the headline minimum wage. This would suggest that the subminimum wage program successfully preserves the employment of its targeted workers, although this paper makes no claim as to the overall financial wellness of households under this program. Although, again, this is conditional on the belief that $\mathbb{K}$ & Ω are unneeded.

If one assumes these treatment arms remove a confounding effect, it suggests then there is a null unemployment effect of the subminimum wage. If one assumes these treatment arms are separate, it suggests there is a negative employment effect. Deciding which model is more credible is a matter for debated professional opinion, which is not a new issue for those who study the minimum wage.

8.2 CAVEATS

Why are the results from the multi-minimum stacked estimator so sensitive? One cause of this sensitivity is due to multicollinearity. The stacked estimator as specified by CDLZ requires several semi parametric effects and their preferred specification includes two way fixed effects with respect to bin-time and bin-state. Even though they defined unit of treatment bin shelves and waves that are broader than the bins and quarters of their unit of analysis, allowing up to 16 cells to contribute information to a single treatment binary, there are still many cases of multicollinearity in bins below the subminimum wage prior to the treatment.

The empirical issue is not about degrees of freedom, even switching from a model with binary time terms versus a quadratic still often yields multicollinear cells among low wages. Broadening the treatment shelf size from $1 and the treatment wave size from one year might produce enough variation to identify these low wage bin effects, similarly expanding the unit of analysis from 25 cent wage bins and quarterly periods may allow for fewer cases of multicollinearity, however the accuracy of these treatment arms become more unclear the larger each unit becomes.

Furthermore, although not included here, limiting treatment distances in $\mathbb{J}$ to those bins that are only below the headline minimum produces similar effects for shelves above the subminimum, but still leaves shelves below unidentified or noisy.

The issue may be far simpler. Studying outcomes of any small population using real world data is an inherently noisy process. It is difficult to ensure a representative sample, accurate completion of the survey, and assumes there are no competing factors that might influence your measurements. The Current Population Survey is conducted with 60,000 households every month, but if only about 5% of the respondents earns near the minimum wage, and only a fraction is allowed to be paid less than that, then results may be driven by a handful of observations.

This problem is not limited to the Current Population Survey. The Minnesota Department of Employment and Economic Development employment data from the unemployment insurance program used to calibrate the results of CDLZ demonstrates similar oddities. From 2005 through 2015 according to the version of this data provided by the CDLZ replication code, there were around 100,000 Minnesotans earning less than the headline minimum wage, but still 50,000 Minnesotans earning less than the subminimum wage, and around 25,000 Minnesotans earning less than $2 an hour. Even going back to when the federal minimum wage was higher than the Minnesota state minimum wage, where the tip credit might be binding, the federal tipped wage was higher than $2. It remains a mystery if these observations accurately reflect a mass of workers with a disability with no minimum, or if even this administrative data reflects incorrectly reported firm level employment data.

Outliers have the possibility to drastically change the results of any regression, many researchers seek to limit the influence of these outliers on their regression estimates with a number of different control factors, but this is an especially difficult problem to face when researching exemptions to the minimum wage because those eligible for the subminimum rate are by their very nature defined by their dissimilarity to their peers. These cases are the exceptions to the rules of labor policy.

8.3 Future Developments

Based on the administrative data from the Minnesota Department of Employment and Economic Development, a many workers still report wages that are even below the subminimum wage, suggesting that even administrative data is not free from data ambiguity. This could be due to other specialized exemptions, it could be due to following nonstandard accounting practices, or (less likely) it could be due to employers shirking the minimum wage law but for some reason choosing to continue to report their payments. Ideally the researcher would have access to individual level information about wages, employment, and timing, but such detailed data is often restricted to the public. Furthermore, such records would still rely on an accurate reporting of wages from the employer's perspective and could still conflict with the list of firms that are, for example, eligible to pay below the minimum wage due to the federal 14-C commensurate wage policy.

The solution to this problem according to CDLZ is to utilize the deconvolution approach similar to Autor et al (2023), this presumes the kernel distribution of the natural logarithm of wages is common among the population and that fluctuations in a given wage bin may be disproportionately due to misreported wages based on patterns in the overall wage distribution. Furthermore Dube, Manning, and Naidu discuss that there are some wage bins that will have higher density than their neighboring bins due to their proximity to round numbers (2020).

Both of these methods rely on the assumption that the residuals of wage misreporting are still log-normally distributed and independent of each other. While some level of independence is necessary for any regression, I hope my paper has demonstrated the possibility that those below the headline minimum wage and those between the headline and subminimum wages may face a different distribution of employment factors and therefore should not be thrown into the same deconvolution algorithm as the rest of the wage distribution. To address this issue, future research could utilize the use of a general method of moments approach with shooting constants at the edges of each bin distribution to estimate the Taylor series approximation of the underlying wage distribution to get an estimate of misreported wages in the sample.

I also hope to include another test using a Two Stage Fuzzy Regression Discontinuity Design model, but I am currently wary about this method due to the possible violation of the Stable Unit Treatment Value Assumption when using the stacked estimator, given that each wage bin is by design supposed to have an equilibrium ripple effect on each other.

Finally, I hope to include the coefficient outlier technique which identifies 1% of the sample that is most responsible for contributing to the estimation of the slope, then drops these units as a robustness test for the overall distribution of observations with similar relationship to the dependent variable (Broderick, Giordano, & Meager, 2020). Yet the use of this tool on a stacked difference in difference estimator is so far uncommon.

8.4 POLICY IMPLICATIONS

The null effects of the subminimum wage may suggest that total labor demand is inelastic under monopsony bargaining power. Another possibility is that the subminimum wage policy as implemented is not intended solely as employment policy, and its effect is narrow. Early evidence of the student subminimum wage suggests it is seldom utilized by the private sector (Spriggs, 1993). If the subminimum wage is intended as a way to subsidize the employment of a class of worker, and if the employment of that worker is considered a public interest, then the use of the subminimum wage may be treated as a way to finance their employment.

Although there is no statistically significant change in employment, this paper does not address changes in output prices in response to a subminimum wage increase. If firms pass through these new costs onto their customers, and if the utilization of this policy is primarily from organizations with a nonprofit objective (such as Community Rehabilitation Programs for people with disabilities), or organizations with some other social good (like the State offering visa permits), then the cost burden may still fall upon the administrators, donors, and grant providers of that organization. How these social objectives are met is based on the relative bargaining power of the stakeholders in the organization and their influence on management (Montias, Ben-Ner, & Neuberger, 1994).

In its most sinister interpretation, the subminimum wage may also be implemented to preserve a subaltern class of worker with intentionally lower economic power than the majority. Whether these lower rates are designed to prevent exclusion or as a tool of it depends on the constellation of other policy factors that contribute to the well-being of these classes of workers.

The employment effects in this paper might be due to monopsony power from the employer, or could be due to a fundamental difference in the factors that determine employment in those sectors that utilize the subminimum wage. Many firms that are listed as subminimum wage eligible by the Department of Labor are community rehabilitation centers for workers with disabilities, and employment situations that may not be directly comparable to their peers. Although my results suggest the subminimum wage may not be a necessary accommodation to this sector, perhaps the more telling takeaway is that my multitiered approach with a new estimator was only necessary due to a lack of full information about such workers. Policymakers would benefit from more proactive study of the classes of workers affected by this policy, but this data dearth requires its own policy action.

8.5 Final Thoughts

The research literature on the minimum wage has changed drastically in the last 10 years. The ongoing debate over regression discontinuity, synthetic control, robust difference in difference, or the stacked estimator has yielded a generation of researchers who both demonstrate great econometric talent while aware that a difference in methodology may yield drastically different results. Despite the numerous papers about the minimum wage there is still great debate over its effect on employment, and although there is strong and convincing evidence using many estimators many researchers are drawn to estimators that yield different results.

Using the stacked estimator and accounting for necessary confounders, my results generally suggest that the subminimum wage has little effect on employment at the headline minimum, and ambiguous results at the subminimum.

The reason I chose a stacked difference in difference estimator is because it gave me a parametric way to plausibly identify workers earning below the headline minimum wage without resulting to the assumption that these workers are subject to wage theft or reporting recall bias. Although it is generous to assume the accuracy of records of the subminimum wage, and it is strong to assume that the count of workers between two wage tiers provide enough information to yield clear results, I would offer that ignoring this possibility is an equally generous assumption, only it assumes the opposite extreme.

If researchers had access to detailed employment demographic information about low wage workers, and if there was a similar popular interest in understanding the employment situation of these exempt workers, perhaps more direct identification would be possible. Until then, results from the stacked estimator based on theory introduce a step to further discussion of the need for this policy. If exemptions to the minimum wage serves a policy goal for community rehabilitation centers, visa workers, or youths, it is worth further discussion over what problem the subminimum wage is intended to address and if other policy actions may more directly serve these communities.

After a minimum wage increase, the added labor cost comes not only from a reduction in firm profit margins but from an increase in output prices. If programs for those with a disability, visa workers, and children are the subject of this policy, the pass-through cost of raising the subminimum wage may still be passed on to the public or onto the family and community members of those who support these workers. The subminimum wage does not exist in a vacuum and a comprehensive policy approach will be necessary to address its larger social objectives.

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10 Results Tables <10/2023 Results>

Change in Employment / Population Ratio by Treatment
Model 6.1 - Alpha = 0%, Trend = Binary

	Headline - $\mathbb{I}$			Subminimum - $\mathbb{J}$		
		95% Interval			95% Interval	
	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper
-4	-0.000001 (0.000147)	-0.000296	0.000294	0.000502 * (0.000279)	-0.000058	0.001063
-3	-0.000165 * (0.000093)	-0.000352	0.000022	0.000332 (0.000224)	-0.000118	0.000781
-2	0.000310 (0.000270)	-0.000233	0.000853	-0.001021 ** (0.000400)	-0.001824	-0.000218
-1	0.000072 (0.000125)	-0.000180	0.000323	0.000004 (0.000116)	-0.000229	0.000237
0	-0.000462 *** (0.000136)	-0.000736	-0.000188	-0.000372 * (0.000216)	-0.000805	0.000062
1	0.000397 *** (0.000112)	0.000173	0.000621	-0.000013 (0.000170)	-0.000355	0.000329
2	0.000094 (0.000094)	-0.000094	0.000282	-0.000116 (0.000199)	-0.000515	0.000282
3	0.000028 (0.000086)	-0.000145	0.000202	-0.000133 (0.000158)	-0.000451	0.000185
4	0.000109 (0.000099)	-0.000090	0.000308	-0.000416 *** (0.000133)	-0.000682	-0.000149
5	0.000084 (0.000106)	-0.000128	0.000297	0.000010 (0.000122)	-0.000236	0.000255
6	-0.000012 (0.000081)	-0.000175	0.000152	0.000009 (0.000128)	-0.000248	0.000265
7	0.000118 (0.000076)	-0.000034	0.000270	0.000128 (0.000111)	-0.000096	0.000351
8	0.000068 (0.000091)	-0.000114	0.000250	-0.000201 * (0.000114)	-0.000429	0.000027
9	0.000038 (0.000077)	-0.000117	0.000194	-0.000149 (0.000116)	-0.000382	0.000084
10	-0.000067 (0.000081)	-0.000230	0.000097	-0.000042 (0.000092)	-0.000227	0.000143
11	-0.000075 (0.000086)	-0.000247	0.000098	-0.000054 (0.000146)	-0.000348	0.000240
12	-0.000041 (0.000078)	-0.000197	0.000115	-0.000035 (0.000085)	-0.000205	0.000135
13	0.000043 (0.000064)	-0.000086	0.000171	-0.000088 (0.000103)	-0.000294	0.000118
14	0.000047 (0.000082)	-0.000119	0.000212	-0.000077 (0.000108)	-0.000293	0.000139
15	-0.000009 (0.000055)	-0.000120	0.000102	0.000067 (0.000107)	-0.000148	0.000282
16	-0.000038 (0.000073)	-0.000183	0.000108	-0.000145 (0.000091)	-0.000328	0.000039
17	0.000010 (0.000091)	-0.000172	0.000192	0.000034 (0.000086)	-0.000140	0.000207

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given shelf and the difference between the treatment waves just after and just before the minimum wage increase,
\ where the after treatment waves +0 through +4 are weighted by 1/5 and the before treatment

Change in Employment / Population Ratio by Treatment Wave : Model 6.1 - Alpha = 0%, Trend = Binary

Referent		Slope (S.E.)	95% Interval Lower	95% Interval Upper	Slope (S.E.)	95% Interval Lower	95% Interval Upper
Headline - 𝕀	-3	-0.000052 (0.00008)	-0.000219	0.000116	0.000211 * (0.00012)	-0.000036	0.000457
	-2	0.000044 (0.00008)	-0.000123	0.000212	0.000176 * (0.00010)	-0.000030	0.000383
	-1	zero zero	zero	zero	zero zero	zero	zero
	0	0.000129 * (0.00007)	-0.000016	0.000274	-0.000081 (0.00010)	-0.000274	0.000112
	1	0.000011 (0.00006)	-0.000106	0.000128	0.000241 ** (0.00010)	0.000046	0.000436
	2	0.000037 (0.00008)	-0.000128	0.000201	0.000087 (0.00009)	-0.000084	0.000259
	3	-0.000050 (0.00010)	-0.000246	0.000145	0.000057 (0.00013)	-0.000196	0.000309
	4	0.000040 (0.00007)	-0.000110	0.000191	-0.000034 (0.00013)	-0.000290	0.000221
Subminimum - 𝕁	-3	-0.00027 ** (0.00011)	-0.00049	-0.00005	-0.00013 (0.00012)	-0.00037	0.00011
	-2	-0.00013 (0.00012)	-0.00036	0.00010	0.00003 (0.00019)	-0.00034	0.00041
	-1	zero zero	zero	zero	zero zero	zero	zero
	0	-0.00030 *** (0.00010)	-0.00051	-0.00009	-0.00033 (0.00023)	-0.00079	0.00014
	1	-0.00037 *** (0.00010)	-0.00057	-0.00017	-0.00030 * (0.00017)	-0.00063	0.00003
	2	-0.00031 ** (0.00013)	-0.00056	-0.00005	-0.00005 (0.00021)	-0.00047	0.00036
	3	-0.00007 (0.00013)	-0.00033	0.00020	0.00013 (0.00017)	-0.00021	0.00047
	4	0.00000 (0.00011)	-0.00023	0.00023	0.00032 ** (0.00014)	0.00003	0.00060

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given wave and the difference between the treatment shelves above and just below the minimum wage increase, where the after treatment shelves +0 through +3 are weighted by 1/4 and the before treatment shelves -4 through -1 is weighted by 1/4.

Change in Employment / Population Ratio by Treatment Shelves:
Modele 6.2 - Alpha = 10%, Trend = Binaries

	Headline - $\mathbb{I}$			Subminimum - $\mathbb{J}$			Between - $\mathbb{K}$		
		95% Interval			95% Interval			95% Interval	
	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper
-4	-0.006481 *** (0.001048)	-0.008586	-0.004377	Dropped			-0.091160 (0.065343)	-0.222406	0.040086
-3	0.001061 *** (0.000082)	0.000896	0.001226	Dropped					
-2	Dropped			Dropped					
-1	Dropped			-0.002229 *** (0.000552)	-0.003336	-0.001121			
0	0.002605 *** (0.000563)	0.001475	0.003735	0.000791 (0.000609)	-0.000433	0.002015			
1	0.000502 *** (0.000156)	0.000189	0.000815	-0.000177 (0.000440)	-0.001061	0.000707			
2	0.000081 (0.000122)	-0.000164	0.000326	0.000354 ** (0.000170)	0.000013	0.000695			
3	-0.000028 (0.000129)	-0.000286	0.000230	-0.000220 (0.000192)	-0.000605	0.000165			
4	0.000004 (0.000109)	-0.000216	0.000223	-0.000428 ** (0.000212)	-0.000853	-0.000003			
5	0.000065 (0.000113)	-0.000161	0.000291	0.000047 (0.000202)	-0.000358	0.000453			
6	0.000029 (0.000106)	-0.000183	0.000241	-0.000108 (0.000152)	-0.000413	0.000198			
7	-0.000083 (0.000107)	-0.000298	0.000133	0.000067 (0.000133)	-0.000199	0.000334			
8	-0.000041 (0.000104)	-0.000250	0.000168	-0.000093 (0.000149)	-0.000393	0.000207			
9	-0.000024 (0.000084)	-0.000193	0.000145	-0.000205 (0.000123)	-0.000453	0.000042			
10	-0.000126 (0.000089)	-0.000305	0.000052	-0.000143 (0.000133)	-0.000410	0.000124			
11	0.000102 (0.000129)	-0.000158	0.000362	-0.000119 (0.000134)	-0.000388	0.000151			
12	-0.000009 (0.000130)	-0.000271	0.000253	0.000174 (0.000144)	-0.000115	0.000464			
13	-0.000031 (0.000118)	-0.000269	0.000207	-0.000083 (0.000134)	-0.000353	0.000187			
14	0.000019 (0.000130)	-0.000242	0.000280	-0.000061 (0.000124)	-0.000310	0.000187			
15	-0.000030 (0.000135)	-0.000300	0.000241	-0.000265 * (0.000135)	-0.000537	0.000006			
16	-0.000327 (0.000216)	-0.000761	0.000106	0.000116 (0.000148)	-0.000181	0.000413			
17	0.000005 (0.000153)	-0.000301	0.000311	0.000012 (0.000164)	-0.000318	0.000342			

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given shelf and the difference

Change in Employment / Population Ratio by Treatment Wave :
Model 6.2 - Alpha = 10%, Trend = Binaries

Referent		Slope (S.E.)		95% Interval Lower	95% Interval Upper	Slope (S.E.)		95% Interval Lower	95% Interval Upper
Headline - 𝕀	-3	0.000027		-0.000282	0.000337	-0.001208	***	-0.001950	-0.000466
		(0.00015)				(0.00037)			
	-2	0.000198		-0.000190	0.000586	-0.000966	*	-0.002065	0.000134
		(0.00019)				(0.00055)			
	-1	zero		zero	zero	zero		zero	zero
		zero				zero			
	0	0.001065	***	0.000636	0.001493	-0.001367	***	-0.001892	-0.000841
		(0.00021)				(0.00026)			
	1	0.000487	**	0.000012	0.000963	-0.000302		-0.000852	0.000248
		(0.00024)				(0.00027)			
	2	0.000641	***	0.000282	0.001000	-0.003080	***	-0.003772	-0.002389
		(0.00018)				(0.00034)			
	3	0.000422	**	0.000094	0.000751	-0.000816	**	-0.001452	-0.000180
		(0.00016)				(0.00032)			
	4	0.000549	**	0.000041	0.001056	-0.001210	***	-0.001855	-0.000566
		(0.00025)				(0.00032)			
Subminimum - 𝕁	-3	0.00025		-0.00049	0.00099	-0.00010		-0.00117	0.00098
		(0.00037)				(0.00054)			
	-2	-0.00052		-0.00146	0.00042	-0.00174	***	-0.00267	-0.00081
		(0.00047)				(0.00046)			
	-1	0.00000	***	0.00000	0.00000	0.00000	***	0.00000	0.00000
		0.0000				0.0000			
	0	-0.00020		-0.00101	0.00060	-0.00058	***	-0.00078	-0.00037
		(0.00040)				(0.00010)			
	1	0.00044		-0.00011	0.00100	-0.00058	***	-0.00078	-0.00037
		(0.00028)				(0.00010)			
	2	-0.00017		-0.00077	0.00044	-0.00058	***	-0.00078	-0.00037
		(0.00030)				(0.00010)			
	3	-0.00006		-0.00051	0.00038	-0.00048		-0.00138	0.00043
		(0.00022)				(0.00045)			
	4	0.00031		-0.00021	0.00083	-0.00058	***	-0.00078	-0.00037
		(0.00026)				(0.00010)			
Between - 𝕂	-3	-0.00973		-0.03990	0.02044				
		-0.0150		0.00000	0.00000				
	-2	-0.02568	*	-0.05345	0.00210				
		-0.0138		0.00000	0.00000				
	-1	zero		zero	zero				
		zero							
	0	-0.15531	***	-0.19348	-0.11713				
		-0.0190		0.00000	0.00000				
	1	0.09055	***	0.02632	0.15479				
		-0.0320		0.00000	0.00000				
	2	-0.00290		-0.06313	0.05733				
		-0.0300		0.00000	0.00000				
	3	-0.02299		-0.06252	0.01653				
		-0.0197		0.00000	0.00000				
	4	-0.02330		-0.06265	0.01605				
		-0.0196		0.00000	0.00000				

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given wave and the difference between the treatment shelves above and just below the minimum wage increase, where the after treatment shelves +0 through +3 are weighted by 1/4 and the before treatment shelves -4 through -1 is weighted by 1/4.

Change in Employment / Population Ratio by Treatment Shelves:
Model 6.3 - Alpha = 10%, Trend = Quadratic

	Headline - 𝕀			Subminimum - 𝕁			Between - 𝕂		
		95% Interval			95% Interval			95% Interval	
	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper
-4	2.412661 (2.407022)	-2.421985	7.247306	2.557064 (2.221488)	-1.904925	7.019054	-0.091160 (0.065343)	-0.222406	0.040086
-3	-1.000348 (2.932327)	-6.890102	4.889405	-0.663785 (2.825607)	-6.339183	5.011613			
-2	0.386457 (2.384850)	-4.403656	5.176570	-0.116170 (2.049239)	-4.232188	3.999849			
-1	0.484427 (2.021079)	-3.575029	4.543882	0.092867 (1.941089)	-3.805925	3.991658			
0	0.002279 (0.001440)	-0.000613	0.005172	-0.003104 ** (0.001468)	-0.006052	-0.000155			
1	0.000397 (0.000819)	-0.001247	0.002041	0.000624 (0.000819)	-0.001021	0.002269			
2	-0.000151 (0.000649)	-0.001454	0.001151	0.000372 (0.000652)	-0.000938	0.001681			
3	-0.000908 *** (0.000215)	-0.001340	-0.000477	0.001090 *** (0.000229)	0.000629	0.001550			
4	-0.000037 (0.000194)	-0.000427	0.000352	0.000605 *** (0.000219)	0.000165	0.001044			
5	-0.000387 *** (0.000119)	-0.000626	-0.000147	0.000383 *** (0.000134)	0.000114	0.000653			
6	-0.000368 (0.000586)	-0.001544	0.000809	0.000463 (0.000625)	-0.000792	0.001717			
7	-0.000154 (0.000649)	-0.001457	0.001150	0.000034 (0.000663)	-0.001299	0.001366			
8	0.000096 (0.000580)	-0.001069	0.001262	-0.000128 (0.000599)	-0.001331	0.001075			
9	-0.000456 (0.000519)	-0.001498	0.000587	0.000271 (0.000532)	-0.000797	0.001340			
10	-0.000336 (0.000363)	-0.001065	0.000393	-0.000156 (0.000410)	-0.000980	0.000668			
11	0.000299 (0.000559)	-0.000824	0.001422	-0.000268 (0.000584)	-0.001441	0.000904			
12	0.001024 ** (0.000443)	0.000135	0.001914	-0.000889 * (0.000464)	-0.001820	0.000043			
13	0.001511 *** (0.000380)	0.000748	0.002274	-0.001320 *** (0.000399)	-0.002123	-0.000518			
14	0.000638 * (0.000348)	-0.000060	0.001337	-0.000419 (0.000366)	-0.001153	0.000316			
15	0.000771 *** (0.000259)	0.000251	0.001291	-0.001010 *** (0.000282)	-0.001576	-0.000445			
16	0.000672 (0.000701)	-0.000737	0.002081	-0.001162 ** (0.000576)	-0.002320	-0.000005			
17	-0.000015 (0.000316)	-0.000650	0.000620	0.000225 (0.000301)	-0.000379	0.000830			

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given shelf and the difference between the treatment waves just after and just before the minimum wage increase, where the after treatment waves +0 through +4 are weighted by 1/5 and the before treatment wave -1 is weighted by 1.

Change in Employment / Population Ratio by Treatment Wave :
Model 6.3 - Alpha = 10%, Trend = Quadratic

Referent		Slope (S.E.)		95% Interval Lower	95% Interval Upper	Slope (S.E.)	95% Interval Lower	95% Interval Upper
Headline - 𝕀	-3	-0.0000968 -0.0004567		-0.0010141	0.0008205	-0.0926709 -1.0920890	-2.2861960	2.1008550
	-2	0.0000830 -0.0005793		-0.0010805	0.0012464	0.9344201 -3.3903250	-5.8752470	7.7440880
	-1	Baseline				Baseline		
	0	0.0006925 -0.0006127		-0.0005381	0.0019232	0.2859521 -2.4267590	-4.5883360	5.1602400
	1	-0.0001137 -0.0002850		-0.0006861	0.0004586	1.1358070 -2.1164770	-3.1152610	5.3868760
	2	0.0002456 -0.0003832		-0.0005240	0.0010152	0.5394206 -1.6470800	-2.7688370	3.8476780
	3	-0.0000233 -0.0002812		-0.0005881	0.0005415	0.5419734 -1.4367330	-2.3437900	3.4277370
	4	0.0007790 -0.0007217		-0.0006705	0.0022286	0.3508414 -1.2792470	-2.2186020	2.9202850
Subminimum - 𝕁	-3	0.0002561 -0.0004500		-0.0006477	0.0011599	-0.2603449 -0.9651563	-2.1989180	1.6782290
	-2	0.0003014 -0.0005992		-0.0009021	0.0015048	0.7771884 -2.6804810	-4.6067160	6.1610930
	-1	Baseline				Baseline		
	0	-0.0005649 -0.0006290		-0.0018283	0.0006986	0.2591763 -2.0324680	-3.8231560	4.3415080
	1	0.0002311 -0.0003130		-0.0003975	0.0008597	1.0821010 -1.5304930	-1.9919850	4.1561860
	2	0.0000039 -0.0003898		-0.0007791	0.0007868	0.2428605 -1.3869650	-2.5429420	3.0286630
	3	-0.0000705 -0.0002802		-0.0006333	0.0004922	1.0543710 -1.2113490	-1.3786940	3.4874360
	4	-0.0000140 -0.0007411		-0.0015026	0.0014745	-0.3010385 -1.1860200	-2.6832290	2.0811520
Between - 𝕂	-3	0.2603155 0.2439122		-0.2295966	0.7502276			
	-2	0.0180094 0.0874082		-0.1575552	0.1935740			
	-1	-0.0261313 0.1565892		-0.3406499	0.2883873			
	0	0.5248420 0.0719586	***	0.3803088	0.6693752			
	1	-0.4143518 0.1330393	***	-0.6815691	-0.1471346			
	2	0.1047959 0.2959516		-0.4896404	0.6992321			
	3	-0.1184775 0.1644837		-0.4488526	0.2118976			
	4	-0.5709372 0.3974338		-1.3700000	0.2273321			

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given wave and the difference between the treatment shelves above and just below the minimum wage increase, where the after treatment shelves +0 through +3 are weighted by 1/4 and the before treatment shelves -4 through -1 is weighted by 1/4.

Change in Employment / Population Ratio by Treatment
Model 6.4 - Alpha = 0%, Trend = Binary (with Pooled Bins)

	Headline - 𝕀			Subminimum - 𝕁		
		95% Interval			95% Interval	
	Slope (S.E.)	Lower	Upper	Slope (S.E.)	Lower	Upper
-4	-0.000602 * (0.000326)	-0.001257	0.000052	0.000309 (0.000330)	-0.000354	0.000973
-3	0.000688 ** (0.000267)	0.000153	0.001224	-0.000617 ** (0.000284)	-0.001188	-0.000047
-2	-0.001155 * (0.000581)	-0.002322	0.000012	0.001415 ** (0.000633)	0.000144	0.002686
-1	-0.000876 *** (0.000193)	-0.001263	-0.000488	0.000146 (0.000199)	-0.000253	0.000545
0	0.000355 (0.000353)	-0.000355	0.001065	-0.000413 (0.000372)	-0.001159	0.000333
1	0.000572 *** (0.000122)	0.000328	0.000817	0.000079 (0.000144)	-0.000211	0.000368
2	0.000059 (0.000151)	-0.000243	0.000362	0.000065 (0.000135)	-0.000206	0.000336
3	0.000131 (0.000303)	-0.000479	0.000740	0.000078 (0.000266)	-0.000457	0.000612
4	0.000074 (0.000193)	-0.000315	0.000462	-0.000054 (0.000204)	-0.000464	0.000356
5	0.000257 (0.000321)	-0.000388	0.000901	0.000083 (0.000350)	-0.000620	0.000785
6	0.000352 (0.000321)	-0.000294	0.000997	0.000058 (0.000340)	-0.000624	0.000740
7	0.000480 (0.000298)	-0.000118	0.001078	0.000034 (0.000303)	-0.000575	0.000643
8	0.000399 (0.000307)	-0.000217	0.001016	-0.000134 (0.000311)	-0.000759	0.000491
9	0.000170 (0.000220)	-0.000273	0.000612	0.000234 (0.000225)	-0.000217	0.000686
10	0.000492 *** (0.000080)	0.000331	0.000652	-0.000273 *** (0.000082)	-0.000437	-0.000109
11	0.000396 ** (0.000187)	0.000021	0.000772	-0.000315 * (0.000185)	-0.000685	0.000056
12	0.000262 (0.000225)	-0.000189	0.000713	-0.000169 (0.000220)	-0.000610	0.000273
13	0.000351 * (0.000178)	-0.000006	0.000708	-0.000219 (0.000184)	-0.000588	0.000151
14	0.000112 (0.000202)	-0.000293	0.000517	0.000070 (0.000209)	-0.000350	0.000489
15	0.000279 (0.000250)	-0.000223	0.000782	-0.000057 (0.000260)	-0.000579	0.000466
16	0.000068 (0.000113)	-0.000160	0.000295	0.000110 (0.000112)	-0.000116	0.000336
17	0.000044 (0.000147)	-0.000252	0.000339	0.000263 * (0.000145)	-0.000029	0.000555

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given shelf and the difference between the treatment waves just after and just before the minimum wage increase, where the after treatment waves +0 through +4 are weighted by 1/5 and the before treatment wave -

Change in Employment / Population Ratio by Treatment Wave :
Model 6.4 - Alpha = 0%, Trend = Binary (with Pooled Bins)

Referent		Slope (S.E.)	95% Interval Lower	95% Interval Upper	Slope (S.E.)	95% Interval Lower	95% Interval Upper
Headline - 𝕀	-3	0.000638 (0.00052)	-0.000417	0.001692	0.000130 (0.00027)	-0.000412	0.000671
	-2	0.000051 (0.00018)	-0.000306	0.000408	-0.000300 (0.00039)	-0.001086	0.000486
	-1	zero zero	zero	zero	zero zero	zero	zero
	0	0.000163 (0.00024)	-0.000319	0.000645	-0.000532 (0.00046)	-0.001456	0.000392
	1	0.000104 * (0.00006)	-0.000012	0.000219	-0.000586 ** (0.00025)	-0.001086	-0.000086
	2	0.000027 (0.00025)	-0.000472	0.000527	-0.000447 (0.00036)	-0.001168	0.000274
	3	0.000256 (0.00018)	-0.000115	0.000626	-0.000289 (0.00023)	-0.000749	0.000171
	4	0.000642 ** (0.00031)	0.000010	0.001274	-0.000576 *** (0.00014)	-0.000852	-0.000300
Subminimum - 𝕁	-3	-0.00042 (0.00053)	-0.00149	0.00065	-0.00013 (0.00029)	-0.00070	0.00045
	-2	0.00022 (0.00019)	-0.00016	0.00059	0.00026 (0.00041)	-0.00056	0.00108
	-1	zero zero	zero	zero	zero zero	zero	zero
	0	-0.00007 (0.00024)	-0.00054	0.00041	0.00009 (0.00045)	-0.00081	0.00099
	1	-0.00005 (0.00006)	-0.00018	0.00008	0.00058 ** (0.00026)	0.00007	0.00110
	2	0.00010 (0.00025)	-0.00041	0.00060	0.00029 (0.00037)	-0.00045	0.00104
	3	-0.00009 (0.00020)	-0.00049	0.00030	0.00017 (0.00024)	-0.00032	0.00066
	4	-0.00013 (0.00033)	-0.00080	0.00053	0.00043 *** (0.00016)	0.00012	0.00075

Statistical Significance 0.10 * 0.05 ** 0.01 ***
Results combined using the delta method and the lincom command. Based on a given wave and the difference between the treatment shelves above and just below the minimum wage increase, where the after treatment shelves +0 through +3 are weighted by 1/4 and the before treatment shelves -4 through -1 is weighted by 1/4.

11 FUTURE TESTS <PENDING DISSERTATION>

11.1 TESTING SENSITIVITY TO ALPHA

R1-R10 will include $\mathbb{I}$, $\mathbb{J}$, $\mathbb{K}$ and Ω. I assume different shares of workers in each wage bin are eligible for the subminimum wage to adjust $\mathbb{K}$ accordingly. Model one assumes a 1% share of workers are eligible, model 2 assumes 5%, model 3 assumes 10%, model 4 assumes 90%, model 5 assumes 100%, and model 6 assumes that each wage bin has a assumed density based on the number of people who report one of a few class indicators which are often used for this sub-minimum wage. This includes disability status to address the commensurate wage, recent immigration status to address the J1 visa status, age and schooling to address the training period exemption, and firm head count to address the size distinction. I display a distribution of results below only for the wage shelves just below, at, and just above each respective minimum wage for the sake of brevity.

Since there are many ways to specify this problem, there are even more ways to test for the robustness of its results. I have included a series of other robustness tests that I do not report in the body of the paper for the sake of brevity. This includes some geographic specification tests in states that changed the sub minimum without changing the headline minimum, such as Pennsylvania, and include tests for states that increase the headline minimum without necessarily increasing the sub minimum, such as Minnesota. In each of these cases however there are so few observations that it's requires the researcher to drastically change the specification of the model to glean any useful insights. This would require either using broader fixed effects, wider treatment shelves, or using a data set with more observations. Below I include a list of specification tests I have designed but are not included. They are scheduled to run over the winter <2023>.

Models **R5-R7** will include $\mathbb{I}$, $\mathbb{J}$, and $\mathbb{K}$ but not Ω. This produces regression results that are based on the difference in the share of marginal costs associated with workers of each class type in each bin, but does not have the additional control on small minimum wage changes or changes made in close succession.

11.2 TESTING SENSITIVITY TO DISABILITY STATUS

H1 will include 𝕀 but assumes that anchor to 𝕁 is one half of headline minimum. This is based on the statutory limit of the commensurate subminimum wage for a person with a disability in the 1980's, which could not go lower than one half the wage of their peers. Later, this backstop was lifted, providing no backstop. For this reason, **H2** uses zero as the subminimum. The dependent variable also changes to the employment to population ratio of those who report a work limiting disability in the CPS.

N will include 𝕀 and 𝕁 but only includes states where the subminimum wage is different from the headline minimum. This result will exclude "never treated" units who do not have a separate subminimum wage policy from the control group, and variation in the treatment reflects early vs late adopters.

11.3 TESTING ISOLATED CHANGES IN ONE RATE WITHOUT THE OTHER

L1, L2, and L3 limit the sample to Pennsylvania and New Jersey. Pennsylvania has a subminimum wage policy that has changed independent of its headline minimum wage. This is one of the rare examples where the subminimum and headline minimum do not change simultaneously. Each treatment will be defined separately for the three different timings of a change in the Pennsylvania minimum wage regime.

M1 and M2 will limit the sample to Minnesota and Wisconsin. The headline minimum wage in Minnesota was below the federal minimum of $7.25. Even though the state minimum was less than this, the effective minimum wage in Minnesota at that time would be the federal wage rate. From the late 90s to the late 2000s, the Minnesota headline minimum wage increased well above the federal minimum to a rate of $9.50, but the subminimum also increased to $7.75, this is a rare occurrence when both the headline and subminimum were below the federal rate and then both were above it after a change in the regime. Furthermore, in 2007 the headline minimum wage increased after and increase in the federal rate without a change in the effective subminimum wage, providing another rare example of change in one wage rate without a change in the other.

11.4 OTHER TESTS

H2 will include the full set of regressors, but limit the sample to only the employment among workers who are plausibly affected by the subminimum wage directly. This would include workers who report having a work limiting disability under the commensurate wage, recent immigrants under the J1 visa program, student workers under the schooling exemption, small business workers according to the March supplement of the CPS, and tipped workers.

R1, R2, R3, and R4 will include $\mathbb{I}$, $\mathbb{J}$, $\mathbb{K}$, and Ω. This is similar to **E**, but each version of this regression presumes a different share of a wage bin α_j^1 that belongs to the exempt policy class and therefore changes the weighted average used as a control. Since we do not have administrative data on the subminimum wage eligibility status of respondents to the Current Population Survey, this section models what the results would be under a set of feasible values of alpha. R1 assumes the rate of exempt workers is 10% uniformly across all wage bins. R2 Assumes a 1% rate. R3 Assumes the percentage is based on the share of the workforce who in this CPS state they are either a recent immigrant, a student in training, eligible for tips, or have a work limiting disability.[21] R4 Assumes alpha is based on the percentage change in a wage bins density after a change in the minimum wage according to data from the Minnesota Department of Employment and Economic Development, and estimating this alpha with a GMM approach using shooting constants to address corner solutions. This is pending funding from

[21] Firm size according to head count is also possible in the CPS, but only during the March supplement, which reduces the sample size and requires separate population weights and therefore is left for future research.

the Roy Wilkins Center for Human Relations and Social Justice for the cost of data.

W will include a limitation of the treatment to only instances that have an increase in the minimum wage during a single time wave. If a minimum wage regime includes treatment over multiple adjacent waves, only the initial treatment is counted as treated, all other instances are treated as a zero.

V will include an expansion of the treatment. Instead of defining each minimum wage increase as a binary event that only is equal to 1 based on the wage shelf and treatment wave distance to the initial change in the minimum wage, this estimator allows for the treatment to be equal to 1 following that change as well. This captures treatment that may attenuate over time.

X will include the definition of the subminimum wage to only those states that have been verified by Vaghul and Zipperer. All other states are presumed to have a subminimum that is equal to their headline minimum unless otherwise stated.

11.5 THE CONTROLLED DIRECT EFFECT

P will include $\mathbb{I}$, $\mathbb{J}$, and $\mathbb{K}$, but this regression will use the subminimum wage as a moderator of the effect of the headline minimum wage. When the effect of a treatment is confounded by other terms due to a non random assignment of the treatment, the researcher can still identify the effect of the treatment through a particular mechanism if they have a term that captures this mechanism and if the mechanism changes in response to the treatment in a way that is separable from other controls. Instead of a linear approach however, the researcher must use the controlled direct effect and model there inference based on a two stage approach designed by Acharya, Blackwell, and Sen.

Since the control direct effect can only de-mediate the effect of a treatment if the mediator changes in response to the treatment status, the treatment must precede the mediator. In the case of the subminimum wage however, there are few clear mediators that respond to the treatment that aren't also in response to the headline. This means I cannot use the controlled direct effect to use the headline minimum wage to de-mediate the effect of the subminimum wage on employment. The inverse, however, poses an opportunity for researchers that is worthwhile. The subminimum wage is by definition in response to the headline minimum wage, it is reasonable to use the subminimum wage as a mediator for the headline minimum wage, although not the other way around. This estimator changes the focus from the employment effect of the subminimum wage itself onto the controlled indirect effect of the effect of the headline minimum wage on employment after controlling for the subminimum wage mechanically.

Equation 22 : CDE Phase 1

$$Y_{jst} = \int_{\tau=-3}^{+4} \left(\int_{k=-4}^{+17} \beta_{\bar{m}}^{\bar{k}\,\tau}\, \mathbb{I}_{j+\bar{k}\;s\;t+\tau}^{k\tau}\, \partial\bar{k} + \int_{\underline{k}=-4}^{+17} \beta_{\underline{m}}^{\underline{k}\,\tau}\, \mathbb{J}_{j+\underline{k}\;s\;t+\tau}^{k\tau}\, \partial\underline{k} \right) + \left(\beta_{\underline{m},\bar{m}}^{\tau}\, \mathbb{K}_{j\;s\;t+\tau}^{\tau} \right) \partial\tau + \Omega_{jst} + \mu_{js} + \psi_{jt} + \left(\epsilon_{jst} + v_s \right)$$

Then replace the employment to population ratio from Y with the predicted values from this regression, and use the predicted values for the dependent variable of the following regression:

Equation 23 : CDE Phase 2

$$\widehat{Y_{jst}} = \int_{\tau=-3}^{+4} \left(\int_{k=-4}^{+17} \beta_{\bar{m}}^{\bar{k}\,\tau}\, \mathbb{I}_{j+\bar{k}\;s\;t+\tau}^{k\tau}\, \partial\bar{k} \right) \partial\tau + \mu_{js} + \psi_{jt} + \left(\epsilon_{jst} + v_s \right)$$

Q will include a similar controlled direct effect approach as model P, but drops the term Ω and experiments with different degrees of alpha.