1 Transversality

Theorem 1.1

Proof.

Claim 1.2. If s is a regular value of π. then $f_s \pitchfork Z$.

Let s be a regular value of π. Then $F^{-1}(Z) \pitchfork X \times \{s\}$. Let $p \in F^{-1}(Z) \cap X \times \{s\}$. Then

$$\begin{aligned} T_pF^{-1}(Z) + T_p(X \times \{s\}) &= T_p(X \times S) \\ T_{f_s(p)}Z + (df_s)_p(T_pX) &= T_{F(p,s)}Y && \text{apply } dF_p \end{aligned}$$

By Sard's, almost every $s \in S$ is a regular value, so we are done. similar to homework question. $\square$

Applications of intersection theory mod 2"

Example 1.3 (Jordan-Brouwer Separation Theorem)

Let $X \subseteq \mathbb{R}^{n+1}$ be a closed hypersurface, then $\mathbb{R}^{n+1} - X^n$ is not connected.

Proof. Suppose $\mathbb{R}^{n+1} - X^n$ is connected, then it is path-connected. Then using local coordinate we can find an arc, one inside the surface and one outside, intersecting once. By path-connected, we can complete a loop with this arc without intersection so intersection number is 1. But we can pull the loop apart and have 0 intersection, a contradiction.

Moreover, $U_\varepsilon(X) - X$ has 2 components. We wish to show these two components are all the components of $\mathbb{R}^{n+1} - X^n$. Take closure and see intersection. $\square$

A consequence of $X = \partial W$.

Corollary 1.4 (No-Retract)

Suppose there exists a retraction $r : X \to \partial X$. Let $i : \partial X \to \partial X$ be the identity. Then r is the extension of i to X. Then $I_2(i, \{x\}) = 0$, a contradiction that it should be 1 since $i^{-1}(\{x\}) = 1$.

Remark 1.5 Locally constant map is globally constant if it is connected because $f^{-1}(c)$ is clopen so it is X.