1 History

Hadamard wanted to minimize real-valued functions on "infinite dimensional sets".

Example 1.1

Let $D:A \to [0,\infty]$ by

$$D[u] = \int_a^b |u'(x)|^2 dx$$

where $A = \{u \in \mathcal{C}^1[a,b] : u(a) = u_a, u(b) = u_b\}$. What function in A minimizes the Dirichlet energy D?

Lemma 1.2 (Fundamental Lemma of the Calculus of Variations)

Let $f \in L^1_{loc}$. If $\int f \phi = 0$ for all $\phi \in V$, then f = 0.