

Homework 1

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We know that between two objects with masses M_1 and M_2 , gravitational force exerted by M_1 onto M_2 satisfies the equation

$$\mathbf{F} = -\frac{GM_1M_2}{\|\mathbf{r}\|^3}\mathbf{r}$$

where $G = 6.67408 \times 10^{-11} m^3/(kg s^2)$ is the gravitational constant, \mathbf{r} is the position vector of center of mass M_2 related to center of mass of M_1 . Then by Newton's second law,

$$\begin{aligned} M_2 \mathbf{g} &= \mathbf{F} \\ \mathbf{g} &= -\frac{GM_1}{\|\mathbf{r}\|^3}\mathbf{r} \end{aligned}$$

In the case of a much smaller spacecraft orbiting the moon, we can treat M_2 as a point mass and use $M_m = 7.342 \times 10^{22} kg$ for M_1 . Using Cartesian coordinate, by assuming that the center of mass of the moon is at the origin, we can rewrite the vector equation above into two scalar components

$$\begin{aligned} g_x &= -\frac{GM_m}{\|\mathbf{r}\|^3}x \\ g_y &= -\frac{GM_m}{\|\mathbf{r}\|^3}y \end{aligned}$$

Together with $\dot{x} = u$, $\dot{y} = v$, $\dot{u} = g_x$, and $\dot{v} = g_y$, we have a system of ODEs to completely describe the motion of the spacecraft:

$$\begin{cases} \dot{x} = u \\ \dot{y} = v \\ \dot{u} = -\frac{GM_m}{\|\mathbf{r}\|^3}x \\ \dot{v} = -\frac{GM_m}{\|\mathbf{r}\|^3}y \end{cases}$$

which can then be solved by MATLAB with the following initial conditions: the spacecraft starts at height h above the moon surface in the y -axis with initial velocity u_0 in the x -direction.

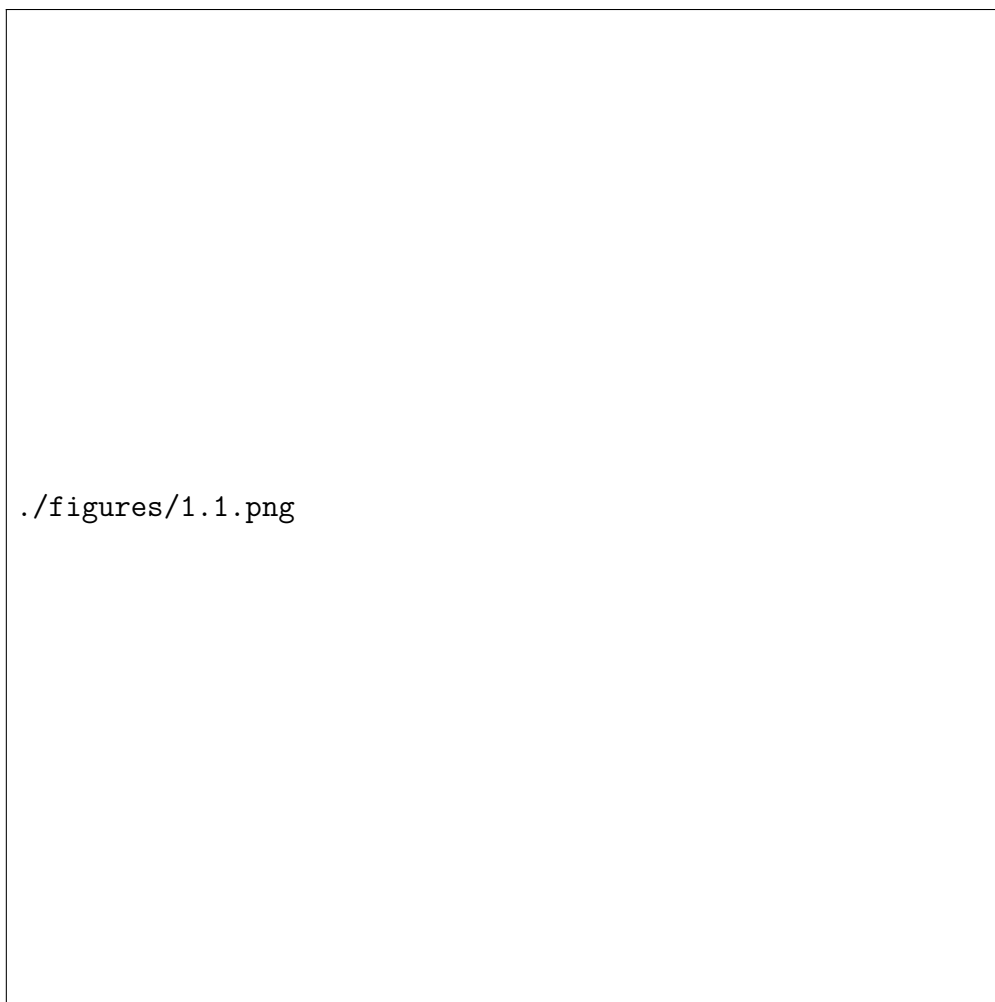


Figure 1

We can verify the simulation by checking conservation of energy.

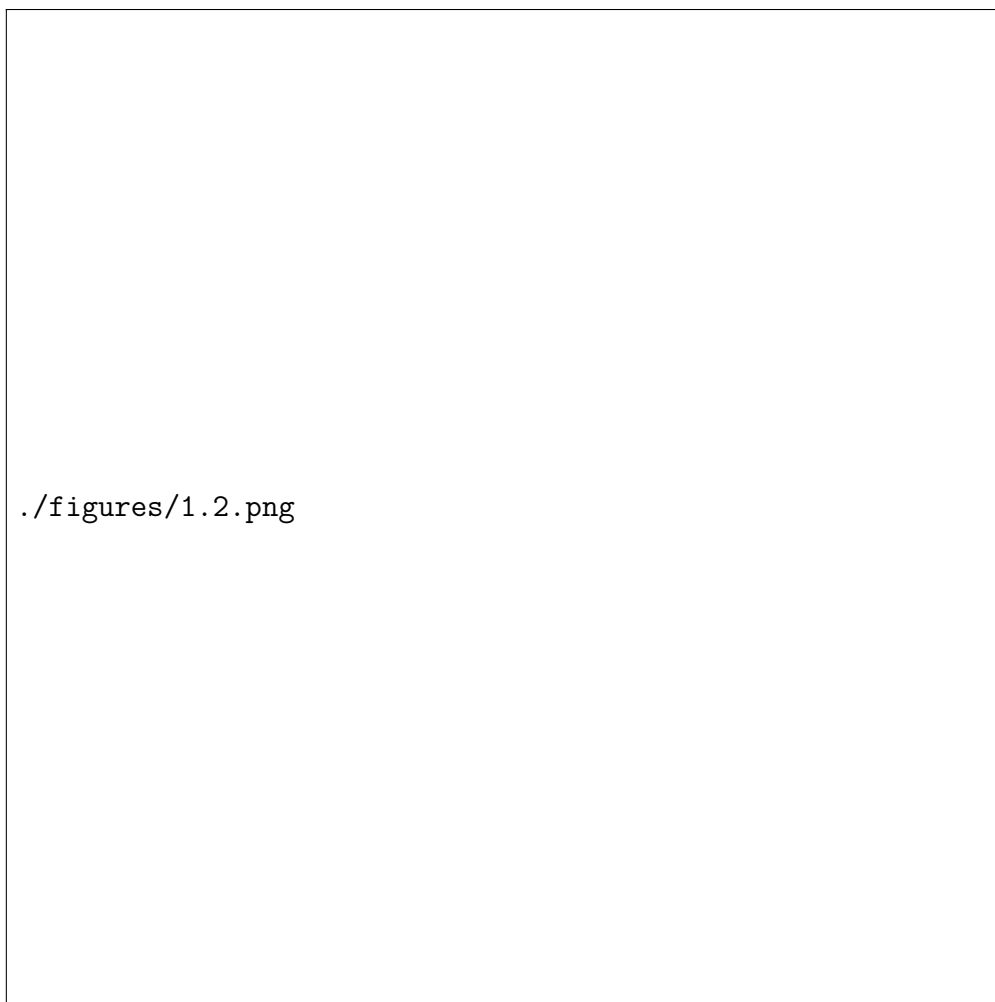


Figure 2: 1.2.png

Once the initial speed u_0 is reduced, we see that the orbit becomes a spiral. This indicates that the ODE solver is not accurate enough.

./figures/1.2.png

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By using ODE89, the orbit looks normal again. We see that

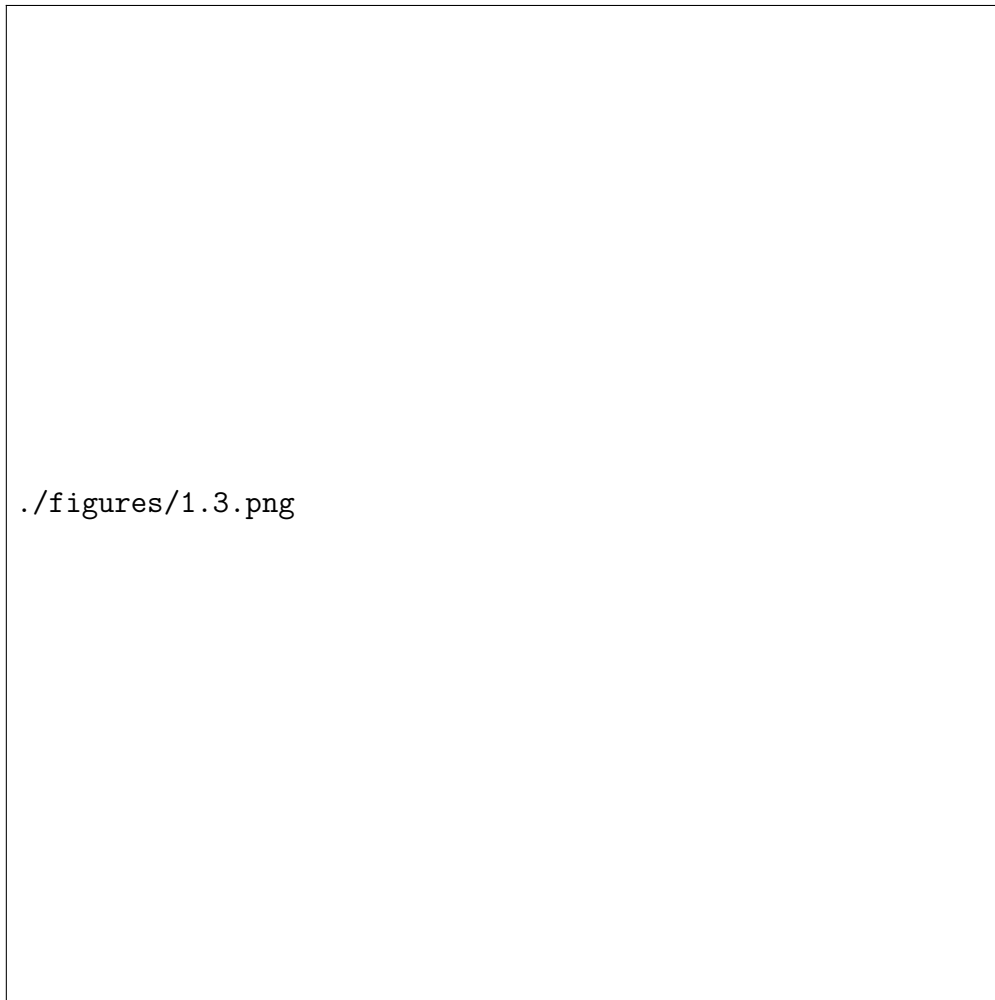


Figure 4: 1.3.png