Homework 1

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We know that between two objects with masses M_1 and M_2, gravitational force exerted by M_1 onto M_2 satisfies the equation

$$\mathbf{F} = -\frac{GM_1M_2}{\|\mathbf{r}\|^3}\mathbf{r}$$

where $G = 6.67408 \times 10^{-11} m^3/(kgs^2)$ is the gravitational constant, $\mathbf{r}$ is the position vector of center of mass M_2 related to center of mass of M_1. Then by Newton's second law,

$$\begin{aligned} M_2\mathbf{g} &= \mathbf{F} \\ \mathbf{g} &= -\frac{GM_1}{\|\mathbf{r}\|^3}\mathbf{r} \end{aligned}$$

In the case of a much smaller spacecraft orbiting the moon, we can treat M_2 as a point mass and use $M_m = 7.342 \times 10^{22} kg$ for M_1. Using Cartesion coordinate, by assuming that the center of mass of the moon is at the origin, we can rewrite the vector equation above into two scalar components

$$\begin{aligned} g_x &= -\frac{GM_m}{\|\mathbf{r}\|^3}x \\ g_y &= -\frac{GM_m}{\|\mathbf{r}\|^3}y \end{aligned}$$

Together with $\dot{x} = u$, $\dot{y} = v$, $\dot{u} = g_x$, and $\dot{v} = g_y$, we have a system of ODEs to completely describe the motion of the spacecraft:

$$\begin{cases} \dot{x} = u \\ \dot{y} = v \\ \dot{u} = -\frac{GM_m}{\|\mathbf{r}\|^3}x \\ \dot{v} = -\frac{GM_m}{\|\mathbf{r}\|^3}y \end{cases}$$

which can then be solved by MATLAB with the following initial conditions: the spacecraft starts at height h above the moon surface in the y-axis with initial velocity u_0 in the x-direction.

./figures/1.1.png

Figure 1

We can verify the simulation by checking conservation of energy.

./figures/1.2.png

Figure 2: 1.2.png

Once the initial speed u_0 is reduced, we see that the orbit becomes a spiral. This indicates that the ODE solver is not accurate enough.

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./figures/1.2.png

By using ODE89, the orbit looks normal again. We see that

./figures/1.3.png

Figure 4: 1.3.png