Note: In this course, smooth is C^∞. Manifolds are assumed to be smooth, without boundary, and finite dimensional.

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Definition 1.1 — $\mathrm{Diff}(M)$ is a topological group under smooth convergence on compact sets.

Assume M is compact.

Proposition 1.2

$\mathrm{Diff}(M)$ is locally homeomorphic to ℓ^2, *i.e.* separable Hilbert space. In particular, it is locally contractible.

Example 1.3

(1) $\mathrm{Diff}_0(M)$ is the path-component of the identity. It is a normal subgroup.

(2) $\mathrm{MCG}(M)$.

(3) $\mathrm{Diff}_C(M) = \{\phi \in \mathrm{Diff}(M) : \phi = \mathrm{id} \text{ outside some compact set} K_\phi\}$.

Motivation:

(1) Classifying fiber bundles.

(2)

Definition 1.4 — The dublde is **flat** if it is isomorphic to $\widetilde{X} \times F/\pi_1(X)$, where $\widetilde{X}$ is the universal cover.