Fourier PDE

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Contents

Polar Coordinates

$P(r, \theta) = P(x, y)$

Spherical Coordinates

$P(\rho, \theta, \phi) = P(x, y, z)$

Definition: Vector Field

Let E be a subset of $\mathbb{R}^3$. A **vector field** on $\mathbb{R}^3$ if function F that assigns to each point (x, y, z) in E a three dimensional vector $F(x, y, z)$.

Definition: Divergence

Let $F(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$ be a vector field on $\mathbb{R}^3$ and suppose the partial derivatives exist,

$$\nabla \cdot F = \langle \partial x, \partial y, \partial z\rangle \cdot \langle P, Q, R\rangle.$$

Theorem: Divergence Theorem

Let E be a simple solid region and let ∂E be the boundary surface of E, given with positive orientation. Let F be a vector field then

$$\int\int\int_E \nabla \cdot F dV = \int\int_{\partial E} F \cdot \mathbf{n} dS.$$

This is a high dimensional version of the FTC

$$\int_a^b \frac{d}{dx} F(x)dx = F(b) - F(a) = -1 \cdot F(a) + 1 \cdot F(b).$$

Note. The LHS of the divergence theorem, we have the integral of a derivative in the region E and the RHS involves the contribution of the original function only on the boundary of the region E in the outward normal direction.

ODE

Definition: General solution

A **general solution** of a linear ODE of order N is a function $y(x)$ such that

- $y(x)$ can be differentiated at least N times.
- $y(x)$ and its first N derivatives satisfy the DE at all x in the domain of the equation.

- $y(x)$ contains exactly N free constants.

Superposition

Example (Function space)**.** The function space $C^n(\mathbb{R})$ consisting of all functions $f(x)$ that have n continuous derivatives defined on $\mathbb{R}$ is a vector space under the usual operations of function addition and scalar multiplication of functions.

Example. For any $y \in C^1(\mathbb{R})$ and non-zero constant α, define $L[y] = y' + \alpha y$, then $L : C^1(\mathbb{R}) \to C^0(\mathbb{R})$ is a linear operator.

Theorem: Superposition Principle

Suppose $v_1, v_2, \ldots, v_n$ are individual solutions to the homogeneous linear equation $L[v] = 0$ then so is any linear combination $c_1v_1 + c_2v_2 + \ldots + c_nv_n$ where $c_i \in \mathbb{R}$.