1 Horner's Method

$$\begin{aligned} P(x) &= (x-z)Q_0(z) + R_0 \\ &= (x-z)[b_n x^{n-1} + \ldots + b_1] + b_0 \\ a_n x^n + \ldots + a_0 &= (b_n x^n + b_{n-1}x^{b-1} + \ldots + b_1 x) - z(b_n x^{n-1} + \ldots + b_1)) + b_0 \end{aligned}$$

$$\begin{aligned} b_n &= a_n \\ b_{n-1} &= a_{n-1} + zb_n \\ \ldots \\ b_1 &= a_1 + zb_2 \\ b_0 &= a_0 \end{aligned}$$

Let $b_{n+1} = 0$
Do this again

$$\begin{aligned} c_n &= b_n \\ \ldots \\ c_1 &= b_1 + zc_2 \end{aligned}$$

Let $c_{n+1} = 0$, $c_k = b_k + zc_{k+1}$.

Root deflation: roots found in the end suffer more from numerical errors.

2 Chapter 3 Interpolation: Lagrange Polynomials

Let

$$P(x) = \sum_{k=1}^{n} P_k(x) = \sum_{k=0}^{n} L_{n,k} f(x_k).$$

where

$$L_{n,k} = \frac{(x-x_0)(x-x_1)\ldots(x-x_{k-1})(x-x_{k+1}\ldots(x-x_n))}{(x_k-x_0)\ldots(x_k-x_{k-1})(x_k-x_{k+1})(x_k-x_n)))}.$$

$$e = \frac{f^{n+1}(\xi)}{n+1}!(x-x_0)(x-x_1)\ldots(x-x_n).$$

3 Neville's Method

4 Cubic Spline

Linear doesn't work because it's not smooth at the junctions. Quadratic misses one condition for the 6 parameters. Cubic is the sweet spot where 8 parameters have 8 reasonable conditions. Conditions for two consecutive cubic polynomials:

- matches function values at two end points, for both functions
- matches each other's values at the middle point
- matches derivatives at the middle point
- matches 2nd derivatives at the overlapped points
-