Note. Let A be a set. THen S_A: symmetric group on A.

Elements of S_A are permutations of A. Operation: composition of functions. S_n: symmetric group on n "letters". This is S_A where $A = \{1, \ldots, n\}$. S_n has order $n!$.

S_n is nonabelian if $n \geq 3$.

Theorem: Cayley

Every group of order n is isomorphic to a subgroup of S_n.

Example. S_4 is a nonabelian group of order 24. Does S_4 have a subgroup isomorphic to V_4. Yes!

Example. What about $\mathbb{Z}_4$? Yes.

Example (8.4)**.** S_5. The two-row notation gives:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$$

where the first row is the input and the second row is the output.

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}.$$

Then

$$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 3 & 2 & 4 \end{pmatrix}.$$

Note that we go from right to left.

See iPad. There are $2n$ choices where n is number of vertices.

Definition: dihedral group

The **dihedral group** D_n (or D_{2n}) if order $2n$ consists of the $2n$ symmetries of a regular n-gon, under the composition of maps. (This can be regarded as a subgroup of S_n).

See iPad for a fact of geometry: two reflections is equivalent to a rotation.

Claim. If α is an acute angle, then reflections will not commute.

Theorem

For $n \geq 3$, D_n is a nonabelian group of order $2n$.

Is there a nonabelian group of order 2020? Yes D_{1010}.

Claim. The identity is a rotation, not a reflection. Rotation makes a subgroup since rotation composite rotation is still a rotation, but reflection does not. Also the determinant of both rotation and identity is 1, but that of reflection is -1.