1 Project: Cooling of Flat Plate

$$\eta = \frac{y}{\sqrt{\frac{V_x}{U_\infty}}}$$

Does heat diffuse faster than momentum? High viscosity $\implies$ low heat diffusivity, high momentum diffusivity.

$$G(\eta) = \frac{t - t_\infty}{t_w - t_\infty}.$$

We can transform the PDEs into two coupled ODEs.

$$F''' + \frac{1}{2}FF'' = 0, F(0) = 0, F'(0) = 0, F'(\infty) = 1.$$

$$G'' + \frac{P_r FG')}{2} = 0, G(0) = 1, G(\infty) = 0.$$

where

$$P_r = \frac{V}{\alpha}.$$

This is a boundary value problem, not an IVP. We need to convert it to an IVP.

Using the garden hose technique, we adjust the value of $f''(0)$ until $F'(\infty) \to 1$. Then we can transform a third-order ODE into a system of three first order ODEs.

Let $P_r = 5$, guess $F''(0) = 0.332057, G'(0) = -0.576689$, use step size of 0.1 with η from 0 to 10 (a number large enough to be considered as infinity),

$$\begin{aligned}
F &= y_1 \\
F' &= y_1' = y_2 \\
F'' &= y_2' = y_3 \\
F''' &= y_3' = \frac{1}{2}FF'' \\
G &= y_4 \\
G' &= y_4' = y_5 \\
G'' &= y_5' = \frac{1}{2}P_r FG'
\end{aligned}$$

where $y_1(0) = 0, y_2(0) = 0, y_3(0) = 332057, y_4(0) = 1, y_5(0) = 0.576689$.

We need $F'(10) \to 1, G'(10) \to 0$ by adjusting $F''(0), G''(0)$. And notice that F"(0) is positive, G'(0) is negative and bounded regardless of P_r.

Plot $F(\eta), G(\eta)$ vs η, and η vs P_r.