Homework 5

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Problem (9.1)**.** Let's start tracing the orbit from 1: $\{1, 5, 2\}$.
3: $\{3\}$.
4: $\{4, 6\}$
The orbits are: $\{1, 5, 2\}, \{3\}, \{4, 6\}$.

Problem (9.2)**.** 1: $\{1, 5, 8, 7\}$.
2: $\{2, 6, 3\}$.
4: $\{4\}$.
The orbits are: $\{1, 5, 8, 7\}, \{2, 6, 3\}, \{4\}$.

Problem (9.7)**.**

$$\begin{aligned}(1\ 4\ 5)(7\ 8)(2\ 5\ 7) &= (1\ 4\ 5\ 8\ 7\ 2)\\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 3 & 5 & 8 & 6 & 2 & 7 \end{pmatrix}\end{aligned}$$

Problem (9.8)**.**

$$\begin{aligned}(1\ 2)(4\ 7\ 8)(2\ 1)(7\ 2\ 8\ 1\ 5) &= (1\ 5\ 8)(2\ 4\ 7)\\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 3 & 7 & 8 & 6 & 2 & 1 \end{pmatrix}\end{aligned}$$

Problem (9.10)**.** By tracing the orbits we can rewrite the permutation as a product of disjointed cycles:

$(1\ 8)(3\ 6\ 4)(5\ 7) = (1\ 8)(3\ 4)(3\ 6)(5\ 7)$.

Problem (9.11)**.** $(1\ 3\ 4)(2\ 6)(5\ 8\ 7) = (1\ 4)(1\ 3)(2\ 6)(5\ 7)(5\ 8)$

Problem (9.23)**.**

a) False. A cycle requires at least one orbit with more than one elements. So the identity permutation is not a cycle.

b) True. By definition.

c) False. 9.15 eliminated a case when a permutation is both a product of even number and product of odd number of transpositions, so we don't have to consider such case in 9.18.

d) True. By Corollary 9.12, since a nontrivial subgroup of S_9 contains permutations of at least two elements, they must be a product of transpositions. Hence some transpositions must be in the subgroup.

e) False. $|A_5| = 5!/2 = 60$.

f) False. $S_1 = \{id\}$ is trivially cyclic with id as the generator.

g) True. $|A_3| = 3!/2 = 3$. We know that all groups with order 3 are isomorphic to $\mathbb{Z}_3$ and it is commutative. So A_3 must be isomorphic to that group and be commutative too.

h) True. By fixing 8 we are only permuting 7 elements. This is isomorphic to S_7.

i) True. Same as above.

j) False. The identity is an even permutation so this set of only odd permutations cannot be a subgroup.

Problem (9.24)**.** A_3 are:

$$\begin{aligned}\rho^0 &= \ (1\ 2)(1\ 2)\\ \rho^1 &= (1\ 2\ 3) = (1\ 3)(1\ 2)\\ \rho^2 &= (1\ 3\ 2) = (1\ 2)(1\ 3)\end{aligned}$$

which are all products of even number of transpositions.

	ρ^0	ρ^1	ρ^2
ρ^0	ρ^0	ρ^1	ρ^2
ρ^1	ρ^1	ρ^2	ρ^0
ρ^2	ρ^2	ρ^0	ρ^1