

# The relative prices of investment and capital

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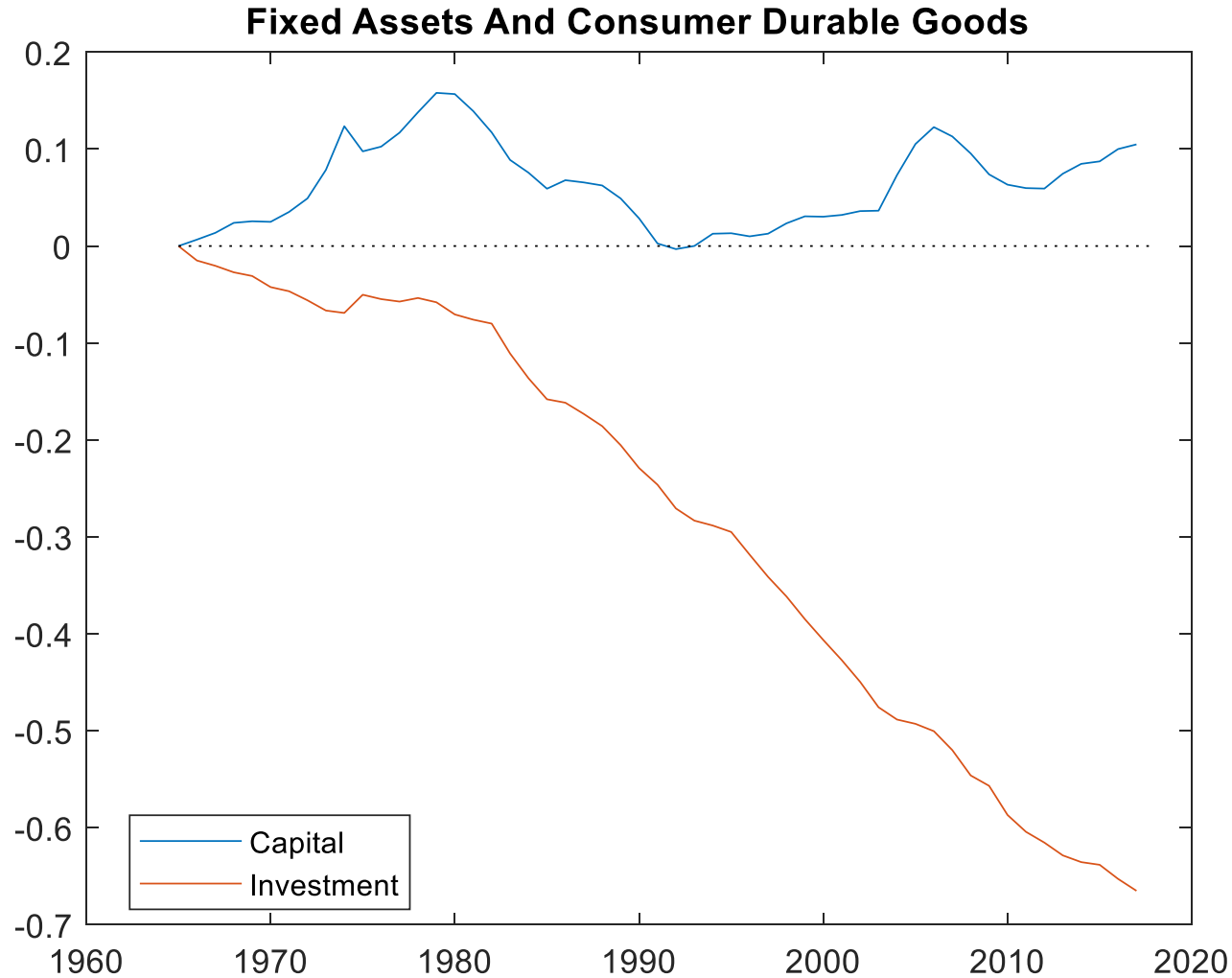
# Overview

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- Observed (US BEA) prices of **capital** are **increasing** in units of consumption.
- Observed (US BEA) prices of **investment** are **falling** in units of consumption.
- The BEA calculates these prices using the Fisher Index to aggregate across capital types and prices.
  - For non-durable goods, this is (roughly) the right thing to do. See e.g. Diewert (1993).
  - For durable goods this may lead to biases. See e.g. Jorgenson & Griliches (1972), Diewert & Lawrence (2000) for discussion of biases relating to depreciation schedules.
- We explain the gap between observed relative capital price growth and observed relative investment price growth through a new measurement bias.
  - The investment bundle is more skewed towards new varieties than the capital bundle.
  - New varieties experience faster productivity growth rates, so have faster falling prices.
- Correctly measured ...
  - Capital and investment prices are identical. (Definition of investment units!)
  - US GDP (investment) growth is  $\approx 0.10$  (0.47) percentage points per year lower.
  - Investment specific technological change only explains 6.9% of aggregate growth.

# Capital & inv. prices, units of cons. goods

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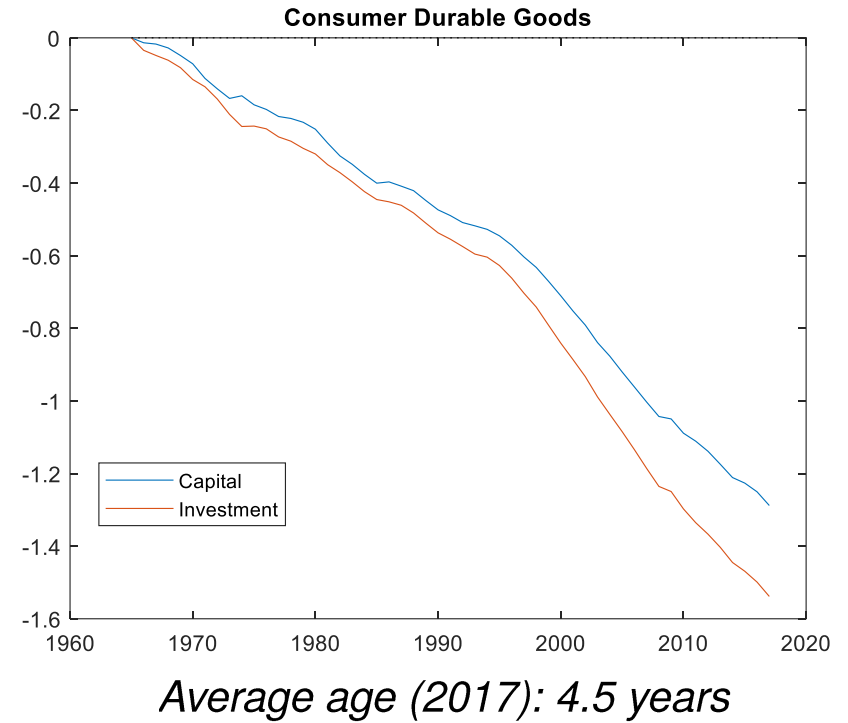
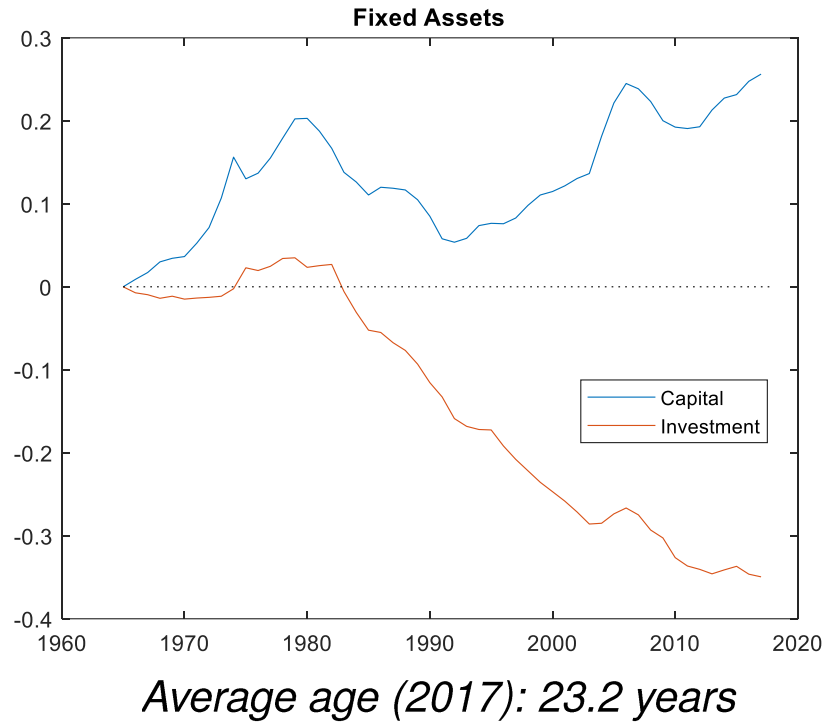
*Average age (2017): 21.7 years*

# Digression: Individual good price measurement

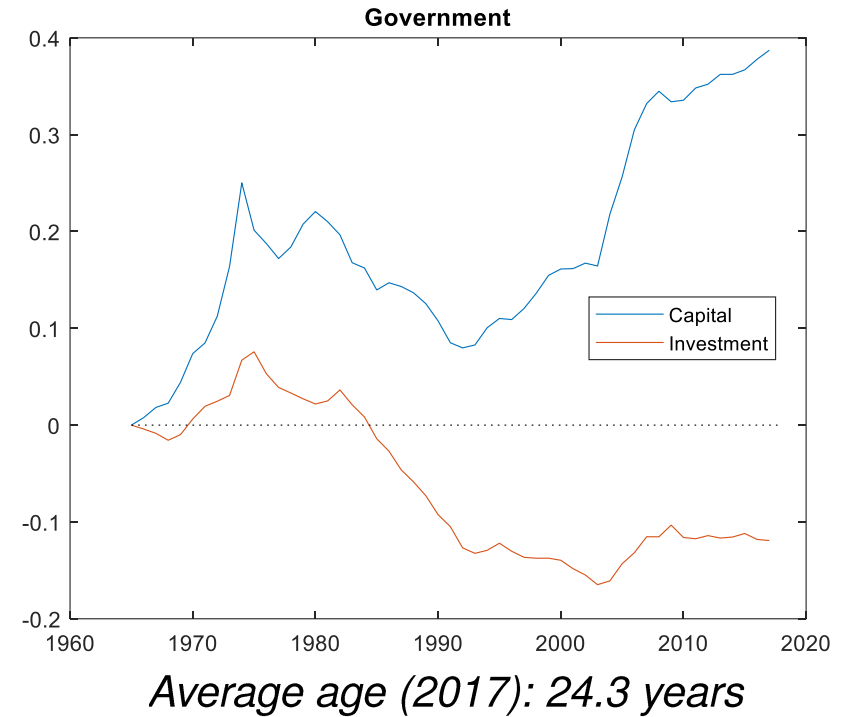
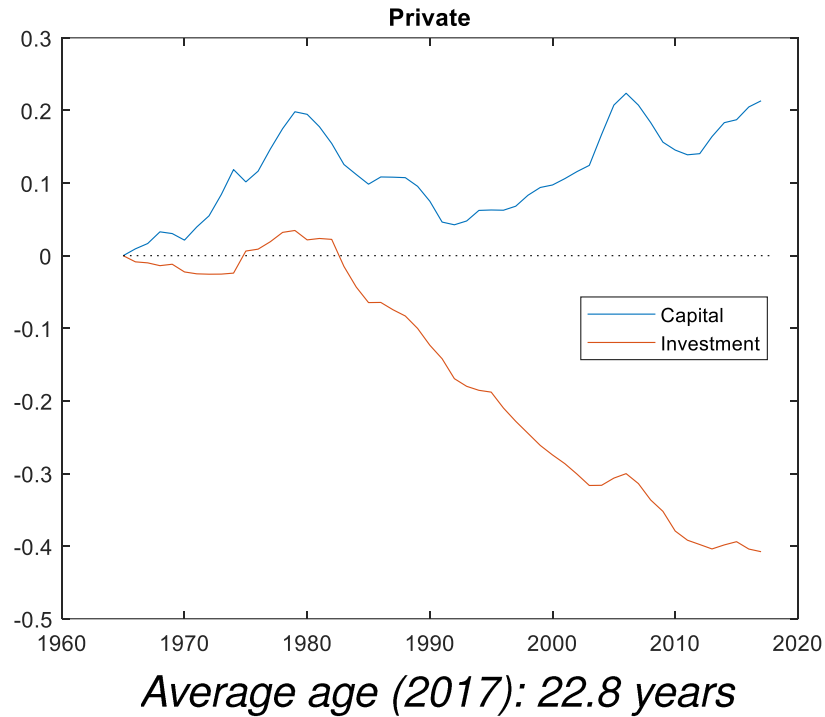
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- This is data from the US BEA Fixed Asset tables.
  - Note that this data enters into standard (NIPA) national accounts via CFC.
- The literature from the 90s was concerned with mismeasurement of individual investment goods.
  - Gordon (1990) produced a new measure which was partly extended by Greenwood, Hercowitz & Krusell (1997) (GHK), Cummins & Violante (2002) and Basu, Fernald, Fisher & Kimball (2013).
- Since then the national accounts have been thoroughly revised, and the problems raised by Gordon are mostly solved.
  - In part using Gordon's methodology.
- For the duration, we take the NIPA measures of the prices of individual goods as correct.
  - Our issue is entirely one of aggregation.

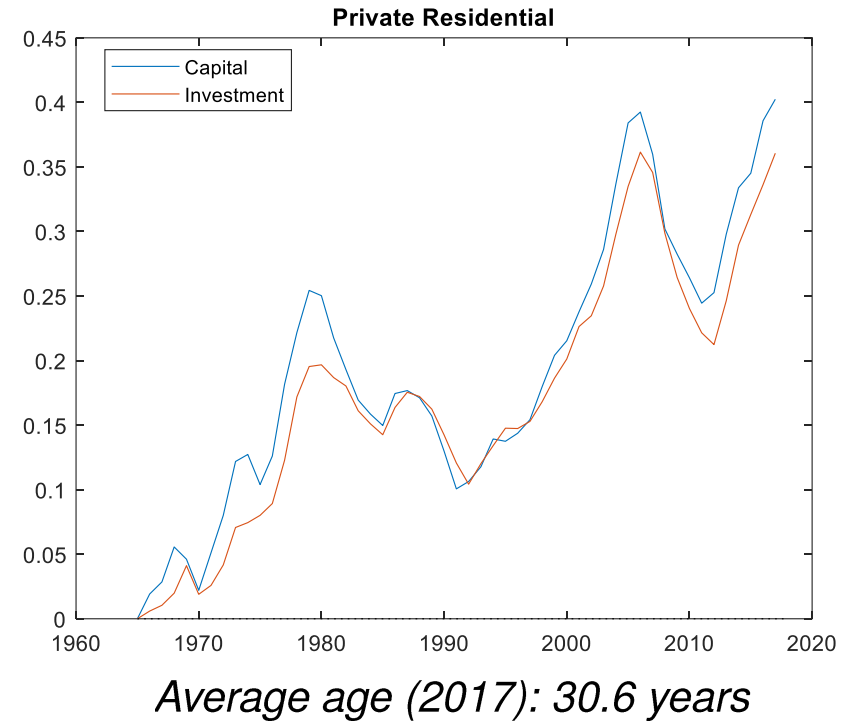
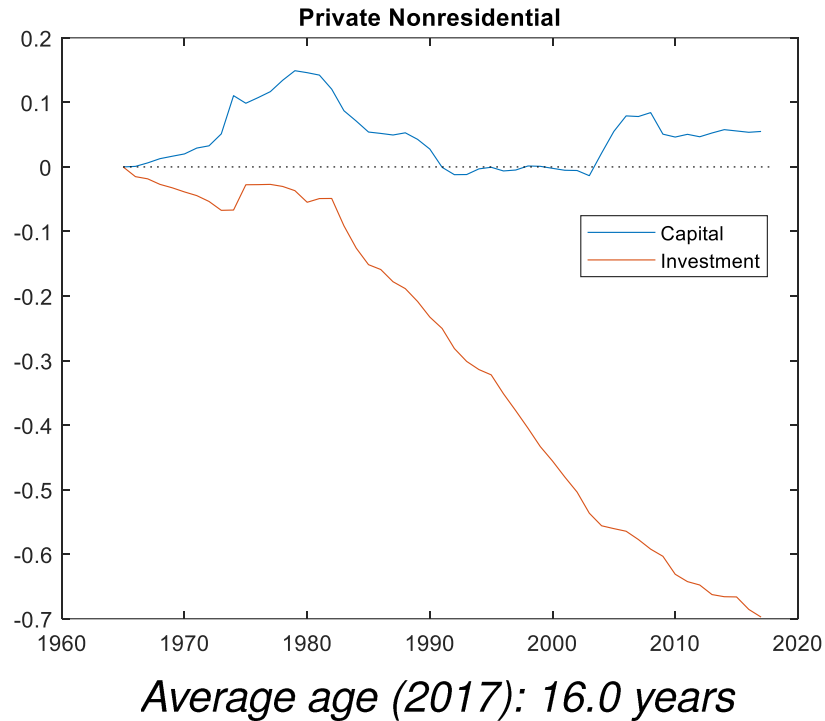
# Capital & inv. prices, units of cons. goods



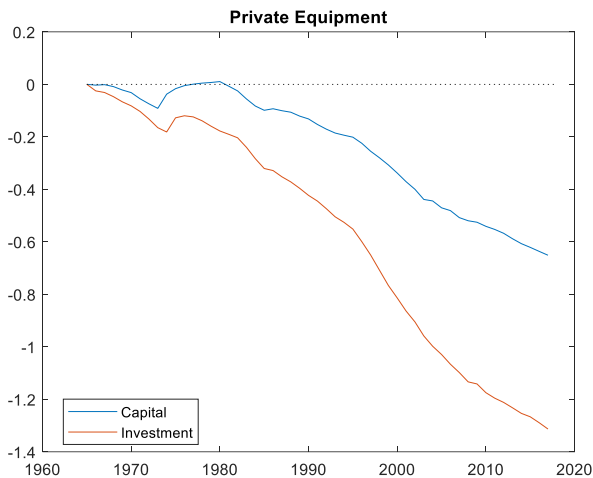
# Capital & inv. prices, units of cons. goods



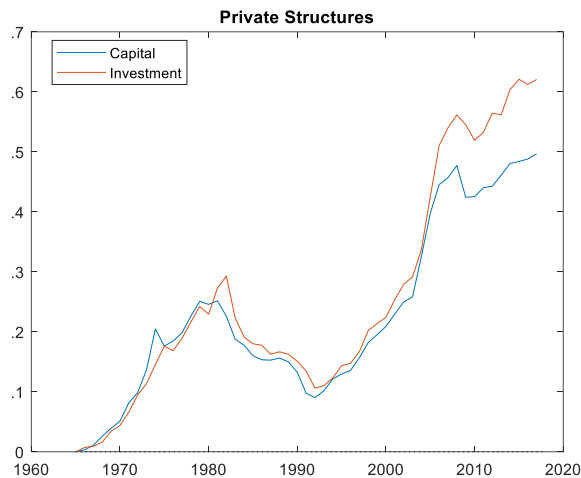
# Capital & inv. prices, units of cons. goods



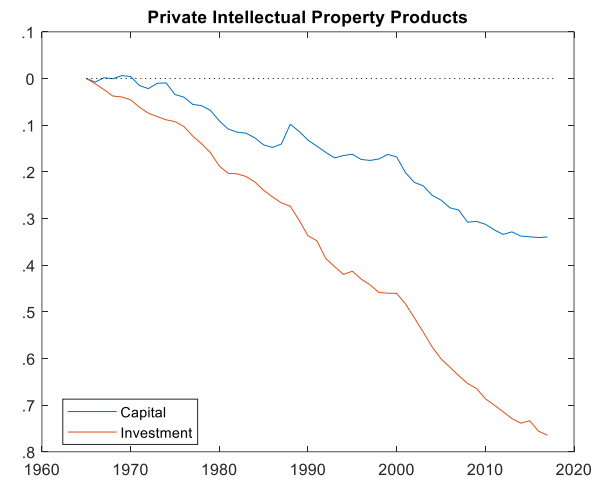
# Capital & inv. prices, units of cons. goods



*Average age (2017): 7.0 years*



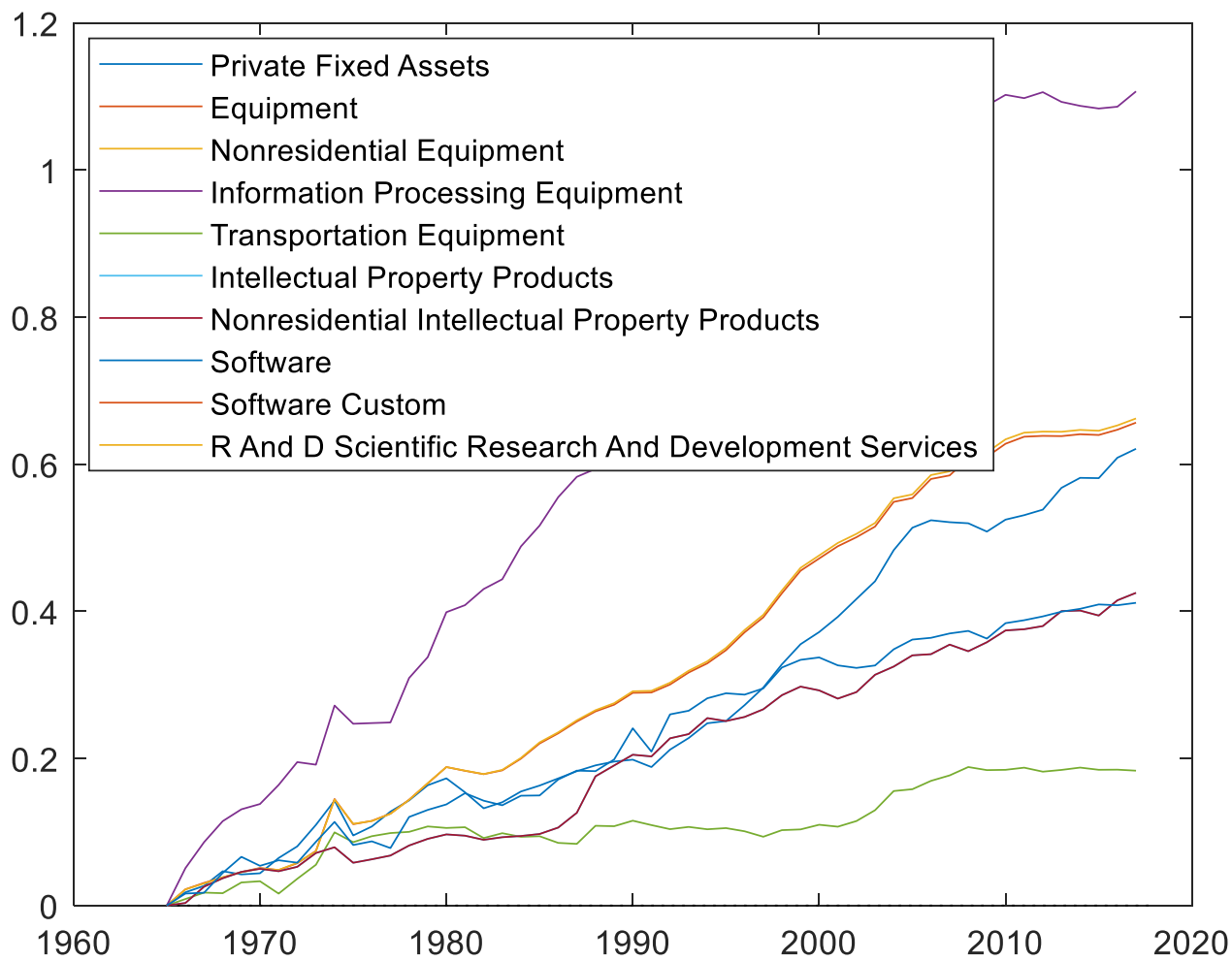
*Average age (2017): 22.9 years*



*Average age (2017): 4.5 years*



# Price of capital in units of inv. by type



Types are plotted for which the graph has a slope statistically significantly different from 0 at 10% (HAC adjusted)

# Lessons from these graphs

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- Across all but one of these broad categories, the price of capital is growing faster than the price of investment.
- The price of fixed assets in units of consumption goods is increasing.
- The categories with the highest average ages have experienced the fastest increases in capital prices.
  - Only the categories with average ages below 10 years have experienced capital price declines.
- Particularly for capital types with relatively low shelf lives, the existing capital stock appears more valuable than you would expect from investment prices.
- Intuition:
  - For a capital type with high product turn-over, producing old varieties soon becomes relatively costly.
  - If new varieties are not perfect substitutes for old varieties, then the stock of old varieties can become valuable.

# Additional related Literature

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- **Cambridge capital controversy:** Robinson, Sraffa, Samuelson, Solow, etc.
- **Vintage capital:** See e.g. Boucekkine, de la Croix & Licandro (2008; 2011).
  - Note that unlike the standard vintage capital literature, in the model here it will be possible to produce old varieties as well as new ones.
- **Relative price of investment & investment specific technological change:** Greenwood, Hercowitz & Krusell (1997), Krusell (1998), Licandro, Ruiz-Castillo & Duran (2002), Justiniano, Primiceri & Tambalotti (2011).
- **Biases in the national accounts:** Broda & Weinstein (2006; 2010), Redding & Weinstein (2016).
- **Micro capital prices:** Lanteri (2018).
- **Variety specific growth:** Adam & Weber (2019).

# Capital in the model

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- The aggregate capital good is produced by a perfectly competitive industry with the technology:

$$K(t) = \left[ \int_{-\infty}^t K_s(t)^{\frac{1}{1+\lambda}} ds \right]^{1+\lambda}.$$

- Note: the set of varieties/vintages evolves exogenously.
- Capital producers choose demand for specific vintages to maximise their profits (zero in equilibrium):

$$R(t)K(t) - \int_{-\infty}^t R_s(t)K_s(t) ds.$$

- $R(t)$  and  $R_s(t)$  are aggregate and vintage  $s$  rental rates, respectively.
- In equilibrium:

$$K_s(t) = K(t) \left( \frac{R_s(t)}{R(t)} \right)^{-\frac{1+\lambda}{\lambda}}, \quad R(t) = \left[ \int_{-\infty}^t R_s(t)^{-\frac{1}{\lambda}} ds \right]^{-\lambda}.$$

# Growth in rental rates

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- For now we assume  $R_t(t)K_t(t) = 0$ .
  - This will be true in the model as new varieties will start off with zero productivity.
  - Removes biases coming from love of (new) varieties.
  - See e.g. Broda & Weinstein (2010).
- Then, we have:

$$\frac{\dot{R}(t)}{R(t)} = \frac{\int_{-\infty}^t R_s(t)K_s(t) \frac{\dot{R}_s(t)}{R_s(t)} ds}{\int_{-\infty}^t R_s(t)K_s(t) ds}.$$

# Digression: Price indices in continuous time

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- The BEA use Fisher aggregators, for which (for an arbitrary good):

$$\frac{P(t)}{P(t-\zeta)} = \sqrt{\frac{\int_{-\infty}^t P_s(t)Y_s(t) ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta)Y_s(t) ds} \frac{\int_{-\infty}^{t-\zeta} P_s(t)Y_s(t-\zeta) ds}{\int_{-\infty}^{t-\zeta} P_s(t-\zeta)Y_s(t-\zeta) ds}}.$$

- Thus if  $P_t(t)Y_t(t) = 0$ :

$$\lim_{\zeta \rightarrow 0} \left[ \frac{1}{\zeta} \log \frac{P(t)}{P(t-\zeta)} \right] = \frac{\int_{-\infty}^t P_s(t)Y_s(t) \frac{\dot{P}_s(t)}{P_s(t)} ds}{\int_{-\infty}^t P_s(t)Y_s(t) ds}.$$

- I.e. if applied to capital quantities and rental rates, this gives the theoretically correct aggregate price.
  - This is not what the BEA does as rental rates are hard to infer.
- In general, the biases we find in this paper do not occur for non-durable goods.
  - Capital services are a non-durable good with price equal to the rental rate of capital.

# Further model primitives (1/2)

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- The law of motion for capital good vintage  $s$  is:

$$\dot{K}_s(t) = I_s(t) - \delta K_s(t).$$

- Capital and labour are combined to produce an intermediate good  $X_t$ , the numeraire. (Details later.)
- One unit of the intermediate good may be converted into  $e^{\gamma t}$  units of the consumption good in period  $t$ .
  - Produced under perfect competition.
  - I.e. the price of a unit of consumption is  $P_C(t) := e^{-\gamma t}$ .
  - Effectively:  $\gamma$  is the TFP growth of the consumption good producing sector.

# Further model primitives (2/2)

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- One unit of the intermediate good becomes:

$$\psi(t-s)\phi e^{\kappa s + \theta(t-s)}$$

units of capital variety  $s$  in period  $t$ . Assume  $\kappa > \theta$ .

- Again produced under perfect competition.
- I.e. the price of investment goods of vintage  $s$  is:

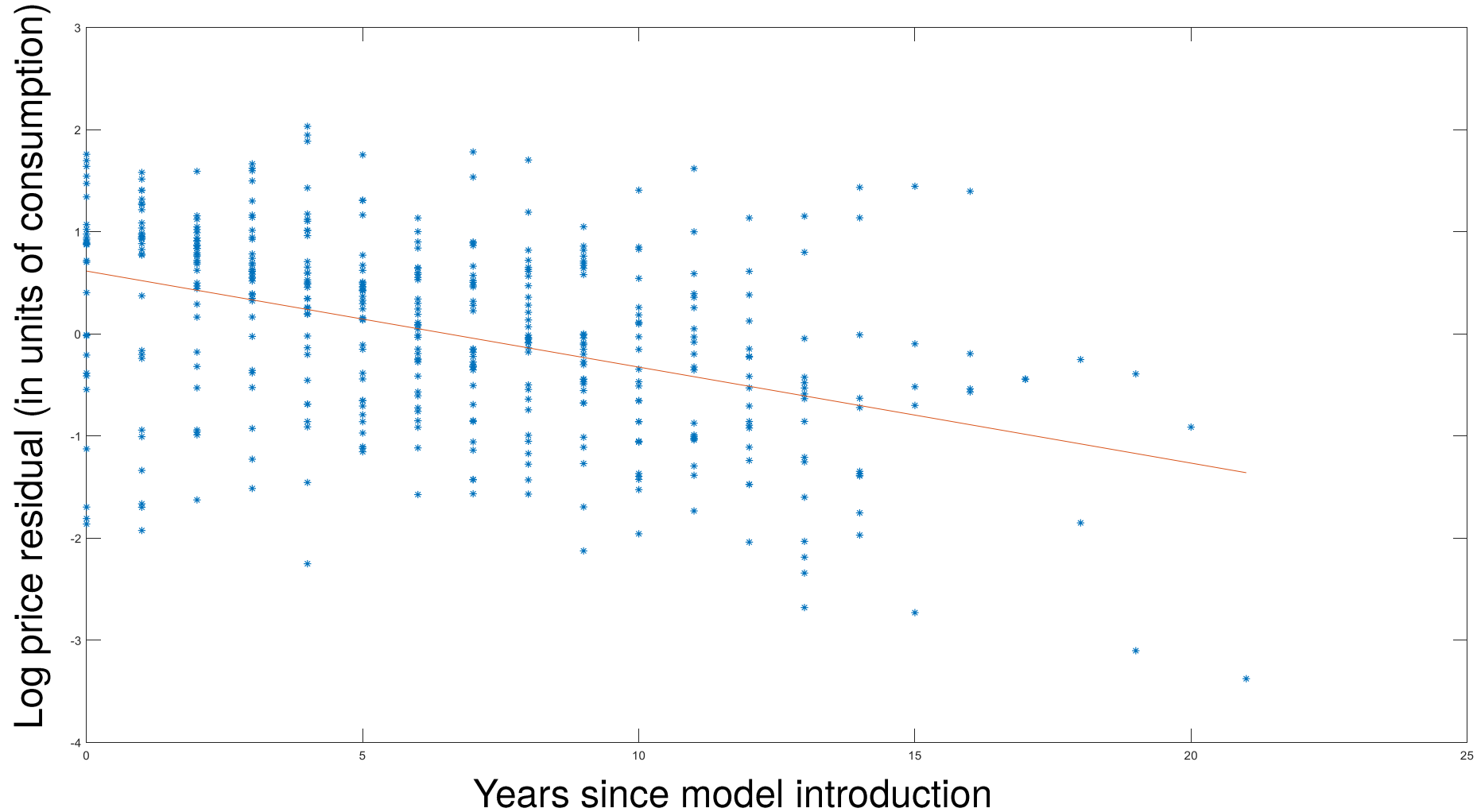
$$Q_s(t) := \frac{1}{\psi(t-s)\phi e^{\kappa s + \theta(t-s)}} \Rightarrow \frac{\dot{Q}_s(t)}{Q_s(t)} = -\theta - \frac{\phi}{t-s}.$$

- $\psi > 0$  just scales productivity.
- $\phi > 0$  determines the growth rate of young products. (All products start with zero productivity, but then experience rapid growth.)
- $\theta$  (possibly negative) determines the growth rate of old products.
- $\kappa > 0$  determines how productivity grows over time, holding fixed variety age  $(t-s)$ .



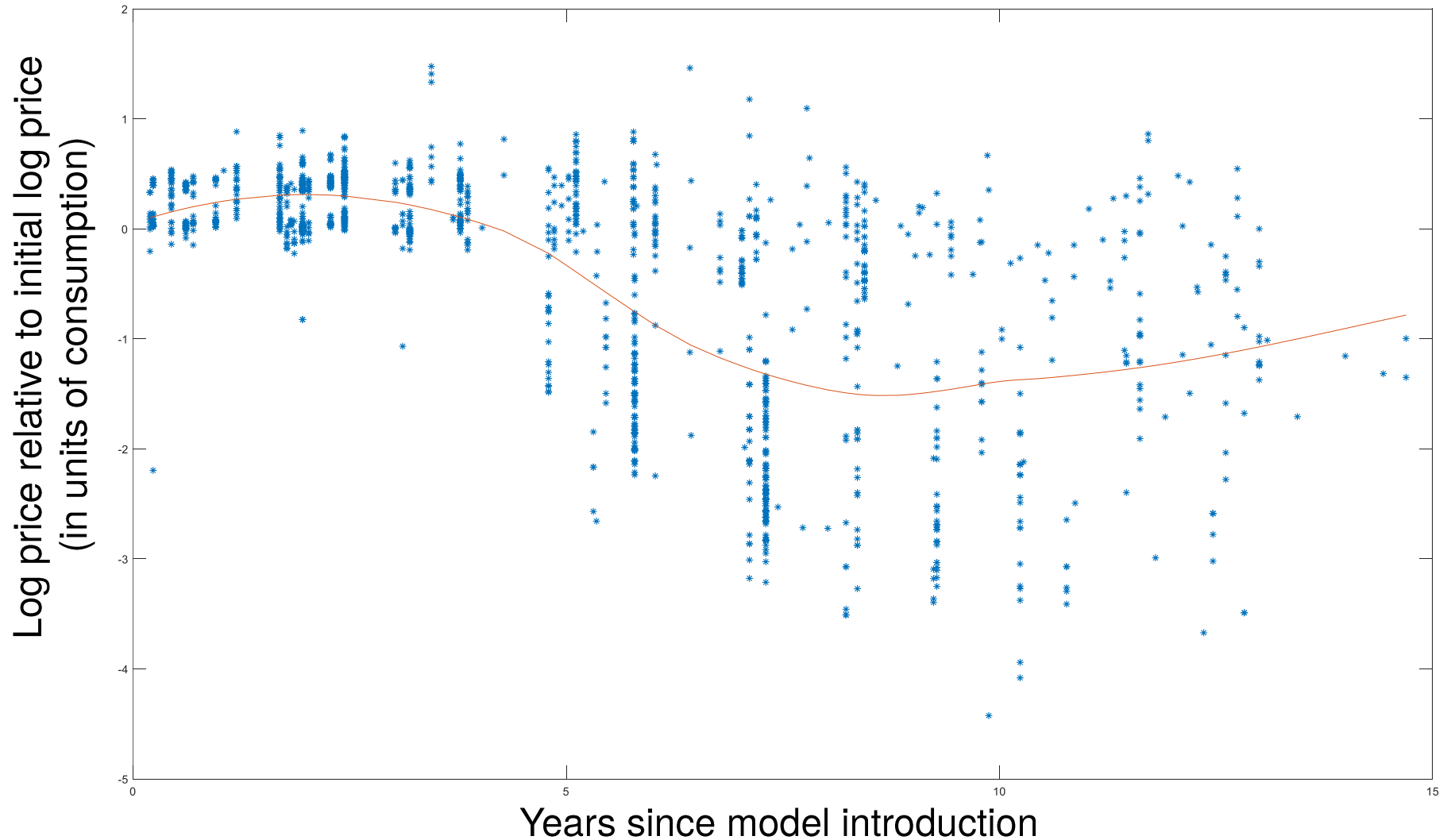
# New aircraft prices

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Residuals after standardizing at the good level. Data kindly provided by A. Lanteri.

# Intel CPU Prices



Current price data web scraped from NewEgg.com. Initial price data web scraped from Wikipedia.

# Optimal capital variety holdings (1/2)

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- Suppose a firm holds capital vintage  $s$ .
- If  $r(t)$  is the deterministic interest rate at  $t$ , the firm's value is:

$$\int_0^{\infty} e^{-\int_0^{\xi} r(t+\tau) d\tau} [R_s(t+\xi)K_s(t+\xi) - Q_s(t+\xi)I_s(t+\xi)] d\xi.$$

- Optimisation implies:

$$\frac{R_s(t)}{Q_s(t)} = \delta + r(t) + \theta + \frac{\phi}{t-s}.$$

- (Unsurprisingly) Price of capital good  $s$  is:

$$\int_0^{\infty} e^{-\int_0^{\xi} r(t+\tau) d\tau - \delta\xi} R_s(t+\xi) d\xi = Q_s(t).$$

# Optimal capital variety holdings (2/2)

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- If we define:

$$R^*(\tau) := \frac{\delta + r^* + \theta + \frac{\phi}{\tau}}{\psi \tau \phi e^{-(\kappa - \theta)\tau}}, \quad R^* := \left[ \int_0^\infty R^*(\tau)^{-\frac{1}{\lambda}} d\tau \right]^{-\lambda},$$

- Then if  $r(t) \equiv r^*$  (i.e. we are on the BGP):

$$R(t) = e^{-\kappa t} R^*, \quad R_s(t) = e^{-\kappa t} R^*(t - s).$$

- Both individual and aggregate rental rates are declining at rate  $\kappa$ .

- Note:

$$\frac{R^{*'}(\tau)}{R^*(\tau)} = \kappa - \theta - \frac{\phi}{(\delta + r^* + \theta)\tau^2 + \phi\tau} - \frac{\phi}{\tau}.$$

- We assume  $\theta > -(\delta + r^*)$ , so  $R^*(\tau)$  is U-shaped.

# Some notation

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- Suppose throughout that the random variable  $T$  has p.d.f.:

$$\tilde{\tau} \mapsto \frac{Q_{t-\tilde{\tau}}(t)K_{t-\tilde{\tau}}(t)}{\int_0^\infty Q_{t-\tau}(t)K_{t-\tau}(t) d\tau} = \frac{\frac{R^*(\tilde{\tau})^{-\frac{1}{\lambda}}}{\delta + r^* + \theta + \frac{\phi}{\tilde{\tau}}}}{\int_0^\infty \frac{R^*(\tau)^{-\frac{1}{\lambda}}}{\delta + r^* + \theta + \frac{\phi}{\tau}} d\tau} = \frac{\frac{e^{-\frac{\kappa-\theta}{\lambda}\tilde{\tau}} \tilde{\tau}^{\frac{1+\lambda+\phi}{\lambda}}}{((\delta + r^* + \theta)\tilde{\tau} + \phi)^{\frac{1+\lambda}{\lambda}}}}{\int_0^\infty \frac{e^{-\frac{\kappa-\theta}{\lambda}\tau} \tau^{\frac{1+\lambda+\phi}{\lambda}}}{((\delta + r^* + \theta)\tau + \phi)^{\frac{1+\lambda}{\lambda}}} d\tau}.$$

- $\mathbb{E}T$  is the average years since variety introduction of the capital stock with weights given by value shares of that age.
  - Not the same as average age from the national accounts.
  - Chiefly convenient for stating results.
- Lemma 1 (exercise in integration by parts):

$$\kappa - \theta = \mathbb{E} \left[ (1 + \lambda) \frac{\phi}{(\delta + r^* + \theta)T^2 + \phi T} + \frac{\phi}{T} \right].$$

# The true price of capital

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- The true price of the aggregate capital stock satisfies:

$$P_K(t) = \frac{\int_{-\infty}^t Q_s(t) K_s(t) ds}{K(t)} = \frac{e^{-\kappa t} R^*}{\delta + r^* + \theta + \phi \mathbb{E} T^{-1}},$$

- **Shrinking at rate  $\kappa$ .**

- By Lemma 1:

$$P_K(t) > \frac{e^{-\kappa t} R^*}{\delta + r^* + \kappa} = \int_0^\infty e^{-\int_0^\xi r(t+\tau) d\tau - \delta \xi} R(t + \xi) d\xi,$$

- With equality in limit  $\kappa - \theta \rightarrow 0$ .
- Buying a unit of the aggregate capital stock at  $t$  gets you period  $t + \xi$  returns of:

$$\int_{-\infty}^t e^{-\delta \xi} R_s(t + \xi) \frac{K_s(t)}{K(t)} ds \neq R(t + \xi)$$

- I.e. a total value of  $P_K(t)$  as expected.

# Investment and effective depreciation

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- Define  $\Delta(t)$  ( $= \Delta^*$  on BGP) to solve:

$$\begin{aligned}\frac{e^{-\kappa t} R^*}{\delta + r^* + \theta + \phi \mathbb{E} T^{-1}} = P_K(t) &= \int_0^\infty e^{-\int_0^\xi (r(t+\tau) + \Delta(t+\tau)) d\tau} R(t + \xi) d\xi \\ &= e^{-\kappa t} R^* \int_0^\infty e^{-(\Delta^* + r^* + \kappa)\xi} d\xi = \frac{e^{-\kappa t} R^*}{\Delta^* + r^* + \kappa}\end{aligned}$$

- I.e.:

$$\Delta^* = \delta - (\kappa - \theta - \phi \mathbb{E} T^{-1}).$$

- We then **define** aggregate investment by:

$$I(t) := \dot{K}(t) + \Delta(t)K(t).$$

- These definitions will ensure:
  1. The aggregate capital good is equivalent to a homogeneous capital good with depreciation  $\Delta(t)$  and rental rate  $R(t)$ .
  2. The aggregate price of investment equals that of capital.

# The true price of investment

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- Let  $z(t) := \frac{I(t)}{K(t)}$  ( $= z^*$  on the BGP), so  $\frac{\dot{K}(t)}{K(t)} = z^* - \Delta$  on the BGP.

- Lemma 2:  $\mathbb{E} \left[ z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda} \frac{R^{*'}(T)}{R^*(T)} \right] = z^*$ .

- True price of aggregate investment is given by:

$$P_I(t) := \frac{\int_{-\infty}^t Q_s(t) I_s(t) ds}{I(t)} = \frac{P_K(t)}{z^*} \mathbb{E} \left[ z^* + \delta - \Delta^* - \frac{1+\lambda}{\lambda} \frac{R^{*'}(T)}{R^*(T)} \right] = P_K(t).$$

- The true investment price is **equal** to the capital price.



# The observed price of capital

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- The observed price of capital grows at rate:

$$\begin{aligned}\frac{\dot{P}_K^{\text{OBS}}(t)}{P_K^{\text{OBS}}(t)} &:= \frac{\int_{-\infty}^t Q_s(t) K_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)} ds}{\int_{-\infty}^t Q_s(t) K_s(t) ds} = -\theta - \mathbb{E} \frac{\phi}{T} \\ &= -\kappa + \mathbb{E} \left[ (1 + \lambda) \frac{\phi}{(\delta + r^* + \theta)T^2 + \phi T} \right] \\ &\geq -\kappa.\end{aligned}$$

- Using Lemma 1.
- The observed price of capital is growing more quickly than the true price!

# The observed price of investment

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- The observed price of investment grows at rate:

$$\begin{aligned} \frac{\dot{P}_I^{\text{OBS}}(t)}{P_I^{\text{OBS}}(t)} &:= \frac{\int_{-\infty}^t Q_s(t) I_s(t) \frac{\dot{Q}_s(t)}{Q_s(t)} ds}{\int_{-\infty}^t Q_s(t) I_s(t) ds} = -\theta - \frac{\mathbb{E} \frac{\phi}{T} \left[ z^* + \delta - \Delta^* - \frac{1 + \lambda R^{*'}(T)}{\lambda R^*(T)} \right]}{\mathbb{E} \left[ z^* + \delta - \Delta^* - \frac{1 + \lambda R^{*'}(T)}{\lambda R^*(T)} \right]} \\ &\leq -\theta - \mathbb{E} \frac{\phi}{T} = \frac{\dot{P}_K^{\text{OBS}}(t)}{P_K^{\text{OBS}}(t)}. \end{aligned}$$

- Thus observed investment growth is below observed capital growth!
- Providing  $\Delta^* + r^* + \kappa - z^* > 0$  (which will hold automatically in our GE setup), we can prove that for sufficiently large  $\lambda$ :

$$\frac{\dot{P}_I^{\text{OBS}}(t)}{P_I^{\text{OBS}}(t)} \leq -\kappa \leq \frac{\dot{P}_K^{\text{OBS}}(t)}{P_K^{\text{OBS}}(t)}$$

- The observed price of investment is growing more slowly than the true price!

# “ $\phi$ bias”

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- “ $\phi$  bias” disappears when  $\phi = 0$ .
- If  $\phi = 0$  then:

$$\frac{\dot{P}_I^{\text{OBS}}(t)}{P_I^{\text{OBS}}(t)} = \frac{\dot{P}_I(t)}{P_I(t)} = -\kappa = \frac{\dot{P}_K(t)}{P_K(t)} = \frac{\dot{P}_K^{\text{OBS}}(t)}{P_K^{\text{OBS}}(t)}.$$

- With  $\phi$  positive, a wedge is driven between observed and true price growth, pushing down observed investment price growth, and pushing up observed capital price growth.
- $\phi$  bias comes from the facts that:
  1. The investment bundle is skewed towards newer goods than the capital bundle.
  2. New goods experience rapid productivity growth.

# Closing the model

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- Let  $C(t)$  be aggregate consumption,  $N(t)$  population,  $w(t)$  labour income per capita in units of consumption goods,  $A(t)$  household total asset holdings.
- Assume on the BGP  $w(t) = w^* e^{gt}$ , where we will solve for  $g$ .

- At  $t$ , households maximise:

$$\int_0^\infty e^{-\rho\tau} N(t+\tau) \frac{\left(\frac{C(t+\tau)}{N(t+\tau)}\right)^{1-\sigma} - 1}{1-\sigma} d\tau,$$

- Subject to:

$$\dot{A}(t) = r(t)A(t) + P_C(t)w(t)N(t) - P_C(t)C(t).$$

- Assume the intermediate good is produced under perfect competition with the technology:

$$\begin{aligned} K(t)^\alpha N(t)^{1-\alpha} &= X(t) = P_C(t)C(t) + P_I(t)I(t) + E(t) \\ &= R(t)K(t) + P_C(t)w(t)N(t). \end{aligned}$$

# In equilibrium

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- Consumption per capita grows at rate:

$$g := \gamma + \frac{\alpha}{1 - \alpha} \kappa.$$

- Capital and investment per capita grow at rate:

$$h := \frac{\kappa}{1 - \alpha}.$$

- We have:

$$\begin{aligned} r^* &= \rho + \sigma g - \gamma = \rho + \frac{\alpha}{1 - \alpha} \sigma \kappa - (1 - \sigma) \gamma, \\ z^* &= h + n + \delta. \end{aligned}$$

- Investment bias condition holds:

$$\Delta^* + r^* + \kappa - z^* = \rho - n - g(1 - \sigma) > 0,$$

- As the RHS of the equality is the effective discount rate.

# Calibration (1/3)

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- Fix:
  - $\lambda \leftarrow \frac{1}{2.2-1} \approx 0.83$ . Broda & Weinstein (2006), 2.2 is median e.o.s. at SITC-3 level, 1990-2001, also e.o.s. for “automatic data process machines”, quite similar to e.o.s. for vehicles, 3.0.
  - $\sigma \leftarrow \frac{1}{0.594} \approx 1.68$ . Havranek et al. (2015), unimportant for results.
- Calibrate (LHS are observed geometric means 1965 to now):
  - $n \leftarrow \frac{d}{dt} \log N(t) \approx 0.0098$ .
  - $\alpha \leftarrow \frac{Y^{\text{GDP}}(t) - Y^{\text{PROP}}(t) - Y^{\text{LAB}}(t)}{Y^{\text{GDP}}(t) - Y^{\text{PROP}}(t)} \approx 0.36$ .
    - $Y^{\text{PROP}}(t)$  is proprietor’s income.
    - $Y^{\text{LAB}}(t)$  is compensation of employees, plus government social benefits to persons, less contributions for government social insurance.
  - $g \leftarrow \frac{d}{dt} \log \left( \frac{Y^{\text{GDP}}(t)}{N(t)P_C(t)} \right) \approx 0.017$ .
  - $z^* \leftarrow \frac{P_I(t)I(t)}{P_K(t)K(t)} \approx 0.073$ .
  - $\mathcal{R} \leftarrow \frac{R(t)K(t)}{P_K(t)K(t)} \approx 0.12$ .

# Calibration (2/3)

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- Calibrate (LHS are observed geometric means 1965 to now):
  - $\mathcal{G}_{PK} \leftarrow \frac{d}{dt} \log \left( \frac{P_K^{\text{OBS}}(t)}{P_C(t)} \right) \approx 0.0049.$
  - $\mathcal{G}_{PI} \leftarrow \frac{d}{dt} \log \left( \frac{P_I^{\text{OBS}}(t)}{P_C(t)} \right) \approx -0.0067.$
  - $\delta^{DATA} \leftarrow \frac{CFC(t)}{P_K(t)K(t)} \approx 0.050.$ 
    - Only used to check the over-identifying restriction:  $\delta^{DATA} = z^* - n - g + \mathcal{G}_{PK}.$
    - We use  $\delta \approx 0.052$  (the RHS of the previous restriction).
    - Error: 0.15 percentage points, 2.9 percent.
- Fix a grid of possible  $\phi$ .
  - Will consider  $0 \leq \phi \leq 100.$
  - Data found to require  $\phi \geq 0.3.$
  - Only calibrated  $\theta$  is at all sensitive to  $\phi.$
  - Main results are not at all sensitive to  $\phi.$

# Calibration (3/3)

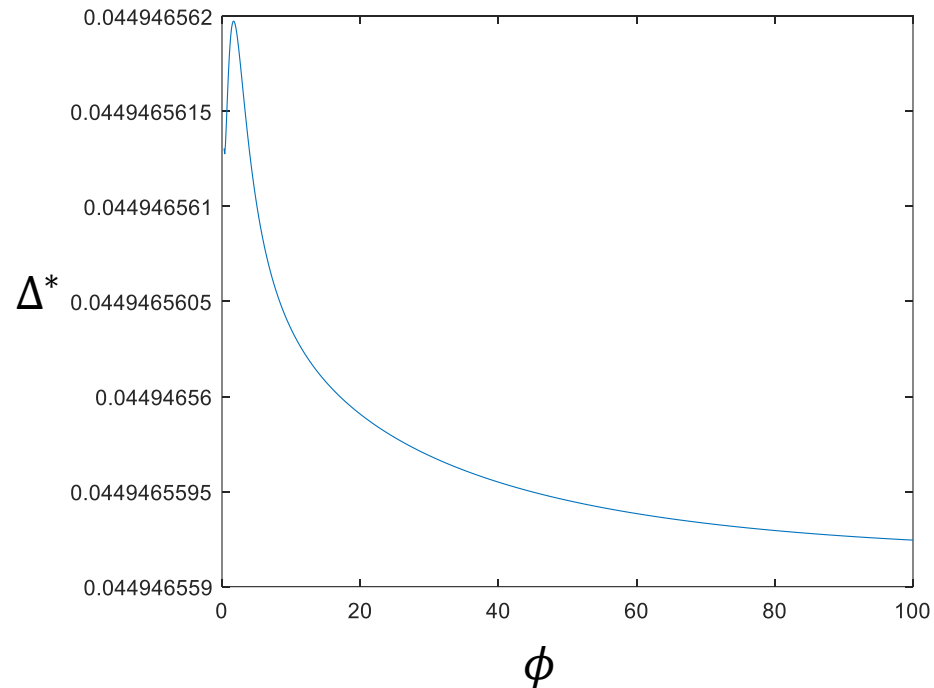
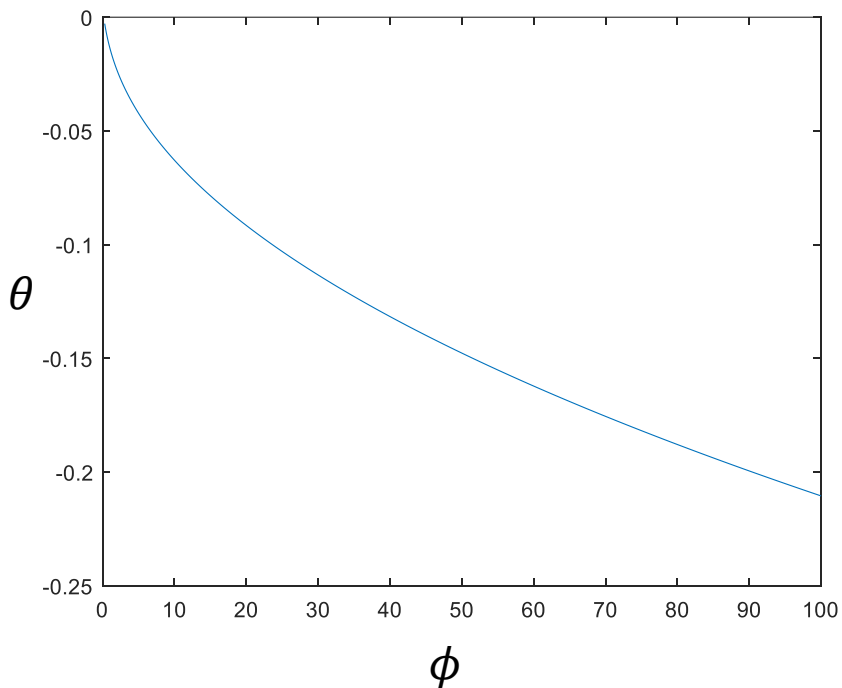
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- Guess  $\theta$ ,  $\Delta^*$ , then evaluate (where values vary with  $\phi$ , a range is given):
  - $h \leftarrow z^* - n - \Delta^* \approx 0.019$ .
  - $\kappa \leftarrow h(1 - \alpha) \approx 0.012$ .
  - $\gamma \leftarrow g - \alpha h \approx 0.010$ .
  - $\phi \mathbb{E} T^{-1} \leftarrow -\mathcal{G}_{PK} + \gamma - \theta \approx 0.0078 \dots 0.22$ .
  - $\frac{\mathbb{E} \frac{\phi}{T} \left[ z^* + \delta - \Delta^* - \frac{1 + \lambda R^{*'}(T)}{\lambda R^*(T)} \right]}{\mathbb{E} \left[ z^* + \delta - \Delta^* - \frac{1 + \lambda R^{*'}(T)}{\lambda R^*(T)} \right]} \leftarrow -\mathcal{G}_{PI} + \gamma - \theta \approx 0.019 \dots 0.23$
  - $r^* \leftarrow \mathcal{R} - \Delta^* - \kappa \approx 0.066$ .
  - $\rho \leftarrow r^* + \gamma - \sigma g \approx 0.048$ .
- Calculate residuals from directly evaluating the expectations above.
  - We evaluate the required integrals numerically, by treating them as expectations of appropriate gamma distributed random variables, mapping these into uniforms, and then using a Fejer Type 1 rule.
  - If they are not zero, adjust our guesses of  $\theta$ ,  $\Delta^*$ .
  - See next slide for results of solving for  $\theta$ ,  $\Delta^*$ .



# Effects of varying $\phi$

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# Robust (to $\phi$ ) calibration consequences

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- Annual growth of capital price in units of consumption goods:
  - Observed: 0.49%.
  - True:  $-0.21\%$ .
  - Bias:  $-0.70$  percentage points
- Annual growth of investment price in units of consumption goods:
  - Observed:  $-0.67\%$
  - True:  $-0.21\%$
  - Bias:  $0.47$  percentage points
- Annual growth of output price in units of consumption goods:
  - Observed:  $-0.15\%$
  - True:  $-0.05\%$
  - Bias:  $0.10$  percentage points
- Annual real output growth is overstated by  $0.10$  percentage points in the NIPA!
- Annual real investment growth is overstated by  $0.47$  percentage points!

# Further consequences

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- Lower role for investment specific technological change as an engine of growth.
  - IST ( $\kappa > \gamma$ ) explains 6.9% of growth in our calibration.
  - Compared to 60% in Greenwood, Hercowitz & Krusell (1997).
  - IST may still drive growth in particular sectors (e.g. equipment, IPP, durable cons.).
- Unlikely can explain the decline in the labour share with falling capital prices and capital/labour substitution (Neiman and Karabarbounis 2014).
  - In fact, capital and labour appear to be gross compliments:
    - Best practice estimates: León-Ledesma, McAdam & Willman (2010).
    - Surveys: Chirinko (2008), Klump, McAdam & Willman (2012).
    - Meta-study: Knoblach, Rößler & Zwerschke (2016)
  - Were capital prices falling rapidly, a rise in the labour share would be expected.
  - We at least reduce this puzzle.
- Piketty (2014) / Piketty & Zucman (2014) via Rognlie (2014).
  - With lower real investment growth, we should expect  $\frac{P_K K}{P_Y Y} \rightarrow \frac{s}{g_I + \delta}$  to be larger.

# What should the BEA do?

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- To (roughly) correctly measure the price of capital and investment, the BEA could proceed as follows:

1. Guess an interest rate  $r(t)$ .
2. Measure individual good prices and depreciation rates as at present.
3. From the good prices  $Q_s(t)$ , depreciation rates  $\delta_s(t)$  and guessed interest rate  $r(t)$ , construct implied rental rates  $R_s(t)$  using:

$$\frac{R_s(t)}{Q_s(t)} = \delta_s(t) + r(t) - \frac{\dot{Q}_s(t)}{Q_s(t)}.$$

3. Aggregate  $R_s(t)$  and  $K_s(t)$  to obtain  $R(t)$  and  $K(t)$  using the Fisher index formula.

- Note that this is implicitly using second derivatives of  $Q_s(t)$ !

4. Evaluate  $P_K(t) = \frac{\int_{-\infty}^t Q_s(t) K_s(t) ds}{K(t)}$  and  $I(t) = \frac{\int_{-\infty}^t Q_s(t) I_s(t) ds}{P_K(t)}$  (so  $P_I(t) = P_K(t)$ ).

5. Evaluate  $\Delta(t) = \frac{I(t) - \dot{K}(t)}{K(t)}$ , and check  $\frac{R(t)}{P_K(t)} - \Delta(t) = r(t) - \frac{\dot{P}_K(t)}{P_K(t)}$ , if not, adjust  $r(t)$ .

# Conclusions

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- We built a simple model to explain the apparent divergence of capital and investment prices.
  - All of this divergence can be explained by incorrect aggregation of the prices of fixed assets.
  - Traditional price aggregates are inappropriate for durable goods.
  - We suggest an alternative.
- Model implies a greatly reduced role for IST.
- Model predicts output growth in units of consumption goods is overstated by about 0.10 percentage points per year.

# Appendix: IST Literature

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- Nominal capital LOM:

$$\begin{aligned}P_{K,t}K_t &= (1 - \delta)P_{K,t}K_{t-1} + P_{I,t}I_t \\ \Rightarrow K_t &= (1 - \delta)K_{t-1} + \frac{P_{I,t}}{P_{K,t}}I_t\end{aligned}$$

- Greenwood, Hercowitz & Krusell (1997) call  $\frac{P_{I,t}}{P_{K,t}}$  “investment specific technological change” (IST).
  - GHK identify it with  $\frac{P_{C,t}}{P_{I,t}}$ , the price of consumption in units of investment.
  - This ratio is rising in the US, so could be an engine of growth.
  - $\frac{P_{I,t}}{P_{K,t}}$  is roughly inversely proportional to Tobin's  $Q = \frac{\text{market}}{\text{replacement}}$ .
- BUT  $\frac{P_{I,t}}{P_{K,t}}$  is the price of investment in units of capital goods, not  $\frac{P_{C,t}}{P_{I,t}}$ !
  - Even if  $\frac{P_{I,t}}{P_{K,t}}$  and  $\frac{P_{I,t}}{P_{C,t}}$  are falling,  $\frac{P_{K,t}}{P_{C,t}}$ , the price of capital in units of consumption goods, may still be rising, in which case IST would not be an engine of growth.