Robust Real Rate Rules

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Motivation: Fragility of the Taylor principle

- The Taylor principle requires the response of nominal rates to inflation to be greater than one.
  - Sufficient for determinacy in simple models. (Guarantees no belief-driven fluctuations / sunspots.)
- Insufficient if there is e.g.:
  - A fraction of hand-to-mouth households (Gali, Lopez-Salido & Valles 2004).
  - Firm-specific capital (Sveen & Weinke 2005).
  - High government spending (Natvik 2009).
  - A positive inflation target (Ascari & Ropele 2009),
  - …particularly with trend growth + sticky wages (Khan, Phaneuf & Victor 2019).
- Inverts if there are e.g.:
  - Enough hand-to-mouth households (Bilbiie 2008).
  - Certain financial frictions (Manea 2019).
  - Non-rational expectations (Branch & McGough 2010; 2018).
  - Active fiscal policy (Leeper & Leith 2016; Cochrane 2022).

This paper

- Monetary rules with a unit response to real rates guarantee determinate inflation under the weakest possible assumptions on the rest of the economy.
    - Robust to household heterogeneity, non-rational households/firms, missing transversality conditions, existence/slope of the Phillips curve, active fiscal policy, etc.

- With a time-varying inflation target: enable the determinate robust implementation of an arbitrary path for inflation.
    - So can match observed inflation dynamics, or any model's optimal policy.

- Easy to implement in practice. Use TIPS to infer real rates. Works with bonds of any maturity.

- Reveal: Fisher equation is key to monetary transmission.

A first example

- Nominal bond: $\$1$ bond purchased at t returns $\$(1+i_t)$ at $t+1$.
- Real bond (e.g., TIPS): $\$1$ bond purchased at t returns $\$(1+r_t+\pi_{t+1})$ at $t+1$.

- Arbitrage $\Rightarrow$ the Fisher equation:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}$$

  - Abstracting from inflation risk / term / liquidity premia for now.

- Central bank uses the “real rate rule”:

$$i_t = r_t + \phi \pi_t$$

- With $\phi > 1$. Then:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t$$

- Unique non-explosive solution, $\pi_t = 0$. Determinate inflation!

Why is this robust? No need for Euler equation!

- Does not require an aggregate Euler equation to hold.
  - Robust to household heterogeneity and hand-to-mouth agents.
  - Robust to non-rational household expectations.

- For the Fisher equation to hold just need either:
  - Two deep pocketed, fully informed, rational agents in the economy, OR,
  - …a large market of rational agents with dispersed information (Hellwig 1980; Lou et al. 2019).

- Much more plausible financial market participants have rational expectations than households.
  - Can even partially relax the rationality requirement for financial market participants.
  - E.g.: Global convergence under learning from arbitrary initial beliefs.

Why is this robust? No need for Phillips curve!

- Does not require an aggregate Phillips curve to hold.
  - Robust to slope of the Phillips curve (if it exists).
  - Robust to forward/backward looking degree of Phillips curve equation.
  - Robust to non-rational firm expectations.

- Under this monetary rule, the Phillips curve is irrelevant for inflation dynamics.
  - The Phillips curve (if it exists) determines the output gap, given inflation.
  - If CB is unconcerned with output and unemployment, they do not need to care about the Phillips curve or its slope.

- Only require that at least some prices are adjusted each period using current information.

Implications for monetary economics

- Which model features lead to amplification or dampening of monetary shocks?
- Under a real rate rule: no change in the model can amplify/dampen monetary shocks other than changing rule.
  - Prior amplification/dampening results were sensitive to the monetary rule. May reverse with a response to r_t of > 1.
- Which shocks drive inflation?
- Under a real rate rule: only monetary policy shocks or shocks to the Fisher equation.
- CB has ultimate responsibility for inflation.

How does monetary policy work under a real rate rule?

- Under a real rate rule, monetary policy does not work via the real rate.
- In fact: This is a general property of NK models even with standard monetary rules.
- Rupert & Šustek (2019) show that with endogenous capital and sufficient monetary shock persistence:
  - Contractionary (positive) monetary shocks lead to falls in output, inflation **and real rates**. Contrary to standard story.
- Instead: Monetary policy operates as under flexible prices. (Exactly under a real rate rule, approximately in general.)
  - Following a monetary shock, inflation jumps to the unique level consistent with non-explosive inflation. More intuition to come.
- Outcomes under a real rate rule can also be replicated with a standard monetary rule with infinite coefficient on π_t.

Real rate rules elsewhere in the literature

- Papers with rules responding to real rates, for analytic convenience e.g.:
  - Adão, Correia & Teles (2011), Holden (2019), Lubik, Matthes & Mertens (2019).
- Closest prior work: Cochrane (2017; 2018; 2022) on spread targeting.
  - Cochrane briefly considers a rule of the form $i_t = r_t + \phi\pi_t$ before setting $\phi = 0$.
  - Determinacy in Cochrane's world comes from the Fiscal Theory of the Price Level.
- Other related work:
  - Hetzel (1990) proposes using nominal bond, real bond spread to guide policy.
  - Dowd (1994) proposes targeting the price of price level futures contracts.
  - Hall & Reis (2016) propose making interest on reserves a function of price level deviations from target, e.g. nominal return from $\$1$ of $\$(1 + r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1 + i_t)\frac{p_t}{p_t^*}$.
- Large literature on rules tracking efficient ("natural") real interest rate.
  - E.g., Cúrdia et al. (2015). Very different idea.

Generalizations and generality

Monetary policy shocks

- Suppose the CB uses the rule:

$$i_t = r_t + \phi \pi_t + \zeta_t$$

- with $\phi > 1$, and ζ_t drawn from an AR(1) process with persistence ρ.

- Then from the Fisher equation:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t + \zeta_t \qquad \Rightarrow \qquad \pi_t = -\frac{1}{\phi - \rho} \zeta_t.$$

- Contractionary (positive) monetary policy shocks reduce inflation.
  - Intuition: Define $\pi_t^* := -\frac{1}{\phi-\rho}\zeta_t$, then $i_t = r_t + \mathbb{E}_t \pi_t^* + \phi(\pi_t - \pi_t^*)$. A contractionary monetary shock lowers the inflation target.
- If the CB is more aggressive (ϕ is larger) inflation is less volatile.
- Can understand inflation dynamics without knowing the rest of the economy.

Explaining observed inflation dynamics

- Large literature finds no role for the Phillips curve in forecasting inflation.
  - Post-1984: IMA$(1,1)$ model beats Phillips curve based forecasts (conditionally & unconditionally) (Dotsey, Fujita & Stark 2018).
  - +: Atkeson & Ohanian (2001), Ang, Bekaert & Wei (2007), Stock & Watson (2009). One explanation: McLeay & Tenreyro (2019).

- Also: Miranda-Agrippino & Ricco (2021):
  - Contractionary monetary policy shock causes immediate fall in the price level.
  - Delayed impact on unemployment.

- All supportive of models in which causation in PC only runs in one direction: *from inflation to the output gap*.
  - As under a real rate rule! [Not saying the CB necessarily follows a real rate rule. Just that outcomes would not be so different.]

Output dynamics in 3 equation NK model

- As before: CB sets $i_t = r_t + \phi\pi_t + \zeta_t$, so $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$.

- Rest of model 1: Phillips curve (PC), with mark-up shock ω_t:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + \kappa\omega_t$$

- Rest of model 2: Discounted/compounded Euler equation (EE) (Bilbiie 2019), n_t exogenous natural rate ($\delta = 1$, $\varsigma =$ EIS recovers standard Euler equation):

$$x_t = \delta\mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t)$$

- PC implies:

$$x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$$

  - x_t does not help forecast inflation as $\mathbb{E}_t\pi_{t+1} = \rho\pi_t$. Once you know π_t, there is no extra useful information in x_t.

Real rate dynamics in 3 equation NK model

- In the model of the last slide, if ω_t is IID, EE implies:

$$r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right]$$

- Derived without solving EE forward!
  - Implies robustness to missing transversality conditions.
  - Also implies degree of discounting/compounding (δ) has no impact on determinacy.
  - Contrasts with Bilbiie (2019): if $\varsigma > 0$, $\beta \leq 1$, with a standard Taylor rule, $\phi > 1$ is only sufficient for determinacy if $\delta \leq 1$.
  - Contrasts with Bilbiie (2008): if $\delta = 1$, $\varsigma < 0$, with a standard Taylor rule, $\phi > 1$ is neither necessary nor sufficient for determinacy.

- Under real rate rule, $\phi > 1$ is always necessary and sufficient! (Given $\phi \geq 0$.)
  - Robust to lags in EE and PC. (PC lag may reduce persistence of effect of monetary shocks on x_t!)

Responding to other endogenous variables

- In the model:

$$i_t = r_t + \phi_\pi \pi_t + \phi_x x_t + \zeta_t$$

$$\pi_t = \tilde{\beta}(1-\varrho_\pi)\mathbb{E}_t \pi_{t+1} + \tilde{\beta}\varrho_\pi \pi_{t-1} + \kappa x_t + \kappa \omega_t, \qquad x_t = \tilde{\delta}(1-\varrho_x)\mathbb{E}_t x_{t+1} + \tilde{\delta}\varrho_x x_{t-1} - \varsigma(r_t - n_t)$$

  - If $\kappa > 0$, $\phi_x \geq 0$ and $\tilde{\beta} \in [0,1]$, then $\phi_\pi > 1$ is sufficient for determinacy!
  - Real rate rule still helps robustness as it disconnects EE from prices.

- In any model: $\phi_\pi > 1$ sufficient for determinacy if responses to other endogenous variables are small enough.
  - Implies robustness to non-unit responses to real rates. Other variables (e.g., output growth) may proxy real rates.

- For greater robustness: Replace other endogenous vars in rule with structural shocks.
  - If structural shocks not observed, can infer from structural equations.
  - If equation parameters not known, can learn in real time, still with determinacy!

Implementing arbitrary inflation dynamics

- Suppose CB uses the rule:

$$i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t)$$

- π^*_t: an arbitrary stochastic process, possibly a function of economy's other endogenous variables and shocks.

- From the Fisher equation: $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. With $\phi > 1$, unique, determinate solution: $\pi_t = \pi^*_t$.
- The CB can hit an arbitrary path for inflation!
  - E.g., optimal policy. So real rate rules can attain highest possible welfare.
  - And real rate rules can explain any observed inflation dynamics.

- Related literature on implementation of optimal policy:
  - Svensson & Woodford (2003), Dotsey & Hornstein (2006), Evans & Honkapohja (2006), Evans & McGough (2010).

Interest rate smoothing

- A fully smoothed real rate rule with time-varying target:

$$i_t - r_t = i_{t-1} - r_{t-1} + \mathbb{E}_t \pi^*_{t+1} - \mathbb{E}_{t-1}\pi^*_t + \theta(\pi_t - \pi^*_t)$$

- From the Fisher equation:

$$\theta(\pi_t - \pi^*_t) = \mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) - \mathbb{E}_{t-1}(\pi_t - \pi^*_t).$$

- Define: $p_t := \sum_{s=1}^{t}(\pi_t - \pi^*_t)$ and $\hat{p}_t := p_t + \frac{1}{\theta}\mathbb{E}_0 p_1$. Then summing over time gives:

$$(1+\theta)\hat{p}_t = \mathbb{E}_t \hat{p}_{t+1}$$

- With $\theta > 0$: Unique equilibrium $\hat{p}_t = 0$, so $\pi_t = \pi^*_t$.

- Produces the same π_t as unsmoothed rule.
- Difference: Only need $\theta > 0$, not $\phi > 1$.
  - Likely much easier for CB to convince agents of the former than of the latter.

Challenges to real rate rules

Real rate rules in non-linear models

- Nominal and real bond pricing:

$$I_t \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} = 1, \qquad R_t \mathbb{E}_t \Xi_{t+1} = 1$$

- Non-linear real rate rule:

$$I_t = R_t \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi}$$

- So:

$$\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \frac{\Pi^*}{\Pi_{t+1}} = \left(\frac{\Pi^*}{\Pi_t} \right)^{\phi}$$

- $\Pi_t = \Pi^*$ is always one solution of this equation! Always locally unique.
- (Approximately) Globally unique under weak assumptions:
  - There exists $\overline{Z} \geq 1$ such that for all sufficiently high ϕ, $1 \leq \frac{\Pi^*}{\Pi_t} \leq \overline{Z^{\frac{1}{\phi-1}}} \to 1$ as $\phi \to \infty$. So, for large ϕ, $\Pi_t \approx \Pi^*$.
  - Under slightly stronger restrictions on the SDF, $\Pi_t = \Pi^*$ is globally unique solution for all sufficiently high ϕ.

Wedges in the Fisher equation

- Many potential sources of a Fisher equation wedge:
  - Liquidity premia on nominal bonds (Fleckenstein, Longstaff & Lustig 2014). Deflation protection on real bonds.
  - Risk premia (already considered). Non-rational expectations. Etc.

- Generalized Fisher equation (ν_t: stationary endogenous wedge):

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu_t$$

- Assume: Exist $\bar{\mu}_0, \bar{\mu}_1, \bar{\mu}_2, \bar{\gamma}_0, \bar{\gamma}_1, \bar{\gamma}_2 \geq 0$ such that for any stationary solution for π_t:

$$|\mathbb{E}\nu_t| \leq \bar{\mu}_0 + \bar{\mu}_1 |\mathbb{E}\pi_t| + \bar{\mu}_2 \operatorname{Var} \pi_t \,, \qquad \operatorname{Var} \nu_t \leq \bar{\gamma}_0 + \bar{\gamma}_1 |\mathbb{E}\pi_t| + \bar{\gamma}_2 \operatorname{Var} \pi_t$$

- Then under a real rate rule: $|\mathbb{E}\pi_t| = O\left(\frac{1}{\phi}\right)$ and $\operatorname{Var} \pi_t = O\left(\frac{1}{\phi^2}\right)$ as $\phi \to \infty$. Wedges are not a problem with large ϕ!

- If liquidity premia are the main distortion, may be better for CB to intervene in inflation swap market.

Fiscal Theory of the Price Level and “over determinacy”

- If price level is determinate independent of MP, then $\phi > 1$ can mean explosive π_t.
    - E.g., true if fiscal policy is active (real surpluses do not respond to debt).
    - With one period debt, active fiscal policy & flexible prices: $\pi_t - \mathbb{E}_{t-1}\pi_t = -\varepsilon_{s,t}$.
    - Inconsistent with standard real rate rule solution: $\pi_t = -\frac{1}{\phi}\varepsilon_{\zeta,t}$ (IID monetary shock) as long as $\varepsilon_{\zeta,t} \neq \phi\varepsilon_{s,t}$.
    - Only explosive solution remains under real rate rule: $\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1} - \varepsilon_{s,t}$.

- This is a knife edge result! With multi-period (geometric coupon) debt: stable π_t solution under a real rate rule.
    - Still consistent with transversality even with active fiscal, active monetary!
    - ↑ bubble in debt price balanced by ↓ quantity. Initial debt price jumps. “Fiscal theory of the debt price”.
    - With passive MP this implies multiplicity, so FTPL does not guarantee uniqueness.

- General result: Always a stable solution under a real rate rule if plausible condition satisfied:
    - Potentially explosive variables (e.g., bond prices) must not feed back to the real economy, and are not too forward looking.

Setting nominal rates out of equilibrium

- Apparent issue: If for $t > 0$, $i_t = r_t + \phi\pi_t$, then $\pi_t = 0$ for $t > 0$, so by Fisher $i_0 = r_0$. CB cannot set $i_0 \neq r_0$!

- Resolution: $\pi_t = 0$ iff $\pi_s = 0$ for all $s \in \{0, 1, \dots, t-1\}$, else $\pi_t = \phi\pi_{t-1}$. If $\pi_0 \neq 0$, Fisher states $i_0 - r_0 = \phi\pi_0$.

- May reappear under bounded rationality. Suppose agents have learned $\pi_t = 0$, then $i_t = r_t$ even out of equilibrium.

- One fix: Modified real rate rule:

$$i_t = r_t + \phi\pi_t + \psi\pi_{t-1}$$

- ψ small. Determinate if $\phi > |1 - \psi|$. Solution: $\pi_t = \varrho\pi_{t-1}$, where $\varrho = \frac{\phi - \sqrt{\phi^2 + 4\psi}}{2} \in (-1, 1)$. Agents learn $\pi_t \approx \varrho\pi_{t-1}$.

- Alternative fix: Price level real rate rule rules.

Practical implementation of real rate rules

Practical implementation: Set-up

- Markets in short maturity TIPS may be illiquid, unavailable or unreliable. So, use longer maturity bonds.
    - But: Long maturities may have substantial risk/term/liquidity premia.
    - Extra complications: Inflation may be observed with a lag. 1 month for US CPI. TIPS may have indexation lag. 3 months in US.

- Notation:
    - S: information lag. Market participants use the $t - S$ information set in period t.
    - L: indexation lag in return of inflation protected bonds.
    - $i_{t|t-S}$: nominal yield per period on a T-period nominal bond at t.
    - $r_{t|t-S}$: real yield per period on a T-period inflation protected bond at t.
    - $\nu_{t|t-S}$ Fisher equation wedge (risk premia etc.) for T-period nominal bonds relative to T-period real bonds at t.
    - $\bar{\nu}_{t|t-S}$ central bank's period t belief about level of $\nu_{t|t-S}$ (possibly correlated with $\nu_{t|t-S}$).

Practical implementation: Fisher equation and rule

- Fisher equation (need $T - L \geq 0$):

$$i_{t|t-S} - r_{t|t-S} = \nu_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k-L}$$

- Monetary rule:

$$i_{t|t-S} - r_{t|t-S} = \bar{\nu}_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi^{*}_{t+k-L} + \phi(\pi_{t-S} - \pi^{*}_{t-S})$$

- Combining implies:

$$\mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T} (\pi_{t+k+S-L} - \pi^{*}_{t+k+S-L}) + (\nu_{t+S|t} - \bar{\nu}_{t+S|t}) = \phi(\pi_t - \pi^{*}_t)$$

- Solution (A_j unique for $\phi > 1$):

$$\pi_t = \pi^{*}_t + \mathbb{E}_t \sum_{j=-\infty}^{\infty} A_j \left(\nu_{t+j+S|t+j} - \bar{\nu}_{t+j+S|t+j}\right)$$

Practical implementation: Discussion

- CB's inflation error $\pi_t - \pi_t^*$ is stationary as long as $\nu_{t+S|t} - \bar{\nu}_{t+S|t}$ is stationary.
- If ϕ is large enough, $\pi_t \approx \pi_t^*$ (under the same assumptions as in previous discussion of Fisher wedges).
- If aggressive enough, endogenous wedges, indexation & information lags do not matter!

- Note: CB's trading desk should hold $i_t - r_t$ constant between meetings.
  - This requires i_t to move between meetings, in response to observed changes in r_t.
  - No reason this should be significantly harder than holding i_t fixed.

- CB could also offer to exchange $\$1$ face value of real debt for $\$(1 + i_t - r_t)$ face value of nominal, as proposed by Cochrane (2017; 2018).
- Or to buy/sell portfolios with $+\$1$ nominal debt, $-\$1$ real debt for $\$(i_t - r_t)$.
- Or trade inflation swaps (which pay $\Pi_{t+1} - K_t$ at $t + 1$, with no payments at t).

The zero lower bound

Problems caused by the ZLB

- With the ZLB, simplest real rate rule means:

$$\max\{0, r_t + \phi\pi_t\} = i_t = r_t + \mathbb{E}_t\pi_{t+1}$$

- So:

$$\mathbb{E}_t\pi_{t+1} = \max\{-r_t, \phi\pi_t\}$$

- Real rates no longer cancel out completely! Euler equation still matters for π_t.

- Extra steady state with $\pi = -r$ (Benhabib, Schmitt-Grohé & Uribe 2001).

- Still multiplicity and/or non-existence conditional on convergence to the standard steady state (Holden 2021).
  - E.g., suppose r_t exogenous, $r_t = 0$ for $t \neq 1$, and we assume that $\pi_t \to 0$ as $t \to \infty$.
  - Multiple solutions if $r_1 = 0$. No solution if $r_1 < 0$. General problem in NK models.

Modified inflation targets

- Non-existence comes from implicitly targeting an infeasibly low level of inflation.

- Easy to fix. Use the rule:

$$i_t = \max\{0, r_t + \mathbb{E}_t \breve{\pi}^*_{t+1} + \phi(\pi_t - \breve{\pi}^*_t)\}, \qquad \breve{\pi}^*_t := \max\{-r_{t-1} + \epsilon, \pi^*_t\}$$

  - π^*_t is the original inflation target. $\breve{\pi}^*_t$ is the modified target. $\epsilon > 0$ is a small constant.

- With modified rule: $\pi_t = \breve{\pi}^*_t$ for all t is an equilibrium. Locally determinate.
  - Closed form solution (rare with ZLB!) makes coordination easy.
  - No deflationary bias as $\pi_t > -r_{t-1}$. Instead: small inflationary bias as $\mathbb{E}\pi_t \geq \mathbb{E}\pi^*_t$.

- Solution is unique conditional on $\breve{\pi}^*_t$ and on a terminal condition ruling out explosions or permanent ZLB traps.
  - Multiple solutions for $\breve{\pi}^*_t$ do not occur for standard NK models.

Equilibrium selection with perpetuities: Idea

- Cochrane (2011) argues no reason to rule out explosive NK equilibria.

- Suppose geometric coupon bonds (GCBs) are traded in the economy. (Later specialise to perpetuities.)
  - Could be approximated by portfolio of different maturity debt. Long-term government contracts (defence,…) also perpetuity like.

- 1 unit of period t GCB bought at t returns $\$1$ at $t+1$, along with $\omega \in (0,1]$ units of period $t+1$ GCB.
  - Suppose stock: $B_t \geq \underline{B}\omega^t$. Then transversality implies GCB price: $Q_t = \mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=0}^{s} \frac{1}{I_{t+k}} \right] \omega^s$.
  - If $I_{t+k} = 1$ for high k, then $Q_{t+k} = \frac{1}{1-\omega}$ for high k. Transversality then requires $0 = \lim_{s\to\infty} \frac{\omega^s}{1-\omega}$, i.e., $|\omega| < 1$. Violated with $\omega = 1$!

- Permanent ZLB ⇒ Infinite perpetuity price ⇒ Infinite nominal wealth ⇒ Infinite inflation ⇒ Physically impossible.

Equilibrium selection with perpetuities: Use

- With sticky prices, explosions are generally ruled out.
  - Standard sticky prices specifications imply Π_t is bounded above. + Real costs of inflation explode as inflation explodes.
  - Prices may become more flexible as $\Pi_t \uparrow$, but seems plausible there is a limit on how often prices can be changed.
- So, under sticky prices the previous rule produces uniqueness if households hold perpetuities.
- Non-linear version (with a target known one period in advance, $\mathcal{E} := \exp \epsilon > 1$):

$$I_t = \max\left\{1, R_t \breve{\Pi}_t^* \left(\frac{\Pi_t}{\breve{\Pi}_{t-1}^*}\right)^{\phi}\right\}, \qquad \breve{\Pi}_t^* := \max\left\{\frac{\mathcal{E}}{R_t}, \Pi_t^*\right\}$$

- Without sticky prices, have to send deviations to the ZLB. E.g., with following ($\bar{I} > 1$, $\phi > 1$ and $\mathcal{E} \in \left(1, \sqrt{\bar{I}}\right)$):

$$I_t = \begin{cases} \max\left\{1, R_t \breve{\Pi}_t^* \left(\frac{\Pi_t}{\breve{\Pi}_{t-1}^*}\right)^{\phi}\right\}, & \text{if } I_{t-1} \in \left(1, \bar{I}\right) \\ 1, & \text{otherwise} \end{cases}, \qquad \breve{\Pi}_t^* := \max\left\{\frac{\mathcal{E}}{R_t}, \min\left\{\frac{\bar{I}}{\mathcal{E} R_t}, \Pi_t^*\right\}\right\}$$

Conclusion

- The Taylor principle does not guarantee determinacy with standard monetary rules.
- Real rate rules ensure determinacy no matter the rest of the economy & give CB almost perfect control of inflation.
- They can be easily implemented using pre-existing assets (nominal and real bonds, or inflation swaps).

- Under a real rate rule:
  - Monetary policy works in spite of, not because of, real rate movements.
  - Causation in the Phillips curve runs exclusively from inflation to the output gap.
  - Household and firm decisions, constraints and inflation expectations are irrelevant for inflation dynamics.
  - Only changes in the rule can amplify the impact of shocks on inflation.

- With a time-varying target, real rate rules can implement optimal monetary policy, or match observed dynamics.
- Real rate rules continue to work in the presence of the ZLB, wedges in the Fisher equation, or active fiscal policy.

Extra slides

Welfare

- Recap: Real rate rules can determinately implement an arbitrary path for inflation, including optimal policy. Automatic that they can attain high welfare!

- Makes sense to limit to "simple" real rate rules though.
  - "Simple" here means simple dynamics of targeted inflation.
  - Claim: Looking for optimal simple inflation dynamics is a useful approach to policy.

- Two exercises follow:
  - MA(0), MA(1) and ARMA$(1,1)$ inflation policy in a simple NK model. Latter is sufficient to attain unconditional optimal.
  - Examination of optimal policy in the Justiniano, Primiceri & Tambalotti (2013) model. Multiple shock ARMA$(1,2)$ inflation policy is very close to fully optimal.

A simple NK model for policy analysis

- Look at welfare in a simple model with the Phillips curve (ω_t IID):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t$$

- And the policy objective to minimise:

$$(1-\beta)\mathbb{E}\sum_{k=0}^{\infty}\beta^k\left(\pi_{t+k}^2 + \lambda x_{t+k}^2\right) = \mathbb{E}\left(\pi_t^2 + \lambda x_t^2\right)$$

  - Equality under the constraint that policy must be time-invariant.

- Optimal policy must have an MA(∞) representation ($\theta_1, \theta_2, \ldots$ TBD):

$$\pi_t = \kappa \sum_{k=0}^{\infty} \theta_k \omega_{t-k}$$

  - Implies objective is:

$$\mathbb{E}\left(\pi_t^2 + \lambda x_t^2\right) = \mathbb{E}\left[\omega_t^2\right]\sum_{k=0}^{\infty}\left[\kappa^2\theta_k^2 + \lambda(\theta_k - \beta\theta_{k+1} - \mathbb{1}[k=0])^2\right]$$

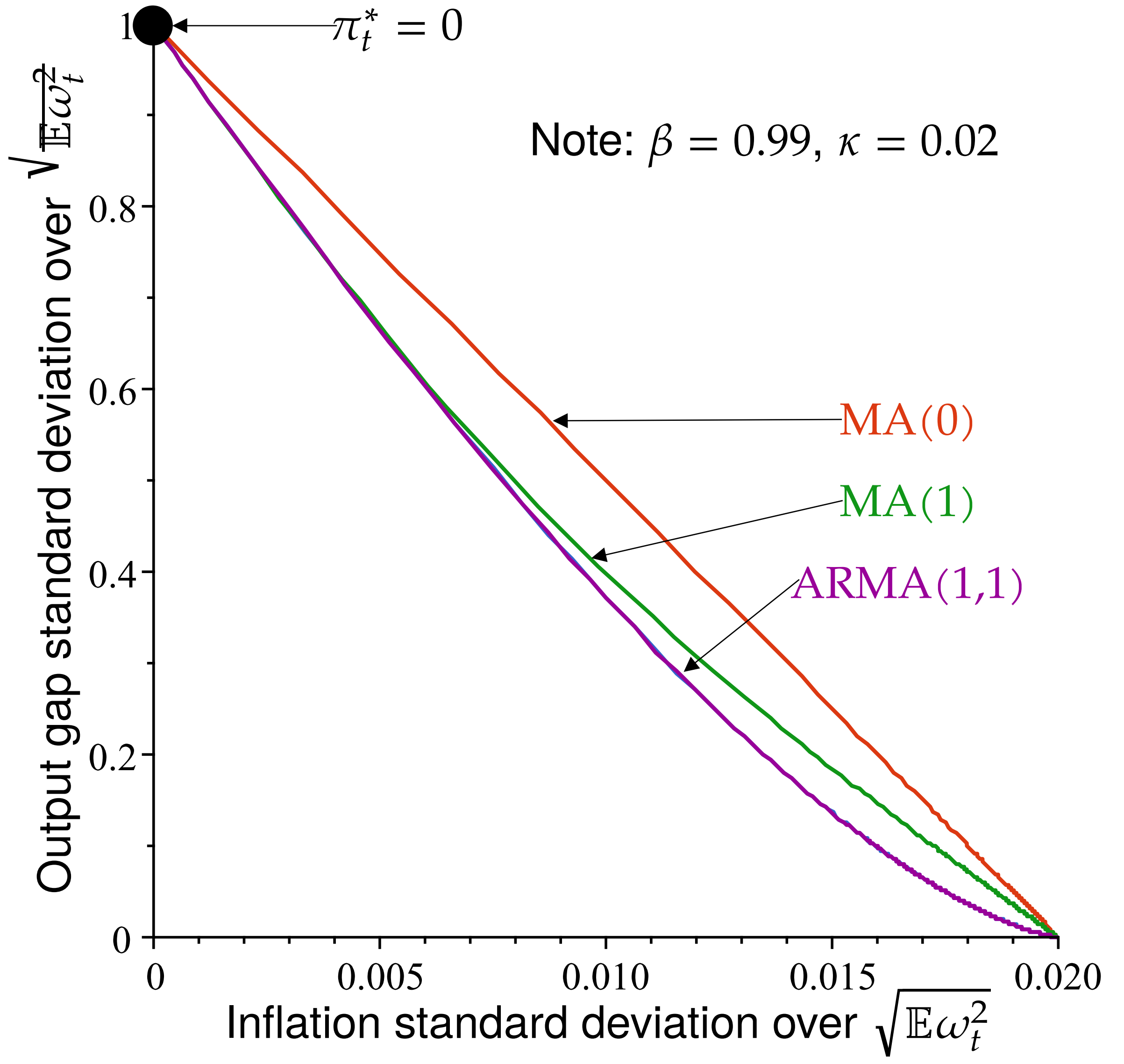
Welfare of real rate rules in a simple model

- Optimising subject to $\pi_t = \pi_t^*$ being MA(0) gives the discretionary optimum with $\pi_t = \kappa \frac{\lambda}{\lambda+\kappa^2} \omega_t$ and $\pi_t + \frac{\lambda}{\kappa} x_t = 0$.

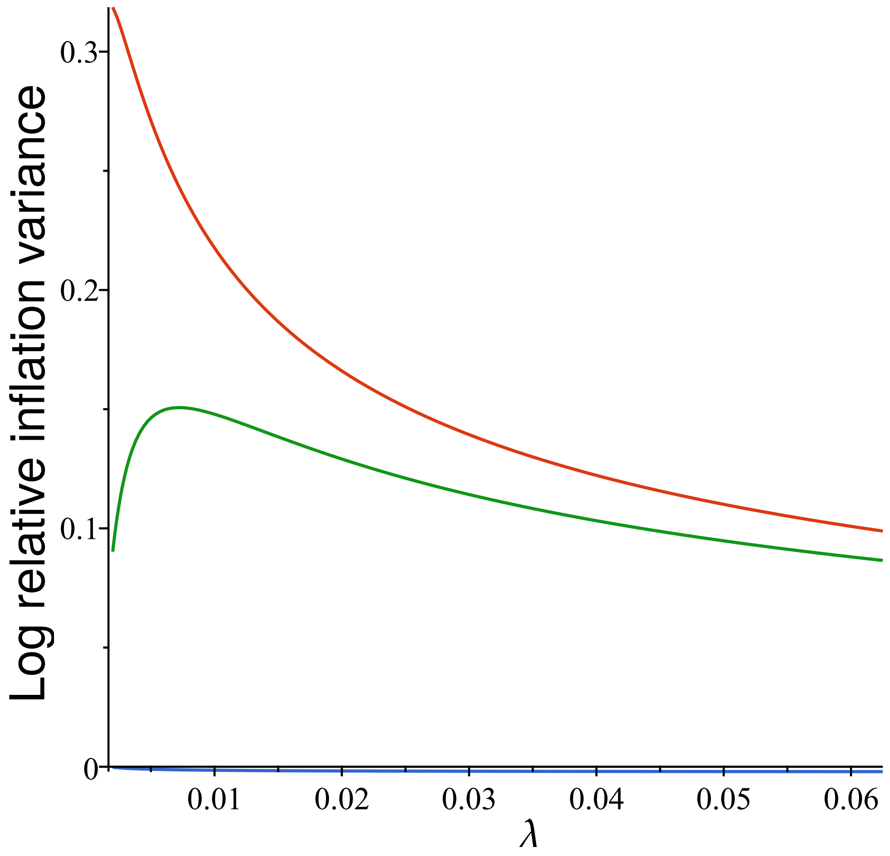
- Optimising subject to $\pi_t = \pi_t^*$ being an MA(1) gives a solution with $\pi_t = \kappa\theta_0\omega_t + \kappa\theta_1\omega_{t-1}$ where $\theta_0 \geq 0$ and $\theta_1 \leq 0$.
  - Thus ω_t increases π_t while reducing $\mathbb{E}_t \pi_{t+1}$, lessening output gap movements.

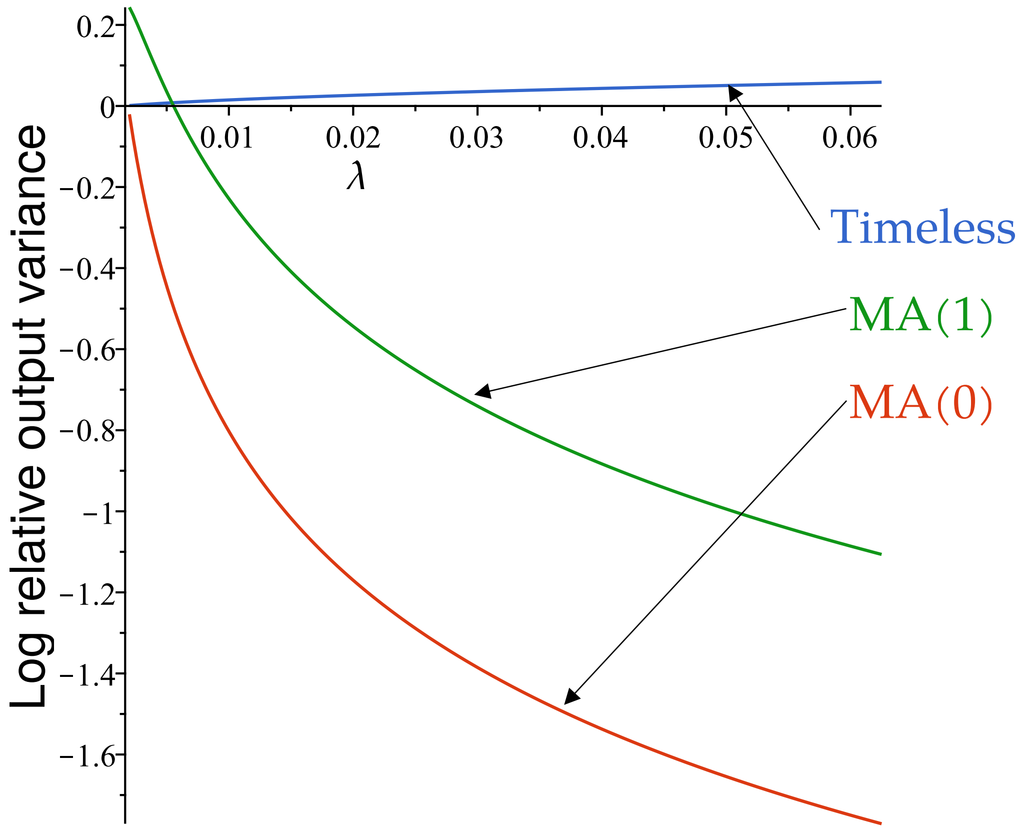
- Optimising subject to $\pi_t = \pi_t^*$ being an ARMA$(1,1)$ give the unconditionally optimal solution from the prior literature (e.g. Damjanovic, Damjanovic & Nolan (2008)) with $\pi_t + \frac{\lambda}{\kappa}(x_t - \beta x_{t-1}) = 0$.
  - Optimal MA coefficient equals $-\beta \approx -0.99$. Close to the IMA$(1,1)$ processes estimated by Dotsey, Fujita & Stark (2018) for the post-1984 period.

Policy frontiers (varying λ)



Log relative variances to ARMA(1,1) policy

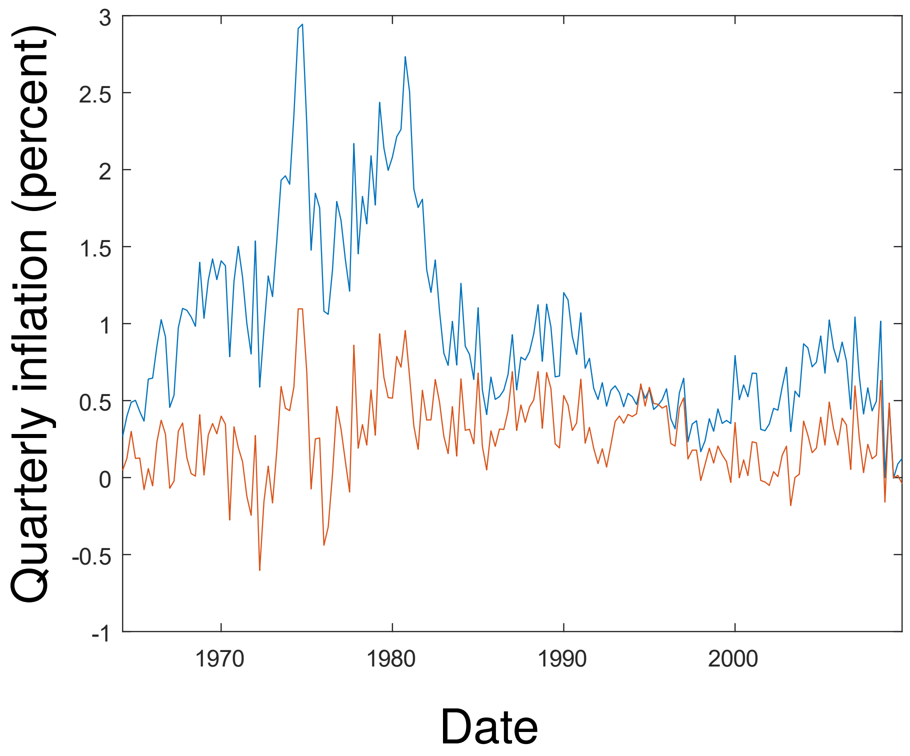


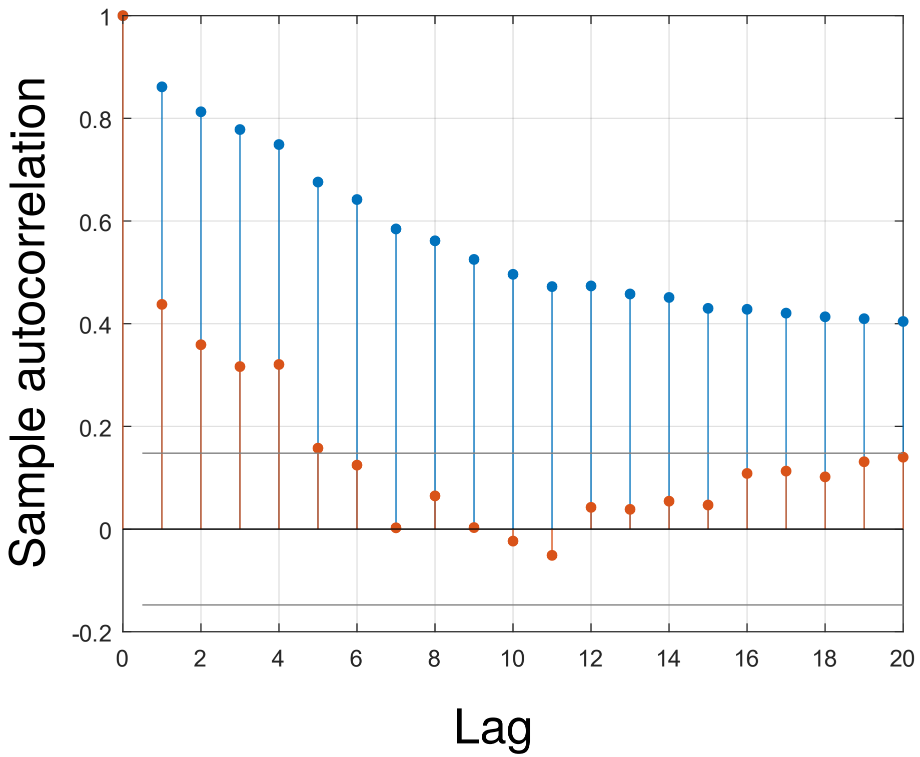


Note: $\beta = 0.99$, $\kappa = 0.02$.

MA(0) and MA(1) policies generate too much inflation variance.

Optimal inflation dynamics in a richer model





Using the Justiniano, Primiceri & Tambalotti (2013) model and replication files.

Blue: actual US inflation dynamics.

Red: inflation dynamics under optimal policy and US historical shocks. Less persistent!

Simple approximation to optimal policy 1/2

- For any $\rho \in (-1,1)$, the solution for optimal inflation has a multiple shock, ARMA$(1,\infty)$ representation:

$$\pi_t - \pi = \rho(\pi_{t-1} - \pi) + \sum_{k=0}^{\infty} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$

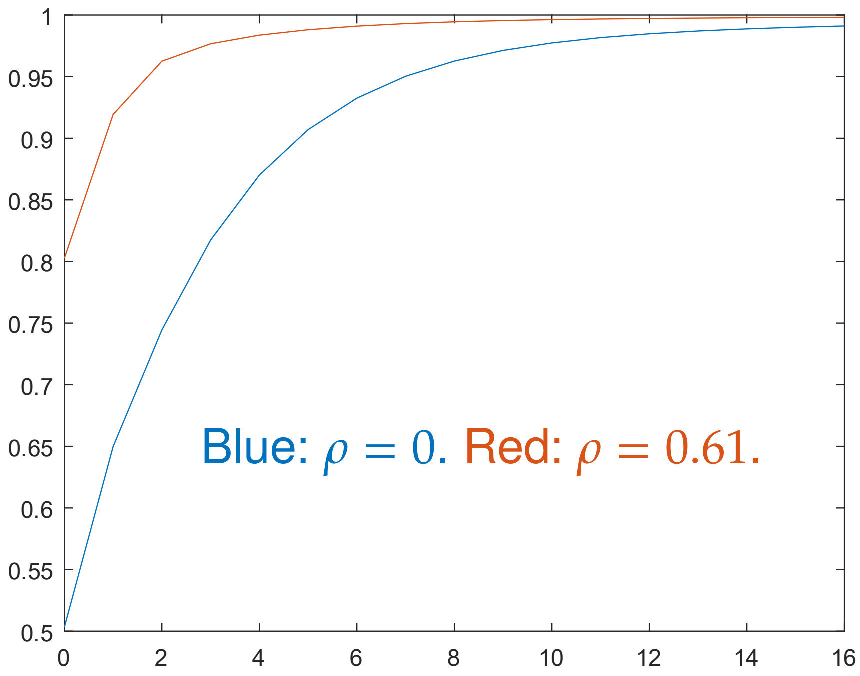
  - $\varepsilon_{1,t}, \dots, \varepsilon_{N,t}$ are the model's structural shocks.

- Approximate by truncating MA terms at some point: E.g. multiple shock ARMA$(1,K)$:

$$\pi_t^{(K)} - \pi = \rho\left(\pi_{t-1}^{(K)} - \pi\right) + \sum_{k=0}^{K} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$

- Henceforth: “multiple shock ARMA” = “MSARMA”.

Simple approximation to optimal policy 2/2



Number of MA lags

Proportion of the variance of optimal inflation in the Justiniano, Primiceri & Tambalotti (2013) model explained by truncating the number of MA lags.

MSARMA$(1,1)$ explains $> 90\%$ of optimal inflation variance, MSARMA$(1,2)$ $> 95\%$!

References

Adão, Bernardino, Isabel Correia & Pedro Teles. 2011. ‘Unique Monetary Equilibria with Interest Rate Rules’. *Review of Economic Dynamics* 14 (3) (July): 432–442.

Ang, Andrew, Geert Bekaert & Min Wei. 2007. ‘Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?’ *Journal of Monetary Economics* 54 (4) (May 1): 1163–1212.

Ascari, Guido & Tiziano Ropele. 2009. ‘Trend Inflation, Taylor Principle, and Indeterminacy’. *Journal of Money, Credit and Banking* 41 (8): 1557–1584.

Atkeson, Andrew & Lee E Ohanian. 2001. ‘Are Phillips Curves Useful for Forecasting Inflation?’ Edited by Edward J Green & Richard M Todd. *Federal Reserve Bank of Minneapolis Quarterly Review* (Winter 2001): 12.

Bilbiie, Florin. 2008. ‘Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic’. *Journal of Economic Theory* 140 (1) (May 1): 162–196.

———. 2019. *Monetary Policy and Heterogeneity: An Analytical Framework*. 2019 Meeting Papers. Society for Economic Dynamics.

Branch, William A. & Bruce McGough. 2010. ‘Dynamic Predictor Selection in a New Keynesian Model with Heterogeneous Expectations’. *Journal of Economic Dynamics and Control* 34 (8) (August 1): 1492–1508.

———. 2018. ‘Chapter 1 - Heterogeneous Expectations and Micro-Foundations in Macroeconomics’. In *Handbook of Computational Economics*, edited by Cars Hommes & Blake LeBaron, 4:3–62. Handbook of Computational Economics. Elsevier.

Cochrane, John H. 2011. ‘Determinacy and Identification with Taylor Rules’. *Journal of Political Economy* 119 (3): 565–615.

———. 2017. ‘The Grumpy Economist: Target the Spread’. *The Grumpy Economist*.

———. 2018. ‘The Zero Bound, Negative Rates, and Better Rules’ (March 2): 27.

———. 2022. *The Fiscal Theory of the Price Level*. Princeton University Press.

Cúrdia, Vasco, Andrea Ferrero, Ging Cee Ng & Andrea Tambalotti. 2015. ‘Has U.S. Monetary Policy Tracked the Efficient Interest Rate?’ *Journal of Monetary Economics* 70: 72–83.

Damjanovic, Tatiana, Vladislav Damjanovic & Charles Nolan. 2008. ‘Unconditionally Optimal Monetary Policy’. *Journal of Monetary Economics* 55 (3) (April 1): 491–500.

Dotsey, Michael, Shigeru Fujita & Tom Stark. 2018. ‘Do Phillips Curves Conditionally Help to Forecast Inflation?’ *International Journal of Central Banking*: 50.

Dotsey, Michael & Andreas Hornstein. 2006. ‘Implementation of Optimal Monetary Policy’. *Economic Quarterly*: 113–133.

Dowd, Kevin. 1994. ‘A Proposal to End Inflation’. *The Economic Journal* 104 (425): 828–840.

Evans, George W. & Seppo Honkapohja. 2006. ‘Monetary Policy, Expectations and Commitment’. *The Scandinavian Journal of Economics* 108 (1): 15–38.

Evans, George W. & Bruce McGough. 2010. ‘Implementing Optimal Monetary Policy in New-Keynesian Models with Inertia’. *The B.E. Journal of Macroeconomics* 10 (1).

Fleckenstein, Matthias, Francis A. Longstaff & Hanno Lustig. 2014. ‘The TIPS-Treasury Bond Puzzle’. *The Journal of Finance* 69 (5): 2151–2197.

Gali, Jordi, J. David Lopez-Salido & Javier Valles. 2004. ‘Rule-of-Thumb Consumers and the Design of Interest Rate Rules’. *Journal of Money, Credit, and Banking* 36 (4) (July 28): 739–763.

Hall, Robert E & Ricardo Reis. 2016. *Achieving Price Stability by Manipulating the Central Bank’s Payment on Reserves*. Working Paper. National Bureau of Economic Research.

Hellwig, Martin F. 1980. ‘On the Aggregation of Information in Competitive Markets’. *Journal of Economic Theory* 22 (3) (June 1): 477–498.

Hetzel, Robert L. 1990. ‘Maintaining Price Stability: A Proposal’. *Economic Review of the Federal Reserve Bank of Richmond* (March): 3.

Holden, Tom D. 2019. *Existence and Uniqueness of Solutions to Dynamic Models with Occasionally Binding Constraints*. EconStor Preprints. ZBW - Leibniz Information Centre for Economics.

Justiniano, Alejandro, Giorgio E. Primiceri & Andrea Tambalotti. 2013. ‘Is There a Trade-Off between Inflation and Output Stabilization?’ *American Economic Journal: Macroeconomics* 5 (2) (April): 1–31.

Khan, Hashmat, Louis Phaneuf & Jean Gardy Victor. 2019. ‘Rules-Based Monetary Policy and the Threat of Indeterminacy When Trend Inflation Is Low’. *Journal of Monetary Economics* (March): S0304393219300479.

Leeper, E. M. & C. Leith. 2016. ‘Chapter 30 - Understanding Inflation as a Joint Monetary–Fiscal Phenomenon’. In *Handbook of Macroeconomics*, edited by John B. Taylor & Harald Uhlig, 2:2305–2415. Elsevier.

Lou, Youcheng, Sahar Parsa, Debraj Ray, Duan Li & Shouyang Wang. 2019. ‘Information Aggregation in a Financial Market with General Signal Structure’. *Journal of Economic Theory* 183 (September 1): 594–624.

Lubik, Thomas A., Christian Matthes & Elmar Mertens. 2019. *Indeterminacy and Imperfect Information*. Federal Reserve Bank of Richmond Working Papers.

Manea, Cristina. 2019. ‘Collateral-Constrained Firms and Monetary Policy’: 66.

McLeay, Michael & Silvana Tenreyro. 2019. ‘Optimal Inflation and the Identification of the Phillips Curve’. In . NBER Chapters. National Bureau of Economic Research, Inc.

Miranda-Agrippino, Silvia & Giovanni Ricco. 2021. ‘The Transmission of Monetary Policy Shocks’. *American Economic Journal: Macroeconomics* 13 (3) (July): 74–107.

Natvik, Gisle James. 2009. ‘Government Spending and the Taylor Principle’. *Journal of Money, Credit and Banking* 41 (1): 57–77.

Stock, James & Mark W. Watson. 2009. ‘Phillips Curve Inflation Forecasts’. In *Understanding Inflation and the Implications for Monetary Policy*, edited by Jeffrey Fuhrer, Yolanda Kodrzycki, Jane Little & Giovanni Olivei, 99–202. Cambridge: MIT Press.

Sveen, Tommy & Lutz Weinke. 2005. ‘New Perspectives on Capital, Sticky Prices, and the Taylor Principle’. *Journal of Economic Theory* 123 (1). Monetary Policy and Capital Accumulation (July 1): 21–39.

Svensson, Lars E. O & Michael Woodford. 2003. ‘Implementing Optimal Policy through Inflation-Forecast Targeting’ (June).