Robust Real Rate Rules

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Paper and slides available at https://www.tholden.org/.

Motivation: Fragility of the Taylor principle

- The **Taylor principle** requires the response of nominal rates to inflation to be greater than one.
  - Sufficient for determinacy in simple models. (Guarantees no belief-driven fluctuations / sunspots.)
- **Insufficient** if there is e.g.:
  - A fraction of hand-to-mouth households (Galí, Lopez-Salido & Valles 2004).
  - Firm-specific capital (Sveen & Weinke 2005).
  - High government spending (Natvik 2009).
  - A positive inflation target (Ascari & Ropele 2009),
  - …particularly with trend growth + sticky wages (Khan, Phaneuf & Victor 2019).
- **Inverts** if there are e.g.:
  - Enough hand-to-mouth households (Bilbiie 2008).
  - Financial frictions (Lewis & Roth 2018; Manea 2019).
  - Non-rational expectations (Branch & McGough 2010; 2018).
  - Active fiscal policy (Leeper 1991; Leeper & Leith 2016; Cochrane 2023).

This paper

- Interest rate rules with a **unit response to real rates** guarantee **determinacy** under the weakest possible assumptions on the rest of the economy.
  - Robust to household heterogeneity, non-rational households/firms, existence/slope of the Phillips curve, active fiscal policy, etc.

- With a **time-varying short-term inflation target**: enable determinate implementation of an arbitrary inflation path.
  - So can match observed inflation dynamics, or any model's optimal policy.

- Easy to implement in practice. **Use TIPS** to infer real rates. Works with bonds of any maturity.

- Reveal: **Fisher equation** is key to monetary transmission.

Main idea

- **Nominal** bond: $\$1$ bond purchased at t returns $\$(1+i_t)$ at $t+1$.
- **Real** bond (e.g., TIPS): $\$1$ bond purchased at t returns $\$(1+r_t+\pi_{t+1})$ at $t+1$.
- Arbitrage $\Rightarrow$ the **Fisher equation** (abstracting from risk / liquidity / etc premia for now):
$$i_t = r_t + \mathbb{E}_t \pi_{t+1}$$
- Central bank uses the "**real rate rule**": $\boxed{i_t = r_t + \phi \pi_t}$
- With $\phi > 1$. Then: $$\mathbb{E}_t \pi_{t+1} = \phi \pi_t$$
- Unique non-explosive solution: $$\pi_t = 0!$$
- **Determinate inflation!** (Using standard NK equilibrium selection rule.)

Why is this robust? No need for Euler equation!

- Does not require an **aggregate Euler equation** to hold.
  - Robust to household heterogeneity and hand-to-mouth agents.
  - Robust to non-rational household expectations.

- For the **Fisher equation** to hold just need either:
  - Two deep pocketed, fully informed, rational agents in the economy, OR,
  - …a large market of rational agents with dispersed information (Hellwig 1980; Lou et al. 2019).

- Much more plausible financial market participants have **rational expectations** than households.
  - Can even partially relax the rationality requirement for financial market participants.

Why is this robust? No need for Phillips curve!

- Does not require an **aggregate Phillips curve** to hold.
  - Robust to slope of the Phillips curve (if it exists).
  - Robust to forward/backward looking degree of Phillips curve equation.
  - Robust to non-rational firm expectations.

- Under this monetary rule, the Phillips curve is **irrelevant for inflation dynamics**.
  - The Phillips curve (if it exists) determines the output gap, given inflation.
  - If CB is unconcerned with output and unemployment, they do not need to care about the Phillips curve or its slope.

- Only require that at least some prices are adjusted each period using current information.

Implications for monetary economics

- Which model features lead to **amplification** or **dampening** of monetary shocks?
- Under a real rate rule: **no change** in the model can amplify/dampen monetary shocks **other than changing rule**.
    - Prior amplification/dampening results were sensitive to the monetary rule. May reverse with a response to r_t of > 1.
- Which shocks **drive inflation**?
- Under a real rate rule: only **monetary policy shocks** or **shocks to the Fisher equation**.
- **CB has ultimate responsibility** for inflation.

How does monetary policy work?

- Under flexible prices, monetary policy **does not work via the real rate**. Real rates are **exogenous**.
- Under a real rate rule, monetary policy **does not work via the real rate**. Real rates are **irrelevant**.

- Outcomes under a real rate rule are qualitatively similar to outcomes under a traditional rule.
- Is it reasonable to believe that monetary policy (mostly) works via the real rate with traditional rules?
  - Rupert & Šustek (2019) show that with endogenous capital and sufficient monetary shock persistence:
  - …contractionary (positive) monetary shocks lead to falls in output, inflation and real rates. Contrary to standard story.

- Instead: Monetary policy works **as under flexible prices.** (Exactly under a real rate rule, approximately in general.)
  - Following a monetary shock, inflation jumps to the unique level consistent with non-explosive inflation.

Real rate rules elsewhere in the literature

- Papers with rules responding to real rates, for **analytic convenience** e.g.:
  - Adão, Correia & Teles (2011), Holden (2019), Lubik, Matthes & Mertens (2019).
- **Closest prior work**: Cochrane (2017; 2018; 2023) on spread targeting.
  - Cochrane briefly considers a rule of the form $i_t = r_t + \phi\pi_t$ before setting $\phi = 0$ (under FTPL).
- Other related work:
  - Hall & Reis (2016): vary interest on reserves with price level deviations, e.g. nominal return from $\$1$ of $\$(1+r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1+i_t)\frac{p_t}{p_t^*}$.
  - Hetzel (1990): Use nominal/real bond spread to guide policy. Dowd (1994): target the price of price level futures contracts.
  - Forecast targeting: Hall & Mankiw (1994), Svensson (1997), ….
  - Bernanke & Woodford (1997): Responding to private inflation forecasts leads to indeterminacy (LHS vs RHS). Bilbiie (2008; 2011) uses a special case with $i_t = \mathbb{E}_t\pi_{t+1} + \cdots$. These are called real rate rules by Beaudry, Preston & Portier (2022).
- Large literature on rules tracking efficient ("natural") real interest rate.
  - E.g., Woodford (2003). Very different idea. $i_t = n_t + \phi\pi_t$ implies $\pi_t = \mathbb{E}_t \sum_{k=0}^{\infty} \phi^{-k-1}(r_{t+k} - n_{t+k})$.

Time-varying short-term inflation targets

Allowing inflation to move

- Zero inflation is **not always desirable**: E.g., with sticky prices want inflation ↑ in response to mark-ups ↑.

- Traditional solution: Respond to output gaps in the rule. **Reduces robustness**.

- Better solution: **Time-varying short-term inflation target**. Target can respond to endogenous variables or shocks.

- Suppose CB uses the **rule**: $\boxed{i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t)}$

  - π^*_t: an arbitrary stochastic process, possibly a function of economy's other endogenous variables and shocks.

- From the Fisher equation: $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. With $\phi > 1$, **unique, determinate solution**: $\pi_t = \pi^*_t$.

Communication

- CB **announces target** π_t^*. May prefer to announce one period in advance, $\pi_{t+1|t}^*$.
  - If announced one period in advance, rule becomes: $i_t = r_t + \pi_{t+1|t}^* + \phi(\pi_t - \pi_{t|t-1}^*)$.
  - One period ahead target means agents do not need to understand law of motion of $\pi_{t+1|t}^*$ to form $\mathbb{E}_t \pi_{t+1}$.

- CB needs to be clear that any change in targeted inflation is **temporary**. Long run target remains at 2% (say).
  - Not really different to current practice. Not raising rates when inflation is above target also needs careful communication.

- The Fed **already announces** a target path for inflation through the **Summary of Economic Projections** (SEP).
  - SEP gives monetary policy makers' forecasts for inflation conditional on their beliefs about "appropriate monetary policy".
  - *"Each participant's projections [are] based on … her or his assessment of appropriate monetary policy … defined as the future path of policy that each participant deems most likely to foster outcomes for economic activity and inflation that best satisfy his or her individual interpretation of the statutory mandate to promote maximum employment and price stability."*

Benefits

- A real rate rule with appropriate π_t^* can **replicate** the outcome of any other monetary regime.
  - Includes rules responding to additional endogenous variables.
  - Includes observed outcomes for inflation. Without evidence on π_t^* cannot rule out that CB follows a real rate rule.

- Also means: Real rate rules can **implement** optimal policy.
  - E.g., with Phillips curve, $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t$, and objective, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right]$, set $\pi_t^* := -\kappa^{-1}\lambda(x_t - x_{t-1})$ (timeless).
  - Or if announced in advance, optimal to set $\pi_{t+1|t}^* := -\kappa^{-1}\lambda \mathbb{E}_t(x_{t+1} - x_t)$.
  - Prior lit.: Svensson & Woodford (2003), Dotsey & Hornstein (2006), Evans & Honkapohja (2006), Evans & McGough (2010).

- **Splits** monetary policy decision in two. Combines benefits of **flexibility** and rigid **commitment**.
  - Governor and board announce target level of inflation for the next month(s). Full benefits of flexibility.
  - Trading desk mechanically sets nominal rates via the rule to hit this level. Full benefits of commitment for determinacy.

Interest rate smoothing

- A fully **smoothed real rate rule** with time-varying short-term target:

$$\boxed{i_t - r_t = i_{t-1} - r_{t-1} + \mathbb{E}_t \pi^*_{t+1} - \mathbb{E}_{t-1} \pi^*_t + \theta(\pi_t - \pi^*_t)}$$

- One benefit: Removes impact of **static wedges** in the Fisher equation. E.g., suppose: $i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu$.
- With $\theta > 0$, $\pi_t \equiv \pi^*_t$ is the **unique equilibrium** even if $\nu \neq 0$.
  - Proof: Let $e_t := \mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1})$. Then $\pi_t - \pi^*_t = \theta^{-1}(e_t - e_{t-1})$, so $\mathbb{E}_t e_{t+1} = (1+\theta) e_t$, and hence $e_t \equiv 0$ and $\pi_t \equiv \pi^*_t$.
  - Also ensures inflation is stationary even if Fisher wedge ν_t has a unit root.
- Produces the same π_t as unsmoothed rule, and hence same i_t. Smoothing is **not observable**.
- Key difference: **Only need $\boldsymbol{\theta > 0}$**, not $\phi > 1$.
  - Easier for CB to convince agents of the former than of the latter.
- Further benefit: Prevents the existence of **sunspot solutions** when there is a ZLB.

Monetary shocks and Fisher equation wedges

Monetary policy shocks

- Suppose the CB uses the **rule**: $$i_t = r_t + \phi\pi_t + \zeta_t$$

- with $\phi > 1$, and ζ_t drawn from an $\mathrm{AR}(1)$ process with persistence ρ.

- Then from the Fisher equation: $\mathbb{E}_t\pi_{t+1} = \phi\pi_t + \zeta_t, \quad \Rightarrow \quad \pi_t = -\dfrac{1}{\phi - \rho}\zeta_t$

- **Contractionary** (positive) monetary policy shocks **reduce** inflation.
  - Define $\pi_t^* := -\frac{1}{\phi-\rho}\zeta_t$, then $i_t = r_t + \mathbb{E}_t\pi_t^* + \phi(\pi_t - \pi_t^*)$. Contractionary monetary shock = temporary fall in inflation target.

- If the CB is more **aggressive** (ϕ is larger) inflation is **less volatile**.

- Inflation dynamics are **independent** of the rest of the economy.

Output and inflation dynamics in 3 equation NK world

- As before: CB sets $i_t = r_t + \phi\pi_t + \zeta_t$, so $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$. Rest of model:

- **Phillips curve** (PC), mark-up shock ω_t: $\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + \kappa\omega_t$

- Discounted/compounded **Euler equation** (EE) (Bilbiie 2019), exogenous natural rate n_t ($\delta = 1$, $\varsigma =$ EIS standard):

$$x_t = \delta\mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t)$$

- PC implies: $$x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$$

- With ω_t IID, EE implies:

$$r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right], \qquad i_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)-\kappa\varsigma\rho}{\phi-\rho}\zeta_t + \omega_t\right]$$

Illustrating robustness in the 3 equation NK world

- Solution for r_t derived **without** solving EE forward!
- Implies **robustness** to **missing transversality** conditions.
- Also implies degree of **discounting/compounding** (δ) has **no impact** on determinacy.
- Contrasts with Bilbiie (2019): if $\varsigma > 0$, $\beta \leq 1$, with standard rule, $\phi > 1$ is only sufficient for determinacy if $\delta \leq 1$.
- And with Bilbiie (2008): if $\delta = 1$, $\varsigma < 0$, with standard rule, $\phi > 1$ is neither necessary nor sufficient for determinacy.

- Under real rate rule, $\phi > 1$ is **always necessary and sufficient**! (Given $\phi \geq 0$.)
  - Continues to hold with lags in EE and PC. (PC lag may reduce persistence of effect of monetary shocks on x_t.)

Wedges in the Fisher equation

- Many sources of a **Fisher equation wedge**:
  - Liquidity premia on nominal bonds (Fleckenstein, Longstaff & Lustig 2014). Deflation protection on real bonds.
  - Risk premia. Convenience yields. Non-rational expectations. Etc.

- **Fisher** equation (ν_t: stationary wedge): $$i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu_t$$
- + Simple **real rate rule**: $$i_t = r_t + \phi \pi_t$$
- $\Rightarrow \mathbb{E}_t \pi_{t+1} + \nu_t = \phi \pi_t$. Exogenous Fisher shocks **act like monetary shocks**, with opposite sign.

- **Endogenous case**: Assume: Exist $\bar{\mu}_0, \bar{\mu}_1, \bar{\mu}_2, \bar{\gamma}_0, \bar{\gamma}_1, \bar{\gamma}_2 \geq 0$ such that for any stationary solution for π_t:
$$|\mathbb{E}\nu_t| \leq \bar{\mu}_0 + \bar{\mu}_1 |\mathbb{E}\pi_t| + \bar{\mu}_2 \operatorname{Var} \pi_t\,, \qquad \operatorname{Var} \nu_t \leq \bar{\gamma}_0 + \bar{\gamma}_1 |\mathbb{E}\pi_t| + \bar{\gamma}_2 \operatorname{Var} \pi_t$$
- Then under a real rate rule: $|\mathbb{E}\pi_t| = O\left(\frac{1}{\phi}\right)$ and $\operatorname{Var} \pi_t = O\left(\frac{1}{\phi^2}\right)$ as $\phi \to \infty$. Wedges are **not a problem** with large ϕ!

The zero lower bound

Problems caused by the ZLB

- **With the ZLB**, simplest real rate rule means:

$$\max\{0, r_t + \phi\pi_t\} = i_t = r_t + \mathbb{E}_t\pi_{t+1}$$

- So: $\max\{-r_t, \phi\pi_t\} = \mathbb{E}_t\pi_{t+1}$. Real rates **no longer cancel out** completely! **Euler equation** still **matters** for π_t.

- **Extra steady state** with $\pi = -r$ (Benhabib, Schmitt-Grohé & Uribe 2001).

- **Multiplicity** and/or non-existence **even conditional on convergence** to the standard steady state (Holden 2021).
    - Suppose r_t exogenous, $r_t = 0$ for $t \neq 1$, and we assume $\pi_t \to 0$ as $t \to \infty$. Multiple solutions if $r_1 = 0$. No solution if $r_1 < 0$.

- **Sunspot solutions**. With PC & EE as before, $\kappa\varsigma > 0$, $\phi > 1$, $n + \pi^* > 0$, $(1-\beta)(1-\delta) - \kappa\varsigma \leq 0$, for q large enough:
    - Sunspot solution that: Starts at ZLB. Stays there with probability q. Otherwise returns to intended steady state (for ever).

Modified inflation targets

- Non-existence comes from implicitly **targeting infeasibly low inflation**. Easy to fix.
- And add **smoothing** to help rule out sunspot equilibria. So: use the rule:

$$\boxed{i_t = \max\{0, r_t + (i_{t-1} - r_{t-1}) + (\mathbb{E}_t \breve{\pi}_{t+1}^* - \mathbb{E}_{t-1} \breve{\pi}_t^*) + \theta(\pi_t - \breve{\pi}_t^*)\}}, \qquad \breve{\pi}_t^* := \max\{\pi_t^*, \epsilon - r_{t-1}\}$$

  - π_t^* is the original inflation target. $\breve{\pi}_t^*$ is the modified target. $\epsilon > 0$ is a small constant. $\theta \in (0,1)$.

- With modified rule: $\pi_t \equiv \breve{\pi}_t^*$ is an equilibrium. **Locally determinate**.
  - Closed form solution (rare with ZLB!) makes coordination easy.
  - No deflationary bias as $\pi_t > -r_{t-1}$. Instead: small inflationary bias as $\mathbb{E}\pi_t \geq \mathbb{E}\pi_t^*$.

- Under perfect foresight: Solution is **unique** conditional on $\breve{\pi}_t^*$ + terminal condition (no permanent ZLB or explosions).
  - Multiple solutions for $\breve{\pi}_t^*$ do not occur for standard NK models.

Sunspots and history dependent strategies

- **Standard argument** for why history dependent strategies help avoid multiplicity:
  - *History dependence leads to higher inflation after exiting the ZLB, raising inflation expectations even while at the ZLB.*
- This channel **cannot help** to rule out sunspot equilibria with a sufficiently **persistent ZLB state**.
  - *In the fully persistent limit, inflation in the non-ZLB state(s) has no impact on inflation in the ZLB state!*
- **Correct channel**: History dependence helps prevent transitions **into** the ZLB state.
  - It permits a weak contemporaneous response to inflation ($\theta \in (0,1)$), so the monetary rule is flatter than the Fisher equation.
- If $\theta \in (0,1)$ then the smoothed modified inflation target rule **does not have** persistent two-state sunspot equilibria.
  - Under the parameter restrictions with which the simple rule does.

Practical implementation of real rate rules

Set-up

- Markets in **short maturity** TIPS may be illiquid, unavailable or unreliable. So, **use longer maturity** bonds.
  - Long bonds are also less likely to hit the ZLB.
  - But: Long maturities may have substantial risk/term/liquidity premia.
  - Extra complications: Inflation may be observed with a lag. 1 month for US CPI. TIPS may have indexation lag. 3 months in US.

- **Notation**:
  - S: information lag. Market participants and CB use the $t-S$ information set in period t. E.g.: $S=1$.
  - L: indexation lag in return of inflation protected bonds. E.g.: $L=3$. Assume $L \geq S$.
  - $i_{t|t-S}$: nominal yield per period on a T-period nominal bond at t.
  - $r_{t|t-S}$: real yield per period on a T-period inflation protected bond at t.
  - $\nu_{t|t-S}$ endogenous Fisher equation wedge (risk premia etc.) for T-period nominal bonds relative to T-period real bonds at t.
  - $\bar{\nu}_{t|t-S}$ central bank's endogenous period t belief about level of $\nu_{t|t-S}$ (possibly correlated with $\nu_{t|t-S}$). Can also include m. shock.

Fisher equation and rule

- **Fisher equation** (assume $T - L \geq -S$):

$$i_{t|t-S} = r_{t|t-S} + \nu_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k-L}$$

- **Monetary rule**:

$$\begin{aligned} i_{t|t-S} = \max \Bigg\{ 0, r_{t|t-S} + \bar{\nu}_{t|t-S} + \left(i_{t-1|t-1-S} - r_{t-1|t-1-S} - \bar{\nu}_{t-1|t-1-S}\right) + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \breve{\pi}^*_{t+k-L} - \mathbb{E}_{t-1-S} \frac{1}{T} \sum_{k=1}^{T} \breve{\pi}^*_{t-1+k-L} \\ + \theta(\pi_{t-S} - \breve{\pi}^*_{t-S}) + \frac{1}{T} \left[(\pi_{t-S} - \breve{\pi}^*_{t-S}) - (\pi_{t-L} - \breve{\pi}^*_{t-L}) \right] \Bigg\} \end{aligned}$$

$$\breve{\pi}^*_t := \max\{\breve{\pi}^{(j)}_t | j \in \{1, \dots, T\}\}$$

$$\breve{\pi}^{(j)}_t := \pi^*_t + \max \left\{ 0, \frac{T}{j} \left(\epsilon - r_{t+j-1-T+L|t+j-1-T+L-S} - \bar{\nu}_{t+j-1-T+L|t+j-1-T+L-S} \right) - \frac{1}{j} \sum_{k=1}^{T-j} \breve{\pi}^*_{t-k} - \mathbb{E}_t \frac{1}{j} \sum_{k=0}^{j-1} \pi^*_{t+k} \right\}$$

- Not super simple! But with the CB announcing $\breve{\pi}^*_t$ (or better $\breve{\pi}^*_{t+1|t}$), agents **do not need to understand LOM** of $\breve{\pi}^*_t$.

Solution

- **Define** $\Delta_t := (\nu_{t+S|t} - \bar{\nu}_{t+S|t}) - (\nu_{t-1+S|t-1} - \bar{\nu}_{t-1+S|t-1})$ and $e_t := \mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T-L+S} (\pi_{t+k} - \breve{\pi}^*_{t+k})$.

- Then: $\pi_t = \breve{\pi}^*_t + \theta^{-1}(e_t - e_{t-1} + \Delta_t)$ and $\theta T e_t = \mathbb{E}_t \sum_{k=1}^{T-L+S} (e_{t+k} - e_{t+k-1} + \Delta_{t+k})$ (**purely forward looking**!).

- If Δ_t is **exogenous**, then we have **determinacy** as long as $\theta > 0$ (Δ_t can be $I(1)$). If $\bar{\nu}_t \equiv \nu_t$ then $\pi_t \equiv \breve{\pi}^*_t$.

- If Δ_t is **endogenous**, then $e_t = \mathbb{E}_t \sum_{j=1}^{\infty} (1 + \theta T)^{-\left\lceil \frac{j}{T-L+S} \right\rceil} \Delta_{t+j}$, and under weak conditions, for large θ, $\boldsymbol{\pi_t} \approx \boldsymbol{\breve{\pi}^*_t}$.

- If θ is large enough, then endogenous non-stationary wedges, indexation & information lags **do not matter**!

Empirical test

Background

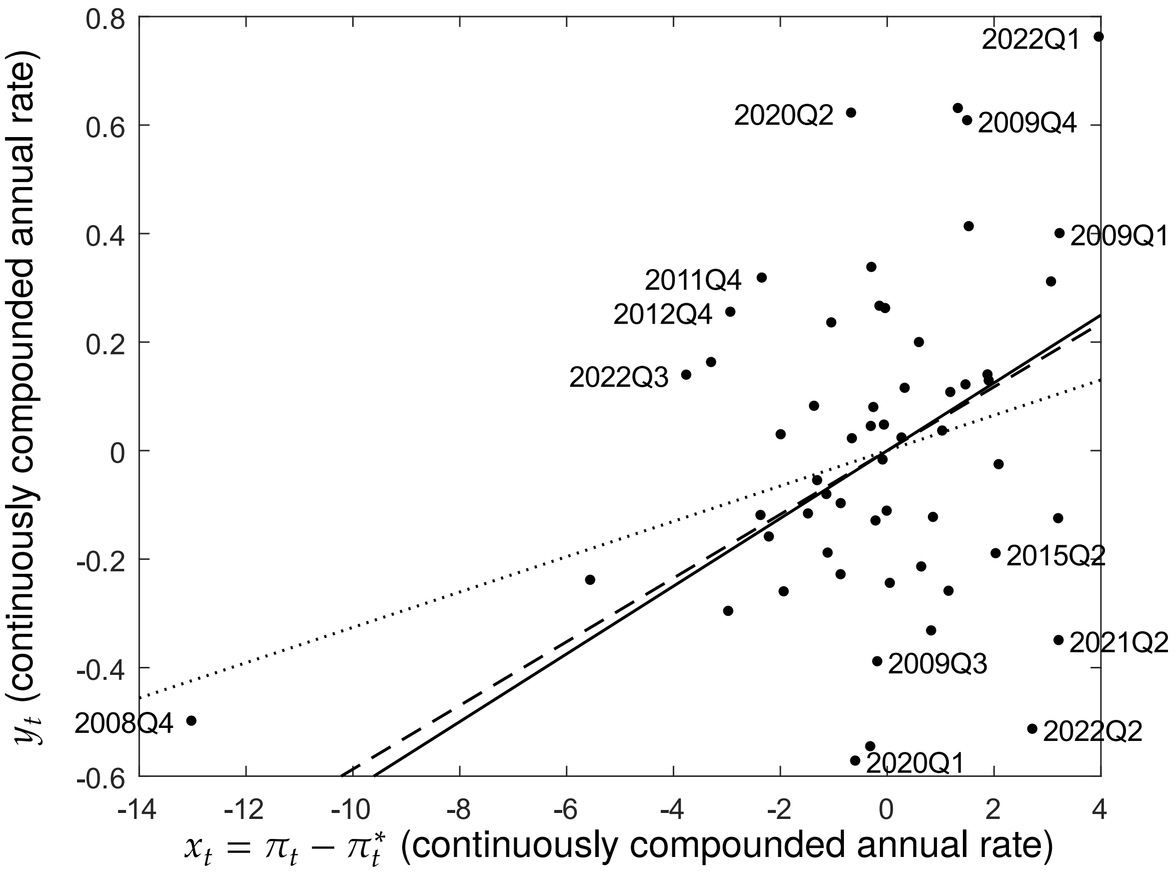
- Would the behaviour of the US Federal Reserve have been drastically **different** if it were following a **real rate rule**?
- To answer: Need to **discipline** π_t^*, else a real rate rule can trivially explain the data.
- I use the US **Summary of Economic Projections** (SEP) to estimate a quarterly time series for π_t^*.
  - SEP projections reflect what Fed board members & regional presidents think inflation ought to be, given conditions.
- Then **estimate** the practical rule above (ignoring ZLB) with quarterly data on US CPI and TIPS breakeven inflation.
  - Data is Q4 2008 to Q4 2022. Take indexation lag $L = 1$ (quarter) and information lag $S = 0$ (quarters).
- Set $x_t := \pi_t - \pi_t^*$ and $\varepsilon_{\bar{\nu},t} := \bar{\nu}_t - \bar{\nu}_{t-1}$ and **estimate**:

$$y_t := \mathbb{E}_t \frac{1}{T} \sum_{k=0}^{T-1} (\pi_{t+k} - \pi_{t+k}^*) - \mathbb{E}_{t-1} \frac{1}{T} \sum_{k=0}^{T-1} (\pi_{t-1+k} - \pi_{t-1+k}^*) + \nu_t - \nu_{t-1} - \frac{1}{T} [(\pi_t - \pi_t^*) - (\pi_{t-1} - \pi_{t-1}^*)] = \theta x_t + \varepsilon_{\bar{\nu},t}$$

Estimates

- Estimating $y_t = \theta x_t + \varepsilon_{\bar{\nu},t}$ by **OLS** gives $\theta \approx 0.033$. HAC p-value of 0.002 for test of $\theta = 0$.
  - Likely biased due to correlation of x_t with $\varepsilon_{\bar{\nu},t}$, and due to measurement error in π_t^*.
- Fix by **instrumenting** x_t with the oil supply news shocks of Känzig (2021).
  - Constructed in a tight window around OPEC announcements so should not be driven by monetary shock.
  - Känzig shows the shocks are correlated with inflation and uncorrelated with standard monetary surprises or the Fed Funds rate.
  - If Fed were following a real rate rule, then above implies $\nu_t - \nu_{t-1}$ and hence y_t must be correlated with the Känzig shocks.
  - Observed correlation: 36%. Significant at 1%.
- **IV** estimates give $\theta \approx 0.062$. HAC p-value of 0.003 for test of $\theta = 0$.
- **Robustness**: Find θ maximising correlation of residuals with Bauer & Swanson (2023) monetary shocks.
  - Available Q4 2008 to Q4 2019. Result: $\theta \approx 0.059$.

Figure 1: Data and linear fits



The dotted line is the OLS estimate. The dashed line is the monetary shock-based estimate.

The solid line is the IV estimate.

Does a real rate rule explain the data?

- What percentage of the variance of various rates can be explained by terms other than direct effect of $\varepsilon_{\bar{\nu},t}$?
  - Set $\mathrm{RSS} := (y_t - \theta x_t)^2$ and then evaluate $1 - \frac{\mathrm{RSS}}{\mathrm{TSS}}$ where TSS is the total sum of squares from a rate of interest.

- IV estimates explain:
  - 49.1% of the variance of changes in five-year breakeven inflation expectations,
  - 53.3% of the variance of changes in five-year treasury yields,
  - 97.4% of the variance of levels of five-year breakeven inflation expectations,
  - $\mathbf{97.5\%}$ **of the variance of levels of five-year treasury yields.**

Challenges to real rate rules

Real rate rules in non-linear models

- Nominal and real bond **pricing**: $$I_t \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} = 1, \qquad R_t \mathbb{E}_t \Xi_{t+1} = 1$$

- **Non-linear real rate rule** with gross $t+1$ inflation target announced at t of $\Pi^*_{t+1|t}$:

$$I_t = R_t \Pi^*_{t+1|t} \left(\frac{\Pi_t}{\Pi^*_{t|t-1}} \right)^{\phi} \qquad \Rightarrow \qquad \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \frac{\Pi^*_{t+1|t}}{\Pi_{t+1}} = \left(\frac{\Pi^*_{t|t-1}}{\Pi_t} \right)^{\phi}$$

- $\Pi_t \equiv \Pi^*_{t|t-1}$ is always one solution of this equation! Always locally unique with $\phi > 1$.

- (Approximately) **Globally unique** under weak assumptions:

  - There exists $\overline{Z} \geq 1$ such that for all sufficiently high ϕ, $1 \leq \frac{\Pi^*_{t|t-1}}{\Pi_t} \leq \overline{Z}^{\frac{1}{\phi-1}} \to 1$ as $\phi \to \infty$. So, for large ϕ, $\Pi_t \approx \Pi^*_{t|t-1}$.

  - Under slightly stronger restrictions on the SDF, $\Pi_t = \Pi^*_{t|t-1}$ is globally unique solution for all sufficiently high ϕ.

How do real rate rules work under bounded rationality?

- With the rule $i_t = r_t + \phi\pi_t + \zeta_t$ (ζ_t $\mathrm{AR}(1)$ with persistence $\rho \in (-1,1)$) the paper looks at the following varieties:
- **Adaptive, naïve and extrapolative** expectations, modelled following Branch & McGough (2009).
  - ϕ large enough is always sufficient for stability. $\phi > 1$ will do in the adaptive and naïve cases. As $\phi \to \infty$, $\operatorname{var} \pi_t \to 0$.
- **Diagnostic** expectations, modelled following Bianchi, Ilut & Saijo (2023).
  - Again ϕ large enough is sufficient for stability. As $\phi \to \infty$, $\operatorname{var}\big((\phi - \rho)\pi_t + \zeta_t\big) \to 0$, meaning rapid convergence to RE.
- **Finite horizon** planning, modelled following Woodford (2019).
  - $\phi > 1$ is stronger than necessary. Again, as $\phi \to \infty$, $\operatorname{var}\big((\phi - \rho)\pi_t + \zeta_t\big) \to 0$.
- **Least squares learning** with perceived law of motion $\pi_t = a_t + b_t\zeta_t + \varepsilon_t$.
  - If $\phi > 1$, then with probability one, a_t converges to 0 and b_t converges to $-\frac{1}{\phi-\rho}$. Global stability independent of initial conditions.
- **Constant gain learning** with same perceived law of motion, but with $\rho = 0$.
  - If $\phi > 1$, and the gain is low enough, then a_t and b_t converge in probability to 0 and $-\frac{1}{\phi-\rho}$. Exact learning despite constant gain.

Fiscal Theory of the Price Level and “over determinacy”

- If price level is determinate independent of MP, then $\phi > 1$ can mean **explosive** π_t.
  - E.g., true if fiscal policy is active (real government primary surpluses do not respond to debt).
  - With one period debt, active fiscal policy, flexible prices, constant real interest rates: $\pi_t - \mathbb{E}_{t-1}\pi_t = -\varepsilon_{s,t}$.
  - Inconsistent with standard real rate rule solution: $\pi_t = -\frac{1}{\phi}\varepsilon_{\zeta,t}$ (IID monetary shock) as long as $\varepsilon_{\zeta,t} \neq \phi\varepsilon_{s,t}$.
  - Only explosive solution remains under real rate rule: $\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1} - \varepsilon_{s,t}$.

- This is a **knife edge result**! With multi-period (geometric coupon) debt: **stable** π_t solution under a real rate rule.
  - Still consistent with transversality even with active fiscal, active monetary!
  - ↑ bubble in debt price balanced by ↓ quantity. Initial debt price jumps. “Fiscal theory of the debt price”.
  - With passive MP this implies multiplicity, so FTPL does not guarantee uniqueness.

- General result: Except in knife edge cases: **Stable solution** under a real rate rule if plausible condition satisfied:
  - Potentially explosive variables (e.g., bond prices) do not feed back to the real economy, and are not too forward looking.

Conclusion

- The Taylor principle does not guarantee determinacy with standard monetary rules.
- Real rate rules **ensure determinacy** no matter the rest of the economy & give CB almost perfect **control** of inflation.
- Under a real rate rule (RRR):
  - Monetary policy works in spite of, not because of, real rate movements.
  - Causation runs exclusively from inflation to the output gap.
  - Household and firm decisions, constraints and inflation expectations are irrelevant for inflation dynamics.
  - Only changes in the rule can amplify the impact of monetary shocks on inflation.

- With a time-varying short-term target, RRRs can implement optimal monetary policy, or match observed dynamics.
- RRRs work in the presence of the ZLB, Fisher equation wedges, bounded rationality, or active fiscal policy.
- They can be practically implemented using pre-existing assets (five-year nominal and real bonds, say).
- Current Fed behaviour is very close to following a RRR with a time-varying short-run inflation target from the SEP.

Extra slides

Setting nominal rates out of equilibrium

- Apparent issue: If for $t > 0$, $i_t = r_t + \phi\pi_t$, then $\pi_t = 0$ for $t > 0$, so by Fisher $i_0 = r_0$. CB cannot set $i_0 \neq r_0$!

- Resolution: $\pi_t = 0$ iff $\pi_s = 0$ for all $s \in \{0,1,\dots,t-1\}$, else $\pi_t = \phi\pi_{t-1}$. If $\pi_0 \neq 0$, Fisher states $i_0 - r_0 = \phi\pi_0$.

- May reappear under bounded rationality. Suppose agents have learned $\pi_t = 0$, then $i_t = r_t$ even out of equilibrium.

- One fix: Modified real rate rule:
$$i_t = r_t + \phi\pi_t - \varrho(\phi - \varrho)\pi_{t-1}$$

- With $\varrho \in (-1,1)$ and $\phi > 1 + \varrho$. Determinate solution: $\pi_t = \varrho\pi_{t-1}$. Agents learn $\pi_t \approx \varrho\pi_{t-1}$.

- Alternative fix: Price level real rate rule rules.

Explaining observed inflation dynamics

- Large literature finds no role for the Phillips curve in forecasting inflation.
  - Post-1984: IMA$(1,1)$ model beats Phillips curve based forecasts (conditionally & unconditionally) (Dotsey, Fujita & Stark 2018).
  - +: Atkeson & Ohanian (2001), Ang, Bekaert & Wei (2007), Stock & Watson (2009). One explanation: McLeay & Tenreyro (2019).

- Also: Miranda-Agrippino & Ricco (2021):
  - Contractionary monetary policy shock causes immediate fall in the price level.
  - Delayed impact on unemployment.

- All supportive of models in which causation only runs in one direction: *from inflation to the output gap*.
  - As under a real rate rule! [Not saying the CB follows a real rate rule. Just that outcomes may not be so different.]

Responding to other endogenous variables

- In the model:

$$i_t = r_t + \phi_\pi \pi_t + \phi_x x_t + \zeta_t$$

$$\pi_t = \tilde{\beta}(1 - \varrho_\pi)\mathbb{E}_t \pi_{t+1} + \tilde{\beta}\varrho_\pi \pi_{t-1} + \kappa x_t + \kappa \omega_t, \qquad x_t = \tilde{\delta}(1 - \varrho_x)\mathbb{E}_t x_{t+1} + \tilde{\delta}\varrho_x x_{t-1} - \varsigma(r_t - n_t)$$

  - If $\kappa > 0$, $\phi_x \geq 0$ and $\tilde{\beta} \in [0,1]$, then $\phi_\pi > 1$ is sufficient for determinacy!
  - Real rate rule still helps robustness as it disconnects EE from prices.

- In any model: $\phi_\pi > 1$ sufficient for determinacy if responses to other endogenous variables are small enough.
  - Implies robustness to non-unit responses to real rates. Other variables (e.g., output growth) may proxy real rates.

- For greater robustness: Replace other endogenous vars in rule with structural shocks.
  - If structural shocks not observed, can infer from structural equations.
  - If equation parameters not known, can learn in real time, still with determinacy!

Equilibrium selection with perpetuities: Idea

- Cochrane (2011) argues no reason to rule out explosive NK equilibria.

- Suppose geometric coupon bonds (GCBs) are traded in the economy. (Later specialise to perpetuities.)
    - Could be approximated by portfolio of different maturity debt. Long-term government contracts (defence,…) also perpetuity like.

- 1 unit of period t GCB bought at t returns $\$1$ at $t+1$, along with $\omega \in (0,1]$ units of period $t+1$ GCB.
    - Suppose stock: $B_t \geq \underline{B}\omega^t$. Then transversality implies GCB price: $Q_t = \mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=0}^{s} \frac{1}{I_{t+k}} \right] \omega^s$.
    - If $I_{t+k} = 1$ for high k, then $Q_{t+k} = \frac{1}{1-\omega}$ for high k. Transversality then requires $0 = \lim_{s \to \infty} \frac{\omega^s}{1-\omega}$, i.e., $|\omega| < 1$. Violated with $\omega = 1$!

- Permanent ZLB $\Rightarrow$ Infinite perpetuity price $\Rightarrow$ Infinite nominal wealth $\Rightarrow$ Infinite inflation $\Rightarrow$ Physically impossible.

Equilibrium selection with perpetuities: Use

- With sticky prices, explosions are generally ruled out.
  - Standard sticky prices specifications imply Π_t is bounded above. + Real costs of inflation explode as inflation explodes.
  - Prices may become more flexible as $\Pi_t \uparrow$, but seems plausible there is a limit on how often prices can be changed.
- So, under sticky prices the modified inflation target rule produces uniqueness if households hold perpetuities.
- Non-linear version (with a target known one period in advance, $\mathcal{E} := \exp \epsilon > 1$):

$$I_t = \max\left\{1, R_t \breve{\Pi}^*_{t+1|t} \left(\frac{\Pi_t}{\breve{\Pi}^*_{t|t-1}}\right)^{\phi}\right\}, \qquad \breve{\Pi}^*_{t+1|t} := \max\left\{\frac{\mathcal{E}}{R_t}, \Pi^*_{t+1|t}\right\}$$

- Without sticky prices, have to send deviations to the ZLB. E.g., with following ($\bar{I} > 1$, $\phi > 1$ and $\mathcal{E} \in \left(1, \sqrt{\bar{I}}\right)$):

$$I_t = \begin{cases} \max\left\{1, R_t \breve{\Pi}^*_{t+1|t} \left(\frac{\Pi_t}{\breve{\Pi}^*_{t|t-1}}\right)^{\phi}\right\}, & \text{if } I_{t-1} \in (1, \bar{I}) \\ 1, & \text{otherwise} \end{cases}, \qquad \breve{\Pi}^*_t := \max\left\{\frac{\mathcal{E}}{R_t}, \min\left\{\frac{\bar{I}}{\mathcal{E} R_t}, \Pi^*_{t+1|t}\right\}\right\}$$

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