Robust Real Rate Rules

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Paper and slides available at https://www.tholden.org/.

Motivation: Fragility of the Taylor principle

- The Taylor principle requires the response of nominal rates to inflation to be greater than one.

  - Sufficient for determinacy in simple models. (Guarantees no belief-driven fluctuations / sunspots.)

- Insufficient if there is e.g.:

  - A fraction of hand-to-mouth households (Gali, Lopez-Salido & Valles 2004).

  - Firm-specific capital (Sveen & Weinke 2005).

  - High government spending (Natvik 2009).

  - A positive inflation target (Ascari & Ropele 2009),

  - …particularly with trend growth + sticky wages (Khan, Phaneuf & Victor 2019).

- Inverts if there are e.g.:

  - Enough hand-to-mouth households (Bilbiie 2008).

  - Certain financial frictions (Manea 2019).

  - Non-rational expectations (Branch & McGough 2010; 2018).

  - Active fiscal policy (Leeper & Leith 2016; Cochrane 2022).

This paper

- Monetary rules with a unit response to real rates guarantee determinate inflation under the weakest possible assumptions on the rest of the economy.

  o Robust to household heterogeneity, non-rational households/firms, active fiscal policy, missing transversality conditions, existence/slope of the Phillips curve, etc.

- With a time-varying inflation target: enable the determinate robust implementation of an arbitrary path for inflation.

  o So can match observed inflation dynamics, or any model's optimal policy.

- Easy to implement in practice. Use TIPS to infer real rates. Works with bonds of any maturity.

- Reveal: Fisher equation is key to monetary transmission.

A first example

- Nominal bond: $1 bond purchased at t returns $\$(1 + i_t)$ at $t + 1$.

- Real bond (e.g., TIPS): $1 bond purchased at t returns $\$(1 + r_t + \pi_{t+1})$ at $t + 1$.

- Arbitrage $\Rightarrow$ the Fisher equation:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}$$

  ○ Abstracting from inflation risk / term / liquidity premia for now.

- Central bank uses the "real rate rule":

$$i_t = r_t + \phi \pi_t$$

- With $\phi > 1$. Then:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t$$

- Unique non-explosive solution, $\pi_t = 0$. Determinate inflation!

Why is this robust? No need for Euler equation!

- Does not require an aggregate Euler equation to hold.

  ○ Robust to household heterogeneity and hand-to-mouth agents.

  ○ Robust to non-rational household expectations.

- For the Fisher equation to hold just need either:

  ○ Two deep pocketed, fully informed, rational agents in the economy, OR,

  ○ …a large market of rational agents with dispersed information (Hellwig 1980; Lou et al. 2019).

- Much more plausible financial market participants have rational expectations than households.

  ○ Can even partially relax the rationality requirement for financial market participants.

  ○ E.g.: Global convergence under learning from arbitrary initial beliefs.

Why is this robust? No need for Phillips curve!

- Does not require an aggregate Phillips curve to hold.

  - Robust to slope of the Phillips curve (if it exists).

  - Robust to forward/backward looking degree of Phillips curve equation.

  - Robust to non-rational firm expectations.

- Under this monetary rule, the Phillips curve is irrelevant for inflation dynamics.

  - The Phillips curve (if it exists) determines the output gap, given inflation.

  - If CB is unconcerned with output and unemployment, they do not need to care about the Phillips curve or its slope.

- Only require that at least some prices are adjusted each period using current information.

Implications for monetary economics

- Which model features lead to amplification or dampening of monetary shocks?

- Under a real rate rule: no change in the model can amplify/dampen monetary shocks other than changing rule.

  - Prior amplification/dampening results were sensitive to the monetary rule. May reverse with a response to r_t of > 1.

- Which shocks drive inflation?

- Under a real rate rule: only monetary policy shocks or shocks to the Fisher equation.

- CB has ultimate responsibility for inflation.

Real rate rules elsewhere in the literature

- Papers with rules responding to real rates, for analytic convenience e.g.:

  - Adão, Correia & Teles (2011), Holden (2019), Lubik, Matthes & Mertens (2019).

- Closest prior work: Cochrane (2017; 2018; 2022) on spread targeting.

  - Cochrane briefly considers a rule of the form $i_t = r_t + \phi \pi_t$ before setting $\phi = 0$.
  - Determinacy in Cochrane's world comes from the Fiscal Theory of the Price Level.

- Other related work:

  - Hetzel (1990) proposes using nominal bond, real bond spread to guide policy.
  - Dowd (1994) proposes targeting the price of price level futures contracts.
  - Hall & Reis (2016) propose making interest on reserves a function of price level deviations from target, e.g. nominal return from $1 of $\$(1 + r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1 + i_t)\frac{p_t}{p_t^*}$.

- Large literature on rules tracking efficient ("natural") real interest rate.

  - E.g., Cúrdia et al. (2015). Very different idea.

Generalizations and generality

Monetary policy shocks

- Suppose the CB uses the rule:

$$i_t = r_t + \phi \pi_t + \zeta_t$$

- with $\phi > 1$, and ζ_t drawn from an AR(1) process with persistence ρ.

- Then from the Fisher equation:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t + \zeta_t \qquad \Rightarrow \qquad \pi_t = -\frac{1}{\phi - \rho} \zeta_t.$$

- Contractionary (positive) monetary policy shocks reduce inflation.

- If the CB is more aggressive (ϕ is larger) inflation is less volatile.

- Can understand inflation dynamics without knowing the rest of the economy.

Explaining observed inflation dynamics

- Large literature finds no role for the Phillips curve in forecasting inflation.

  ○ Post-1984: IMA(1,1) model beats Phillips curve based forecasts (conditionally & unconditionally) (Dotsey, Fujita & Stark 2018).

  ○ +: Atkeson & Ohanian (2001), Ang, Bekaert & Wei (2007), Stock & Watson (2009). One explanation: McLeay & Tenreyro (2019).

- Also: Miranda-Agrippino & Ricco (2021):

  ○ Contractionary monetary policy shock causes immediate fall in the price level.

  ○ Delayed impact on unemployment.

- All supportive of models in which causation in PC only runs in one direction: *from inflation to the output gap*.

  ○ As under a real rate rule!

Output dynamics in 3 equation NK model

- As before: CB sets $i_t = r_t + \phi\pi_t + \zeta_t$, so $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$.

- Rest of model 1: Phillips curve (PC), with mark-up shock ω_t:
$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + \kappa\omega_t$$

- Rest of model 2: Discounted/compounded Euler equation (EE) (Bilbiie 2019), n_t exogenous natural rate ($\delta = 1$, $\varsigma =$ EIS recovers standard Euler equation):
$$x_t = \delta\mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t)$$

- PC implies: $x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$. x_t does not help forecast inflation as $\mathbb{E}_t\pi_{t+1} = \rho\pi_t$.

  o Once you know π_t, there is no extra useful information in x_t.

Real rate dynamics in 3 equation NK model

- In the model of the last slide, if ω_t is IID, EE implies:

$$r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right]$$

- Derived without solving EE forward!

  o Implies robustness to missing transversality conditions.

  o Also implies degree of discounting/compounding (δ) has no impact on determinacy.

  o Contrasts with Bilbiie (2019): if $\varsigma > 0$, $\beta \leq 1$, with a standard Taylor rule, $\phi > 1$ is only sufficient for determinacy if $\delta \leq 1$.

  o Contrasts with Bilbiie (2008): if $\delta = 1$, $\varsigma < 0$, with a standard Taylor rule, $\phi > 1$ is neither necessary nor sufficient for determinacy.

- Under real rate rule, $\phi > 1$ is always necessary and sufficient! (Given $\phi \geq 0$.)

  o Robust to lags in EE and PC. (PC lag may reduce persistence of effect of monetary shocks on x_t!)

Responding to other endogenous variables

- In the model:

$$i_t = r_t + \phi_\pi \pi_t + \phi_x x_t + \zeta_t$$

$$\pi_t = \tilde{\beta}(1 - \varrho_\pi)\mathbb{E}_t \pi_{t+1} + \tilde{\beta}\varrho_\pi \pi_{t-1} + \kappa x_t + \kappa \omega_t, \qquad x_t = \tilde{\delta}(1 - \varrho_x)\mathbb{E}_t x_{t+1} + \tilde{\delta}\varrho_x x_{t-1} - \varsigma(r_t - n_t)$$

  - If $\kappa > 0$, $\phi_x \geq 0$ and $\tilde{\beta} \in [0,1]$, then $\phi_\pi > 1$ is sufficient for determinacy!
  - Real rate rule still helps robustness as it disconnects EE from prices.

- In any model: $\phi_\pi > 1$ sufficient for determinacy if responses to other endogenous variables are small enough.

  - Implies robustness to non-unit responses to real rates. Other variables (e.g., output growth) may proxy real rates.

- For greater robustness: Replace other endogenous vars in rule with structural shocks.

  - If structural shocks not observed, can infer from structural equations.
  - If equation parameters not known, can learn in real time, still with determinacy!

Implementing arbitrary inflation dynamics

- Suppose CB uses the rule:

$$i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t)$$

- π^*_t: an arbitrary stochastic process, possibly a function of economy's other endogenous variables and shocks.

- From the Fisher equation: $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. With $\phi > 1$, unique, determinate solution: $\pi_t = \pi^*_t$.

- The CB can hit an arbitrary path for inflation!

  o E.g., optimal policy. So real rate rules can attain highest possible welfare.

  o And real rate rules can explain any observed inflation dynamics.

- Related literature on implementation of optimal policy:

  o Svensson & Woodford (2003), Dotsey & Hornstein (2006), Evans & Honkapohja (2006), Evans & McGough (2010).

Interest rate smoothing

- A fully smoothed real rate rule with time-varying target:

$$i_t - r_t = i_{t-1} - r_{t-1} + \mathbb{E}_t \pi_{t+1}^* - \mathbb{E}_{t-1} \pi_t^* + \theta(\pi_t - \pi_t^*)$$

- From the Fisher equation:

$$\theta(\pi_t - \pi_t^*) = \mathbb{E}_t(\pi_{t+1} - \pi_{t+1}^*) - \mathbb{E}_{t-1}(\pi_t - \pi_t^*).$$

- Define: $p_t := \sum_{s=1}^{t}(\pi_t - \pi_t^*)$ and $\hat{p}_t := p_t + \frac{1}{\theta}\mathbb{E}_0 p_1$. Then summing over time gives:

$$(1 + \theta)\hat{p}_t = \mathbb{E}_t \hat{p}_{t+1}$$

- With $\theta > 0$: Unique equilibrium $\hat{p}_t = 0$, so $\pi_t = \pi_t^*$.

- Produces the same π_t as unsmoothed rule.

- Difference: Only need $\theta > 0$, not $\phi > 1$.

  o Likely much easier for CB to convince agents of the former than of the latter.

Challenges to real rate rules

- Nominal and real bond pricing:

$$I_t \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} = 1, \qquad R_t \mathbb{E}_t \Xi_{t+1} = 1$$

- Non-linear real rate rule:

$$I_t = R_t \Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^\phi$$

- So:

$$\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \frac{\Pi^*}{\Pi_{t+1}} = \left(\frac{\Pi^*}{\Pi_t}\right)^\phi$$

- $\Pi_t = \Pi^*$ is always one solution of this equation! Always locally unique.

- (Approximately) Globally unique under weak assumptions:

  ○ There exists $\overline{Z} \geq 1$ such that for all sufficiently high ϕ, $1 \leq \frac{\Pi^*}{\Pi_t} \leq \overline{Z}^{\frac{1}{\phi-1}} \to 1$ as $\phi \to \infty$. So, for large ϕ, $\Pi_t \approx \Pi^*$.

  ○ Under slightly stronger restrictions on the SDF, $\Pi_t = \Pi^*$ is globally unique solution for all sufficiently high ϕ.

Wedges in the Fisher equation

- Many potential sources of a Fisher equation wedge:

  - Liquidity premia on nominal bonds (Fleckenstein, Longstaff & Lustig 2014). Deflation protection on real bonds.
  - Risk premia (already considered). Non-rational expectations. Etc.

- Generalized Fisher equation (ν_t: stationary endogenous wedge):

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu_t$$

- Assume: Exist $\bar{\mu}_0, \bar{\mu}_1, \bar{\mu}_2, \bar{\gamma}_0, \bar{\gamma}_1, \bar{\gamma}_2 \geq 0$ such that for any stationary solution for π_t:

$$|\mathbb{E}\nu_t| \leq \bar{\mu}_0 + \bar{\mu}_1|\mathbb{E}\pi_t| + \bar{\mu}_2 \operatorname{Var}\pi_t, \qquad \operatorname{Var}\nu_t \leq \bar{\gamma}_0 + \bar{\gamma}_1|\mathbb{E}\pi_t| + \bar{\gamma}_2 \operatorname{Var}\pi_t$$

- Then under a real rate rule: $|\mathbb{E}\pi_t| = O\left(\frac{1}{\phi}\right)$ and $\operatorname{Var}\pi_t = O\left(\frac{1}{\phi^2}\right)$ as $\phi \to \infty$. Wedges are not a problem with large ϕ!

- If liquidity premia are the main distortion, may be better for CB to intervene in inflation swap market.

Fiscal Theory of the Price Level and "over determinacy"

- If price level is determinate independent of MP, then $\phi > 1$ can mean explosive π_t.

  - E.g., true if fiscal policy is active (real surpluses do not respond to debt).
  - With one period debt, active fiscal policy & flexible prices: $\pi_t - \mathbb{E}_{t-1}\pi_t = -\varepsilon_{s,t}$.
  - Inconsistent with standard real rate rule solution: $\pi_t = -\frac{1}{\phi}\varepsilon_{\zeta,t}$ (IID monetary shock) as long as $\varepsilon_{\zeta,t} \neq \phi\varepsilon_{s,t}$.
  - Only explosive solution remains under real rate rule: $\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1} - \varepsilon_{s,t}$.

- This is a knife edge result! With multi-period (geometric coupon) debt: stable π_t solution under a real rate rule.

  - Still consistent with transversality even with active fiscal, active monetary!
  - ↑ bubble in debt price balanced by ↓ quantity. Initial debt price jumps. "Fiscal theory of the debt price".
  - With passive MP this implies multiplicity, so FTPL does not guarantee uniqueness.

- General result: Always a stable solution under a real rate rule if plausible condition satisfied:

  - Potentially explosive variables (e.g., bond prices) must not feed back to the real economy, and are not too forward looking.

Setting nominal rates out of equilibrium

- Apparent issue: If for $t > 0$, $i_t = r_t + \phi\pi_t$, then $\pi_t = 0$ for $t > 0$, so by Fisher $i_0 = r_0$. CB cannot set $i_0 \neq r_0$!

- Resolution: $\pi_t = 0$ iff $\pi_s = 0$ for all $s \in \{0, 1, \ldots, t-1\}$, else $\pi_t = \phi\pi_{t-1}$. If $\pi_0 \neq 0$, Fisher states $i_0 - r_0 = \phi\pi_0$.

- May reappear under bounded rationality. Suppose agents have learned $\pi_t = 0$, then $i_t = r_t$ even out of equilibrium.

- One fix: Modified real rate rule:

$$i_t = r_t + \phi\pi_t + \psi\pi_{t-1}$$

- ψ small. Determinate if $\phi > |1 - \psi|$. Solution: $\pi_t = \varrho\pi_{t-1}$, where $\varrho = \frac{\phi - \sqrt{\phi^2 + 4\psi}}{2} \in (-1, 1)$. Agents learn $\pi_t \approx \varrho\pi_{t-1}$.

- Alternative fix: Price level real rate rule rules.

Practical implementation of real rate rules

Practical implementation: Set-up

- Markets in short maturity TIPS may be illiquid, unavailable or unreliable. So, use longer maturity bonds.

  - But: Long maturities may have substantial risk/term/liquidity premia.

  - Extra complications: Inflation may be observed with a lag. 1 month for US CPI. TIPS may have indexation lag. 3 months in US.

- Notation:

  - S: information lag. Market participants use the $t - S$ information set in period t.

  - L: indexation lag in return of inflation protected bonds.

  - $i_{t|t-S}$: nominal yield per period on a T-period nominal bond at t.

  - $r_{t|t-S}$: real yield per period on a T-period inflation protected bond at t.

  - $v_{t|t-S}$ Fisher equation wedge (risk premia etc.) for T-period nominal bonds relative to T-period real bonds at t.

  - $\bar{v}_{t|t-S}$ central bank's period t belief about level of $v_{t|t-S}$ (possibly correlated with $v_{t|t-S}$).

Practical implementation: Fisher equation and rule

- Fisher equation (need $T - L \geq 0$):

$$i_{t|t-S} - r_{t|t-S} = \nu_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k-L}$$

- Monetary rule:

$$i_{t|t-S} - r_{t|t-S} = \bar{\nu}_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi^*_{t+k-L} + \phi(\pi_{t-S} - \pi^*_{t-S})$$

- Combining implies:

$$\mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T} (\pi_{t+k+S-L} - \pi^*_{t+k+S-L}) + (\nu_{t+S|t} - \bar{\nu}_{t+S|t}) = \phi(\pi_t - \pi^*_t)$$

- Solution (A_j unique for $\phi > 1$):

$$\pi_t = \pi^*_t + \mathbb{E}_t \sum_{j=-\infty}^{\infty} A_j (\nu_{t+j+S|t+j} - \bar{\nu}_{t+j+S|t+j})$$

Practical implementation: Discussion

- CB's inflation error $\pi_t - \pi_t^*$ is stationary as long as $\nu_{t+S|t} - \bar{\nu}_{t+S|t}$ is stationary.

- If ϕ is large enough, $\pi_t \approx \pi_t^*$ (under the same assumptions as in previous discussion of Fisher wedges).

- If aggressive enough, endogenous wedges, indexation & information lags do not matter!

- Note: CB's trading desk should hold $i_t - r_t$ constant between meetings.

    o This requires i_t to move between meetings, in response to observed changes in r_t.

    o No reason this should be significantly harder than holding i_t fixed.

- CB could also offer to exchange \$1 face value of real debt for \$$(1 + i_t - r_t)$ face value of nominal, as proposed by Cochrane (2017; 2018).

- Or to buy/sell portfolios with $+$\$1 nominal debt, $-$\$1 real debt for \$$(i_t - r_t)$.

- Or trade inflation swaps (which pay $\Pi_{t+1} - K_t$ at $t + 1$, with no payments at t).

The zero lower bound

Problems caused by the ZLB

- With the ZLB, simplest real rate rule means:

$$\max\{0, r_t + \phi\pi_t\} = i_t = r_t + \mathbb{E}_t\pi_{t+1}$$

- So:

$$\mathbb{E}_t\pi_{t+1} = \max\{-r_t, \phi\pi_t\}$$

- Real rates no longer cancel out completely! Euler equation still matters for π_t.

- Extra steady state with $\pi = -r$ (Benhabib, Schmitt-Grohé & Uribe 2001).

- Still multiplicity and/or non-existence conditional on convergence to the standard steady state (Holden 2021).

  - E.g., suppose r_t exogenous, $r_t = 0$ for $t \neq 1$, and we assume that $\pi_t \to 0$ as $t \to \infty$.

  - Multiple solutions if $r_1 = 0$. No solution if $r_1 < 0$. General problem in NK models.

Modified inflation targets

- Non-existence comes from implicitly targeting an infeasibly low level of inflation.

- Easy to fix. Use the rule:

$$i_t = \max\{0, r_t + \mathbb{E}_t \check{\pi}^*_{t+1} + \phi(\pi_t - \check{\pi}^*_t)\}, \qquad \check{\pi}^*_t := \max\{-r_{t-1} + \epsilon, \pi^*_t\}$$

  o π^*_t is the original inflation target. $\check{\pi}^*_t$ is the modified target. $\epsilon > 0$ is a small constant.

- With modified rule: $\pi_t = \check{\pi}^*_t$ for all t is an equilibrium. Locally determinate.

  o Closed form solution (rare with ZLB!) makes coordination easy.
  o No deflationary bias as $\pi_t > -r_{t-1}$. Instead: small inflationary bias as $\mathbb{E}\pi_t \geq \mathbb{E}\pi^*_t$.

- Solution is unique conditional on $\check{\pi}^*_t$ and on a terminal condition ruling out explosions or permanent ZLB traps.

  o Multiple solutions for $\check{\pi}^*_t$ do not occur for standard NK models.

Equilibrium selection with perpetuities: Idea

- Cochrane (2011) argues no reason to rule out explosive NK equilibria.

- Suppose geometric coupon bonds (GCBs) are traded in the economy. (Later specialise to perpetuities.)

  - Could be approximated by portfolio of different maturity debt. Long-term government contracts (defence,…) also perpetuity like.

- 1 unit of period t GCB bought at t returns \$1 at $t+1$, along with $\omega \in (0,1]$ units of period $t+1$ GCB.

  - Suppose stock: $B_t \geq \underline{B}\omega^t$. Then transversality implies GCB price: $Q_t = \mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=0}^{s} \frac{1}{I_{t+k}}\right]\omega^s$.

  - If $I_{t+k} = 1$ for high k, then $Q_{t+k} = \frac{1}{1-\omega}$ for high k. Transversality then requires $0 = \lim_{s\to\infty} \frac{\omega^s}{1-\omega}$, i.e., $|\omega| < 1$. Violated with $\omega = 1$!

- Permanent ZLB $\Rightarrow$ Infinite perpetuity price $\Rightarrow$ Infinite nominal wealth $\Rightarrow$ Infinite inflation $\Rightarrow$ Physically impossible.

Equilibrium selection with perpetuities: Use

- With sticky prices, explosions are generally ruled out.

  - Standard sticky prices specifications imply Π_t is bounded above. + Real costs of inflation explode as inflation explodes.
  - Prices may become more flexible as $\Pi_t \uparrow$, but seems plausible there is a limit on how often prices can be changed.

- So, under sticky prices the previous rule produces uniqueness if households hold perpetuities.

- Non-linear version (with a target known one period in advance, $\mathcal{E} := \exp \epsilon > 1$):

$$I_t = \max\left\{1, R_t \breve{\Pi}_t^* \left(\frac{\Pi_t}{\breve{\Pi}_{t-1}^*}\right)^\phi\right\}, \qquad \breve{\Pi}_t^* := \max\left\{\frac{\mathcal{E}}{R_t}, \Pi_t^*\right\}$$

- Without sticky prices, have to send deviations to the ZLB. E.g., with following ($\bar{I} > 1$, $\phi > 1$ and $\mathcal{E} \in \left(1, \sqrt{\bar{I}}\right)$):

$$I_t = \begin{cases} \max\left\{1, R_t \breve{\Pi}_t^* \left(\frac{\Pi_t}{\breve{\Pi}_{t-1}^*}\right)^\phi\right\}, & \text{if } I_{t-1} \in (1, \bar{I}) \\ 1, & \text{otherwise} \end{cases}, \qquad \breve{\Pi}_t^* := \max\left\{\frac{\mathcal{E}}{R_t}, \min\left\{\frac{\bar{I}}{\mathcal{E}R_t}, \Pi_t^*\right\}\right\}$$

Conclusion

- The Taylor principle does not guarantee determinacy with standard monetary rules.

- Real rate rules ensure determinacy no matter the rest of the economy & give CB almost perfect control of inflation.

- They can be easily implemented using pre-existing assets (nominal and real bonds, or inflation swaps).

- Under a real rate rule:
  - Monetary policy works in spite of, not because of, real rate movements.
  - Causation in the Phillips curve runs exclusively from inflation to the output gap.
  - Household and firm decisions, constraints and inflation expectations are irrelevant for inflation dynamics.
  - Only changes in the rule can amplify the impact of shocks on inflation.

- With a time-varying target, real rate rules can implement optimal monetary policy, or match observed dynamics.

- Real rate rules continue to work in the presence of the ZLB, wedges in the Fisher equation, or active fiscal policy.

Extra slides

Welfare

- Recap: Real rate rules can determinately implement an arbitrary path for inflation, including optimal policy. Automatic that they can attain high welfare!

- Makes sense to limit to "simple" real rate rules though.

  - "Simple" here means simple dynamics of targeted inflation.

  - Claim: Looking for optimal simple inflation dynamics is a useful approach to policy.

- Two exercises follow:

  - $MA(0)$, $MA(1)$ and $ARMA(1,1)$ inflation policy in a simple NK model. Latter is sufficient to attain unconditional optimal.

  - Examination of optimal policy in the Justiniano, Primiceri & Tambalotti (2013) model. Multiple shock $ARMA(1,2)$ inflation policy is very close to fully optimal.

A simple NK model for policy analysis

- Look at welfare in a simple model with the Phillips curve (ω_t IID):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t$$

- And the policy objective to minimise:

$$(1-\beta)\mathbb{E} \sum_{k=0}^{\infty} \beta^k \left(\pi_{t+k}^2 + \lambda x_{t+k}^2 \right) = \mathbb{E} \left(\pi_t^2 + \lambda x_t^2 \right)$$

  o Equality under the constraint that policy must be time-invariant.

- Optimal policy must have an MA(∞) representation ($\theta_1, \theta_2, ...$ TBD):

$$\pi_t = \kappa \sum_{k=0}^{\infty} \theta_k \omega_{t-k}$$

  o Implies objective is:

$$\mathbb{E}\left(\pi_t^2 + \lambda x_t^2 \right) = \mathbb{E}\left[\omega_t^2 \right] \sum_{k=0}^{\infty} \left[\kappa^2 \theta_k^2 + \lambda (\theta_k - \beta \theta_{k+1} - \mathbb{1}[k=0])^2 \right]$$

Welfare of real rate rules in a simple model

- Optimising subject to $\pi_t = \pi_t^*$ being $MA(0)$ gives the discretionary optimum with $\pi_t = \kappa \frac{\lambda}{\lambda + \kappa^2} \omega_t$ and $\pi_t + \frac{\lambda}{\kappa} x_t = 0$.

- Optimising subject to $\pi_t = \pi_t^*$ being an $MA(1)$ gives a solution with $\pi_t = \kappa \theta_0 \omega_t + \kappa \theta_1 \omega_{t-1}$ where $\theta_0 \geq 0$ and $\theta_1 \leq 0$.

  - Thus ω_t increases π_t while reducing $\mathbb{E}_t \pi_{t+1}$, lessening output gap movements.

- Optimising subject to $\pi_t = \pi_t^*$ being an $ARMA(1,1)$ give the unconditionally optimal solution from the prior literature (e.g. Damjanovic, Damjanovic & Nolan (2008)) with $\pi_t + \frac{\lambda}{\kappa}(x_t - \beta x_{t-1}) = 0$.

  - Optimal MA coefficient equals $-\beta \approx -0.99$. Close to the $IMA(1,1)$ processes estimated by Dotsey, Fujita & Stark (2018) for the post-1984 period.
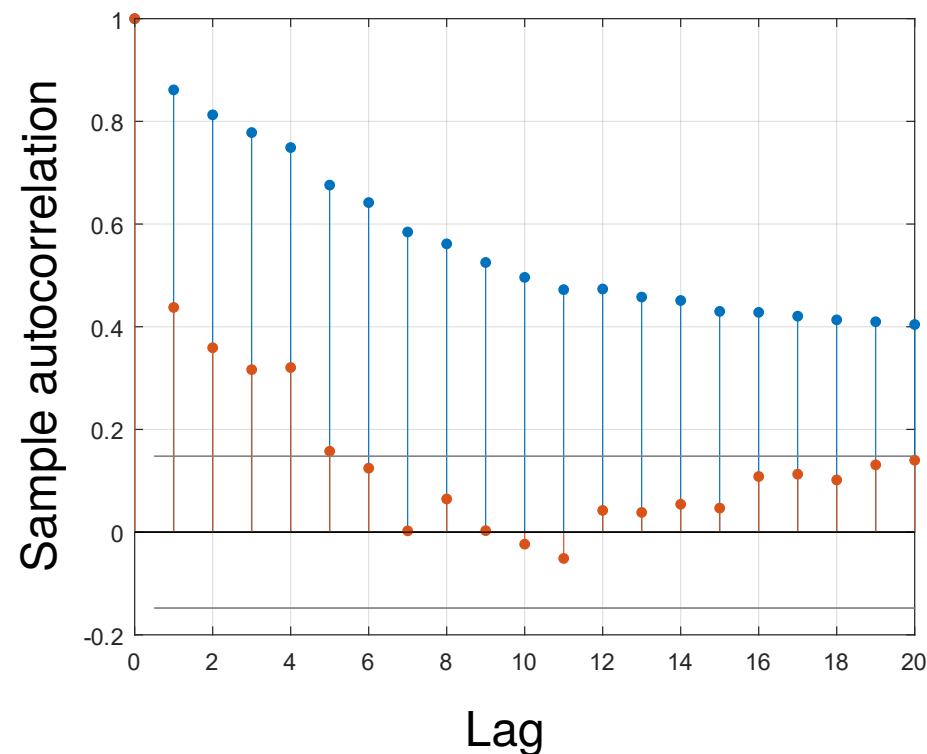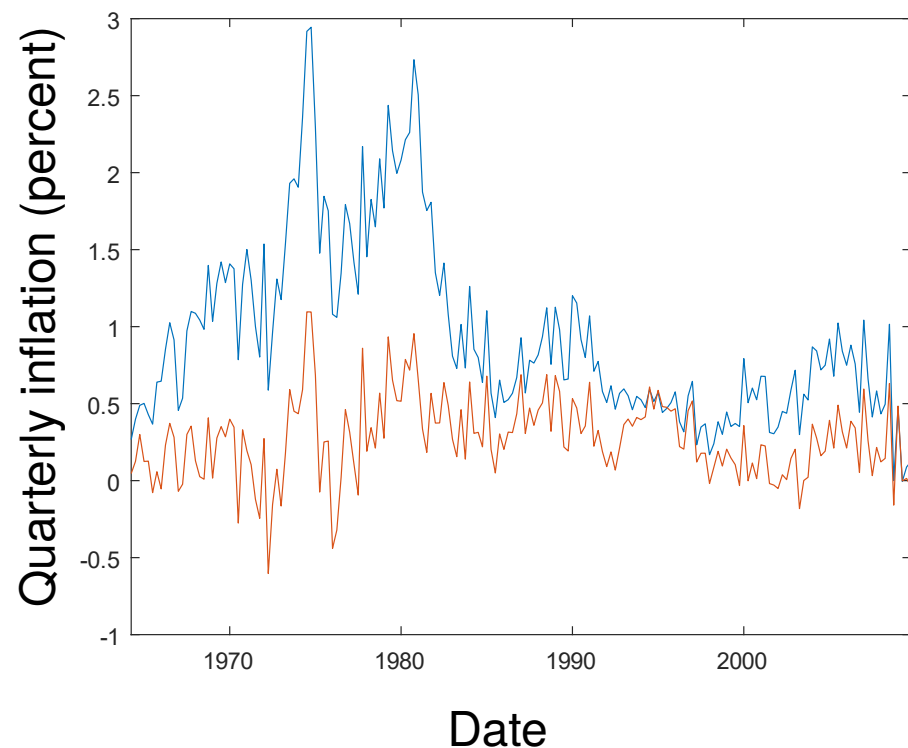
Policy frontiers (varying λ)
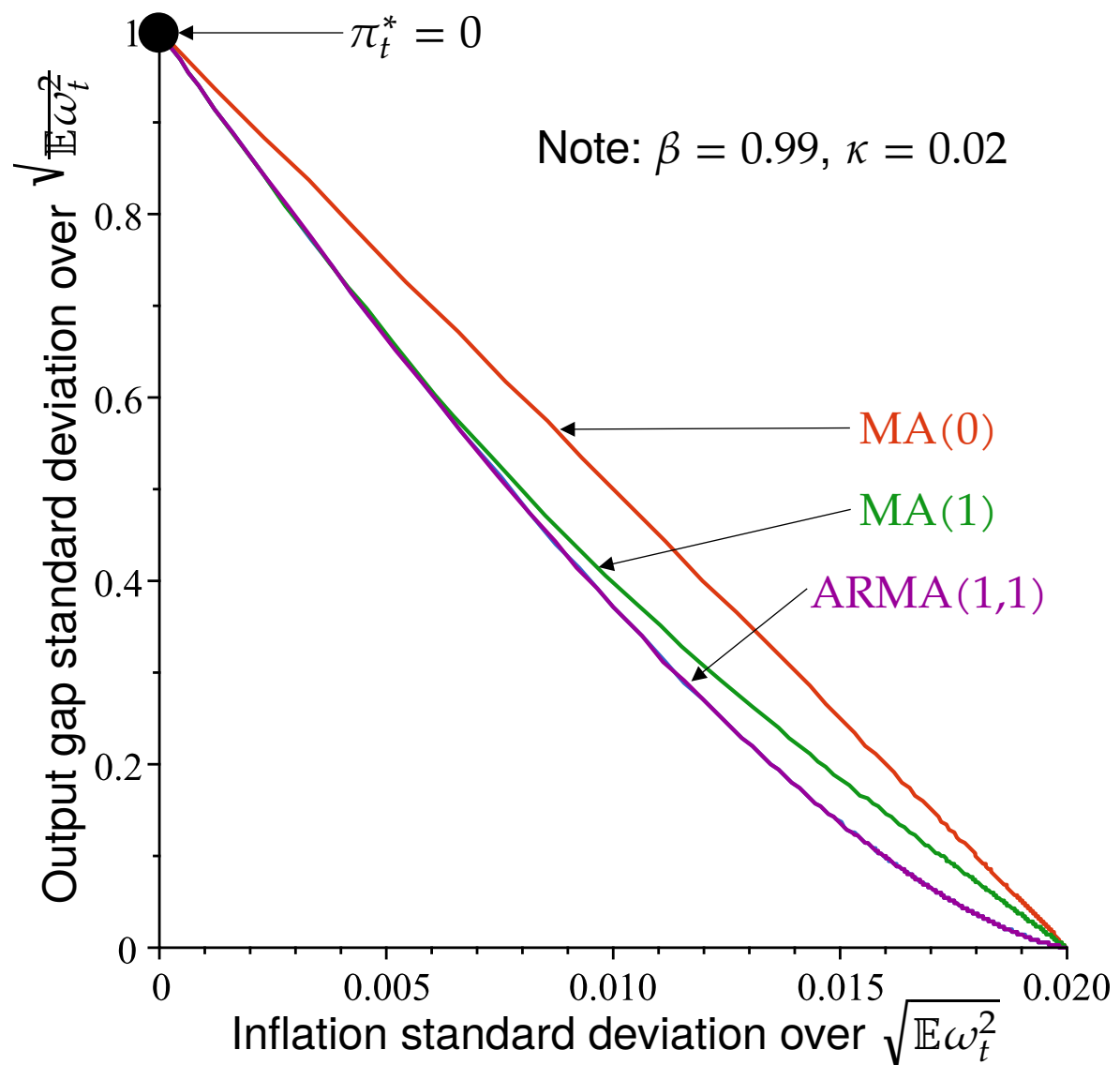
Log relative variances to ARMA(1,1) policy
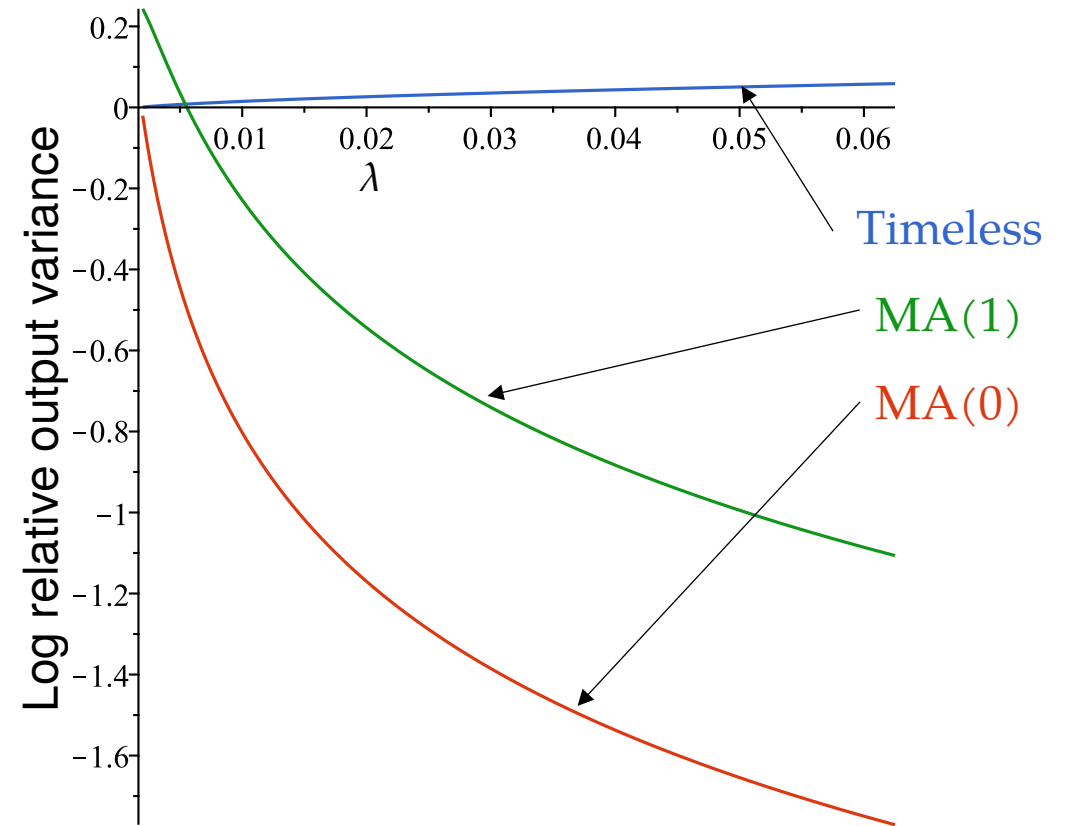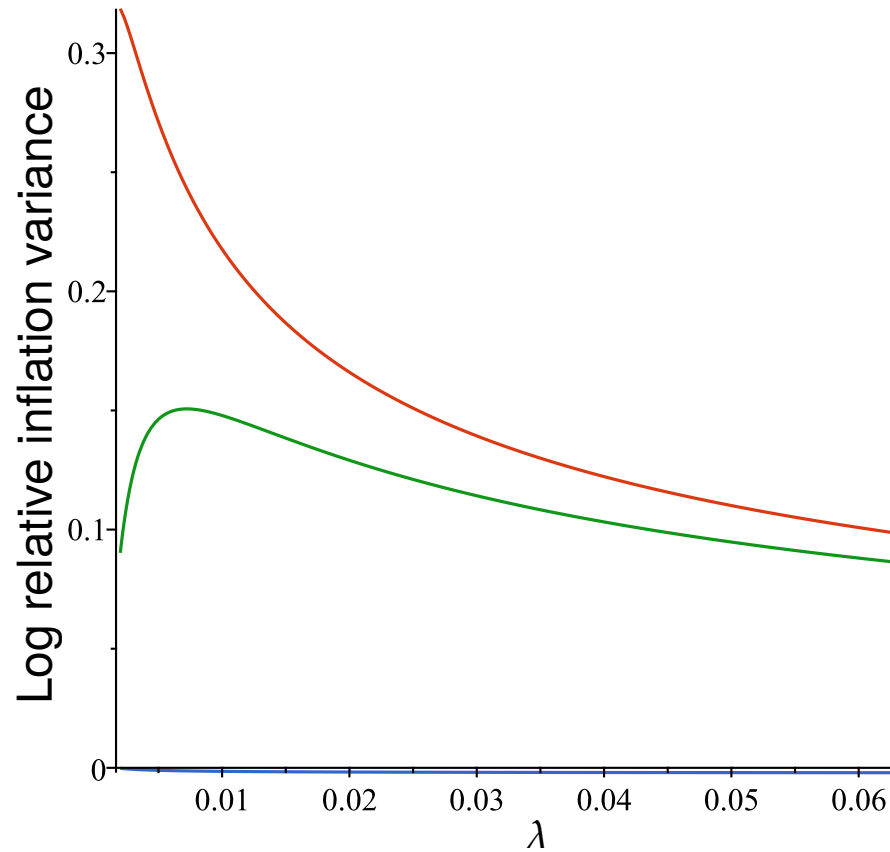
Note: $\beta = 0.99$, $\kappa = 0.02$.

MA(0) and MA(1) policies generate too much inflation variance.

Optimal inflation dynamics in a richer model

Using the Justiniano, Primiceri & Tambalotti (2013) model and replication files.

Blue: actual US inflation dynamics.

Red: inflation dynamics under optimal policy and US historical shocks. Less persistent!

Simple approximation to optimal policy 1/2

- For any $\rho \in (-1,1)$, the solution for optimal inflation has a multiple shock, ARMA$(1, \infty)$ representation:

$$\pi_t - \pi = \rho(\pi_{t-1} - \pi) + \sum_{k=0}^{\infty} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$
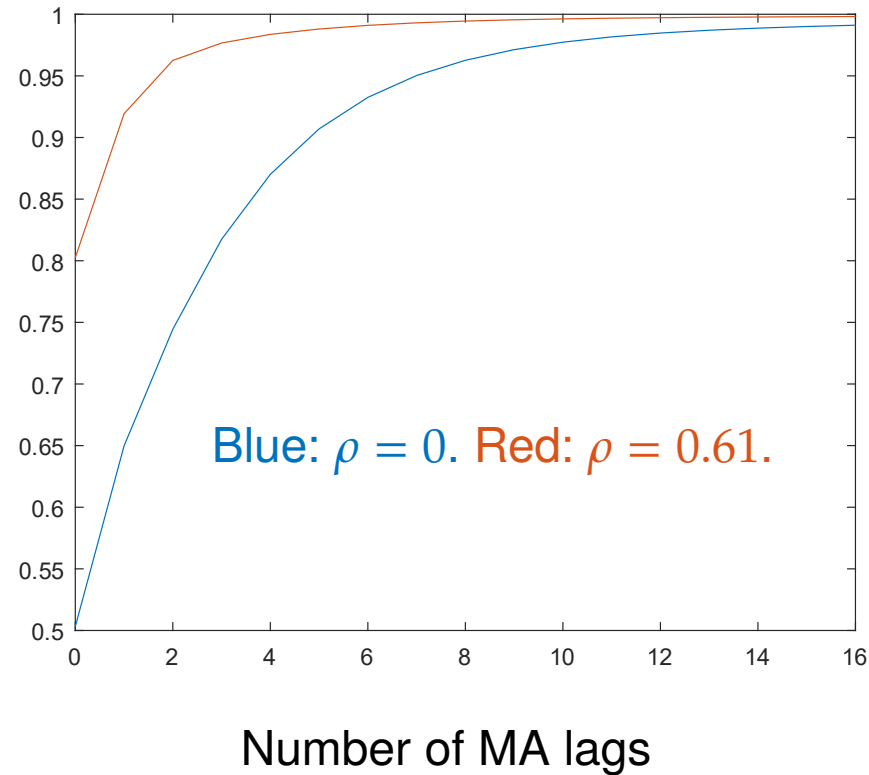
  - $\varepsilon_{1,t}, \ldots, \varepsilon_{N,t}$ are the model's structural shocks.

- Approximate by truncating MA terms at some point: E.g. multiple shock ARMA$(1, K)$:

$$\pi_t^{(K)} - \pi = \rho\big(\pi_{t-1}^{(K)} - \pi\big) + \sum_{k=0}^{K} \sum_{n=1}^{N} \theta_{n,k}^{(\rho)} \varepsilon_{n,t-k}$$

- Henceforth: "multiple shock ARMA" = "MSARMA".

Blue: $\rho = 0$. Red: $\rho = 0.61$.

Number of MA lags

Proportion of the variance of optimal inflation in the Justiniano, Primiceri & Tambalotti (2013) model explained by truncating the number of MA lags.

MSARMA(1,1) explains $> 90\%$ of optimal inflation variance, MSARMA(1,2) $> 95\%$!

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