Robust Real Rate Rules

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Paper and slides available at https://www.tholden.org/.

Motivation: Fragility of the Taylor principle

- The Taylor principle requires the response of nominal rates to inflation to be greater than one.
  - Sufficient for determinacy in simple models. (Guarantees no belief-driven fluctuations / sunspots.)
- Insufficient if there is e.g.:
  - A fraction of hand-to-mouth households (Galí, Lopez-Salido & Valles 2004).
  - Firm-specific capital (Sveen & Weinke 2005).
  - High government spending (Natvik 2009).
  - A positive inflation target (Ascari & Ropele 2009),
  - …particularly with trend growth + sticky wages (Khan, Phaneuf & Victor 2019).
- Inverts if there are e.g.:
  - Enough hand-to-mouth households (Bilbiie 2008).
  - Certain financial frictions (Manea 2019).
  - Non-rational expectations (Branch & McGough 2010; 2018).
  - Active fiscal policy (Leeper & Leith 2016; Cochrane 2023).

This paper

- Monetary rules with a unit response to real rates guarantee determinate inflation under the weakest possible assumptions on the rest of the economy.
  - Robust to household heterogeneity, non-rational households/firms, missing transversality conditions, existence/slope of the Phillips curve, active fiscal policy, etc.

- With a time-varying inflation target: enable the determinate robust implementation of an arbitrary path for inflation.
  - So can match observed inflation dynamics, or any model's optimal policy.

- Easy to implement in practice. Use TIPS to infer real rates. Works with bonds of any maturity.

- Reveal: Fisher equation is key to monetary transmission.

A first example

- Nominal bond: $\$1$ bond purchased at t returns $\$(1 + i_t)$ at $t + 1$.
- Real bond (e.g., TIPS): $\$1$ bond purchased at t returns $\$(1 + r_t + \pi_{t+1})$ at $t + 1$.

- Arbitrage $\Rightarrow$ the Fisher equation:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}$$

  - Abstracting from inflation risk / term / liquidity premia for now.

- Central bank uses the “real rate rule”:

$$i_t = r_t + \phi \pi_t$$

- With $\phi > 1$. Then:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t$$

- Unique non-explosive solution, $\pi_t = 0$. Determinate inflation!

Why is this robust? No need for Euler equation!

- Does not require an aggregate Euler equation to hold.
  - Robust to household heterogeneity and hand-to-mouth agents.
  - Robust to non-rational household expectations.

- For the Fisher equation to hold just need either:
  - Two deep pocketed, fully informed, rational agents in the economy, OR,
  - …a large market of rational agents with dispersed information (Hellwig 1980; Lou et al. 2019).

- Much more plausible financial market participants have rational expectations than households.
  - Can even partially relax the rationality requirement for financial market participants.
  - E.g.: Global convergence under learning from arbitrary initial beliefs.

Why is this robust? No need for Phillips curve!

- Does not require an aggregate Phillips curve to hold.
  - Robust to slope of the Phillips curve (if it exists).
  - Robust to forward/backward looking degree of Phillips curve equation.
  - Robust to non-rational firm expectations.

- Under this monetary rule, the Phillips curve is irrelevant for inflation dynamics.
  - The Phillips curve (if it exists) determines the output gap, given inflation.
  - If CB is unconcerned with output and unemployment, they do not need to care about the Phillips curve or its slope.

- Only require that at least some prices are adjusted each period using current information.

Implications for monetary economics

- Which model features lead to amplification or dampening of monetary shocks?
- Under a real rate rule: no change in the model can amplify/dampen monetary shocks other than changing rule.
  - Prior amplification/dampening results were sensitive to the monetary rule. May reverse with a response to r_t of > 1.
- Which shocks drive inflation?
- Under a real rate rule: only monetary policy shocks or shocks to the Fisher equation.
- CB has ultimate responsibility for inflation.

How does monetary policy work (under a real rate rule)?

- Under a real rate rule, monetary policy does not work via the real rate.
- In fact: This is a general property of NK models even with standard monetary rules.
- Rupert & Šustek (2019) show that with endogenous capital and sufficient monetary shock persistence:
  - Contractionary (positive) monetary shocks lead to falls in output, inflation **and real rates**. Contrary to standard story.
- Instead: Monetary policy operates as under flexible prices. (Exactly under a real rate rule, approximately in general.)
  - Following a monetary shock, inflation jumps to the unique level consistent with non-explosive inflation. More intuition to come.
- Outcomes under a real rate rule can also be replicated with a standard monetary rule with infinite coefficient on π_t.

Real rate rules elsewhere in the literature

- Papers with rules responding to real rates, for analytic convenience e.g.:
  - Adão, Correia & Teles (2011), Holden (2019), Lubik, Matthes & Mertens (2019).
- Closest prior work: Cochrane (2017; 2018; 2023) on spread targeting.
  - Cochrane briefly considers a rule of the form $i_t = r_t + \phi \pi_t$ before setting $\phi = 0$ (under FTPL).
- Other related work:
  - Hall & Reis (2016): vary interest on reserves with price level deviations, e.g. nominal return from $\$1$ of $\$(1 + r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1 + i_t)\frac{p_t}{p_t^*}$.
  - Hetzel (1990): Use nominal/real bond spread to guide policy. Dowd (1994): target the price of price level futures contracts.
  - Forecast targeting: Hall & Mankiw (1994), Svensson (1997), ….
  - Bernanke & Woodford (1997): Responding to private inflation forecasts leads to indeterminacy (LHS vs RHS). Bilbiie (2008; 2011) uses a special case with $i_t = \mathbb{E}_t \pi_{t+1} + \cdots$. These are called real rate rules by Beaudry, Preston & Portier (2022).
- Large literature on rules tracking efficient ("natural") real interest rate.
  - E.g., Woodford (2003). Very different idea. $i_t = n_t + \phi \pi_t$ implies $\pi_t = \mathbb{E}_t \sum_{k=0}^{\infty} \phi^{-k-1}(r_{t+k} - n_{t+k})$.

Generalizations and generality

Monetary policy shocks

- Suppose the CB uses the rule:

$$i_t = r_t + \phi \pi_t + \zeta_t$$

- with $\phi > 1$, and ζ_t drawn from an AR(1) process with persistence ρ.

- Then from the Fisher equation:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t + \zeta_t \qquad \Rightarrow \qquad \pi_t = -\frac{1}{\phi - \rho} \zeta_t.$$

- Contractionary (positive) monetary policy shocks reduce inflation.
    - Intuition: Define $\pi_t^* := -\frac{1}{\phi-\rho}\zeta_t$, then $i_t = r_t + \mathbb{E}_t \pi_t^* + \phi(\pi_t - \pi_t^*)$. A contractionary monetary shock lowers the inflation target.
- If the CB is more aggressive (ϕ is larger) inflation is less volatile.
- Inflation dynamics are independent of the rest of the economy.

Output dynamics in 3 equation NK model

- As before: CB sets $i_t = r_t + \phi\pi_t + \zeta_t$, so $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$.

- Rest of model 1: Phillips curve (PC), with mark-up shock ω_t:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + \kappa\omega_t$$

- Rest of model 2: Discounted/compounded Euler equation (EE) (Bilbiie 2019), n_t exogenous natural rate ($\delta = 1$, $\varsigma =$ EIS recovers standard Euler equation):

$$x_t = \delta\mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t)$$

- PC implies:

$$x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$$

Real rate dynamics in 3 equation NK model

- In the model of the last slide, if ω_t is IID, EE implies:

$$r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right], \qquad i_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)-\kappa\varsigma\rho}{\phi-\rho}\zeta_t + \omega_t\right].$$

- Derived without solving EE forward!
  - Implies robustness to missing transversality conditions.
  - Also implies degree of discounting/compounding (δ) has no impact on determinacy.
  - Contrasts with Bilbiie (2019): if $\varsigma > 0$, $\beta \leq 1$, with a standard Taylor rule, $\phi > 1$ is only sufficient for determinacy if $\delta \leq 1$.
  - Contrasts with Bilbiie (2008): if $\delta = 1$, $\varsigma < 0$, with a standard Taylor rule, $\phi > 1$ is neither necessary nor sufficient for determinacy.

- Under real rate rule, $\phi > 1$ is always necessary and sufficient! (Given $\phi \geq 0$.)
  - Robust to lags in EE and PC. (PC lag may reduce persistence of effect of monetary shocks on x_t!)

Implementing arbitrary inflation dynamics

- Suppose CB uses the rule:

$$i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t)$$

- π^*_t: an arbitrary stochastic process, possibly a function of economy's other endogenous variables and shocks.
- E.g.: Timeless optimal (Woodford 1999): $\pi^*_t := -\frac{\lambda}{\kappa}(x_t - x_{t-1})$. Or if set in advance: $\pi^*_{t+1|t} := -\frac{\lambda}{\kappa}\mathbb{E}_t(x_{t+1} - x_t)$.
- More robust solution than responding to other variables in the rule. In line with existing practice in the SEP.

- From the Fisher equation: $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. With $\phi > 1$, unique, determinate solution: $\pi_t = \pi^*_t$.
- The CB can hit an arbitrary path for inflation!
  - E.g., optimal policy (=highest possible welfare!). And real rate rules can explain any observed inflation dynamics.
- Related literature on implementation of optimal policy:
  - Svensson & Woodford (2003), Dotsey & Hornstein (2006), Evans & Honkapohja (2006), Evans & McGough (2010).

Interest rate smoothing

- A fully smoothed real rate rule with time-varying target:

$$i_t - r_t = i_{t-1} - r_{t-1} + \mathbb{E}_t \pi^*_{t+1} - \mathbb{E}_{t-1} \pi^*_t + \theta(\pi_t - \pi^*_t)$$

- From the Fisher equation:

$$\theta(\pi_t - \pi^*_t) = \mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) - \mathbb{E}_{t-1}(\pi_t - \pi^*_t).$$

- Define: $p_t := \sum_{s=1}^{t}(\pi_s - \pi^*_s)$ and $\hat{p}_t := p_t + \frac{1}{\theta}\mathbb{E}_0 p_1$. Then summing over time gives:

$$(1+\theta)\hat{p}_t = \mathbb{E}_t \hat{p}_{t+1}$$

- With $\theta > 0$: Unique equilibrium $\hat{p}_t = 0$, so $\pi_t = \pi^*_t$.

- Produces the same π_t as unsmoothed rule.
- Difference: Only need $\theta > 0$, not $\phi > 1$.
    - Likely much easier for CB to convince agents of the former than of the latter.

Challenges to real rate rules

Real rate rules in non-linear models

- Nominal and real bond pricing:

$$I_t \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} = 1, \qquad R_t \mathbb{E}_t \Xi_{t+1} = 1$$

- Non-linear real rate rule with gross $t+1$ inflation target announced at t of $\Pi^*_{t+1|t}$:

$$I_t = R_t \Pi^*_{t+1|t} \left(\frac{\Pi_t}{\Pi^*_{t|t-1}} \right)^{\phi}$$

$$\Rightarrow \mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \frac{\Pi^*_{t+1|t}}{\Pi_{t+1}} = \left(\frac{\Pi^*_{t|t-1}}{\Pi_t} \right)^{\phi}$$

- $\Pi_t = \Pi^*_{t|t-1}$ is always one solution of this equation! Always locally unique with $\phi > 1$.

- (Approximately) Globally unique under weak assumptions:
  - There exists $\overline{Z} \geq 1$ such that for all sufficiently high ϕ, $1 \leq \frac{\Pi^*_{t|t-1}}{\Pi_t} \leq \overline{Z^{\frac{1}{\phi-1}}} \to 1$ as $\phi \to \infty$. So, for large ϕ, $\Pi_t \approx \Pi^*_{t|t-1}$.
  - Under slightly stronger restrictions on the SDF, $\Pi_t = \Pi^*_{t|t-1}$ is globally unique solution for all sufficiently high ϕ.

Wedges in the Fisher equation

- Many potential sources of a Fisher equation wedge:
  - Liquidity premia on nominal bonds (Fleckenstein, Longstaff & Lustig 2014). Deflation protection on real bonds.
  - Risk premia (already considered). Non-rational expectations. Etc.

- Generalized Fisher equation (ν_t: stationary endogenous wedge):

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu_t$$

- Assume: Exist $\bar{\mu}_0, \bar{\mu}_1, \bar{\mu}_2, \bar{\gamma}_0, \bar{\gamma}_1, \bar{\gamma}_2 \geq 0$ such that for any stationary solution for π_t:

$$|\mathbb{E}\nu_t| \leq \bar{\mu}_0 + \bar{\mu}_1 |\mathbb{E}\pi_t| + \bar{\mu}_2 \operatorname{Var} \pi_t \,, \qquad \operatorname{Var} \nu_t \leq \bar{\gamma}_0 + \bar{\gamma}_1 |\mathbb{E}\pi_t| + \bar{\gamma}_2 \operatorname{Var} \pi_t$$

- Then under a real rate rule: $|\mathbb{E}\pi_t| = O\left(\frac{1}{\phi}\right)$ and $\operatorname{Var} \pi_t = O\left(\frac{1}{\phi^2}\right)$ as $\phi \to \infty$. Wedges are not a problem with large ϕ!

- If liquidity premia are the main distortion, may be better for CB to intervene in inflation swap market.

Fiscal Theory of the Price Level and "over determinacy"

- If price level is determinate independent of MP, then $\phi > 1$ can mean explosive π_t.
  - E.g., true if fiscal policy is active (real government primary surpluses do not respond to debt).
  - With one period debt, active fiscal policy & flexible prices: $\pi_t - \mathbb{E}_{t-1}\pi_t = -\varepsilon_{s,t}$.
  - Inconsistent with standard real rate rule solution: $\pi_t = -\frac{1}{\phi}\varepsilon_{\zeta,t}$ (IID monetary shock) as long as $\varepsilon_{\zeta,t} \neq \phi\varepsilon_{s,t}$.
  - Only explosive solution remains under real rate rule: $\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1} - \varepsilon_{s,t}$.

- This is a knife edge result! With multi-period (geometric coupon) debt: stable π_t solution under a real rate rule.
  - Still consistent with transversality even with active fiscal, active monetary!
  - ↑ bubble in debt price balanced by ↓ quantity. Initial debt price jumps. "Fiscal theory of the debt price".
  - With passive MP this implies multiplicity, so FTPL does not guarantee uniqueness.

- General result: Except in knife edge cases: Stable solution under a real rate rule if plausible condition satisfied:
  - Potentially explosive variables (e.g., bond prices) do not feed back to the real economy, and are not too forward looking.

Setting nominal rates out of equilibrium

- Apparent issue: If for $t > 0$, $i_t = r_t + \phi\pi_t$, then $\pi_t = 0$ for $t > 0$, so by Fisher $i_0 = r_0$. CB cannot set $i_0 \neq r_0$!

- Resolution: $\pi_t = 0$ iff $\pi_s = 0$ for all $s \in \{0,1,\dots,t-1\}$, else $\pi_t = \phi\pi_{t-1}$. If $\pi_0 \neq 0$, Fisher states $i_0 - r_0 = \phi\pi_0$.

- May reappear under bounded rationality. Suppose agents have learned $\pi_t = 0$, then $i_t = r_t$ even out of equilibrium.

- One fix: Modified real rate rule:
$$i_t = r_t + \phi\pi_t - \varrho(\phi - \varrho)\pi_{t-1}$$

- With $\varrho \in (-1,1)$ and $\phi > 1 + \varrho$. Determinate solution: $\pi_t = \varrho\pi_{t-1}$. Agents learn $\pi_t \approx \varrho\pi_{t-1}$.

- Alternative fix: Price level real rate rule rules.

The zero lower bound

Problems caused by the ZLB

- With the ZLB, simplest real rate rule means:

$$\max\{0, r_t + \phi\pi_t\} = i_t = r_t + \mathbb{E}_t\pi_{t+1}$$

- So:

$$\mathbb{E}_t\pi_{t+1} = \max\{-r_t, \phi\pi_t\}$$

- Real rates no longer cancel out completely! Euler equation still matters for π_t.

- Extra steady state with $\pi = -r$ (Benhabib, Schmitt-Grohé & Uribe 2001).

- Still multiplicity and/or non-existence conditional on convergence to the standard steady state (Holden 2021).
  - E.g., suppose r_t exogenous, $r_t = 0$ for $t \neq 1$, and we assume that $\pi_t \to 0$ as $t \to \infty$.
  - Multiple solutions if $r_1 = 0$. No solution if $r_1 < 0$. General problem in NK models.

Modified inflation targets

- Non-existence comes from implicitly targeting an infeasibly low level of inflation.
- Easy to fix. Use the rule:

$$i_t = \max\{0, r_t + \mathbb{E}_t \breve{\pi}^*_{t+1} + \phi(\pi_t - \breve{\pi}^*_t) - \varrho(\phi - \varrho)(\pi_{t-1} - \breve{\pi}^*_{t-1})\}, \qquad \breve{\pi}^*_t := \max\{\pi^*_t, -r_{t-1} + \epsilon\}$$

  - π^*_t is the original inflation target. $\breve{\pi}^*_t$ is the modified target. $\epsilon > 0$ is a small constant. $\varrho \in (-1,1)$, $\phi > 1 + \varrho$.

- With modified rule: $\pi_t = \breve{\pi}^*_t$ for all t is an equilibrium. Locally determinate.
  - Closed form solution (rare with ZLB!) makes coordination easy.
  - No deflationary bias as $\pi_t > -r_{t-1}$. Instead: small inflationary bias as $\mathbb{E}\pi_t \geq \mathbb{E}\pi^*_t$.

- Perfect foresight solution is unique conditional on $\breve{\pi}^*_t$ + a terminal condition ruling out explosions or permanent ZLB.
  - Multiple solutions for $\breve{\pi}^*_t$ do not occur for standard NK models.
- Setting $\varrho \ll 0$ removes remaining sunspot equilibria. (A crude make-up strategy.)

Practical implementation of real rate rules

Practical implementation: Set-up

- Markets in short maturity TIPS may be illiquid, unavailable or unreliable. So, use longer maturity bonds.
  - Long bonds are also less likely to hit the ZLB.
  - But: Long maturities may have substantial risk/term/liquidity premia.
  - Extra complications: Inflation may be observed with a lag. 1 month for US CPI. TIPS may have indexation lag. 3 months in US.

- Notation:
  - S: information lag. Market participants and CB use the $t-S$ information set in period t. E.g.: $S=1$.
  - L: indexation lag in return of inflation protected bonds. E.g.: $L=3$.
  - $i_{t|t-S}$: nominal yield per period on a T-period nominal bond at t.
  - $r_{t|t-S}$: real yield per period on a T-period inflation protected bond at t.
  - $\nu_{t|t-S}$ endogenous Fisher equation wedge (risk premia etc.) for T-period nominal bonds relative to T-period real bonds at t.
  - $\bar{\nu}_{t|t-S}$ central bank's endogenous period t belief about level of $\nu_{t|t-S}$ (possibly correlated with $\nu_{t|t-S}$).

Practical implementation: Fisher equation and rule

- Fisher equation (need $T - L \geq 0$):

$$i_{t|t-S} = r_{t|t-S} + \nu_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k-L}$$

- Monetary rule:

$$i_{t|t-S} = \max\left\{0, r_{t|t-S} + \bar{\nu}_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \breve{\pi}^*_{t+k-L} + \phi(\pi_{t-S} - \breve{\pi}^*_{t-S})\right\}$$

$$\breve{\pi}^*_t \coloneqq \max\left\{\pi^*_t, T(\epsilon - r_{t-T+L|t-T+L-S} - \bar{\nu}_{t-T+L|t-T+L-S}) - \sum_{j=1}^{T-1} \breve{\pi}^*_{t-j}\right\}$$

- Combining implies that in non-ZLB equilibrium:

$$\mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T} (\pi_{t+k+S-L} - \breve{\pi}^*_{t+k+S-L}) + (\nu_{t+S|t} - \bar{\nu}_{t+S|t}) = \phi(\pi_t - \breve{\pi}^*_t)$$

- Solution ($A_j \geq 0$ unique for $\phi > 1$): $\pi_t = \breve{\pi}^*_t + \mathbb{E}_t \sum_{j=-\infty}^{\infty} A_j \big(\nu_{t+j+S|t+j} - \bar{\nu}_{t+j+S|t+j}\big)$

Practical implementation: Discussion

- CB's inflation error $\pi_t - \breve{\pi}_t^*$ is stationary as long as $\nu_{t+S|t} - \bar{\nu}_{t+S|t}$ is stationary.
- If ϕ is large enough, $\pi_t \approx \breve{\pi}_t^*$ (under the same assumptions as in previous discussion of Fisher wedges).
- If aggressive enough, endogenous wedges, indexation & information lags do not matter!

- Note: CB's trading desk should hold $i_t - r_t$ constant between meetings.
  - This requires i_t to move between meetings, in response to observed changes in r_t.
  - No reason this should be significantly harder than holding i_t fixed.

- CB could also offer to exchange $\$1$ face value of real debt for $\$(1 + i_t - r_t)$ face value of nominal, as proposed by Cochrane (2017; 2018).
- Or to buy/sell portfolios with $+\$1$ nominal debt, $-\$1$ real debt for $\$(i_t - r_t)$.
- Or trade inflation swaps (which pay $\Pi_{t+1} - K_t$ at $t + 1$, with no payments at t).

Conclusion

- The Taylor principle does not guarantee determinacy with standard monetary rules.
- Real rate rules ensure determinacy no matter the rest of the economy & give CB almost perfect control of inflation.
- They can be easily implemented using pre-existing assets (nominal and real bonds, or inflation swaps).

- Under a real rate rule:
  - Monetary policy works in spite of, not because of, real rate movements.
  - Causation in the Phillips curve runs exclusively from inflation to the output gap.
  - Household and firm decisions, constraints and inflation expectations are irrelevant for inflation dynamics.
  - Only changes in the rule can amplify the impact of shocks on inflation.

- With a time-varying target, real rate rules can implement optimal monetary policy, or match observed dynamics.
- Real rate rules continue to work in the presence of the ZLB, wedges in the Fisher equation, or active fiscal policy.

Extra slides

Explaining observed inflation dynamics

- Large literature finds no role for the Phillips curve in forecasting inflation.
  - Post-1984: IMA$(1,1)$ model beats Phillips curve based forecasts (conditionally & unconditionally) (Dotsey, Fujita & Stark 2018).
  - +: Atkeson & Ohanian (2001), Ang, Bekaert & Wei (2007), Stock & Watson (2009). One explanation: McLeay & Tenreyro (2019).

- Also: Miranda-Agrippino & Ricco (2021):
  - Contractionary monetary policy shock causes immediate fall in the price level.
  - Delayed impact on unemployment.

- All supportive of models in which causation in PC only runs in one direction: *from inflation to the output gap*.
  - As under a real rate rule! [Not saying the CB follows a real rate rule. Just that outcomes may not be so different.]

Responding to other endogenous variables

- In the model:

$$i_t = r_t + \phi_\pi \pi_t + \phi_x x_t + \zeta_t$$

$$\pi_t = \tilde{\beta}(1 - \varrho_\pi)\mathbb{E}_t \pi_{t+1} + \tilde{\beta}\varrho_\pi \pi_{t-1} + \kappa x_t + \kappa \omega_t, \qquad x_t = \tilde{\delta}(1 - \varrho_x)\mathbb{E}_t x_{t+1} + \tilde{\delta}\varrho_x x_{t-1} - \varsigma(r_t - n_t)$$

  - If $\kappa > 0$, $\phi_x \geq 0$ and $\tilde{\beta} \in [0,1]$, then $\phi_\pi > 1$ is sufficient for determinacy!
  - Real rate rule still helps robustness as it disconnects EE from prices.

- In any model: $\phi_\pi > 1$ sufficient for determinacy if responses to other endogenous variables are small enough.
  - Implies robustness to non-unit responses to real rates. Other variables (e.g., output growth) may proxy real rates.

- For greater robustness: Replace other endogenous vars in rule with structural shocks.
  - If structural shocks not observed, can infer from structural equations.
  - If equation parameters not known, can learn in real time, still with determinacy!

Equilibrium selection with perpetuities: Idea

- Cochrane (2011) argues no reason to rule out explosive NK equilibria.

- Suppose geometric coupon bonds (GCBs) are traded in the economy. (Later specialise to perpetuities.)
  - Could be approximated by portfolio of different maturity debt. Long-term government contracts (defence,...) also perpetuity like.

- 1 unit of period t GCB bought at t returns $\$1$ at $t+1$, along with $\omega \in (0,1]$ units of period $t+1$ GCB.
  - Suppose stock: $B_t \geq \underline{B}\omega^t$. Then transversality implies GCB price: $Q_t = \mathbb{E}_t \sum_{s=0}^{\infty}\left[\prod_{k=0}^{s} \frac{1}{I_{t+k}}\right]\omega^s$.
  - If $I_{t+k} = 1$ for high k, then $Q_{t+k} = \frac{1}{1-\omega}$ for high k. Transversality then requires $0 = \lim_{s\to\infty} \frac{\omega^s}{1-\omega}$, i.e., $|\omega| < 1$. Violated with $\omega = 1$!

- Permanent ZLB $\Rightarrow$ Infinite perpetuity price $\Rightarrow$ Infinite nominal wealth $\Rightarrow$ Infinite inflation $\Rightarrow$ Physically impossible.

Equilibrium selection with perpetuities: Use

- With sticky prices, explosions are generally ruled out.
  - Standard sticky prices specifications imply Π_t is bounded above. + Real costs of inflation explode as inflation explodes.
  - Prices may become more flexible as $\Pi_t \uparrow$, but seems plausible there is a limit on how often prices can be changed.
- So, under sticky prices the modified inflation target rule produces uniqueness if households hold perpetuities.
- Non-linear version (with a target known one period in advance, $\mathcal{E} := \exp \epsilon > 1$):

$$I_t = \max \left\{ 1, R_t \breve{\Pi}^*_{t+1|t} \left(\frac{\Pi_t}{\breve{\Pi}^*_{t|t-1}} \right)^{\phi} \right\}, \qquad \breve{\Pi}^*_{t+1|t} := \max \left\{ \frac{\mathcal{E}}{R_t}, \Pi^*_{t+1|t} \right\}$$

- Without sticky prices, have to send deviations to the ZLB. E.g., with following ($\bar{I} > 1$, $\phi > 1$ and $\mathcal{E} \in \left(1, \sqrt{\bar{I}}\right)$):

$$I_t = \begin{cases} \max \left\{ 1, R_t \breve{\Pi}^*_{t+1|t} \left(\frac{\Pi_t}{\breve{\Pi}^*_{t|t-1}} \right)^{\phi} \right\}, & \text{if } I_{t-1} \in (1, \bar{I}) \\ 1, & \text{otherwise} \end{cases}, \qquad \breve{\Pi}^*_t := \max \left\{ \frac{\mathcal{E}}{R_t}, \min \left\{ \frac{\bar{I}}{\mathcal{E} R_t}, \Pi^*_{t+1|t} \right\} \right\}$$

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