# RATIONAL POINTS ON HIGHER DIMENSIONAL VARIETIES: PROGRAM

- Dec 2nd (Mon)
  - $-\ 9{:}30$   $10{:}30$  Emanuel Peyre
  - 11:00 12:00 Shihoko Ishii
  - 12:00 14:00 Lunch
  - -14:00 15:00 Tony Várilly-Alvarado
  - 15:30 16:30 Kazuhiro Ito
- Dec 3rd (Tue)
  - $-\,$ 9:30 10:30 Masato Kuwata
  - 11:00 12:00 David McKinnon
  - 12:00 14:00 Lunch
  - 14:00 15:00 Bianca Viray
  - -15:30 16:30 Kevin Destagnol
- Dec 4th (Wed)
  - $-\,$ 9:00 10:00 François Loeser
  - $-\ 10:15$  11:15 Yuta Kusakabe
  - 11:30-12:30 Takashi Suzuki
  - 12:30 14:30 Lunch
  - 14:30 17:00 Free discussion
- Dec 5th (Thu)
  - -9:30 10:30 Kanetomo Sato
  - 11:00 12:00 Khoa Nguyen
  - 12:00 13:30 Lunch
  - 13:30 14:30 Hideo Kojima
  - 14:45 15:45 Kaoru Sano
  - 16:00 17:00 Takahiro Shibata
- Dec 6th (Fri)

- 9:00 10:00 Hiromu Tanaka
- 10:15 11:15 Yuri Yatagawa
- 11:30 12:30 Kazuhiko Yamaki
- 12:30 14:00 Lunch

#### Titles and Abstracts

## • Emanuel Peyre

Title: Rational points of bounded height.

Abstract: In the 80's Manin's program lead to several breakthroughs in the understanding of the behaviour of rational points of bounded height on varieties. In recent years, after the discovery of new phenomena, this field has known significant progress. In particular, the expected behaviour was proven on the complement of accumulating thin subsets in several cases and the work of S. Tanimoto, B. Lehmann and A. Sengupta gave a better understanding of the accumulating thin subsets on Fano varieties. At the same time, new questions were raised regarding extensions of the program, beyond Fano varieties, and when using other invariants. This talk will give a survey of the state of the art in this program.

#### • Shihoko Ishii

Title: Prime divisor with a good liftability of functions.

Abstract: We consider a point 0 of a smooth variety A over a field of characteristic p>0. We conjecture that every prime divisor E with the center at 0 has a good property: "E can be lifted to a prime divisor  $\tilde{E}$  with the center at  $\tilde{0}$  on a variety  $\tilde{A}$  over the field of characteristic 0 with the good property with respect to the liftability of a function." If the conjecture holds, then some statements in characteristic 0 induces the statements in positive characteristic. In the talk, I will give some evidences of the conjecture.

# • Kazuhiro Ito

Title: On a torsion version of the weight-monodromy conjecture and its applications

Abstract: I will talk about a torsion version of the weight-monodromy conjecture for a proper smooth scheme over a local field. I will prove it for surfaces and for the equal characteristic case. In the equal characteristic case, the proof relies on an ultraproduct variant of Weil II recently established by A.Cadoret. As applications, I will discuss some finiteness properties of the Brauer group and the Chow group of codimension two of a proper smooth scheme over a local field.

#### • David McKinnon

Title: Curves of best approximation

Abstract: Imagine a lonely rational point, sitting on a smooth projective variety somewhere. It longs for company, but the other rational points keep their distance? or else they are complicated, and who wants a complicated relationship? Then the point notices a beautiful rational curve, passing

right through it (hard to imagine how the point managed to miss this), with lots of other rational points on it that aren?t that complicated at all! Does this story always have a happy ending like this? Come to my talk so I can tell you that we don't know.

#### • François Loeser

Title: Non-archimedean parametrizations, points counting and applications Abstract: We will provide a general overview on some recent work on non-archimedean parametrizations and their applications. We will start by presenting our work with Cluckers and Comte on the existence of good Yomdin-Gromov parametrizations in the non-archimedean context and a p-adic Pila-Wilkie theorem. We will then explain how this is used in our work with Chambert-Loir to prove bialgebricity results in products of Mumford curves. We will conclude with our recent work with Cluckers and Forey providing uniform bounds for the number of  $\mathbb{F}_q[t]$ -points of bounded degree on algebraic varieties.

#### • Takashi Suzuki

Title: Iwasawa theory for abelian varieties over function fields and Hodge-Witt elliptic surfaces

Abstract: I will discuss Iwasawa theory (in particular, Iwasawa mu-invariants) for abelian varieties over function fields under constant  $\mathbb{Z}_p$ -extensions, mainly from the side of algebraic geometry in positive characteristic. We give formulas for the mu-invariant in terms of the dimensions of the Tate-Shafarevich scheme and the Brauer scheme. When the abelian variety is semistable, we also give a formula for mu in terms of the L-function using Lai-Longhi-Tan-Trihan's Iwasawa main conjecture. The vanishing of mu is related to the Hodge-Witt property of algebraic surfaces. Even though supersingular K3 surfaces provide plenty of elliptic curves with non-zero mu-invariant, we show that elliptic curves with mu=0 form a dense open subvariety in the moduli of elliptic surfaces. This means that generic elliptic curves satisfy an analogue of Iwasawa's mu=0 conjecture. Joint with K. F. Lai, I. Longhi, K.-S. Tan and F. Trihan.

#### Kanetomo Sato

Title: Étale cohomology and a zeta value of arithmetic schemes Abstract: (j/w Takao Yamazaki) Let X be an integral proper regular arithmetic scheme of dimension d. I will start this talk with a result that the étale cohomology groups of X with ' $\mathbb{Q}_p(d)$ '-coefficients are isomorphic to the Selmer groups of Bloch-Kato. This comparison result is based on a certain vanishing of étale 2-extensions, and another comparison result relating the order of Tate-Shafarevich groups with p-adic Abel-Jacobi mappings and local factors (of the L- and the zeta functions) when d=2. I will further give a residue formula of the zeta function  $\zeta(X,s)$  at s=2 modulo a rational number prime to p, when X is a proper regular model of a CM elliptic curve over  $\mathbb{Q}$  and p is a good and regular prime number.

#### • Khoa Nguyen

Title: The Hermite-Joubert problem and a conjecture of Brassil-Reichstein

Abstract: The Hermite-Joubert problem asks for which integers  $n \geq 5$  such that for every characteristic zero field extension E/F of degree n there exists  $\delta \in E^*$  with  $\mathrm{Tr}_{E/F}(\delta) = \mathrm{Tr}_{E/F}(\delta^3) = 0$ . We prove that every n of the form  $3^{k_1} + 3^{k_2} + 3^{k_3}$  for integers  $k_1 > k_2 > k_3 \geq 0$  does not satisfy the above property in the Hermite- Joubert problem. This settles a conjecture of Brassil-Reichstein. The proof involves solving a system of diophantine equations that is an instance of the Mordell-Lang conjecture (i.e. Faltings' theorem).

# • Hideo Kojima

Title: Rational open surfaces of log Kodaira dimension  $\leq 1$  Abstract: Let V be a smooth projective rational surface defined over an algebraically closed field. Let C be a connected curve on V such that S = V - C has log Kodaira dimension  $\leq 1$ . We show that, if the number of irreducible components of C is larger than the Picard number of V, then S contains a simple affine surface. We also give some results on the curve C when  $V = \mathbb{P}^2$ .

#### • Hiromu Tanaka

Title: Rational points and minimal model program in positive characteristic Abstract: It is known that smooth Fano varieties have rational points if the base field is either a finite field or the function field of a curve defined over an algebraically closed field of characteristic zero. In this talk, we summarise some related results from the viewpoint of minimal model program. We also discuss some open problems on this topic.

## • Yuri Yatagawa

Title: Wild ramification and characteristic cycle

Abstract: The characteristic cycle of a constructible sheaf on a smooth variety is an algebraic cycle on the cotangent bundle of the variety, and, when the variety is projective, it computes the Euler characteristic of the sheaf. In this talk, we give a sufficient condition for two sheaves having the same characteristic cycle, which is a weaker one than Deligne-Illusie's for two sheaves having the same Euler characteristic. This is the joint work with T. Saito.