PRACTICAL SUGGESTIONS FOR MATHEMATICAL WRITING

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In the examples below, red is bad and green is good.

1. Important things

(1) If a claim does not follow immediately from the previous sentence alone, explain what it does follow from.
   - Combining the previous two sentences shows that...
   - By Lemma 8.3, ...
(2) If a sentence contains more than one claim, make clear which reason is the justification for each claim. For example, in a chain of equalities, make clear which reason explains which equality. Sometimes this can be effected by placing a reason before or after the chain of equalities, or by using `\begin{align*} \end{align*}` with a reason at the right in each line.
(3) A reader reaching the period of a sentence should know why each claim up to that point is true. If a claim is made whose proof will come only *after* the period, indicate this by ending the sentence with something like ", as we now explain", or put the claim into a formal construction like `\begin{lemma} \end{lemma} \begin{proof} \end{proof}`.
(4) It is not enough to write so that your words *can* be given the meaning you intend; write so that they cannot reasonably be given any other meaning. Ambiguity should be eliminated even if it would cause only a minute of confusion.
(5) Break up long arguments by making parts of them into lemmas, even if the lemmas are used only once. The goal is to minimize what a reader must keep in mind at one time.
(6) If a section contains several theorems, propositions, and lemmas, but only one of them is needed in subsequent sections, mention this. (Again, this can free up memory in the reader's brain!)
(7) Make quantifiers unambiguous: instead of writing
   We have $x^2 + 1 \in S$ for $x \in \mathbb{R}$.   or worse   We have $x^2 + 1 \in S$, $x \in \mathbb{R}$.
   write one of
   We have $x^2 + 1 \in S$ for all $x \in \mathbb{R}$.
   We have $x^2 + 1 \in S$ for some $x \in \mathbb{R}$.
(8) Proofs should usually indicate where the hypotheses are being used.
(9) When citing a book or article, include a theorem number or page number. (This should be omitted only in cases in which you really mean to cite the entire work.)
(10) When citing an arXiv preprint, include the version number or precise date. (This way, even if a new version is uploaded, the reader can figure out what you were referring

to.) When citing a preprint elsewhere on the web, give the URL and date of the manuscript or date downloaded.

(11) Cite "forthcoming work" only if there is a publicly available preprint. (The reason is that sometimes the work ends up not appearing for years or does not appear at all, and the announcement serves to discourage others from working on the problem, which can be damaging to the field: see the third bullet in Section I of the AMS Policy Statement on Ethical Guidelines at `http://www.ams.org/about-us/governance/policy-statements/sec-ethics`.)

2. Other things

(12) Keep theorem statements short. Definitions should *precede* the theorem in which they are used.

(13) Keep sentences short. Combine sentences (with ", and" or with a semicolon) only if it helps to clarify the logic or if the sentences are otherwise closely related.

(14) If an argument involves combining several equalities, string them together in one chain of equalities (in the correct order!), if possible; this can make the logic easier to follow since everyone understands transitivity. The same applies to inequalities.

(15) The introduction to an article should get to new and interesting theorems as soon as possible. It is OK to postpone definitions to a later "Notation" section (and to refer readers in the introduction to this later section) if those definitions are standard enough that most readers will be able to read the introduction without them.

(16) If a proof breaks into parts, do the easier parts first.

(17) When claiming that two objects are isomorphic, specify the map between them explicitly (at least in one direction) and claim that *that* map is an isomorphism.

(18) Start induction arguments with base case $n = 0$ instead of $n = 1$ if it is easier.

(19) Usually when people write that something is "clear", it is because they could not figure out a good explanation of why it is true (and sometimes this is because the statement is wrong!) If it really is clear, it is usually not necessary to say that it is clear. If it is not clear (or even if it is), then try to say a few words to give the reason.

(20) Minimize use of the ", where..." construction, in which explanations of notation come after the notation is used. Instead, define variables *before* they are needed.

(21) The subject of a sentence or clause cannot consist of words outside a formula together with a fraction of the formula. Example: Instead of

For a quadratic polynomial $ax^2 + bx + c$, the discriminant

$$\Delta = b^2 - 4ac.$$

in which the subject is "the discriminant Δ", write

For a quadratic polynomial $ax^2 + bx + c$, the discriminant Δ equals $b^2 - 4ac$.

(22) Do not use abbreviations like WLOG, iff, and s.t. (Those are for blackboard use only, if at all.)

(23) Do not use logical symbols such as $\exists$ and $\forall$, unless you are writing about formal logic and they appear in a logical formula. Instead, write out $\exists$ as "there exists" (or "there exist"), and so on. See [CMOS, 12.5].

(24) Do not start a sentence with a symbol [CMOS, 12.7]. For example,

H denotes the Sylow p-subgroup of G.

should not appear as a full sentence.

(25) Avoid contractions such as "don't" in formal writing.

(26) Do not use a proof by contradiction when a direct proof would be just as easy.

(27) Refer to theorems by *number* instead of writing "the previous theorem", "the proposition above", and the like.

3. LATEX ISSUES

(28) Use a single numbering system for all theorems, lemmas, etc., instead of having both a Theorem 1.1 and a Lemma 1.1 in the same paper. This makes statements easier to find, and can help prevent citation errors. LATEX can do this for you: one solution is to put

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{lemma}[theorem]{Lemma}
\newtheorem{corollary}[theorem]{Corollary}
...
```

in your preamble. (This presumes that you are using `amsart` or some other AMS document class; otherwise precede all this with `\usepackage{amsthm}` in your preamble.)

(29) To get $\mathrm{Gal}(L/K)$ instead of $Gal(L/K)$, put `\DeclareMathOperator{\Gal}{Gal}` once in the preamble and `\Gal` in the body of the paper each time you need it. (This presumes that you are using `amsart` or some other AMS document class; otherwise precede all this with `\usepackage{amsmath}` in your preamble.)

(30) Use `\hfill` before `\begin{enumerate}` if necessary, to prevent misalignment of the first item as in

Theorem 3.1. (a) *First statement.*
(b) *Second statement.*
(c) *Third statement.*

(31) Instead of `f : X \to Y`, use `f \colon X \to Y` for appropriate spacing. Compare $f : X \to Y$ and $f\colon X \to Y$.

(32) Try `\usepackage{fullpage}` in the preamble to fit more on a page without having to set margins manually.

(33) Try `\usepackage{microtype}` just before your `\begin{document}` to allow LATEX to adjust spacing between characters to reduce the number of bad line breaks.

(34) Finally, if you have mastered everything else, try `\usepackage{colonequals}` in the preamble and `\colonequals` as needed in the body. Compare $:=$ and $\coloneqq$ carefully!!!

4. NITPICKS

(35) The words "so that" convey purpose, whereas "such that" just imposes a condition. Each instance of "so that" should be replaceable by at least one of "in order that" and "with the result that".

Good: We include 0 in $\mathbb{N}$ so that $\mathbb{N}$ contains the size of every finite set.
Bad: An abelian group is a group so that every two elements commute.

(36) If you are trying only to indicate a logical implication between statements A and B, write "A, so B" instead of "A, so that B" or "A, and so B".

(37) The word "only" should be placed as close as possible to the word it is modifying. See [Kil07] for examples.

(38) Instead of "Given g an element of G" write "Given an element g of G".

(39) When referring to a theorem (or proposition or section or . . .) by number, capitalize the word "Theorem".
  - "By Section 3.4 and Theorem 5.6, the proposition holds."
  - "By Faltings's theorem, $X(\mathbb{Q})$ is finite."

(40) Displaying a formula does not change the punctuation required. Thus, instead of
    By Theorem 3.2, we have:
$$\mathrm{Gal}(L/K) \simeq \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$$

write
    By Theorem 3.2, we have
$$\mathrm{Gal}(L/K) \simeq \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}.$$

(41) "Assume G is a finite group." should be "Assume that G is a finite group." On the other hand, "Assume Hypothesis A." is OK. Similarly, "Assume $n \geq 1$." is OK, because "$n \geq 1$" serves as a noun unit. The same rules apply to the use of "Suppose". See [Wes15] for more details.

(42) Refer to the 1980s, not the 1980's: see [CMOS, 9.34].

(43) I.e. and e.g. should be followed by a comma, at least in American English. Also, if what come before and after are full sentences, then the i.e. or e.g. should be preceded by a semicolon instead of a comma.

(44) Minimize use of parentheses. For instance, $\log x$ is better than $\log(x)$, and it is traditional to omit parentheses also in some expressions like $\sin 2x$. But in $\sin(x+y)$, parentheses are necessary.

(45) Usually it is not necessary to use a centered dot or other symbol for multiplication. For example, write $(n+1)(n+2)$, not $(n+1) \cdot (n+2)$ or $(n+1) \times (n+2)$. If both factors are numbers, however, juxtaposition would be confusing, so write $3 \cdot 7$.

(46) Filler words like "We remark that" and "Note that" should be omitted, except when their absence would produce a sentence starting with a symbol.

(47) If a proof ends with an end-of-proof symbol, then it is not necessary to write "This concludes the proof." or "Thus we are done."

(48) Fractions in exponents or subscripts should use the slash instead of being stacked vertically [CMOS, 12.47]. Write $x^{3/2}$, not $x^{\frac{3}{2}}$.

(49) Just as an ordered tuple is written $(2,3,5)$ instead of $\{2,3,5\}$, a sequence should be written $(a_i)_{i\geq 0}$ instead of $\{a_i\}_{i\geq 0}$.

(50) The prefix "non" does not need a hyphen when it appears in common words such as nonempty, nonnegative, nonsingular, nontrivial, and nonzero [CMOS, 7.85].

(51) Most numbers in math papers should be written as numerals instead of words [Wes15]. Exception: If it is a single-digit number being used only as a counting number instead of a mathematical quantity, then it may be spelled out as a word. Example: These

two genus 2 curves are isomorphic. (Note: Some authors write small numbers as words in more situations.)

(52) Technically it is OK to use "as" as a synonym for "since", but this usage of "as" sounds stilted and can make sentences harder to parse because of other possible meanings of "as". Therefore, "since" should be preferred [Wes15]. A particularly egregious example: "As x does not tend to $+\infty$, the expression e^x is bounded."

(53) Similarly, do not use "per" when what you mean is "by".

REFERENCES

[CMOS] *The Chicago manual of style, 16th edition*, University Of Chicago Press, 2010.

[Kil07] James Kilpatrick, *If we could only get this one right*, Eugene Register-Guard (January 14, 2007), 32. `https://news.google.com/newspapers?id=4WBWAAAAIBAJ&sjid=tfADAAAAIBAJ&pg=6697,2895111&hl=en` .

[Wes15] Douglas B. West, *The grammar according to West*, September 17, 2015. Available at `http://www.math.illinois.edu/~dwest/grammar.html` .

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