RIMS Workshop
Algebraic Number Theory and Related Topics

Organizers:
Manabu Ozaki (Waseda University)
Iwao Kimura (Toyama University)

Date: December 9 (Mon)– December 13(Fri), 2019
Place: Room 420, Research Institute for Mathematical Sciences (RIMS),
Kyoto University, Kyoto 606-8502, JAPAN

December 9 (Mon)

10:15 – 10:30 Introduction

10:30 – 11:20 **Shunsuke Yamana** (Osaka City University)
Exceptional zeros of p-adic L-functions

p-adic L-functions involve modified p-factors which measure the discrepancy between the p-adic and classical special values in the interpolation formula. It is a puzzling fact that this factor can vanish at the central point. Then the p-adic L-function trivially vanish at the point, and such a zero is called an exceptional zero. The p-adic L-function of an elliptic curve E has an exceptional zero if and only if E has split multiplicative reduction at p. In this case the precise relation between derivative of the p-adic L-function and the algebraic part of the central value was conjectured by Mazur-Tate-Teitelbaum and proved by Greenberg-Stevens. In this talk I will determine the exceptional zeros of cyclotomic p-adic L-functions associated to three ordinary semi-stable elliptic curves and prove an identity between double or triple derivatives of the p-adic L-function and central L-values. This is a joint work with Ming-Lun Hsieh.

11:35 – 12:35 **Kenichi Namikawa*** (Kyushu University)
Recent developments on Iwasawa theory for Asai representations

Twisted tensor products of representations of the absolute Galois group over quadratic fields are called Asai representations. In this talk, I will survey recent developmentson Iwasawa main conjecture (IMC) for Asai representations of degree 4. In particular, I introduce a method of theta correspondences for a study of IMC. This talk is partially based on joint works with Ming-Lun Hsieh.

14:00 – 14:50 **Yoshihiro Ônishi** (Meijo University)
Vanishing elliptic Gauss sums and Bernoulli-Hurwitz type numbers

Replacing the exponential factors in a classical Gauss sum by division values of suitable elliptic function, we obtain what we call an elliptic Gauss sum (EGS). The famous paper "Notes on elliptic curves II" by Birch and Swinnerton-Dyer used EGS's in order to calculate values of L-series at 1 for some elliptic curves with CM by the Gauss integers. Asai investigated EGS's for elliptic curves with CM by Gauss integers (or Eisenstein integers) around 2006. Concerning his work, the speaker gave certain congruence relation connecting EGS's and coefficients (Bernoulli-Hurwitz type numbers) of the lemniscate elliptic function, which gives rise to a congruence relation connecting a square-root of the order of corresponding Tate-Shafarevich group and such a coefficient under assuming the full BSD conjecture. Moreover, the least residue of the coefficient modulo the modulus really gives a square root of the order of Tate-Shafarevich group, which is an analogue of well-known congruence connecting the class number of an imaginary quadratic field and certain Bernoulli (or Euler) number. The former half of this talk will be taken to explain the congruence. The latter half will be for presenting some (numerical evidences of) new congruence of Kummer type involving other Bernoulli-Hurwitz numbers in the case that EGS's vanish, in other words, that the corresponding coefficients are congruent to zero. The new congruence might relate a p-adic L-series for additive reduction case.

15:05 – 15:55 **Masahiro Mine** (Tokyo Institute of Technology)
Densities of cubic fields under some conditions on L-values

Taniguchi–Thorne and Bhargava–Shankar–Tsimerman studied the densities of cubic fields satisfying i) various local specifications or ii) congruence conditions for their discriminants. In this talk, we study the density of cubic fields under the condition that values of Artin L-functions, which are arising from the cubic fields, belong to a given region. We determine the main term of the density by applying a result of Taniguchi–Thorne together with the methods of Probability theory and Equidistribution theory.

16:10 – 17:00 **Kota Yamamoto** (Nagoya Institute of Technology)
On iterated extensions of number fields by quadratic rational maps

Let p be a prime number and ϕ be a rational map of degree p defined over a number field k. For $b_0 \in k$, let $\{b_n\}_{n\geq 0}$ be a sequence defined by $\phi(b_{n+1}) = b_n$ recursively. Sometimes the number fields $k_n = k(b_n)$ form a sequence $\{k_n\}_{n\geq 0}$ of iterated extensions of p-power degree. The number field k_n is contained in a Galois extension $K_n = k(\phi^{-n}(b_0))$ of k. It is known that if ϕ is post-critically finite (i.e. the forward orbit of each critical point of ϕ is finite) then the number of primes of k ramifying in an extension $K_\infty = \bigcup_{n\geq 0} K_n$ of k is finite. Then moreover, the size of the image $\mathrm{Gal}(K_\infty/k)$ of an arboreal Galois representation is small, and sometimes K_∞/k becomes a p-adic Lie extension. In this talk, for various quadratic rational maps ϕ, we consider iterated extensions $\{k_n\}_{n\geq 0}$ of 2-power degree under some conditions with respect to the first term b_0, and give results on Galois groups of iterated extensions and the 2-parts of ideal class groups. In particular, for a situation where K_∞/k is a 2-adic Lie extension of dimension 2, we give examples where the unramified Iwasawa module $X(K_\infty)$ is pseudo-null as an analogue of Greenberg's conjecture. A key tool is a result on p-indivisibility of the orders of T-ideal class groups (this is isomorphic to the Galois group of the maximal unramified abelian extension in which all primes lying over T split completely) of cyclic extensions of degree p^2. This is partially a joint work with Yasushi Mizusawa.

December 10 (Tue)

10:00 – 10:50 **Hikaru Hirano** (Kyushu University)
On abelian arithmetic Chern-Simons invariants for certain real quadratic number fields

In recent years, based on the analogies in arithmetic topology, Minhyong Kim initiated to study an arithmetic analogue for totally imaginary number fields of the Chern-Simons theory for 3-manifolds. His treatment employs some results on e'tale cohomology groups of the integer rings of totally imaginary number fields which no longer hold for number fields with real places. In this talk, we extend Kim's definition for any number field, by using the modified etale cohomology groups and fundamental groups which take real places into account. We then show explicit formulas of mod 2 arithmetic Chern-Simons and Dijgraaf invariants for certain real quadratic fields. We also show a topological analogue of our results.

11:05 – 12:05 **Jun Ueki*** (Tokyo Denki University)
Chebotarev link and idelic class field theory

We formulate an analogue of the idelic class field theory over a 3-manifold equipped with an infinite link obeying the Chebotarev law. A typical example of such an infinite link is obtained from the figure-eight knot in S^3. It contains every type of knot as a component and admits Kopei's product formula. We observe an analogue of a quintic field as an example.

13:30 – 14:20 **Minoru Hirose** (Kyushu University)
Generalized double zeta values and modular forms

Double zeta values are real numbers defined by a certain infinite series which generalize the values of Riemann zeta function at positive integers. Modular forms are holomorphic functions on the upper half plane satisfying certain conditions. It is known that there are interesting

connections between the linear relations of double zeta values and modular forms for $SL(2,\mathbb{Z})$. Generalized double zeta values are certain sums of double zeta values, which appear as regularized iterated integrals or coefficients of KZ-associator. In this talk, we investigate a linear relation among generalized double zeta values and relate them to modular forms.

14:35 – 15:25 **Nobuo Sato** (Kyushu University)

On Deligne type basis for alternating multiple zeta values and the structure of the motivic Galois group of the mixed Tate motives over $\mathbb{Z}[1/2]$.

Deligne gave a basis of the $\mathbb{Q}$-vector space spanned by motivic alternating multiple zeta values (AMZV) using the theory of mixed Tate motives. In my talk, I would like to talk about my recent result that enhances Deligne's work, proving that Deligne's basis (with a slight modification) in fact gives a basis of the $\mathbb{Z}_{(2)}$ (the ring of rationals with odd denominators)-module spanned by motivic AMZV. Moreover, the proof is based on an explicit reduction procedure of a given AMZV into the basis by exploiting its 2-adic property, which in tern gives a complete set of relations among motivic AMZV. This result gives an affirmative answer to the $\mathbb{Z}[1/2]$-analog of the long-standing problem asking whether or not the Grothendieck 宴典eichmller group coincides with the motivic Galois group of the category of mixed Tate motives over $\mathbb{Z}$. This is a joint research work with Minoru Hirose at Kyushu University.

15:40 – 16:30 **Kenichi Bannai** (Keio University)

On the Shintani Generating Class of Algebraic Tori Associated to Totally Real Fields

The specialization at non-trivial torsion pints of the higher logarithmic derivatives of the rational function $G(t) = t/(1-t)$ gives special values of Lerch zeta functions for the rational number field. Since the Dirichlet L-functions for the rational number field can be expressed as a linear sum of Lerch zeta functions, $G(t)$ plays an important role in the study of special values of Dirichlet L-functions. For general totally real fields, a canonical generating function for special values of Lerch zeta function is not known. The speaker in a joint work with Kei Hagihara, Kazuki Yamada and Shuji Yamamoto succeeded by using the generating function for Shintani zeta functions of Lerch type defined by Shintani in defining canonical class which may be regarded as an analogue of $G(t)$ in the cohomology, equivariant with respect to the unit group, of a certain algebraic torus associated to a totally real field. One may define higher logarithmic derivatives of this class, and the specialization at non-trivial torsion points give special values of Lerch zeta functions for the totally real field. Since this class plays the role of special values of Lerch zeta functions, we call this class the Shintani generating class. Our result shows that in the higher dimensional case, it is effective to consider cohomology classes instead of rational functions. In this talk, we will explain the above results as well as the relation of the Shintani generating class to p-adic Hecke L-functions and p-adic polylogarithms of the totally real field.

December 11 (Wed)

9:30 – 10:20 **Noriko Hirata-Kohno** (Nihon University)
Recent Developments on Polylogarithms Conjecture

Let r, m be positive integers. Let $\mathrm{Li}_s(z)$ be the s-th polylogarithmic function with $s = 1, 2, , r$, namely,

$$\mathrm{Li}_s(z) = \sum_{k=0} 6\infty \frac{z^{k+1}}{(k+1)^s}, z \in \mathbb{C},\ |z| < 1.$$

Note $\mathrm{Li}_1(z) = -\log(1-z)$. We call polylogarithms the values of the function. Let $\alpha_1, , \alpha_m \in \mathbb{Q}$ be pairwise distinct rational numbers with $0 < |\alpha_j| < 1$ $(1 \leq j \leq m)$.

In this talk, we show a new criterion for the linear independence of polylogarithms over $\mathbb{Q}$. Whenever the points $\alpha_1, , \alpha_m \in \mathbb{Q}$ are "sufficiently closed" to the origin, then the $rm+1$ numbers: $\mathrm{Li}_1(\alpha_1), \mathrm{Li}_2(\alpha_1), , \mathrm{Li}_r(\alpha_1), \mathrm{Li}_1(\alpha_2), \mathrm{Li}_2(\alpha_2), , \mathrm{Li}_r(\alpha_2), , , \mathrm{Li}_1(\alpha_m), \mathrm{Li}_2(\alpha_m), , \mathrm{Li}_r(\alpha_m)$, and 1 are linearly independent over $\mathbb{Q}$. This is the first result that gives a condition for the linear independence of values of the r polylogarithmic functions $\mathrm{Li}_1(z), , \mathrm{Li}_r(z)$ at several m distinct rational points. We also generalize our results to the s-th Lerch transcendent function and in an algebraic number field case:

$$\Phi_s(x, z) = \sum_{k=0}^{\infty} \frac{z^{k+1}}{(k+x+1)^s}, z \in \overline{\mathbb{Q}},\ z \text{ arithmetically very closed to } 0$$

We note that the relation

$$\log\left(1 - \frac{1}{3}\right) + \log\left(1 + \frac{1}{2}\right) = 0$$

shows the indispensability of the condition that the points are "arithmetically very closed" to the origin, that we will precise in the talk. Our statement is related to the polylogarithms conjecture of R. Murty as well as observations by Y. Andr ' and G. V. Chudnovsky. Our main tool relies on Padé approximation.

10:35 – 11:25 **Yoshiaki Okumura** (Tokyo Institute of Technology)
On K-virtual Drinfeld modules and Drinfeld modular curves

As is well-known, there are many beautiful analogues between number fields and function fields. For an algebraic extension K of the rational function field $\mathbb{F}_q(T)$ over a finite field, we introduce the notion of K-virtual Drinfeld modules as a function field analogue of $\mathbb{Q}$-curves, which are elliptic curves over $\bar{\mathbb{Q}}$ isogenous to all its Galois conjugates. In this talk, we explain that all non-CM rank-two K-virtual Drinfeld modules are parametrized up to isogeny by K-rational points of the quotient curve $Y_*(\mathfrak{n})$ of the Drinfeld modular curve $Y_0(\mathfrak{n})$ with some square-free level $\mathfrak{n}$ by all Atkin-Lehner involutions. This is an analogue of Elkies' well-known result on $\mathbb{Q}$-curves.

11:40 – 12:40 **Seidai Yasuda*** (Osaka University)
The Langlands correspondence for global function fields after Vincent Lafforgue

12:40 – Meeting of the steering committee (mainly for the committee members)

December 12 (Thu)

10:00 – 10:50 **Kazuki Yamada** (Keio University)
Rigid analytic reconstruction of Hyodo-Kato theory

In this talk, I will give a new and intuitive construction of Hyodo-Kato cohomology and Hyodo-Kato map, based on logarithmic rigid geometry. This is useful for explicit computation, and realizes a natural interpretation of the dependence of Hyodo-Kato map. More precisely, our Hyodo-Kato map depends only on the choice of a branch of p-adic logarithm, but not on the choice of a uniformizer. This is a joint work with Veronika Ertl (Regensburg University).

11:05 – 12:05 **Teruhisa Koshikawa*** (RIMS)

Recent developments in integral p-adic Hodge theory

I will give an overview of the recent works of Bhatt-Morrow-Scholze and Bhatt-Scholze on geometric aspects of p-adic Hodge theory. They introduced new cohomology theories of p-adic formal schemes (smooth over the base), and in particular obtained "geometric realization" of Breuil-Kisin modules. Their results may be regarded as integral refinements of previously known comparison theorems in p-adic Hodge theory.

13:30 – 14:20 **Yuya Murakami** (Tohoku University)
Continuity of the values of j-function at real quadratic points and its extension

In 2009, Professor Masanobu Kaneko defined the value "val(w)" of the elliptic modular j-function at any real quadratic point w. The values val(w) when w goes over all real quadratic points satisfy some continuity with respect to the continued fraction expansions of real quadratic points. This kind of continuity was conjectured by Professor Masanobu Kaneko in 2009 and proved by Bengoechea-Imamoglu in 2018. The speaker refined BengoecheaImamoglu 窶冴 method and proved a kind of continuity in a more general setting. As a result, it turns out that val is away from the continuity in the Euclidean topology. In this talk, I will introduce the results as mentioned above.

14:35 – 15:25 **Yuki Yamamoto** (University of Tokyo)
On the types for supercuspidal representations of inner forms of GL_N

Let G be the multiplicative group of a central simple algebra over a non-Archimedean local field. When we consider smooth representations of G, the theory of types is useful. A type is an irreducible smooth representation of some compact open subgroup in G, which can classify irreducible representations of G in a certain sense. I will explain types and discuss the existence and uniqueness of types. In particular, I will show that for an irreducible supercuspidal representation π, $[G, \pi]_G$-types defined over some maximal compact subgroup in G are unique up to G-conjugation under some unramifiedness assumption on a simple stratum for π.

15:40 – 16:30 **Yoshiyasu Ozeki** (Kanagawa University)
Torsion of algebraic groups and Lubin-Tate extensions

In this talk, we give some finiteness properties for torsion points of commutative algebraic groups over a p-adic field with values in a finite extension of the Lubin-Tate extension of a p-adic field. The main theorem is a straightforward generalization of the Imai's theorem shown in 1975, which gives a finiteness of torsion points of abelian varieties with values in cyclotomic $\mathbb{Z}_p$-extensions.

18:00 – 20:00 Conference Banquet

December 13(Fri)

10:00 – 10:50 **Kazumi Kasaura** (University of Tokyo)
On extension of overconvergent log isocrystals on log smooth varieties

By works of Kedlaya and Shiho, it is known that, for a smooth variety over a field of positive characteristic and a simple normal crossing divisor on it, overconvergent isocrystals on the complement of the divisor satisfying a certain monodromy condition can be extended to convergent log isocrystals on the whole variety equipped with the log structure associated to the divisor. In this talk, we explain a generalization of this result: for a log smooth variety satisfying some conditions, overconvergent log isocrystals on the trivial locus of a direct summand of the log structure satisfying a certain monodromy condition can be extended to convergent log isocrystals on the whole log variety.

11:05 – 12:05 **Shun Ohkubo*** (Nagoya University)
Logarithmic growth filtrations for (φ, ∇)-modules over the bounded Robba ring

Around 1970, Dwork proved that for a certain class of p-adic ordinary linear differential equations over p-adic open discs, the coefficients of the power series solutions satisfy a "log-growth"

condition. Around 2010, this theory was reconsidered and developed by Andre, Chiarellotto, Kedlaya, Tsuzuki.... In this talk, we explain recent progresses on this topic due to the above people and the speaker.

13:30 – 14:20 **Daichi Takeuchi** (University of Tokyo)
Epsilon factors and Micro-Local Analysis of l-adic sheaves on Varieties

For l-adic sheaves on varieties over finite fields, the constant terms of the functional equations of the L-functions, called global epsilon factors, are important arithmetic invariants. When the varieties are curves, Deligne and Laumon show that they admit product formulae in terms of local epsilon factors.

In this talk, I will explain that, attaching some coefficients to irreducible components of singular supports, we can define refinements of characteristic cycles.

We will see that, after taking modulo roots of unity, they give product formulae of global epsilon factors for higher dimensional varieties.

I will also explain that these results can be generalized to arbitrary perfect fields of any characteristic.

14:35 – 15:25 **Benjamin Collas** (University of Bayreuth/RIMS)
Arithmetic and Motives for Moduli Stacks of Curves

Due to the arithmetic of the Knudsen-Mumford stratification, the tower of moduli stacks of curves is a key object of geometric Galois theory and of the Tannakian category of mixed Tate motives. The goal of this talk is to present a similar perspective with respect to the stack stratification of the spaces. As a motivation, we will first present how the first cyclic stack inertia strata are endowed with a Galois action of Tate-type, then how Artin-Mazur and Morel-Voevodsky simplicial and homotopical theories provide a fruitful context for some "stack" motivic decomposition and Tannakian results that reflect the arithmetic side. In genus 0, this approach leads in particular to an interpretation of the mixed Tate motivic Galois group as a certain loop-group, and to the definition of computable (hidden) periods of stack nature.

15:40 – 16:30 **Ippei Nagamachi** (University of Tokyo)
On the Hom version of the Grothendieck Conjecture for hyperbolic polycurves of dimension 2

In this talk, we treat the Hom version of the Grothendieck Conjecture for hyperbolic polycurves of dimension 2. We group theoretically characterize dominant morphisms from regular varieties to hyperbolic polycurves of dimension 2 in some sense. Also, we show that any open group homomorphism between their fundamental groups is induced by a morphism of varieties if the Grothendieck Section Conjecture holds. If time permits, we give a counterexample to the pro-p version of the Grothendieck Conjecture for hyperbolic polycurves of dimension 2.

16:30 – 16:40 Closing

* Invited speakers

Program Committee:
Manabu Ozaki (Waseda University),
Iwao Kimura (Toyama University),
Naoki Imai (University of Tokyo)