Dynamic Labor Supply: Human Capital

Thomas H. Jørgensen

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Plan for today

- Dynamic Labor supply w/w.o. **human capital** (HC)
  Keane (2016)
  - Reduced-form estimation, bias from HC
  - Elasticities, HC affect relation between them
  - Age effects
  - Simulate
- Related literature Imai and Keane (2004); Keane and Wasi (2016).
- "Human capital" = "learning by doing" here
  educational choices etc. not the focus *here*

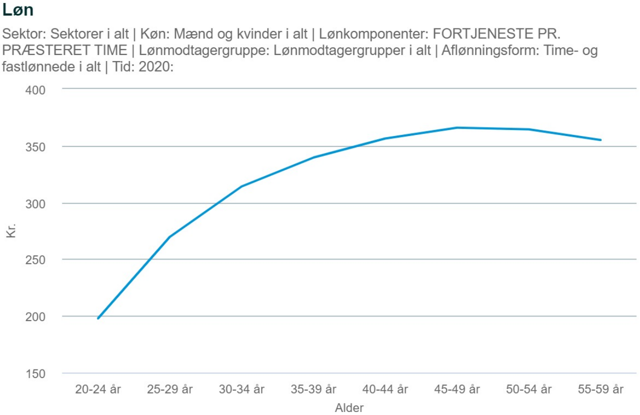
Empirical Motivation: I

- **From Arrow (1962):**
  Lundberg (1961, pp. 129-133) has given the name " Horndal effect" to a very similar phenomenon. The Horndal iron works in Sweden had no new investment (and therefore presumably no significant change in its methods of production) for a period of 15 years, yet productivity (output per manhour) rose on the average close to 2 % per annum. We find again steadily increasing performance which can only be imputed to learning from experience.

Empirical Motivation: II

- **Wages vary** over life...
  could be due to learning by doing (HC)

Figure: Hourly Wage over the Life Cycle in Denmark (LONS50).



Empirical Motivation: III (Motivation for Keane, 2016)

- **Low Frisch** elasticities estimated in large literature (Keane, 2011)

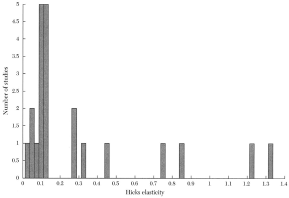


Figure 5. Distribution of Hicks Elasticity of Substitution Estimates

Note: The figure contains a frequency distribution of the twenty-two estimates of the Hicks elasticity of substitution discussed in this survey.

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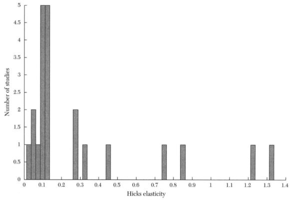

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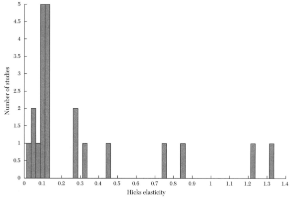

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- **Basic life-cycle model**: Frisch is *upper bound* of Hicks and Marshall
  $\rightarrow$ taxes hardly distort behavior
  $\rightarrow$ optimal tax rates are high
- **Could be erroneous conclusion**: Human capital
  $\rightarrow$ Value of time $\neq$ net-of-tax wage
  $\rightarrow$ Frisch might not be upper bound
  $\rightarrow$ Elasticities vary with age (i.e. estimation sample important)

Simple Life-Cycle Model (Keane, 2016)

- Workers chose throughout life, $t = 1, \dots, T$
  - consumption, $c_t > 0$
  - labor hours worked, $h_t \geq 0$

- To maximize the discounted value

$$\frac{c_1^{1+\eta}}{1+\eta} - \beta \frac{h_1^{1+\gamma}}{1+\gamma} + \sum_{t=2}^{T} \rho^{t-1} \left(\frac{c_t^{1+\eta}}{1+\eta} - \beta \frac{h_t^{1+\gamma}}{1+\gamma} \right)$$

  where
  - $\rho \in (0, 1)$ is the discount factor
  - $\eta \leq 0$ is the CRRA
  - $\gamma \geq 0$ is the curvature wrt. hours worked
  - $\beta > 0$ is the strength of the dis-utility of work

- No uncertainty - perfect foresight

Simple Life-Cycle Model, Keane (2016)

- **Inter-temporal budget** constraint is

$$a_{t+1} = (1+r)\left(a_t + (1-\tau_t)w_t h_t - c_t\right),\ a_0 = 0$$

where

  - r is the real interest rate
  - τ_t is the tax rate

- **Human capital** accumulation,

$$k_{t+1} = k_t + h_t,\ k_0 = 0$$

- **Endogenous wages**, $h_t \rightarrow k_{t+1} \rightarrow w_{t+1}$,

$$\begin{aligned} w_t &= w\left(1 + \alpha k_t\right) \\ &= w\left(1 + \alpha \sum_{s=1}^{t-1} h_s\right) \end{aligned}$$

Optimal Consumption

- We will assume $\rho(1+r)=1$
  $\rightarrow$ consumption is perfectly smoothed across periods, $c_t = C\ \forall t$.
- C can be found from the life-time constraint that the NPV of consumption must equal the NPV of resources,

$$\sum_{t=1}^{T}\frac{c_t}{(1+r)^t} = \sum_{t=1}^{T}\frac{w_t(1-\tau_t)h_t}{(1+r)^t}$$

$$C(1+r)^T\sum_{t=1}^{T}(1+r)^{-t} = (1+r)^T\sum_{t=1}^{T}w_t(1-\tau_t)h_t(1+r)^{-t}$$

$$C = \frac{\sum_{t=1}^{T}w_t(1-\tau_t)h_t(1+r)^{T-t}}{\sum_{t=1}^{T}(1+r)^{T-t}} \tag{1}$$

MRS and Human Capital: General

- **Bellman equation** formulation

$$V_t(a_t, k_t) = \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta \frac{h_t^{1+\gamma}}{1+\gamma} + \rho V_{t+1}(a_{t+1}, k_{t+1})$$

- **First order conditions**

$$c_t^{\eta} - \rho(1+r) V_{t+1}^1(a_{t+1}, k_{t+1}) = 0$$
$$-\beta h_t^{\gamma} + \rho V_{t+1}^2(a_{t+1}, k_{t+1}) + \rho(1+r)(1-\tau_t) w_t V_{t+1}^1(a_{t+1}, k_{t+1}) = 0$$

- **The MRS** is, using $\rho(1+r) = 1$ and thus $c_t = C$,

$$\beta h_t^{\gamma} / C^{\eta} = w_t(1-\tau_t) + \rho V_{t+1}^2(a_{t+1}, k_{t+1}) / C^{\eta}$$

- Last term is related to the endogeneity of wages from human capital

Solution

- **No analytical solution** for optimal hours, $h_t^\star(a_t, k_t)$
- **Our tools can be used** to solve for this function
  Backwards induction
  Interpolation
- **See notebook.**

Elasticities

- **Without human capital** and non-labor income ($\alpha = 0$, $N = 0$), we had

$$\underbrace{e_F}_{\frac{1}{\gamma}} \geq \underbrace{e_H}_{\frac{1}{\gamma-\eta}} \geq \underbrace{e_M}_{\frac{1+\eta}{\gamma-\eta}}$$

- How does the elasticities now look like?
(The inequalities might not hold for all parameter values!)

OCT and Human Capital

- The Marginal Rate of Substitution (MRS) condition is here

$$\underbrace{\beta h_t^{\gamma} / C^{\eta}}_{\text{OCT}} = \underbrace{w_t(1-\tau_t)}_{\text{wage}} + \underbrace{\alpha w F_t}_{\text{HC}} \tag{2}$$

where

$$F_t = \sum_{s=t+1}^{T} \frac{h_s(1-\tau_s)}{(1+r)^{s-t}},\ F_T = 0$$

measures the NPV of future human capital investments.

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- Taking logs of eq. (2) gives

$$\log h_t = \frac{1}{\gamma}\log[w_t(1-\tau_t) + \alpha w F_t] + \frac{\eta}{\gamma}\log C - \frac{1}{\gamma}\log\beta \tag{3}$$

OCT and Human Capital

Figure: Opportunity Cost of Time and Human Capital, Keane (2016).

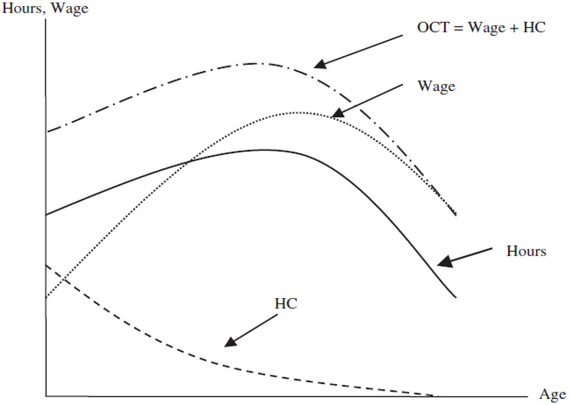

Fig. 1. *Hours, Wages and Price of Time over the Life-cycle*

Notes. This Figure plots the components of the first-order condition for labour supply generated by the life-cycle model with human capital: $\beta h_t^{\gamma}/C^{\eta} = w_t(1-\tau_t) + \alpha w F_t$. Here, *Wage* $\equiv w_t(1-\tau_t)$, *the 'human capital term' HC* $\equiv \alpha w F_t$, *and the 'opportunity cost of time' OCT* $\equiv$ *Wage* + *HC*. Note that the term HC captures the return to an hour of work experience, in terms of increased present value of future wages.

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- **A:** *transitory* tax-changes is a smaller part of the OCT for young $\rightarrow$ **Frisch elasticity is lower** (compared to when $\alpha = 0$)

$$e_{F,t} \equiv \left.\frac{\partial \log h_t}{\partial \log(1-\tau_t)}\right|_{dC=0} = \frac{1}{\gamma} \frac{w_t(1-\tau_t)}{w_t(1-\tau_t) + \alpha w F_t} < \frac{1}{\gamma}$$

with $F_t \rightarrow 0$ for $t \rightarrow T$.

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- **Derivation:** taking logs of eq. (2) and *partial* derivative wrt. $(1-\tau_t)$

$$\log h_t = \frac{1}{\gamma} \log[w_t(1-\tau_t) + \alpha w F_t] + \frac{\eta}{\gamma} \log C - \frac{1}{\gamma} \log \beta$$

such that

$$\frac{\partial \log h_t}{\partial \log(1-\tau_t)} = \frac{\partial \log h_t}{\partial (1-\tau_t)} \underbrace{\frac{\partial(1-\tau_t)}{\partial \log(1-\tau_t)}}_{=(1-\tau_t)} = \frac{1}{\gamma} \frac{w_t}{w_t(1-\tau_t) + \alpha w F_t}(1-\tau_t)$$

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$$e_{M,t} \equiv \left. \frac{\partial \log h_t}{\partial \log(1-\tau)} \right| = \frac{1 + \eta \cdot E_t / C}{\gamma - \eta \cdot C_t^* / C}$$

where

$$E_t = \sum_{s=t}^{T} \frac{w_s(1-\tau)}{(1+r)^{s-t}}$$
$$C_t^* = w_t(1-\tau)h_t + \alpha w h_t F_t = OCT_t \cdot h_t$$

is the PV of after-tax earnings and “effective earnings”, respectively.

- Note $e_{M,t} \rightarrow \frac{1}{\gamma} = e_F$ for $t \rightarrow T$ since $E_T = C_T^* = 0$.

Marshall vs. Frisch Elasticity when $\alpha > 0$

- **We might have a violation** of the old inequality

$$e_F \overset{?}{\geq} e_M$$

- **Frisch is only an upper bound** if

$$\frac{1}{\gamma}\frac{w_t(1-\tau_t)}{w_t(1-\tau_t)+\alpha wF_t} \geq \frac{1+\eta \cdot E_t/C}{\gamma - \eta \cdot C_t^*/C}$$

  which – if there is no income effects ($\eta = 0$) – is satisfied only if

$$\alpha = 0.$$

- **They converge** in age:
  $e_{M,t} \rightarrow \frac{1}{\gamma} = e_F$ for $t \rightarrow T$ since $E_T = C_T^* = 0$.

Reduced-form Estimation

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- Where $F_t < F_{t-1}$.

  $\rightarrow \left[\bullet \right] < \Delta \log(w_t(1-\tau_t))$, which is the included regressor

  $\rightarrow \frac{1}{\hat{\gamma}} < \frac{1}{\gamma}$, as variation $\Delta \log h_t$ is "unchanged"/"fixed" by data

  $\rightarrow \hat{e}_F < e_F$ (downwards bias in Frisch, which might not upper bound)

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- We can do **structural estimation**, but we must specify everything... Reduced form nice since we can remain "agnostic" about wage process.

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  Re-write the "true" equation

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$$= \frac{1}{\gamma}\Delta \log(w_t(1-\tau_t)) + \underbrace{f(w_t, w_{t-1}, \tau_t, \tau_{t-1}, F_t, F_{t-1}) + u_t}_{=\epsilon_t\text{: error-term in standard regression}}$$

  Any *relevant* instrument, Z_t, with $Cov(w_t(1-\tau_t), Z_t) \neq 0$ will also be ***invalid***, as $Cov(\epsilon_t, Z_t) \neq 0$!

Simulated Elasticities

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- **Short run:**
  Current period response
  Like the Frisch, Marshall and Hicks
- **Long run:**
  Average effect from permanent increase throughout life

- See notebook after discussion

Simulated Elasticities: Short Run

- **Short Run**:
  Response in period t to a transitory change in period t

Table 3

Short-run Labour Supply Responses to Taxes in the Imai–Keane Model

	Transitory		Permanent (unanticipated)	
Age	Unanticipated	Anticipated (Frisch)	Uncompensated (Marshall)	Compensated (Hicks)
20	**0.30**	**0.30**	0.14	**0.64**
25	**0.36**	**0.36**	0.12	**0.54**
30	**0.44**	**0.44**	0.12	**0.48**
35	0.52	0.52	0.10	0.46
40	0.64	0.66	0.14	0.46
45	0.76	0.84	0.20	0.56
50	0.94	1.06	0.46	0.84
55	1.24	1.44	1.06	1.44
60	**1.74**	**1.96**	1.88	**2.09**

Notes. All figures are elasticities of current hours with respect to tax changes. The 'transitory' increase only applies for one year at the indicated age. In the 'anticipated' case this has no wealth effect, so it is a pure Frisch effect. The 'permanent' tax increases take effect (unexpectedly) at the indicated age and last until age 65. In the 'compensated' case the proceeds of the tax (in each year) are distributed back to agents in lump sum form. Figures in bold are cases where permanent tax effects exceed transitory tax effects.

- **Bold:** $e_H > e_F$ (unlike baseline model)

Simulated Elasticities: Long Run

- **Long Run**:
  Response in period t to a permanent change from period 20 until 65 ("regime shift". Could alternatively have been from age t to 65)

Table 4

Lifetime Effects of a Permanent Tax Increase on Labour Supply

Age	Uncompensated	Compensated
20	0.14	0.64
30	0.14	0.66
40	0.18	0.84
45	0.24	1.14
50	0.42	1.74
60	1.82	4.00
Lifetime hours (Ages 20–65)	0.40	1.32

Notes. This Table compares the baseline simulation of the Imai and Keane (2004) model with an alternative scenario where the tax rate on earnings is permanently higher. The increase is in effect from the first period (age 20) until the terminal period (age 65). The Table reports both the uncompensated case and the case where the proceeds of the tax (in each year) are distributed back to agents in lump sum form.

Next Time

- **Next time:**
  Labor supply and children.

- **Literature:**
  Adda, Dustmann and Stevens (2017): "The Career Costs of Children"
  - **Read** before lecture
  - **Reading guide:**
    Section 1: Introduction. Key
    Section 2: Data. Skim fast.
    Section 3: Model. *Key*, but complex. Get the idea.
    Section 4: Results. *Simulations in sections E, F and G are key*!

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