TTK4215 - Adaptive Control
Introduction to Reinforcement Learning

Norwegian University of Science and Technology (NTNU)

20. November, 2018

Machine Learning

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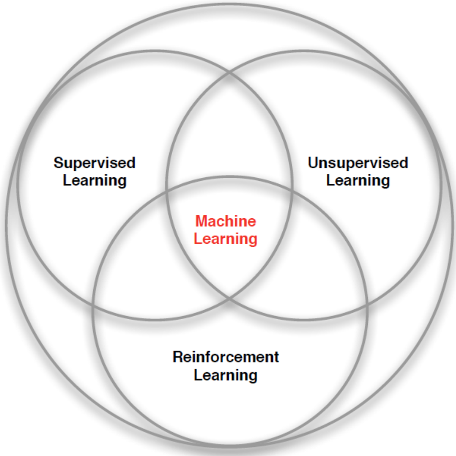

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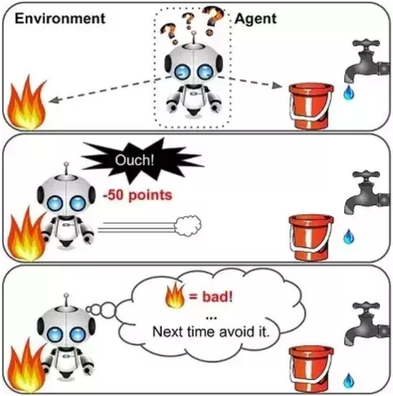

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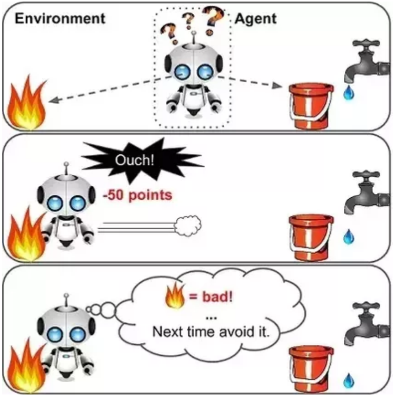
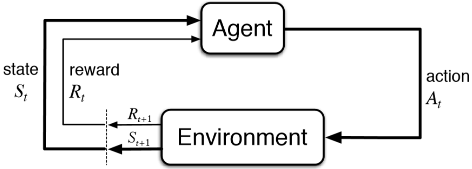


Example Video: Robot walking.

States, Actions and Rewards



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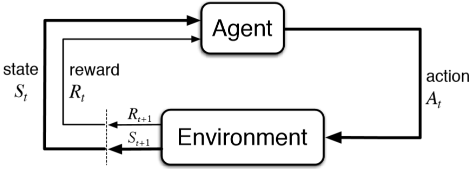

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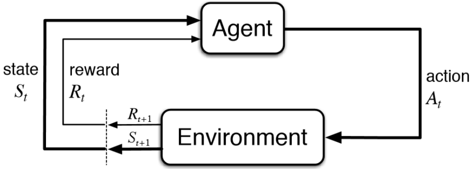

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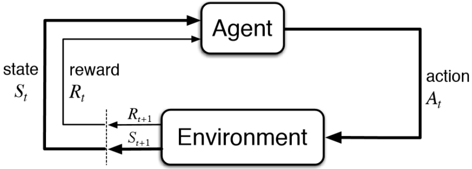

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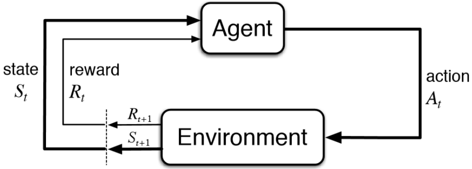

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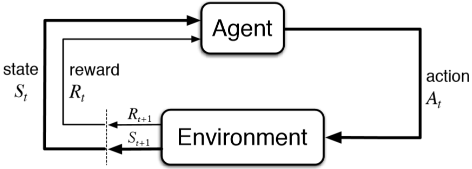

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All goals can be described by the maximization of expected cumulative reward.

The agent's goal in Reinforcement Learning: Maximize total future reward.

States, Actions and Rewards: Examples

Example (Playing Chess)

- State: The organization of pieces on the board
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Example (Making a robot walk)

- State: Orientation (and position) of robot and its limbs
- Action: Moving the joints / limbs
- Reward: +/- for forward motion / falling over

Markov Decision Process

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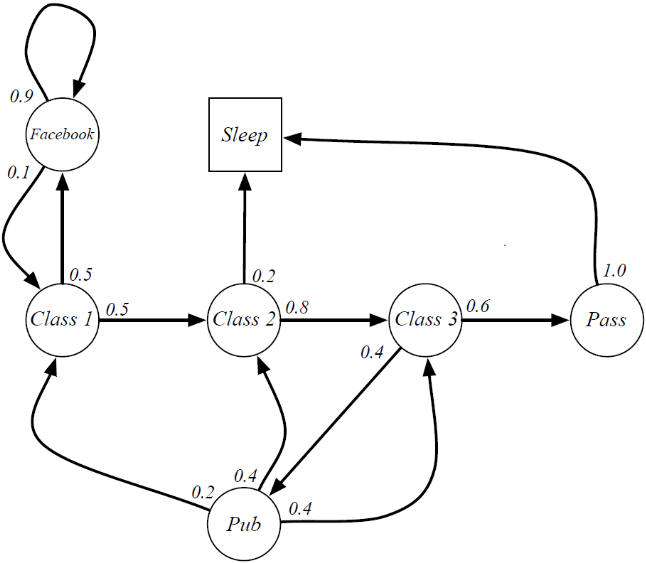
A discrete stochastic process, a *Markov chain* with rewards and actions.

Markov Property

The probability of reaching s' from s dependes only on s and not on the history of earlier states.

Markov Decision Process



(Example from David Silver's "Lecture 2: Markov Decision Processes")

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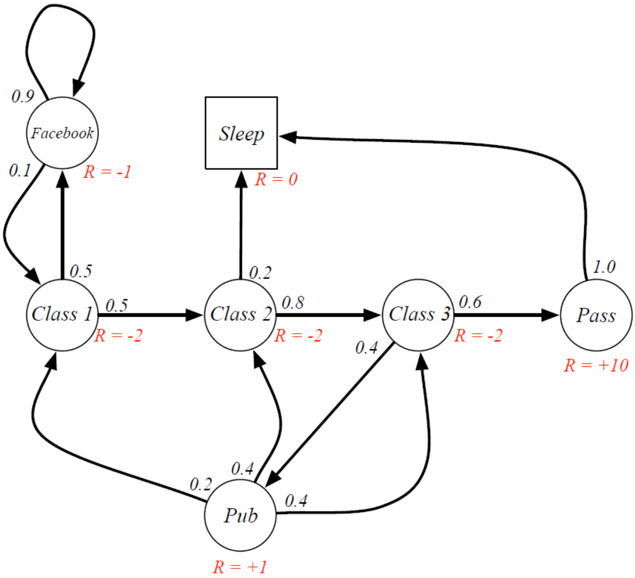

Markov Decision Process

Deterministic or Stochastic

The state transition and reward functions can also be deterministic:

$$s' = \delta(s, a)$$
$$r = R(s, a, s') = R(s, a, \delta(s, a)) \triangleq r(s, a)$$

For now, think about the state transition function/probabilities as a large multi-dimensional table.

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A RL control problem is the problem of finding an optimal policy. Optimal in what sense?

Return and value function

Imagine that you are following some policy $\pi(a \mid s)$ producing a state, action, reward sequence $(s, a, r)_t$.

Definition

The **return** G_t is the total future discounted reward from time-step t:

$$G_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k}. \tag{2.2}$$

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Definition

The **value function** $V^\pi(s)$ is the expected return starting from state s when following policy π:

$$V^\pi(s) = \mathbb{E}_\pi[G_t \mid s_t = s]. \tag{2.4}$$

Optimal Policy and Value Function

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The optimal policy $\pi^*(s)$ is the policy that maximizes $V^\pi(s)$ for all states s:

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Bellman Equation for $V^*(s)$

We can rewrite a little bit:

$$V^*(s) = \max_{a \in A(s)} \left(R(s,a) + \gamma \sum_{s'} P(s' \mid s,a) V^*(s') \right) \tag{2.7}$$

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This is called a **Bellman equation**. Can solve using iterative dynamic programming methods if we know $R(s)$ and $P(s' \mid s, a)$!

Action-Value (Q) Function

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Let's do a little more rewriting..

Definition

The action-value function (Q-function) $Q(s,a)$ is the expected reward of taking action a in state s *plus* the expected return of following the optimal policy $\pi^*(s)$ thereafter:

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The following relations hold:

Definition

$$V^*(s) = \max_a Q(s,a) \tag{2.11}$$

$$\pi^*(s) = \text{argmax}_a Q(s,a) \tag{2.12}$$

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Another Bellman equation:

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Recall:

$$\pi^*(s) = \text{argmax}_a Q(s, a) \tag{2.15}$$

This shows that if we learn $Q(s, a)$, we know how to select optimal actions without knowing the system transition model or the reward function! We will make use of this fact later (Q-Learning).

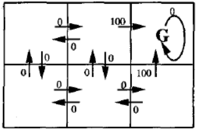
Theorem

For any Markov Decision Process:

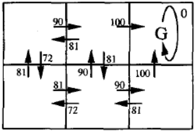
- *There exists an optimal policy $\pi^*(s)$ that is better than or equal to all other policies $\pi(s)$*
- *All optimal policies achieve the optimal value function, $V^{\pi^*}(s) = V^*(s)$*
- *All optimal policies achieve the optimal action-value function, $Q^{\pi^*}(s,a) = Q^*(s,a)$*

- The optimal policy can easily be computed if you know V^* or Q^*.

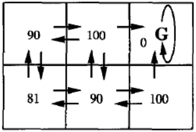
Example



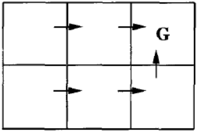
$r(s, a)$ (immediate reward) values



$Q(s, a)$ values



$V^*(s)$ values



One optimal policy

Value iteration

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5. The sequence V_i generated by value iteration converges to the optimal value function V^*.

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3. Note that we have to know the reward function and state transition probabilities.
4. This update step is called a Bellman update.
5. The sequence V_i generated by value iteration converges to the optimal value function V^*.
6. The optimal policy can be found from the optimal value function V^*.

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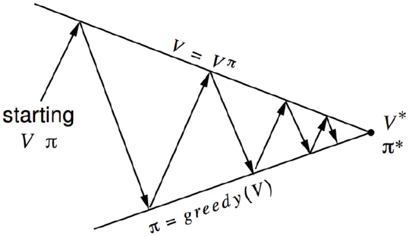

Learning to Act In an Unknown Environment

In the previous section, when doing value iteration and policy iteration to solve the MDP, the state transition function and the reward function where *known*.

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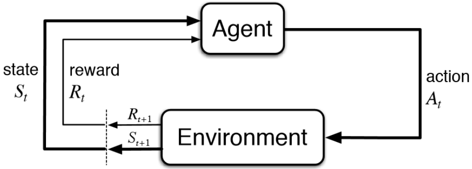

What if the system model and reward is unknown?

What are we going to learn?

Different Agent Designs

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We are going to focus on a *model-free* method, called *Q-Learning* that learns an action-value representation $\hat{Q}(s, a)$ of $Q(s, a)$.

Q-Learning

Recall the Bellman equations for Q:

$$Q(s,a) = r(s,a) + \gamma \max_{a'} Q(s',a') \tag{3.1}$$

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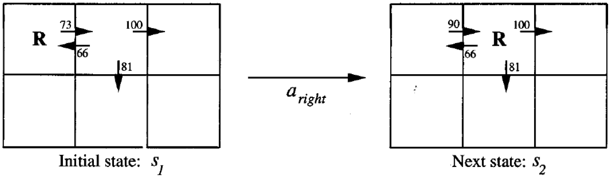
What action to take? Exploration vs exploitation. E.g. act greedy, but leave some probability to explore (random action).

Illustration



$$
\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \ \max\{66, 81, 100\} \\
&\leftarrow 90
\end{aligned}
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Convergence of Q-Learning

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Assumptions

- Bounded rewards.
- Every state-action pair is visited infinitely often.
- Conditions on α_t

Might need *a lot* of iterations to converge!

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So far: $V(s)$ or $Q(s,a)$ stored as a look-up table.

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θ can be updated using e.g. gradient descent or least squares methods.

Example

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- Continuous state-space discretized into *boxes*.

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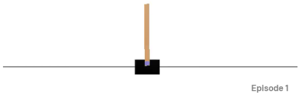
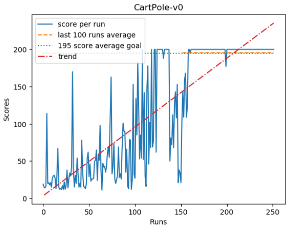
- Learning to control the CartPole (Python code on Blackboard).
- Continuous state-space discretized into *boxes*.
- Guided practical exercise: Exchange Q-table with function approximator (neural net) using TensorFlow and Keras.





Further Reading

RL:

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Deep RL:

- OpenAI Spinning Up
- YouTube videos and slides from Deep Reinforcement Learning course by Sergey Levine at UC Berkeley