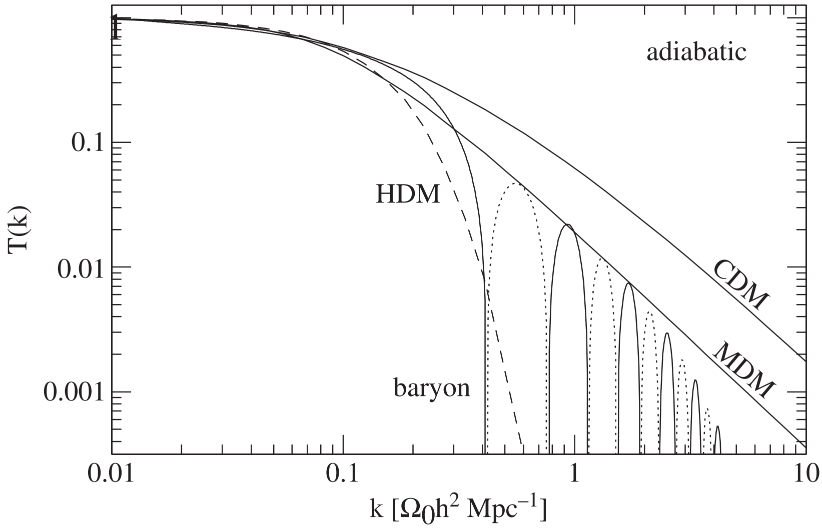

Fig. 4.2. The transfer functions for adiabatic perturbations calculated with the CMBFAST code (Seljak & Zaldarriaga, 1996): Results are shown for a purely baryonic model [the dotted parts of the curve indicate negative values of $T(k)$], for a CDM model, a HDM model, and a so-called mixed dark matter (MDM) model, consisting of 30% HDM and 70% CDM.

$$\lambda_{\rm H} = a \int_0^t \frac{c\,{\rm d}t}{a} = a\tau = 2ct \tag{4.243}$$

(see §3.2.4), we see that all sub-horizon perturbations are smaller than the Jeans length. Therefore, as soon as an adiabatic baryon perturbation enters the horizon, it starts to oscillate due to the large pressure of the photon–baryon fluid. These oscillations continue until recombination, after which the perturbations start to grow via gravitational instability. However, this only applies for fluctuations with sizes larger than the Silk damping scale; fluctuations on smaller scales will have damped out before recombination.

Detailed calculations of the transfer function for adiabatic baryon models (i.e. models with isentropic initial perturbations in the baryons and photons that evolve adiabatically) have been carried out by Peebles (1981). An example of the post-recombination transfer function is shown in Fig. 4.2, along with several other transfer functions to be discussed below. On scales $k > k_{\rm eq}$ the transfer function drops rapidly due to the horizon effect and due to Silk damping. The oscillations on these scales in the post-recombination transfer function reflect the phases at recombination of the perturbations that have not been entirely damped. The deep troughs (between the solid and dotted peaks) reflect the scales on which this phase happens to be such that $\delta_{\rm b} = 0$, and are separated by $\Delta k \sim \pi a(t_{\rm rec})/(c_{\rm s} t_{\rm rec}) \sim 0.3(\Omega_{{\rm b},0} h^2){\rm Mpc}^{-1}$.

Because of Silk damping, structure formation models based on isentropic, baryonic perturbations require large initial fluctuations in order to be able to form structures with masses comparable to the damping scale ($M \sim M_{\rm d} \sim 10^{14}\,{\rm M}_\odot$). As we will see in Chapter 5, nonlinear structures form when their corresponding perturbations have grown to $\delta_{\rm m} \sim 1$. Since $\delta_{\rm m}$ grows with a rate $\propto (1+z)$ in an EdS universe (or slower if $\Omega_0 < 1$) during the matter dominated era, this implies amplitudes of the order of $\delta_{\rm m} \gtrsim 10^{-3}$ at $z \sim z_{\rm rec}$. In the case of isentropic perturbations, the temperature fluctuations in the photon field are related to the density fluctuations as

$$\frac{\delta T}{T} = \frac{1}{4}\delta_\gamma = \frac{1}{3}\delta_{\rm b}. \tag{4.244}$$