

# Morphotactics as Tier-Based Strictly Local Dependencies

Alëna Aksënova   Thomas Graf   Sedigheh Moradi

Department of Linguistics  
Stony Brook University  
Stony Brook, NY 11794, USA

\*mail@thomasgraf.net

alena.aksenova@stonybrook.edu, sedigheh.moradi@stonybrook.edu

## Abstract

It is commonly accepted that morphological dependencies are finite-state in nature. We argue that the upper bound on morphological expressivity is much lower. Drawing on technical results from computational phonology, we show that a variety of morphotactic phenomena are tier-based strictly local and do not fall into weaker subclasses such as the strictly local or strictly piecewise languages. Since the tier-based strictly local languages are learnable in the limit from positive texts, this marks a first important steps towards general machine learning algorithms for morphology. Furthermore, the limitation to tier-based strictly local languages explains typological gaps that are puzzling from a purely linguistic perspective.

## 1 Introduction

Different aspects of language have different levels of complexity. A lot of recent work in phonology (see Graf (2010), Heinz (2011a; 2011b; 2015), Chandlee (2014), Jardine (2015) and references therein) argues that phonological well-formedness conditions are subregular and hence do not require the full power of finite-state automata. This is particularly noteworthy because computational phonology still relies heavily on finite-state methods (Kaplan and Kay, 1994; Frank and Satta, 1998; Riggle, 2004). A similar trend can be observed in computational syntax, where the original characterization as mildly context-sensitive string languages (Huybregts, 1984; Shieber, 1985) is now being reinterpreted in terms of subregular tree languages (Graf, 2012; Graf and Heinz, 2015). Curiously missing from these investigations is morphology.

While linguistic theories sometimes consider morphology a part of syntax, computational morphology recognizes that morphology is much closer to phonology than syntax regarding weak generative capacity. Consequently, computational morphology involves largely the same finite-state methods as computational phonology (Koskeniemi, 1983; Karttunen et al., 1992). This raises the question whether morphology, just like phonology, uses only a fraction of the power furnished by these tools. A positive answer would have important repercussions for linguistics as well as natural language processing. The subregular classes identified in computational phonology are learnable in the limit from positive text (Heinz et al., 2012), so a subregular theory of morphology would greatly simplify machine learning while also explaining how morphological dependencies can be acquired by the child from very little input. A subregular model of morphology would also be much more restricted with respect to what processes are predicted to arise in natural languages. It thus provides a much tighter typological fit than the regular languages. In this paper, we argue that the subregular view of morphology is indeed correct, at least for morphotactics.

Morphotactics describes the syntax of morphemes, that is to say, their linear order in the word and the conditions that license their presence or enforce their absence. One can distinguish *surface morphotactics* from *underlying morphotactics*. The former regulates the shape of the pronounced surface strings, whereas the latter is only concerned with the arrangements of the morphemes in the initial representation rather than how said morphemes are realized in specific environments. We only consider underlying morphotactics in this paper.

The following example may clarify the distinction. In German, the past participle of a verb is

formed via a circumfix. The first part of the circumfix is always the prefix *ge-*, whereas the second part may be the suffix *-en* or *-t* depending on the verb stem. In addition, the suffixes can also occur on their own, e.g. on infinitives or the third person singular form of the verb. Surface morphotactics thus has to ensure that *ge-* always appears with one of these two suffixes, and that the form of the suffix matches the stem. At the same time, it does not need to worry about matching *-en* or *-t* with *ge-* since these forms can occur independently as realizations of different morphemes. Underlying morphotactics, on the other hand, is unaware of the surface realizations and only knows that some abstract prefix *GE-* must always occur with the abstract suffix *-EN*, and the other way round. The fact that *-EN* has a surface realization that is indistinguishable from the infinitival marker, which does not require the prefix *GE-*, is irrelevant for underlying morphotactics. More succinctly: underlying morphotactics regulates the distribution of morphemes, surface morphotactics the distribution of allomorphs.

This paper considers a variety of phenomena — circumfixation, variable affix ordering, unbounded prefixation — and concludes that they all belong to the class of tier-based strictly local languages. We first show that even though many morphotactic dependencies are strictly local, that is not the case for all of them (Sec. 2.1). While some of these outliers are strictly piecewise (Sec. 2.2), tier-based strictly local grammars are needed to handle the full range of data points (Sec. 2.3). This prompts our conjecture that all dependencies that are part of underlying morphotactics stay within the class of tier-based strictly local languages. We then use this hypothesis in Sec. 3 to explain two typological gaps with respect to compounding markers and circumfixation.

## 2 Subregular Patterns in Morphology

The regular languages are one of the best understood language classes, with many attractive properties. Yet it is often forgotten that this class properly includes many weaker ones (McNaughton and Pappert, 1971), some of which have recently attracted much interest in computational phonology. At the very bottom of the hierarchy one finds strictly local and strictly piecewise languages (Rogers et al., 2010), and a little bit higher up the tier-based strictly local languages (Heinz et al.,

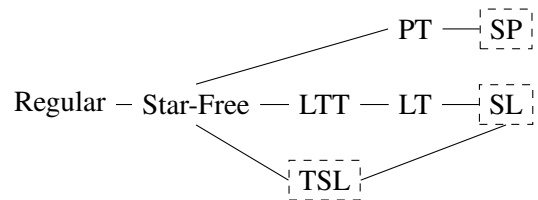


Figure 1: The subregular hierarchy as given in Heinz et al. (2011); language classes in dashed boxes are studied in this paper

2011). The subregular hierarchy includes many other classes (see Fig. 1), but those three are noteworthy because they are conceptually simple and efficiently learnable in the limit from positive data (Heinz et al., 2012; Jardine and Heinz, 2016) while also furnishing sufficient power for a wide range of phonological phenomena (Heinz, 2015; Jardine, 2015).

In this section, we investigate the role of strictly local, strictly piecewise and tier-based strictly local patterns in morphotactics. We show that some not but not all patterns are strictly local or strictly piecewise, whereas all typologically instantiated patterns seem to fit in the class of tier-based strictly local languages.

### 2.1 Strictly Local

A string language  $L$  over alphabet  $\Sigma$  is *strictly local* (SL) iff there is some  $k \in \mathbb{N}$  such that  $L$  is generated by a strictly  $k$ -local grammar  $G$ . Such a grammar consists of a finite set of  $k$ -grams, each one of which describes an illicit substring. More precisely, given a string  $w \in \Sigma^*$ , let  $\hat{w}^k := \bowtie^k w \bowtie^k$  (where  $\bowtie, \bowtie \notin \Sigma$ ) and  $k\text{-grams}(w) := \{s \mid s \text{ is a substring of } \hat{w}^{k-1} \text{ of length } k\}$ . Then  $G$  generates string  $w$  iff  $k\text{-grams}(w) \cap G = \emptyset$ . That is to say,  $G$  generates every string over  $\Sigma$  that does not contain an illicit substring.

Most phonological dependencies can be described in strictly local terms — see Heinz (2015) for numerous examples. Consider for instance the well-known process of *word-final obstruent devoicing* that forces voiced obstruents at the end of the word to be realized as voiceless: *moroz* [maros] ‘frost’ in Russian, *Bad* [bat] ‘bath’ in German). If one considers phonotactics rather than mappings from underlying representations to surface forms, this is tantamount to a ban against word-final voiced obstruents. Said ban, in turn, is captured by a strictly 2-local grammar  $G$  that contains all bigrams of the form  $v\bowtie$ , with  $v$  a voiced

obstruent.

The specification of SL grammars can be simplified by applying mappings. In the case at hand, one could define a function  $f$  that replaces every voiced obstruent by the designated symbol  $\diamond$  so that the grammar  $G$  can be reduced to the single bigram  $\diamond \times$ . One has to be careful, though. The SL languages are not closed under relabelings, in fact, every regular language is the image of a strictly 2-local language under some relabeling. However, the directionality of the  $\diamond$ -relabeling above is the opposite: first the relabeling is applied, and then the grammar filters out strings in the image of that relabeling. As long as the relabeling is a many-to-one map between alphabets (and thus does not introduce distinctions that weren't already part of the original alphabet), this provably does not increase weak generative capacity for any of the formalisms discussed in this paper.

We make use of such relabelings in the following sections in order to convert natural language patterns into more easily described formal languages. For morphotactics, though, this raises the issue how the size of the atomic units should be chosen. One could posit that morphology, just like phonology, treats every phonological segment as a symbol. In that case, stems and morphemes are strings of symbols. Alternatively, one may treat each morpheme, including stems, as an atomic symbol. This is an important decision when it comes to modeling the interactions of morphology and phonology such as phonologically conditioned allomorphy. Fortunately our results are independent of this choice, due to the productive nature of compounding.

To better understand why different representations could in principle affect subregular complexity, note first that whether a stem is represented as a single, atomic symbol or as a sequence of phonological segments determines if prefixes and suffixes might be separated by an unbounded amount of symbols. Consider a circumfix  $u-$   $-v$ , where neither part of the affix may occur without the other. A concrete example is the nominalization circumfix  $ke-$   $-an$  in Indonesian (Mahdi, 2012; Sneddon, 1996):

- (1) a. tingii  
high
- b. ke- tinggi -an  
NMN- high -NMN  
'altitude'

If a stem is a single symbol  $x$ , then  $x$  and  $uxv$  are well-formed whereas  $ux$  and  $xv$  are not due to  $u-$   $-v$  being a circumfix whose subparts cannot occur in isolation. This generalization is easily captured by the strictly 3-local grammar  $\{\times xv, ux \times\}$ . However, if stems are sequences of symbols, then the well-formed patterns are of the form  $x^+$  or  $ux^+v$  (since the length of stems is in principle unbounded). The illicit strings, on the other hand, are of the form  $\times x^+v$  and  $ux^+ \times$ . But no strictly local grammar can generate the former patterns without also generating the latter. That is due to the strictly local languages being closed under suffix substitution closure.

**Suffix Substitution Closure** Language  $L$  is SL iff for all strings  $u_1, v_1, u_2, v_2$ , there exists a  $k \in \mathbb{N}$  such that for any string  $x$  of length  $k-1$ , if  $u_1xv_1, u_2xv_2 \in L$ , then  $u_1xv_2 \in L$ .

If there is no upper bound on the length of stems, then we can infer from  $x^k \in L$  and  $ux^kv \in L$  that both  $x^kv \in L$  and  $ux^k \in L$ . It seems, then, that circumfixes are strictly local only if each stem is an atomic symbol.

But this line of reasoning erroneously assumes that the circumfix can only apply to individual stems, which ignores the availability of compounding. Returning to Indonesian, we see that its nominalization marker isn't restricted to single stems but can also apply to compounds.

- (2) a. maha siswa  
big pupil  
'student'
- b. ke- maha siswa -an  
NMN- big pupil -NMN  
'student affairs'

Compounding is an unbounded process, so even if each stem is mapped to a single symbol  $x$ , one ends up with the same patterns as with the segmental mapping approach: well-formed  $x^+$  and  $ux^+v$  versus ill-formed  $ux^+$  and  $x^+v$ . Since the choice of representation does not affect the subregular complexity results, we opt for the segmental mapping, which does not require us to use compounding in all our natural language data points.

The preceding discussion yielded as a nice side-result that circumfixation is not SL unless every part of the circumfix can also occur on its own. Such circumfixes are very common, wherefore not all aspects of morphotactics are SL. However,

large swaths of morphology still are, with a couple of examples from English given below:

- (3) a. un- do  
a- xx  
b. break -able  
xxxxx -b
- (4) a. punch -ed  
xxxxx -c  
b. put -ε  
xxx -c

Any kind of affix that only consists of one part and whose distribution is determined within a locally bounded domain is part of strictly local morphotactics. While we did not do any quantitative comparisons, we believe the majority of morphological dependencies to belong to this class.

## 2.2 Strictly Piecewise

While SL covers a wide range of phenomena, it isn't just circumfixes that require more power. Problems arise whenever a dependency involves both the domain of prefixes and the domain of suffixes, and such configurations are not limited to circumfixes. In most languages the ordering of affixes is assumed to be fixed, but there are languages in which affixes are ordered relatively freely and do not follow a strict template, thereby creating non-local dependencies.

Let us consider the following data from Swahili (Stump, 2016):

- (5) a. a- vi- soma  
SBJ:CL.1- OBJ:CL.8- read  
-vyo  
-REL:CL.8  
w-w-xxxx-c  
'reads'
- b. a- si- vyo- vi-  
SBJ:CL.1- NEG- REL:CL.8- read  
soma  
-OBJ:CL.8  
w-w-c-w-xxxx  
'doesn't read'
- c. \* a- vyo- vi-  
SBJ:CL.1- REL:CL.8- OBJ:CL.8-  
soma -vyo  
read -REL:CL.8  
w-c-w-xxxx-c  
'reads'

Two very different generalizations could be drawn from this data, only one of which is strictly local. The SL generalization states that *vyo* is only licensed if it follows either a segment that is part of a stem or the prefix *si*. But then one would still have to ensure that *vyo* is a suffix rather than a prefix whenever *si* is not present, which cannot be done with SL grammars. The much more intuitive alternative is that *vyo* may occur at most once in a word. This provably cannot be enforced by any SL grammar, either: if  $w^*cw^*x^+ \subset L$  and  $w^*x^+c \subset L$ , then  $L$  must also contain strings of the form  $w^*cw^*x^+c$ .

The second generalization is similar to the phonological requirement that no word may contain more than one primary stress, which is *strictly piecewise* (SP; Heinz (2010), Rogers et al. (2010)). SP grammars work exactly the same as SL grammar except that they do not list illicit substrings but illicit subsequences. Given a string  $w$ , its set of  $k$ -sequences is  $k\text{-seqs}(w) := \{s \mid s \text{ is a subsequence of } \hat{w}^{k-1} \text{ of length } k\}$ . A strictly  $k$ -piecewise grammar  $G$  is a finite set of  $k$ -grams over  $\Sigma \cup \{\bowtie, \bowtie\}$ , and the language generated by  $G$  is  $L := \{w \mid k\text{-seqs}(w) \cap G = \emptyset\}$ .

The Swahili pattern as generalized above is strictly 2-piecewise — the grammar only need to contain the bigram *vyovy*. But we cannot fully capture the requirement that *vyo* is a suffix only if *si* is not present. Suppose we increase the locality rank from 2 to 3 and include *si x vyo* as an illicit subsequence. This forces *vyo* to be a prefix in the presence of *si*. However, this still incorrectly allows for *vyo* to be a prefix in the absence of *si*, and this cannot be blocked by an SP grammar. The problem is that any word of the form  $u \text{ } vyo \text{ } v \text{ } x$  contains only subsequences that also occur in the well-formed  $u \text{ } sivyov \text{ } x$ . Consequently, any SP grammar that generates the latter also generates the former. Swahili's inflectional morphology thus provides evidence that the long-distance dependencies captured by SP can be useful, but they are not sufficient.

Note that the same kind of reasoning also shows circumfixation not to be SP. All subsequences of  $ux^+$  are subsequences of  $ux^+v$ , so if the latter is well-formed then by definition the former must be, too. In both cases the underlying problem is that SP grammars only enforce the absence of an affix, not its presence. It seems that we must move higher up the subregular hierarchy in order

to accommodate circumfixes and the distribution of Swahili *vyo*.

### 2.3 Tier-Based Strictly Local

As pointed out in the previous section, the Swahili pattern isn't too dissimilar from the phonological requirement that no word has more than one primary stress. However, the distribution of primary stress is more specific than that: every phonological word has exactly one primary stress. Ensuring the presence of at least one primary stress is beyond the capabilities of SP grammars — once again this holds because every subsequence of an ill-formed word without primary stress is also a subsequence of the well-formed counterpart with exactly one primary stress. A better model for primary stress assignment is furnished by *tier-based strictly local* (TSL; Heinz et al. (2011)) grammars, which also happen to be powerful enough for circumfixation.

A TSL grammar is an SL grammar that operates over a *tier*, a specific substructure of the string. Given a tier-alphabet  $T \subseteq \Sigma$ , let  $E_T$  be a mapping that erases all symbols in a string that do not belong to  $T$ . For  $a \in \Sigma$  and  $w \in \Sigma^*$ ,

$$E_T(aw) := \begin{cases} a \cdot E_T(w) & \text{if } a \in T \\ E_T(w) & \text{if } a \notin T \\ \varepsilon & \text{otherwise} \end{cases}$$

The  $T$ -tier of a string  $w$  is its image under  $E_T$ . A tier-based strictly  $k$ -local grammar  $G$  is a pair  $\langle K, T \rangle$  where  $K$  is a strictly  $k$ -local grammar over tier-alphabet  $T$ . The grammar generates the language  $L(G) := \{w \mid E_T(w) \in L(K)\}$ . Note that every SL language is a TSL language with  $T = \Sigma$ .

The distribution of primary stress is tier-based strictly 2-local. Assuming that primary stress is indicated as some diacritic on symbols, the tier-alphabet  $T$  contains all symbols with this diacritic. This is tantamount to projecting a tier that only contains segments with primary stress. The grammar then contains the bigram  $\times \times$  to block words with an empty primary stress tier, i.e. words that contain no primary stress. In addition, every bigram  $uv$  for  $u, v \in T$  is added to rule out words with more than one primary stress. The requirement of having exactly one primary stress thus boils down to having exactly one segment on the primary stress tier, which is a strictly local dependency over that tier.

The Swahili pattern from the previous section can also be analyzed as tier-based strictly 2-local, and the same is true for circumfixation. For Swahili we project a tier that contains only the affix *vyo*, and we do not allow more than one segment on this tier. As a result, *vyo* occurs at most once per word. If we want to ensure that *vyo* is a prefix whenever *si* is present, we need to postulate a marker  $\#$  that indicates the edges of the stem. The projected tier then includes all instances of *vyo*, *si* and the marker  $\#$  (of which there are exactly two). On this tier, the 4-gram *si##vyo* correctly excludes all ill-formed cases of *vyo* as a suffix, whereas  $\times \text{vyo} \# \#$  prevents *vyo* from occurring as a prefix in the absence of *si*. Adapting the ban against two instances of *vyo* to this new type of tier is left as an exercise to the reader.

In order to regulate the distribution of circumfixes such as *u-* *-v*, we have to project a tier that contains only those subparts  $u$  and  $v$ . If the affixes can never occur by themselves, then we block  $\times v$  and  $u \times$ . Removing one of those two bigrams creates asymmetric cases where one of the affixes — but not the other — is sometimes allowed to be present by itself. We also add *uu* and *vv* to block strings where the prefix parts outnumber or are outnumbered by the suffix parts of the circumfix. Note that this has the side effect of also prohibiting unbounded circumfixation, a point we return to in Sec. 3.2.

At this point, we can safely say that natural language morphotactics is at least TSL (barring the discovery of any intermediate classes between SL and TSL). SL is sufficiently powerful for large parts of morphology, but any kind of dependency that involves both prefixes and suffixes is likely not SL. Some patterns that are not SL are SP, but these also turn out to be TSL. To the best of our knowledge, there are no morphological dependencies that are SP but not TSL (even though the two language classes are incomparable). We thus put forward the following conjecture:

**Subregular Morphotactics** All morphotactic dependencies are tier-based strictly local.

### 3 Beyond Tier-Based Strictly Local?

If the subregular hypothesis is correct, then no morphological pattern may exceed the computational power furnished by tier-based strictly local grammars. In particular, whenever the combination of two attested TSL patterns is not TSL, that

combination should not be attested. The subregular hypothesis thus provides a principled explanation for typological gaps. In this section we consider two such cases related to compounding markers and the limits of circumfixation.

### 3.1 Case Study 1: Compounding Markers

Compounding describes the combination of two or more stems to form a compound lexeme, where the stems may belong to different categories. Languages differ with respect to whether compounding is (at least sometimes) explicitly marked. In the following we exhibit two TSL compounding patterns from Turkish and Russian whose combination is not typologically attested. We then explain why this combined pattern is not TSL, deriving the otherwise puzzling typological gap.

Turkish possessive compounds (see Aslan and Altan (1998) for a detailed description) obey the general pattern  $stem-stem^+-o$ , where  $o$  stands for the compounding marker  $-si$ .

- (6) a. bahçe kapı -si  
garden gate -COMP  
xxxx-xxxx-o  
‘garden gate’  
b. türk kahve -si  
turkish coffee -COMP  
xxxx-xxxx-o  
‘Turkish coffee’  
c. türk bahçe kapı -si (\*-si)  
turkish garde gate -COMP (\*-COMP)  
xxxx-xxxx-xxxx-o(\*o)  
‘Turkish garden gate’

The compounding marker is added when two stems are combined. Addition of further stems does not increase the number of compounding markers, it is always a single marker for the whole word. The resulting pattern  $stem-(stem^+-o)$  is tier-based strictly 2-local under the assumption that a designated symbol occurs between stems, say  $\square$ . For then we can project a tier that contains only  $\square$  and  $o$ , with the only illicit bigrams on this tier being  $\times o$ ,  $o\square$ , and  $\square\times$ .

Russian compounding, on the other hand, follows the pattern  $(stem-o)^*-stem$ , which means that the addition of a new stem to the compound requires the appearance of the compounding marker  $-o-$  between the stems:

- (7) a. vod -o- voz  
water -COMP- carry

xxx-o-xxx

‘water-carrier’

- b. vod -o- voz -o- voz  
water -COMP- carry -COMP- carry  
xxx-o-xxx-o-xxx  
‘carrier of water-carriers’

Assuming once again the presence of the special symbol  $\#$  — which marked the edges of stems in the previous section — we can show this pattern to also be tier-based strictly 2-local. In this case, the illicit bigrams are  $\#\#$ ,  $oo$ ,  $\times o$ , and  $o\times$ . Observe that we can remove the first one of these bigrams to allow for cases where the compounding marker is optional.

One may wonder now whether it is possible to have a combination of the compounding patterns seen with Russian and Turkish. From a linguistic perspective, the expected answer is yes. If compounding can be marked by a suffix as in Turkish, and compounding can introduce a marker with each step as in Russian, then it should be possible to introduce a suffix with each compounding step. But to the best of our knowledge, no language instantiates this system. But from a computational perspective, this typological gap is expected because the described system is not TSL — as a matter of fact, it isn’t even regular.

A language  $L$  that suffixes a marker to a compound with each compounding step would produce compounds where the number of compound markers is proportional to the number of stems. Let  $h$  be a map that replaces all stems by  $s$ , all compound markers by  $o$ , and all other material by some other symbol. Intersecting  $h(L)$  with the regular language  $s^+o^+$  yields the language  $s^mo^n$ ,  $m > n$ . This string set is easily shown to be context-free (e.g. via the Myhill-Nerode theorem), and since regular languages are closed under homomorphisms and intersection, it follows that  $L$  cannot be regular. But every TSL language is regular, so the combination of Russian and Turkish outlined above is not TSL. The absence of this compounding pattern in the typology of natural languages thus lends further support to our conjecture that natural language morphotactics is limited to TSL dependencies.

### 3.2 Case Study 2: Unbounded Affixation

Circumfixation already played an important role in motivating TSL as a reasonable lower bound

on how much power is required for natural language morphotactics. We now show that just like compounding markers, circumfixation also suggests that TSL marks the upper bound on required expressivity. In particular, unbounded affixation is widely attested across languages, whereas unbounded circumfixation is not.

A number of languages allow some of their affixes to occur multiple times. For instance, the Russian temporal prefix *posle-* can appear iteratively in the beginning of a word like *zavtra* ‘tomorrow’.

- (8) a. *posle-* *zavtra*  
 AFTER- tomorrow  
 a-xxxxxx  
 ‘the day after tomorrow’  
 b. *posle-* *posle-* *zavtra*  
 AFTER- AFTER- tomorrow  
 a-a-xxxxxx  
 ‘the day after the day after tomorrow’

The very same pattern is also found in German, with *morgen* ‘tomorrow’, *über-morgen* ‘the day after tomorrow’, *über-über-morgen* ‘the day after the day after tomorrow’, and so on. German also has the pattern *ur-groß-vater*, *ur-ur-groß-vater*, which is the analog of English *great grandfather*, *great great grandfather* and its iterations (various linguistic diagnostics show that these are morphological words rather than phrases). Note that in all those cases it is impossible to insert other prefixes between the iterated prefix: *\*ur-groß-ur-ur-groß-vater*. In sum, some affixes can be freely iterated as long as no other affixes intervene.

These patterns are all TSL by virtue of being strictly 2-local. We illustrate this claim with German. We ignore the problem of how *groß* can be restricted to occur only with specific stems (if stems are atomic symbols, this is trivial, otherwise it requires a strictly *k*-local grammar over the string of phonological segments where *k* is large enough to contain both *groß* and the relevant stems). We also assume, as before, that there is some marker # that marks the edges of stems. Then to ensure that the relevant strings of prefixes follow the pattern *ur\*groß#*, the sequences *großur*, *großgroß*, and *ur#* are marked as illicit. Unbounded prefixation thus stays within the class of TSL dependencies.

An interesting counterpart to Russian and German is Ilocano (Galvez Rubino, 1998), which uses

the circumfix *ka-* *-an* with a function similar to *posle* and *über*.

- (9) a. *bigat*  
 morning  
 xxxxx  
 ‘morning’  
 b. *ka-* *bigat* *-an*  
 NEXT- morning -NEXT  
 a-xxxxx-a’  
 ‘the next morning’

Crucially, Ilocano differs from Russian and German in that the circumfix cannot be iterated.

- (10) \* *ka-* *ka-* *bigat* *-an* *-an*  
 NEXT- NEXT- morning -NEXT -NEXT  
 a-a-xxxxx-a’-a’  
 ‘the next morning after the next one’

Given our previous discussion of circumfixation in Sec. 2.3, Ilocano clearly instantiates a tier-based strictly 2-local pattern, too.

As before, there is little linguistic reason why unbounded circumfixation should be blocked. If affixation can be unbounded to construct more and more complex versions of *day after tomorrow*, and *day after tomorrow* can be derived via circumfixation, then one would expect unbounded circumfixation to be a viable option. But once again there is a clear computational reason as to why this does not happen: the corresponding morphotactic system would no longer be TSL.

Let *L* be some minor variant of Russian where *posle-* is instead a circumfix *pos-* *-le*. As before we let *h* be a homomorphism that replaces all stems by *s*, the two parts of the circumfix by *o*, and all other material by some distinct symbol. The intersection of *h(L)* with the regular language *o<sup>+</sup>so<sup>+</sup>* is the context-free string set *o<sup>n</sup>so<sup>n</sup>*. Therefore *L* is supra-regular and cannot be tier-based strictly local. Unbounded circumfixation simply cannot be reconciled with the subregular hypothesis.

## 4 Conclusion

The received view is that all of morphology is easily modeled with finite-state machines (Koskeniemi, 1983; Karttunen et al., 1992). We contend that this view is overly generous and that tighter bounds can be established, at least for specific subparts of morphology. Morphotactics defines the restrictions on the possible orderings of morphological units, and we argued based on data from

typologically diverse languages that the power of natural language morphotactics is severely restricted:

**Subregular Morphotactics** All morphotactic dependencies are tier-based strictly local.

In contrast to regular languages, tier-based strictly local languages are efficiently learnable in the limit from positive text. Our result thus marks a first step towards provably correct machine learning algorithms for natural language morphology.

Admittedly, morphotactics is just one of several parts of morphology. We put aside allomorphy and only considered the distribution of morphemes in the underlying forms. But our result shows that the subregular perspective, which has proven tremendously fruitful in the domain of phonology, can be extended to morphology. There has been a lot of progress in extending the subregular hierarchy from languages to transductions (see Chandlee (2014) and references therein), and we are confident that these results will allow us to expand the focus of investigation from morphotactics to morphology at large.

It will also be interesting to see how uniform the complexity bounds are across different modules of morphology. In phonology, suprasegmental dependencies tend to be more complex than segmental ones (Jardine, 2015). Most constructions in this paper involve derivational morphology, but the affix *vyo* in Swahili is related to inflectional morphology and turned out to have a distribution that is neither SL nor SP. So both derivational and inflectional morphotactics occupy points in  $TSL \setminus (SL \cup SP)$ . In this regard it is also worth noting that some phonological processes such as tone plateauing belong to  $SP \setminus TSL$ , whereas no morphological dependencies seem to be part of this subclass. We hope to address these and related issues in future work.

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