E& M 2 Final Project

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1 Methodology

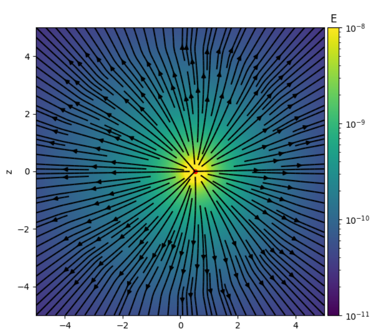
For this project we utilized the python package PyCharge. This package allows us to simulate the E&M fields surrounding moving point charges. For each scenario we wrote individual code that simulates the movement/behavior through plotting the electric field. We then took these numerous plots and compiled them into movies, which you can find on our github repository. The plotting was done using python's matplotlib, pcolormesh, and streamplot.

Throughout the project we plotted the charge's behavior over varying time periods. For each time period though, we plotted the charge at 50 different points. The time period we look at varies based on each simulation and isn't necessarily easy to figure out, however they generally take place over very short time periods. From now on when we express the time in terms of "time steps" we simply are telling you which out of those 50 points in time you are looking at.

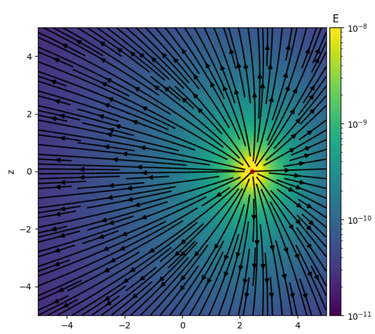
2 Results

2.1 Point Charge with Constant Acceleration

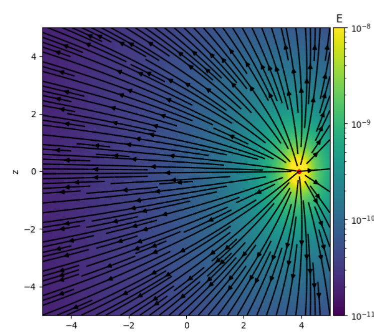
For this scenario we have a charge starting at rest with an acceleration of $5 * 10^{15} m/s^2$ until it reaches 0.999c.



(a) The oscillating point charge at timestep t=12.



(b) The oscillating point charge at timestep t=26.



(c) The oscillating point charge at timestep t=33.

From Griffith's equations 10.72 and 10.73 we know that we can express the electric and magnetic field of moving point charges with the following equations:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0} \frac{r_s}{(\mathbf{r_s} * \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \mathbf{r_s} \times (\mathbf{u} \times \mathbf{a})] \tag{1}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c} \hat{\mathbf{r_s}} \times \mathbf{E}(\mathbf{r},t) \tag{2}$$

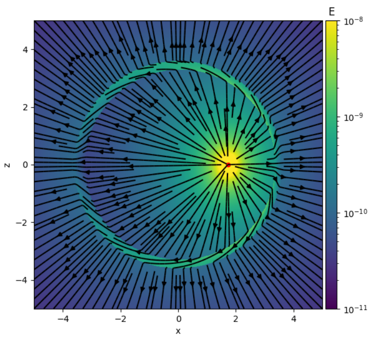
where $\mathbf{u} = c\hat{r} - \mathbf{v}$ and $\mathbf{r_s}$ is the displacement vector.

In this scenario we have both the velocity and acceleration field which results in some radiation. Since the acceleration is constant so is this radiation, and in the images shown you can see that the electric field looks very similar to that of the point charge moving with constant velocity, however there is a slight bending of the field lines.

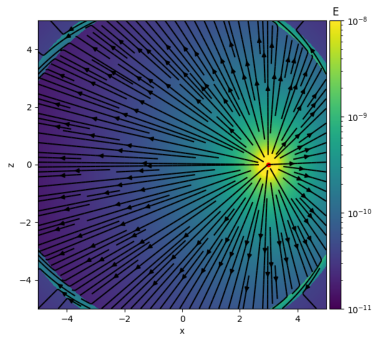
As acceleration is constant, the time derivative of the electric field at the same relative point to the charge is nearly 0, but not fully. Specifically we would find that the only remaining component would be from the velocity field, which makes sense as the time derivative of velocity would be acceleration. It also explains why we only see a slight bending of the field lines because the velocity field contributes by order of $\frac{1}{r^2}$ and so it is only really impactful very close to the charge.

2.2 Point Charge with a burst of Acceleration

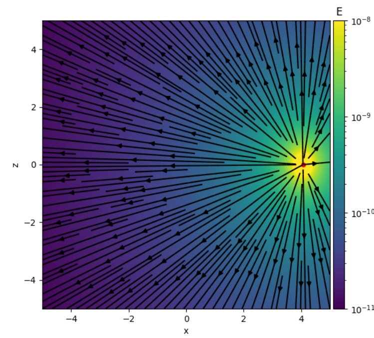
In this case we have a charge initially at rest moving along in the x direction for 1 ns until it reaches a velocity of 0.5c, at which point the particle continues on with constant velocity.



(a) The point charge at timestep t=10.



(b) The point charge at timestep t=17.



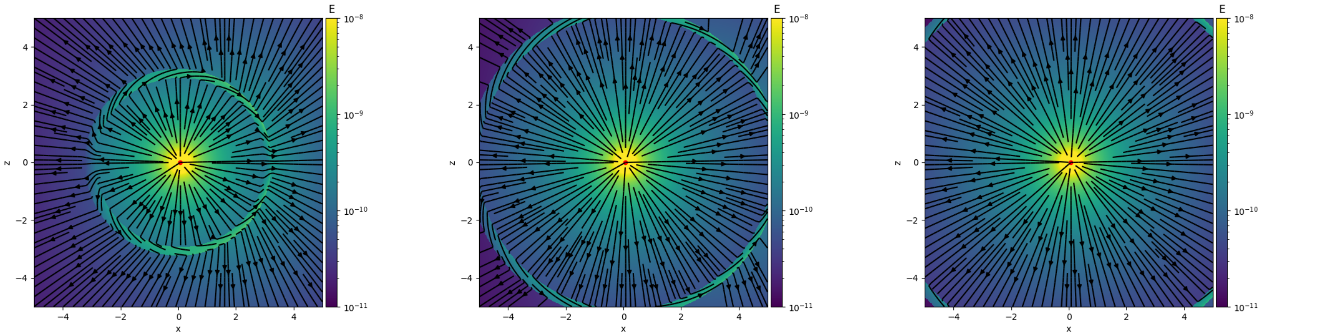
(c) The point charge at timestep t=23.

Look at the point charge with a burst of acceleration we see that initially there appears to be an outgoing bubble of electric field, which is a demonstration of the radiation that occurs due to the acceleration of point charges. This radiation is determined by equation 1. The presence of that bubble like structure is a consequence of the non-zero and now much more pronounced time derivative of the electric field. You no longer have a constant Poynting vector but actively changing radiation, specifically as the acceleration ends you get that the radiation greatly diminishes as the electric field is again dominated by the velocity field.

After the acceleration stops we see that the charge continues going on with the typical electric field of a point charge moving with constant velocity. Compared with the point charge moving with constant acceleration we see that there's areas with much smaller electric field once it reaches this state as there's not constant ongoing radiation being emitted.

2.3 Point Charge coming to a Stop

For this scenario we have a point charge that is moving at 0.5c in the x direction and starting at t=0 it decelerates until it comes to a complete stop after 1 ns.



(a) The point charge at timestep t=9. (b) The point charge at timestep t=15. (c) The point charge at timestep t=19.

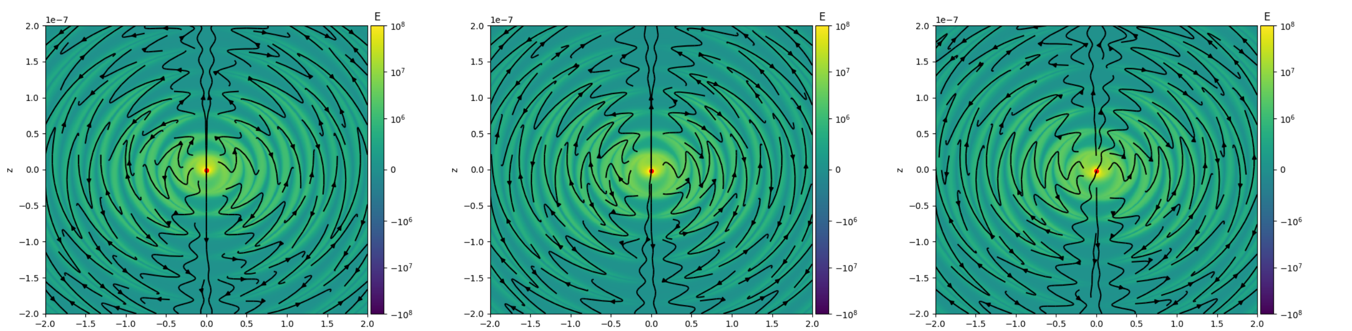
Similar to the case before with the burst of acceleration here we see that there's a burst of apparent radiation as the particle decelerates, however this time we see that the field lines are pointing in the opposite direction within that ring of radiation. On the northern hemisphere in the burst of acceleration image as it crosses that ring width the field lines bent in the counter-clockwise direction, however with the deceleration they bend in the clockwise direction. This indicates how the directionality of acceleration impacts the direction of the electric field lines.

Ultimately though, that is where the differences should largely stop as deceleration is simply acceleration in the direction against motion and so we would expect the same processes to occur.

2.4 Oscillating Charge

For the oscillating charge case we have a charge that is moving up and down with an amplitude of 2 nm in the z direction with a period of 10^{-6} seconds. The frequency of oscillation is therefore $2\pi * 10^{16}$ Hz.

For this case, I decided to zoom in on the electric field within $\pm 2.0 * 10^{-7}$ m.



(a) The oscillating point charge at timestep t=11. (b) The oscillating point charge at timestep t=12. (c) The oscillating point charge at timestep t=13.

Throughout our timeseries of plots you can see that there appear to be "ripples" in the electric field flowing out from the charge. It's important to note that the simulation as we did it assumed that the charge had been oscillating before t=0.

This makes sense as we can represent the point charge's motion using the equation $z = z_0 cos(\omega t)$. The acceleration of the charge would then be expressed as $\vec{a} = -\omega^2 z_0 cos(\omega t)$. Looking at equation 1 we see that we get both a velocity and acceleration field which creates a complex electric field. The result is overall though that we do have some net flux of energy outwards.

Additionally, if you look closely at the center of the image you can see the innermost ring almost trailing behind the point charge. This is due to the retarded time making it so the electric field has to almost "catch up" to the charge.

Comparing to the case of the oscillating dipole field our images also make sense, as in the case of the dipole field you witness monochromatic spherical waves determined by the equation below that we found in class.

$$\vec{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi} * (\frac{sin\theta}{r}) * cos(\omega(t - \frac{r}{c}))\hat{\theta} \tag{3}$$