Project 2 Proposal

Data-drive Fracture Mechanics – Discovering Elastic Constants and Failure Criteria using Physics-informed Neural Networks

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Overview

This project will investigate extending the work of Raissi et al. 2021 [1] to develop a Physics Informed Neural Network (PINN) for discovering the parameters of a linear elastic fracture mechanics (LEFM) material model. The crack propagation of a simple 2-dimensional (2D) single edge notch (SEN) specimen will be considered. Although many researchers have implemented various data-driven techniques for calibrating constitutive models [2-5] of varying complexity and even some fracture- or damage-based models [6-10], there appears to be a lack of basic examples showcasing these novel approaches for discovering LEFM parameters. Many such papers investigate very complex material behavior while others are investigating calibration of novel numerical approaches for fracture mechanics. In this project, literature will be thoroughly reviewed to benchmark current efforts in data-driven fracture mechanics and investigations will seek to implement a simple method for extracting relevant material parameters from a field output of a 2d finite element (FE) simulation of a SEN specimen using a PINN.

Background

LEFM assumes that the formalism of classical Cauchy continuum mechanics for a solid, homogenous solid body is applicable. The general kinematic and dynamic equilibrium equations are given by the following equations using indicial notation, respectively:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$\sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i$$

Where ε_{ij} are the components of strain, σ_{ij} are the components of Cauchy stress, u_i are the components of displacement, ρ is the mass density, and b_i are body forces per unit volume. If body forces, ρb_i, and inertia terms, $\rho\ddot{u}_i$, are ignored, then the following equations describes static equilibrium involving the divergence of stress being equal to zero:

$$\sigma_{ij,j} = 0$$

Initial boundary value problems (IBVP) in solid mechanics are concerned with solving these kinematic and equilibrium partial differential equations. The solution variables include displacements, stresses, and strains. In 3-dimensions (3D), there are a total of 15 unknown field variables. With 15 unknowns and only 9 equations, further closure relations are needed to present a well-posed problem. These closure relations are also referred to as constitutive models. Linear isotropic elasticity allows one to relate the 6 components of strain to the 6 components of stress using the following equations:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$$

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} + \varepsilon_{ij}\right)$$

Where the first equation is the form of linear elasticity with the two material parameters λ and μ which are called the Lamé parameter and the shear modulus, respectively. The second equation is linear elasticity with the two material parameters E and ν which are called the elastic modulus and Poisson's ratio, respectively. These equations are equivalent.

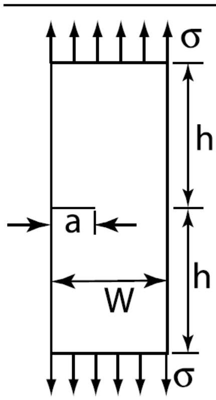
Fracture mechanics investigates the problem of crack propagation following two different, yet equivalent paradigms: stress-based or energy-based approaches [11]. The stress-based approach is concerned with determining the field of stresses in a region near the crack tip where stress concentration occurs. The stress concentration factor at the crack tip is determined and compared to a critical stress concentration factor (also called the "fracture toughness"). Energy-based approaches are concerned with determining the energy release rate associated with crack propagation. The energy release rate is determined and compared to a critical energy release rate (also called the "fracture energy"). The analytical relationship between stress, strain, and displacements and the fracture toughness/energy have been determined for a variety of different geometries and loading scenarios. Further details of these relationships will be provided in the final report.

LEFM deals exclusively with linear elastic materials that fail in a brittle manner in which a material point loses its ability to store any elastic energy once a maximum yield stress has been reached. After one material point has lost its ability to carry any load in a brittle material, adjacent material points must carry the load, often resulting in the unstable failure of each adjacent material point until the structure has been cleaved apart. However, catastrophic failure can be avoided and stable crack growth is possible when a structure is loaded under displacement-control. Further discussion and details of failure criteria will be provided in the final report.

Methodology

For this study, a PINN will be developed to model the crack-propagation in a SEN specimen. Figure 1 shows an empirically derived formula for determining the mode I stress intensity factor for this specimen with a variety of parameters. A similar style of equations will be used for the displacement loading case of interest. A brittle failure criteria will be used in conjunction with a form of the stress intensity factor and input into the 2D equations of linear elasticity and formulated into a PINN. The PINN will be trained on data extracted from a FEM described in the following section. The goal is for the PINN to rediscover the two elastic material properties, fracture toughness, and maximum tensile strength input into the simulation. Further elaboration of mathematical details will be provided in the final report.

Table 5.1 Stress intensity solutions for several fracture test specimen geometries. $E' = E$ (plane stress), $E' = E/(1-\nu^2)$ (plane strain). Adapted from [4] and [8]



Single Edge Notch Tension (SENT)

$h/W > 1$

$K_I = \sigma\sqrt{\pi a}\,F(a/W)$

$F(a/W) = 0.265(1 - a/W)^4 + \frac{0.857+0.265a/W}{(1-a/W)^{3/2}}$

Figure 1: Stress intensity factor for SEN specimen in tension. Excerpt of table 5.1 from Zehnder 2012 [11]

Simulation

Abaqus is a commercial finite element package with a variety of physics capabilities and benchmark problems. The extended finite element method (XFEM) is a promising capability for simulating the fracture and failure behavior of a solid material. One of the Abaqus XFEM benchmark problems is used for generating training data for the PINN [12]. In this benchmark problem, a displacement is applied to the top and bottom edges of a rectangular plate with a pre-existing edge crack, also referred to as a single edge notch (SEN) specimen in tension. It is uncertain at this time what output from the XFEM simulation will be most useful for training the PINN. Options include field variables such as stress, strain, or displacement, and history data such as force vs. displacement. Although the XFEM approach is a recent and exciting numerical method, it is outside of the scope of this project to discuss how it works. One thing to note is that XFEM is a general, numerical approach for modeling fracture and it has a number of features that make it more attractive than other numerical approaches that may be plagued by instability issues and require many prior assumptions. For the purposes of this project, the results of the XFEM simulations will be treated as experimental data.

Additional Goals

If initial goals of this project are achieved for a Mode I loading of the SEN specimen and if time permits, several stretch goals can be investigated:

- Add noise to simulation data
- Investigate Mode II (shear) loading
- Investigate mixed mode crack propagation

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