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LOGICAL SYNTAX OF LANGUAGE

RUDOLF CARNAP

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CONTENTS

PART IV. GENERAL SYNTAX

PAGE

PART V. PHILOSOPHY AND SYNTAX

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PREFACE TO THE ENGLISH EDITION

The present English edition contains some sections which are not found in the German original. These are §§ 16*a*, 34*a–i*, 38*a–c*, 60*a–d*, 71*a–e*. These twenty-two sections were included in the manuscript of the German original when it was sent for publication (in December 1933) but had to be taken out because of lack of space. The content of § 34*a–i* was, in a slightly different formulation, published in German in the paper *Ein Gültigskriterium für die Sätze der klassischen Mathematik*, and the content of §§ 60*a–d* and 71*a–d* in *Die Antinomien und die Unvollständigkeit der Mathematik*. § 60 of the original has been omitted here, since it was only a shortened substitute for § 60*a–d*.

In the Bibliography some less important publications have been deleted, and others, mainly of the last few years, have been added.

Several smaller additions and corrections have been made. The more important of these occur at the following points: § 8, regressive definition; § 12, RI 2 (see footnote); § 14, proofs added to Theorems 3 and 7; § 21, D 29; § 22, two insertions in D 64 (see footnote), D 83; § 29, footnote; § 30, PSII 4 (see footnote to § 12); PSII 19, condition added; § 48 (see footnote); § 51, definition of 'L-consequence'; § 56 (see footnote), Theorems 8 and 9 taken out; § 57, Theorems 2 and 3 corrected, and last paragraph added; § 62, explanation of '$\mathfrak{Q}_2\,[\mathfrak{S}_2]$'; §§ 65 and 66, definitions of 'extensional' restricted to closed partial expressions, and Theorem 65. 8*a* added; § 67, end of second paragraph. The majority of these corrections and a number of further ones have been suggested by Dr. A. Tarski, others by J. C. C. McKinsey and W. V. Quine, to all of whom I am very much indebted for their most helpful criticisms.

The problem of rendering the German terminology was naturally a most difficult one, in some cases there being no English word in existence which corresponded exactly to the original, in others the obvious equivalent being unavailable because of its special associations in some other system. It was necessary sometimes to appropriate for our purposes words which have not previously borne a technical significance, sometimes to coin entirely new ones. If at first sight some of these seem ill at ease or outlandish, I can only ask the reader to bear in mind the peculiar difficulties involved, and assure him that no term was chosen without most careful consideration and the conviction that it would justify itself in use.

To facilitate discussion and reference, the German symbolic abbreviations have been retained in all the strictly formalized portions of the book. English equivalents have been substituted only where they occur in the non-formal text, as mere convenient abbreviations which are not properly symbolic (e.g. "TN" for "term-number" instead of the German "GZ"), or as incidental symbols introduced simply for purposes of illustration (e.g. "fa" for "father" instead of the German "Va"). Wherever a German abbreviation has been used for the first time, the full German word has been inserted in brackets; and in the case of the terms introduced by formal definitions, a complete key to the symbolization is given in a footnote at the beginning of the respective sections.

I wish to express my best thanks to the Countess von Zeppelin for the accomplishment of the difficult task of translating this book, further to Dr. W. V. Quine for valuable suggestions with regard to terminology, and to Dr. E. C. Graham, Dr. O. Helmer, and Dr. E. Nagel for their assistance in checking the proofs.

R. C.

Cambridge, Mass., May 1936

FOREWORD

For nearly a century mathematicians and logicians have been striving hard to make logic an exact science. To a certain extent, their efforts have been crowned with success, inasmuch as the science of logistics has taught people how to manipulate with precision symbols and formulae which are similar in their nature to those used in mathematics. But a book on logic must contain, in addition to the formulae, an expository context which, with the assistance of the words of ordinary language, explains the formulae and the relations between them; and this context often leaves much to be desired in the matter of clarity and exactitude. In recent years, logicians representing widely different tendencies of thought have developed more and more the point of view that in this context is contained the essential part of logic; and that the important thing is to develop an exact method for the construction of these sentences about sentences. The purpose of the present work is to give a systematic exposition of such a method, namely, of the method of "logical syntax". (For further details, see Introduction, pp. 1 and 2.)

In our "Vienna Circle", as well as in kindred groups (in Poland, France, England, U.S.A., and, amongst individuals, even in Germany) the conviction has grown, and is steadily increasing, that metaphysics can make no claim to possessing a scientific character. That part of the work of philosophers which may be held to be scientific in its nature—excluding the empirical questions which can be referred to empirical science—consists of logical analysis. The aim of logical syntax is to provide a system of concepts, a language, by the help of which the results of logical analysis will be exactly formulable. *Philosophy is to be replaced by the logic of science*—that is to say, by the logical analysis of the concepts and sentences of the sciences, for *the logic of science is nothing other than the logical syntax of the language of science*. That is the conclusion to which we are led by the considerations in the last chapter of this book.

The book itself makes an attempt to provide, in the form of an exact syntactical method, the necessary tools for working out the problems of the logic of science. This is done in the first place by the formulation of the syntax of two particularly important types of language which we shall call, respectively, 'Language I' and

'Language II'. Language I is simple in form, and covers a narrow field of concepts. Language II is richer in modes of expression; in it, all the sentences both of classical mathematics and of classical physics can be formulated. In both languages the investigation will not be limited to the mathematico-logical part of language—as is usually the case in logistics—but will be essentially concerned also with synthetic, empirical sentences. The latter, the so-called 'real' sentences, constitute the core of science; the mathematico-logical sentences are analytic, with no real content, and are merely formal auxiliaries.

With Language I as an example, it will be shown, in what follows, how the syntax of a language may be formulated within that language itself (Part II). The usual fear that thereby contradictions—the so-called 'epistemological' or 'linguistic' antinomies—must arise, is not justified.

The treatment of the syntax of Languages I and II will be followed by the outline of a general syntax applicable to any language whatsoever (Part IV); and, although the attempt is very far from attaining the desired goal, yet the task is one of fundamental importance. The range of possible language-forms and, consequently, of the various possible logical systems, is incomparably greater than the very narrow circle to which earlier investigations in modern logic have been limited. Up to the present, there has been only a very slight deviation, in a few points here and there, from the form of language developed by Russell which has already become classical. For instance, certain sentential forms (such as unlimited existential sentences) and rules of inference (such as the Law of Excluded Middle), have been eliminated by certain authors. On the other hand, a number of extensions have been attempted, and several interesting, many-valued calculi analogous to the two-valued calculus of sentences have been evolved, and have resulted finally in a logic of probability. Likewise, so-called intensional sentences have been introduced and, with their aid a logic of modality developed. The fact that no attempts have been made to venture still further from the classical forms is perhaps due to the widely held opinion that any such deviations must be justified—that is, that the new language-form must be proved to be 'correct' and to constitute a faithful rendering of 'the true logic'.

To eliminate this standpoint, together with the pseudo-problems and wearisome controversies which arise as a result of it, is one of the chief tasks of this book. In it, the view will be maintained that we have in every respect complete liberty with regard to the forms of language; that both the forms of construction for sentences and the rules of transformation (the latter are usually designated as "postulates" and "rules of inference") may be chosen quite arbitrarily. Up to now, in constructing a language, the procedure has usually been, first to assign a meaning to the fundamental mathematico-logical symbols, and then to consider what sentences and inferences are seen to be logically correct in accordance with this meaning. Since the assignment of the meaning is expressed in words, and is, in consequence, inexact, no conclusion arrived at in this way can very well be otherwise than inexact and ambiguous. The connection will only become clear when approached from the opposite direction: let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. By this method, also, the conflict between the divergent points of view on the problem of the foundations of mathematics disappears. For language, in its mathematical form, can be constructed according to the preferences of any one of the points of view represented; so that no question of justification arises at all, but only the question of the syntactical consequences to which one or other of the choices leads, including the question of non-contradiction.

The standpoint which we have suggested—we will call it the *Principle of Tolerance* (see p. 51)—relates not only to mathematics, but to all questions of logic. From this point of view, the task of the construction of a general syntax—in other words, of the definition of those syntactical concepts which are applicable to languages of any form whatsoever—is a very important one. In the domain of general syntax, for instance, it is possible to choose a certain form for the language of science as a whole, as well as for that of any branch of science, and to state exactly the characteristic differences between it and the other possible language-forms.

The first attempts to cast the ship of logic off from the *terra firma* of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after 'correctness'. Now, however, that impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities.

In a number of places in the text, reference is made to the most important literature on the subject. A complete list has not, however, been attempted. Further bibliographical information may easily be obtained from the writings specified. The most important references are given on the following pages: pp. 96 ff., comparison of our Language II with other logical systems; pp. 136 ff., on the symbolism of classes; pp. 158 ff., on syntactical designations; pp. 253 f., on the logic of modalities; pp. 280 f. and 320 f. on the logic of science.

For the development of ideas in this book, I owe much to the stimulation I have received from various writings, letters and conversations on logical problems. Mention should here be made of the most important names. Above all, I am indebted to the writings and lectures of Frege. Through him my attention was drawn to the standard work on logistics—namely, the *Principia Mathematica* of Whitehead and Russell. The point of view of the formal theory of language (known as "syntax" in our terminology) was first developed for mathematics by Hilbert in his "metamathematics", to which the Polish logicians, especially Ajdukiewicz, Lesniewski, Lukasiewicz, and Tarski, have added a "metalogic". For this theory, Gödel created his fruitful method of "arithmetization". On the standpoint and method of syntax, I have, in particular, derived valuable suggestions from conversations with Tarski and Gödel. I have much for which to thank Wittgenstein in my reflections concerning the relations between syntax and the logic of science; for the divergences in our points of view, see pp. 282 ff. (Incidentally, *à propos* of the remarks made—especially in § 17 and § 67—in opposition to Wittgenstein's former dogmatic standpoint, Professor Schlick now informs me that for some time past, in writings as yet unpublished, Wittgenstein has agreed that the rules of language may be chosen with complete freedom.) Again, I have learned much from the writings of authors with whom I am not entirely in agreement; these are, in the first place, Weyl, Brouwer, and Lewis. Finally, I wish to express my gratitude to Professor Behmann and Dr. Gödel for having read the manuscript of this book in an earlier draft (1932), and for having made numerous valuable suggestions towards its improvement.

R. C.

Prague, May 1934

INTRODUCTION

§ 1. What is Logical Syntax?

By the **logical syntax** of a language, we mean the formal theory of the linguistic forms of that language—the systematic statement of the formal rules which govern it together with the development of the consequences which follow from these rules.

A theory, a rule, a definition, or the like is to be called *formal* when no reference is made in it either to the meaning of the symbols (for example, the words) or to the sense of the expressions (e.g. the sentences), but simply and solely to the kinds and order of the symbols from which the expressions are constructed.

The prevalent opinion is that syntax and logic, in spite of some points of contact between them, are fundamentally theories of a very different type. The syntax of a language is supposed to lay down rules according to which the linguistic structures (e.g. the sentences) are to be built up from the elements (such as words or parts of words). The chief task of logic, on the other hand, is supposed to be that of formulating rules according to which judgments may be inferred from other judgments; in other words, according to which conclusions may be drawn from premisses.

But the development of logic during the past ten years has shown clearly that it can only be studied with any degree of accuracy when it is based, not on judgments (thoughts, or the content of thoughts) but rather on linguistic expressions, of which sentences are the most important, because only for them is it possible to lay down sharply defined rules. And actually, in practice, every logician since Aristotle, in laying down rules, has dealt mainly with sentences. But even those modern logicians who agree with us in our opinion that logic is concerned with sentences, are yet for the most part convinced that logic is equally concerned with the relations of meaning between sentences. They consider that, in contrast with the rules of syntax, the rules of logic are non-formal. In the following pages, in opposition to this standpoint, the view that logic, too, is concerned with the *formal* treatment of sentences will be presented and developed. We shall see that the logical characteristics of sentences (for instance, whether a sentence is analytic, synthetic, or contradictory; whether it is an existential sentence or not; and so on) and the logical relations between them (for instance, whether two sentences contradict one another or are compatible with one another; whether one is logically deducible from the other or not; and so on) are solely dependent upon the syntactical structure of the sentences. In this way, logic will become a part of syntax, provided that the latter is conceived in a sufficiently wide sense and formulated with exactitude. The difference between syntactical rules in the narrower sense and the logical rules of deduction is only the difference between *formation rules* and *transformation rules*, both of which are completely formulable in syntactical terms. Thus we are justified in designating as 'logical syntax' the system which comprises the rules of formation and transformation.

In consequence of the unsystematic and logically imperfect structure of the natural word-languages (such as German or Latin), the statement of their formal rules of formation and transformation would be so complicated that it would hardly be feasible in practice. And the same difficulty would arise in the case of the artificial word-languages (such as Esperanto); for, even though they avoid certain logical imperfections which characterize the natural word-languages, they must, of necessity, be still very complicated from the logical point of view owing to the fact that they are conversational languages, and hence still dependent upon the natural languages.

For the moment we will leave aside the question of the formal deficiencies of the word-languages, and, by the consideration of examples, proceed to convince ourselves that rules of formation and transformation are of like nature, and that both permit of being formally apprehended. For instance, given an appropriate rule, it can be proved that the word-series "Pirots karulize elatically" is a sentence, provided only that "Pirots" is known to be a substantive (in the plural), "karulize" a verb (in the third person plural), and "elatically" an adverb; all of which, of course, in a well-constructed language—as, for example, in Esperanto—could be gathered from the form of the words alone. The meaning of the words is quite inessential to the purpose, and need not be known. Further, given an appropriate rule, the sentence "A karulizes elatically" can be deduced from the original sentence and the sentence "A is a Pirot"—again provided that the type to which the individual words belong is known. Here also, neither the meaning of the words nor the sense of the three sentences need be known.

Owing to the deficiencies of the word-languages, the logical syntax of a language of this kind will not be developed, but, instead, we shall consider the syntax of two artificially constructed symbolic languages (that is to say, such languages as employ formal symbols instead of words). As a matter of fact, throughout all modern logical investigations, this is the method used; for only in a symbolic language has it proved possible to achieve exact formulation and rigid proofs. And only in relation to a constructed symbolic language of this kind will it be possible to lay down a system of rules at once simple and rigid—which alone will enable us to show clearly the characteristics and range of applicability of logical syntax.

The sentences, definitions, and rules of the syntax of a language are concerned with the forms of that language. But, now, how are these sentences, definitions, and rules themselves to be correctly expressed? Is a kind of super-language necessary for the purpose? And, again, a third language to explain the syntax of this super-language, and so on to infinity? Or is it possible to formulate the syntax of a language within that language itself? The obvious fear will arise that in the latter case, owing to certain reflexive definitions, contradictions of a nature seemingly similar to those which are familiar both in Cantor's theory of transfinite aggregates and in the pre-Russellian logic might make their appearance. But we shall see later that without any danger of contradictions or antinomies emerging it is possible to express the syntax of a language in that language itself, to an extent which is conditioned by the wealth of means of expression of the language in question.

However, we shall not at first concern ourselves with this problem, important though it is. We shall proceed, instead, to construct syntactical concepts relating to the languages we have chosen, and postpone, for a while, the question as to whether we are able or not to express the rules and sentences based on these concepts in that language itself. In the first stages of a theory, such a naïve approach seems always to have proved the most fruitful. For instance, geometry, arithmetic, and the differential calculus all appeared first, and only much later (in some cases, hundreds of years after) did epistemological and logical discussions of the already developed theories ensue. Hence we shall start by constructing the syntax, and then, later on, proceed to formalize its concepts and thereby determine its logical character.

In following this procedure, we are concerned with two languages: in the first place with the language which is the object of our investigation—we shall call this the **object-language**—and, secondly, with the language in which we speak *about* the syntactical forms of the object-language—we shall call this the **syntax-language**. As we have said, we shall take as our object-languages certain symbolic languages; as our syntax-language we shall at first simply use the English language with the help of some additional Gothic symbols.

§ 2. Languages as Calculi

By a **calculus** is understood a system of conventions or rules of the following kind. These rules are concerned with elements—the so-called **symbols**—about the nature and relations of which nothing more is assumed than that they are distributed in various classes. Any finite series of these symbols is called an **expression** of the calculus in question.

The rules of the calculus determine, in the first place, the conditions under which an expression can be said to belong to a certain category of expressions; and, in the second place, under what conditions the transformation of one or more expressions into another or others may be allowed. Thus the system of a language, when only the formal structure in the sense described above is considered, is a calculus. The two different kinds of rules are those which we have previously called the rules of formation and transformation—namely, the syntactical rules in the narrower sense (e.g. "An expression of this language is called a sentence when it consists, in such and such a way, of symbols of such and such a kind, occurring in such and such an order"), and the so-called logical laws of deduction (e.g. "If a sentence is composed of symbols combined in such and such a way, and if another is composed of symbols combined in such and such another way, then the second can be deduced from the first"). Further, every well-determined mathematical discipline is a calculus in this sense. But the system of rules of chess is also a calculus. The chessmen are the symbols (here, as opposed to those of the word-languages, they have no meaning), the rules of formation determine the position of the chessmen (especially the initial positions in the game), and the rules of transformation determine the moves which are permitted—that is to say, the permissible transformations of one position into another.

In the widest sense, logical syntax is the same thing as the construction and manipulation of a calculus; and it is only because languages are the most important examples of calculi that, as a rule, only languages are syntactically investigated. In the majority of calculi (even in those which are not languages in the proper sense of the word), the elements are written characters. The term 'symbol' in what follows will have the same meaning as the word 'character'. It will not be assumed that such a symbol possesses a meaning, or that it designates anything.

When we maintain that logical syntax treats language as a calculus, we do not mean by that statement that language is nothing more than a calculus. We only mean that syntax is concerned with that part of language which has the attributes of a calculus—that is, it is limited to the formal aspect of language. In addition, any particular language has, apart from that aspect, others which may be investigated by other methods. For instance, its words have meaning; this is the object of investigation and study for semasiology. Then again, the words and expressions of a language have a close relation to actions and perceptions, and in that connection they are the objects of psychological study. Again, language constitutes an historically given method of communication, and thus of mutual influence, within a particular group of human beings, and as such is the object of sociology. In the widest sense, the science of language investigates languages from every one of these standpoints: from the syntactical (in our sense, the formal), from the semasiological, from the psychological, and from the sociological.

We have already said that syntax is concerned solely with the formal properties of expressions. We shall now make this assertion more explicit. Assume that two languages (*Sprachen*), S_1 and S_2, use different symbols, but in such a way that a one-one correspondence may be established between the symbols of S_1 and those of S_2 so that any syntactical rule about S_1 becomes a syntactical rule about S_2 if, instead of relating it to the symbols of S_1, we relate it to the correlative symbols of S_2; and conversely. Then, although the two languages are not alike, they have the same *formal structure* (we call them isomorphic languages), and syntax is concerned solely with the structure of languages in this sense. From the syntactical point of view it is irrelevant whether one of two symbolical languages makes use, let us say, of the sign '&', where the other uses '•' (in word-languages: whether the one uses 'and' and the other 'und') so long as the rules of formation and transformation are analogous. For instance, it depends entirely on the formal structure of the language and of the sentences involved, whether a certain sentence is analytic or not; or whether one sentence is deducible from another or not. In such cases the *design* (visual form, *Gestalt*) of the individual symbols is a matter of indifference. In an exact syntactical definition, no allusion will be made to this design. Further, it is equally unimportant from the syntactical point of view, that, for instance, the symbol 'and' should be specifically a thing consisting of printers' ink. If we agreed always to place a match upon the paper instead of that particular symbol, the formal structure of the language would remain unchanged.

It should now be clear that any series of any things will equally well serve as terms or expressions in a calculus, or, more particularly, in a language. It is only necessary to distribute the things in question in particular classes, and we can then construct expressions having the form of series of things, put together according to the rules of formation. In the ordinary languages, a series of symbols (an expression) is either a temporal series of sounds, or a spatial series of material bodies produced on paper. An example of a language which uses movable things for its symbols is a card-index system; the cards serve as the object-names for the books of a library, and the riders as predicates designating properties (for instance, 'lent', 'at the book-binders', and such like); a card with a rider makes a sentence.

The *syntax* of a language, or of any other calculus, is concerned, in general, with the *structures of possible serial orders* (of a definite kind) *of any elements whatsoever*. We shall now distinguish between pure and descriptive syntax. *Pure syntax* is concerned with the possible arrangements, without reference either to the nature of the things which constitute the various elements, or to the question as to which of the possible arrangements of these elements are anywhere actually realized (that is to say, with the possible forms of sentences, without regard either to the designs of the words of which the sentences are composed, or to whether any of the sentences exist on paper somewhere in the world). In pure syntax only definitions are formulated and the consequences of such definitions developed. Pure syntax is thus wholly analytic, and is nothing more than *combinatorial analysis*, or, in other words, the *geometry* of finite, discrete, serial structures of a particular kind. *Descriptive syntax* is related to pure syntax as physical geometry to pure mathematical geometry; it is concerned with the syntactical properties and relations of empirically given expressions (for example, with the sentences of a particular book). For this purpose—just as in the application of geometry—it is necessary to introduce so-called correlative definitions, by means of which the kinds of objects corresponding to the different kinds of syntactical elements are determined (for instance, "material bodies consisting of printers' ink of the form '$\vee$' shall serve as disjunction symbols"). Sentences of descriptive syntax may, for instance, state that the fourth and the seventh sentences of a particular treatise contradict one another; or that the second sentence in a treatise is not syntactically correct.

When we say that pure syntax *is concerned with* the forms of sentences, this 'concerned with' is intended in the figurative sense. An analytic sentence is not actually "concerned with" anything, in the way that an empirical sentence is; for the analytic sentence is without content. The figurative 'concerned with' is intended here in the same sense in which arithmetic is said to be concerned with numbers, or pure geometry to be concerned with geometrical constructions.

We see, therefore, that whenever we investigate or judge a particular scientific theory from the logical standpoint, the results of this *logical analysis* must be formulated as *syntactical sentences*, either of pure or of descriptive syntax. The *logic of science* (logical methodology) is nothing else than the *syntax of the language of science*. This fact will be shown clearly in the concluding chapter of this book. The syntactical problems acquire a greater significance by virtue of the anti-metaphysical attitude represented by the Vienna Circle. According to this view, the sentences of metaphysics are pseudo-sentences which on logical analysis are proved to be either empty phrases or phrases which violate the rules of syntax. Of the so-called philosophical problems, the only questions which have any meaning are those of the logic of science. To share this view is to *substitute logical syntax for philosophy*. The above-mentioned anti-metaphysical attitude will not, however, appear in this book either as an assumption or as a thesis. The inquiries which follow are of a formal nature and do not depend in any way upon what is usually known as philosophical doctrine.

The method of syntax which will be developed in the following pages will not only prove useful in the logical analysis of scientific theories—it will also help in the *logical analysis of the word-languages*. Although here, for the reasons indicated above, we shall be dealing with symbolic languages, the syntactical concepts and rules—not in detail but in their general character—may also be applied to the analysis of the incredibly complicated word-languages. The direct analysis of these, which has been prevalent hitherto, must inevitably fail, just as a physicist would be frustrated were he from the outset to attempt to relate his laws to natural things—trees, stones, and so on. In the first place, the physicist relates his laws to the simplest of constructed forms; to a thin straight lever, to a simple pendulum, to punctiform masses, etc. Then, with the help of the laws relating to these constructed forms, he is later in a position to analyze into suitable elements the complicated behaviour of real bodies, and thus to control them. One more comparison: the complicated configurations of mountain chains, rivers, frontiers, and the like are most easily represented and investigated by the help of geographical co-ordinates—or, in other words, by constructed lines not given in nature. In the same way, the syntactical property of a particular word-language, such as English, or of particular classes of word-languages, or of a particular sub-language of a word-language, is best represented and investigated by comparison with a constructed language which serves as a system of reference. Such a task, however, lies beyond the scope of this book.

Terminological Remarks

The reason for the choice of the term '(logical) syntax' is given in the introduction. The adjective 'logical' can be left out where there is no danger of confusion with linguistic syntax (which is not pure in its method, and does not succeed in laying down an exact system of rules), for example, in the text of this book and in logical treatises in general.

As the word itself suggests, the earliest calculi in the sense described above were developed in mathematics. Hilbert was the first to treat mathematics as a calculus in the strict sense—i.e. to lay down a system of rules having mathematical formulae for their objects. This theory he called *metamathematics*, and his original object in developing it was to attain the proof of the freedom from contradiction of classical mathematics. Metamathematics is—when considered in the widest sense and not only from the standpoint of the task just mentioned—the syntax of the mathematical language. In analogy to the Hilbertian designation, the Warsaw logicians (Lukasiewicz and others) have spoken of the 'meta-propositional calculus', of *metalogic*, and so on. Perhaps the word 'metalogic' is a suitable designation for the sub-domain of syntax which deals with logical sentences in the narrower sense (that is, excluding the mathematical ones).

The term *semantics* is used by Chwistek to designate a theory which he has constructed with the same object as our syntax, but which makes use of an entirely different method (of this we shall say more later). But since, in the science of language, this word is usually taken as synonymous with 'semasiology' (or 'theory of meaning') it is perhaps not altogether desirable to transfer it to syntax—that is, to a formal theory which takes no account of meanings. (Compare: Bréal, *Essai de sémantique. Science des significations*. Paris, 1897. 5th edn. 1921, p. 8: "La science, que j'ai proposé d'appeler la Sémantique", with footnote: "*Σημαντικὴ τέχνη*, la science des significations".)

The designation *sematology* may (following Bühler) be retained for the empirical (psychological, sociological) theory of the application of symbols in the widest sense. The empirical science of language is thus a sub-domain of sematology. But it must be distinguished from *semasiology* which, as a part of the science of language, investigates the meaning of the expressions of the historically given languages.

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BIBLIOGRAPHY AND INDEX OF AUTHORS

The numbers immediately following an author's name indicate the pages of this book on which he is mentioned. The main references are printed in black type.

The shortened forms of titles which precede them in square brackets are those used in citation throughout the book.

* The publications marked with an asterisk have appeared since the writing of the German original, and hence are not mentioned in the text. The most important of these are: Hilbert and Bernays [*Grundl.* 1934]; Quine [*System*] (see the author's review in *Erkenntnis*, **5**, 1935, p. 285); Tarski [*Wahrh.*] (cf. Kokoszynska [*Wahrheit*]).

Exhaustive bibliographies of the literature of logistics and logical syntax are to be found in: Fraenkel [*Mengenlehre*]; Jörgensen [*Treatise*]; and Lewis [*Survey*].

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