LECON2112 Advanced Microeconomics II

– Assignment 2 –

(SOLUTIONS)

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Exercises[1]

8Bd. Consider the following game. There are five players, three male singles 1, 2 and 3, and two female singles, a and b. The preferences $\succ_1$, $\succ_2$, $\succ_3$, $\succ_a$ and $\succ_b$ of those five players vis-à-vis a partner from the other gender and remaining single (which is denoted $\emptyset$) are as follows:

$$\begin{array}{ll} b \succ_1 a \succ_1 \emptyset & \qquad 1 \succ_a 2 \succ_a 3 \succ_a \emptyset \\ a \succ_2 b \succ_2 \emptyset & \qquad 2 \succ_b 1 \succ_b 3 \succ_b \emptyset \\ a \succ_3 b \succ_3 \emptyset & \end{array}$$

In stage 1, players 1, 2 and 3 propose the name of one female player, a or b. In stage 2, each player among a and b, if she got two or three propositions in stage 1, chooses one of them. Then, couples are formed. In case one female player did not get any proposition, she remains single. If she gets one proposition, she is married to the player having made the proposition. If she gets several propositions, she is married to the proposition she has chosen in stage 2.

(e) Does there exist a Nash equilibrium whose outcome is such that 3 gets married with a? If yes, give one, otherwise prove it.

SOLUTION. Here is a strategy profile whose outcome features that male 3 is married to female a:

$$s = \begin{cases} s_1 = b, \\ s_2 = b, \\ s_3 = a, \\ s_a = (1,1,3,3), \\ s_b = (3,2,3,3) \end{cases} \qquad \text{with} \qquad \begin{array}{l} \theta_1(s) = \emptyset \\ \theta_2(s) = b \\ \theta_3(s) = a \\ \theta_a(s) = 3 \\ \theta_b(s) = 2 \end{array}.$$

We have indeed that $\theta_3(s) = a$, as required. We still need to prove that this strategy profile is a Nash equilibrium. That is, we must show that each player plays best response: for all $i \in 1, 2, 3, a, b$ there exists no s_i' such that $\theta_i(s_a, s_{-a}) \succsim_a \theta_i(s_a', s_{-a})$.

[1] Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

In words, we prove for each player i that deviating from the strategy s_i when the others play s_{-i} cannot be profitable.

This is clear for female b that is married with 2, her favorite male. The same holds for male 3 that is married to his favorite female a.

This is also straightforward for female a, since she is married to the only male that proposed to her. Given that neither 1 nor 2 proposed to her in s_{-a}, any other strategy $s'_a \in S_a$ yields the same outcome for her.

Let us consider male 1 that remains single. If he had played $s'_1 = a$ instead of s_1, he would also remain single given s_a. Indeed, s_a is such that, when confronted to the choice set $1, 3$, female a picks male 3. Observe that, even if strategy s_a is a weakly dominated strategy for a, s_a still constitutes a best reply to s_{-a}.

Similarly, male 2 cannot gain by declaring $s'_2 = b$ instead of s_2. Again, s_a is such that, when confronted to the choice set $2, 3$, female a picks male 3.

Remark: consider the following strategy profile:

$$s' = \begin{cases} s_1 = b, \\ s_2 = b, \\ s_3 = a, \\ s'_a = (1, 1, \mathbf{1}, 3), \\ s_b = (3, 2, 3, 3) \end{cases} \qquad \text{with} \qquad \begin{array}{l} \theta_1(s') = \emptyset \\ \theta_2(s') = b \\ \theta_3(s') = a \\ \theta_a(s') = 3 \\ \theta_b(s') = 2 \end{array} .$$

Now, strategy profile s' does not constitute a Nash equilibrium, because male 1 can profitably deviate by declaring $s'_1 = a$.

(f) Does there exist a Nash equilibrium in undominated strategies whose outcome is such that 3 gets married with a? If yes, give one, otherwise prove it.

SOLUTION. We prove that there exists no Nash equilibrium in undominated strategies such that male 3 gets married to female a.

Proof by contradiction: assume that s is a Nash equilibrium in undominated strategies such that $\theta_3(s) = a$.

Given that all strategies in s are undominated, we must have that

$$s_a = (1, 1, 1, 2) \quad \text{and} \quad s_b = (2, 2, 1, 2).$$

Indeed, s_a is a weakly dominant strategy for a. This implies that any other strategy $s'_a \in S_a$ is weakly dominated by s_a. Hence, strategy s_a is the unique undominated strategy in S_a. The same holds for strategy s_b.

Given s_a, the only possibility for $\theta_3(s) = a$ is that

$$s_1 = b, \quad s_2 = b, \quad \text{and} \quad s_3 = a.$$

Given s_b, we have that $\theta_1(s) = \emptyset$. Nevertheless, we have that $\theta_1(s'_1 = a, s_{-1}) = a$. In other words, strategy profile s does not constitute a Nash equilibrium since male 1 can profitably deviate by proposing to a instead of proposing to b. A contradiction to our assumption.

8Ba. Two agents have to contribute to a common pot. They have preferences over the total amount in the pot, not over their own contributions. Each agent has an optimal amount, and moving further away from the optimal amount is worse. Formally let $I = \{1, 2\}$, $S_1 = S_2 = \mathbb{R}_{++}$, and, for all $s_1 \in S_1$, $s_2 \in S_2$,

$$u_1(s_1, s_2) = -|A_1 - (s_1 + s_2)|$$
$$u_2(s_1, s_2) = -|A_2 - (s_1 + s_2)|$$

where A_1 (resp. A_2) is the optimal amount of agent 1 (resp. 2), and we assume $A_1 < A_2$. Which strategies survive iterative deletion of strictly dominated strategies?

SOLUTION. Let us first study the behavior of each agent's utility function. Figure 1 shows agent i's utility as a function of her own strategy s_i, for some fixed values of agent j's contribution, s_j^1 , s_j^2 and s_j^3 , with $s_j^1 > s_j^2 > s_j^3$. Whenever agent $j \in 1, 2$ contributes with s_j, agent's $i \neq j$ utility function can be represented as follows:

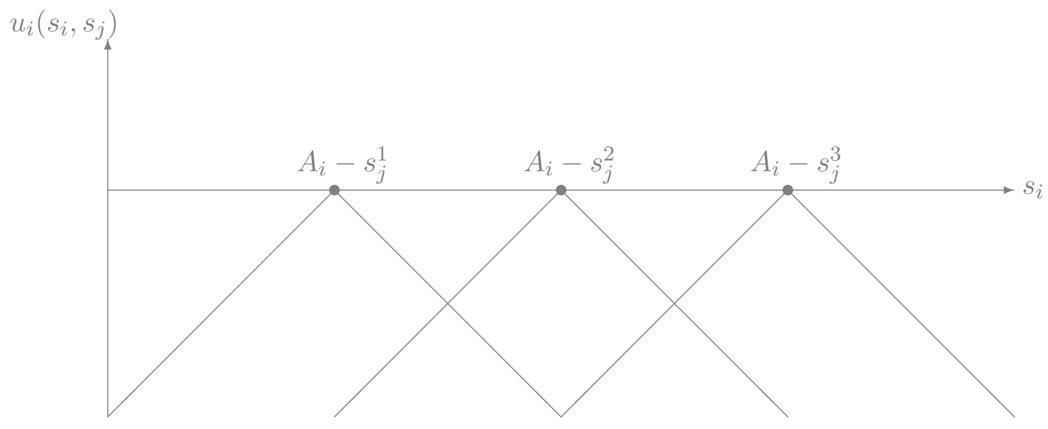

Figure 1: Agent i's utility depending on her own contribution, for some fixed values of s_j

Let us consider player 1. If $s_2 = 0$, then her best reply will be to put A_1. Will it ever be profitable to choose $s_1 > A_1$? Let us consider Figure 2.

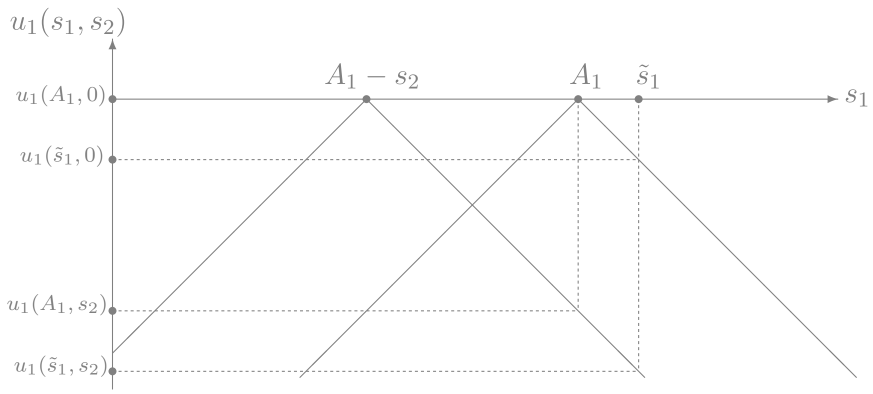

Figure 2: Playing $\tilde{s}_1 > A_1$ is a strictly dominated strategy for player 1

Playing $\tilde{s}_1 > A_1$ when player 2 plays 0 clearly gives a lower payoff than playing A_1. In effect, let $\tilde{s}_1 = A_1 + |k|$ with $|k| > 0$, we have $u_1(\tilde{s}_1, 0) = -|A_1 - (\tilde{s}_1 + 0)| = -|k| < 0 = u_1(A_1, 0)$.

Furthermore, notice that, even if player 2 plays some $s_2 > 0$, playing $s_1 = A_1$ always guarantees a strictly higher utility than $\tilde{s}_2$. Indeed, whenever player 2 contributes with a positive amount, contributing with anything more than her optimal amount is never optimal for player 1. Thus, any strategy s_1 such that $s_1 > A_1$ is strictly dominated by $s_1' = A_1$.

Now consider player 2. Playing $\tilde{s}_2 > A_2$ is a strictly dominated strategy, for symmetric reasons as the one explained above. But there are more strictly dominated strategies that player 2 can put aside when she takes into account that any $s_1 > A_1$ is strictly dominated for player 1. If player 1 plays A_1, then the best reply for player 2 is to play $s_2 = A_2 - A_1$. Will it ever be profitable to play something lower than $A_2 - A_1$?

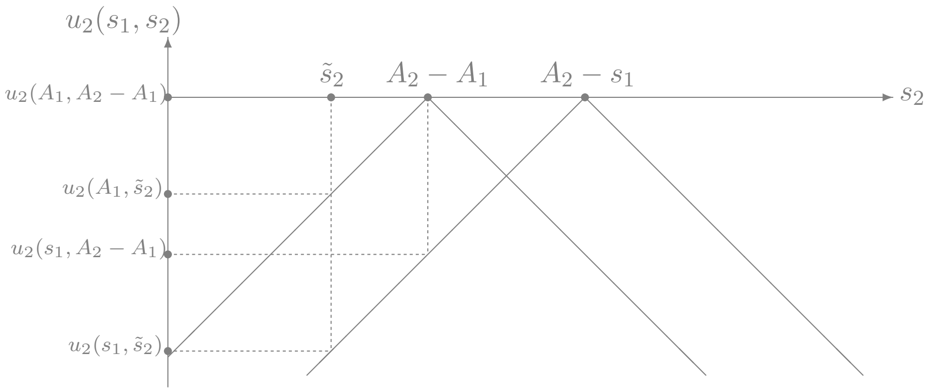

Figure 3: Playing $s_2 < A_2 - A_1$ is a strictly dominated strategy for player 2

As we can see from Figure 3, because $s_1 \leq A_1$ (iterative elimination of strictly dominated strategies), any strategy $\tilde{s}_2 < A_2 - A_1$ is strictly dominated by $s_2 = A_2 - A_1$. Therefore, at this step of the iterative elimination of dominated strategies, any strategy

$s_2 \notin [A_2 - A_1, A_2]$ is strictly dominated. Now, since player 1 knows that player 2 will never play anything lower than $A_2 - A_1$ (these strategies do not survive the iterative deletion process), by the same reasoning we applied above, she will never play any s_1 such that $s_1 > 2A_1 - A_2$; which implies that player 2 will never play any s_2 smaller than $2(A_2 - A_1)$ and so on.

The iterative deletion procedure, starting from $s_1, s_2 \in \mathbb{R}_+$, identifies step by step a larger set of strictly dominated strategies. The strategies surviving this procedure are in each step:

- Step 1 player 1: $s_1 \in [0, A_1]$,
- Step 1 player 2: $s_2 \in [A_2 - A_1, A_2]$,
- Step 2 player 1: $s_1 \in [0, 2A_1 - A_2]$,
- Step 2 player 2: $s_2 \in [2(A_2 - A_1), A_2]$,
- Step 3 player 1: $s_1 \in [0, 3A_1 - 2A_2]$,
- Step 3 player 2: $s_2 \in [3(A_2 - A_1), A_2]$,
- ...
- Step n player 1: $s_1 \in [0, nA_1 - (n-1)A_2]$,
- Step n player 2: $s_2 \in [n(A_2 - A_1), A_2]$,

As the number of steps increases, the upper-bound of the set of player 1's surviving strategies will at some point be smaller than its lower bound 0. In effect, we assumed $A_1 < A_2$ and there exists hence some value of n for which the expression $nA_1 - (n-1)A_2$ becomes negative. Since player 1 cannot play a negative s_1, the iterative deletion process stops at $s_1 = 0$. There exists no accessible strategy strictly dominating $s_1 = 0$. For player 2, the lower bound of her set of undominated strategies $n(A_2 - A_1)$ tends to ∞ and will at some point be larger than its upper-bound A_2. Nevertheless, from the non-negativity constraint on s_1, the iterative deletion process stops when only $s_1 = 0$ survives and hence only $s_2 = A_2$ survives.

In the end, the only strategies that survive the iterative deletion of strictly dominated strategies are $s_1 = 0$ for player 1 and $s_2 = A_2$ for player 2.

8Dg. In a remote country, the national army is composed of three separate forces A, B and C, each of them controlled by a different general. One day, general A judges that the current civil government is taking decisions that are in contradiction with the country's constitution. Therefore, general A considers making a coup in order to replace the current civil government. Knowing that general A is likely to make a coup, generals B and C have to separately and simultaneously decide whether to be loyal and fight the coup in case there is one (L) or join the rebellion and support the coup in case there is one (R).

There are four possible outcomes of that game. First, there could be no coup. That happens if general A decides not to try and make a coup. Second, there could be a successful coup. That happens when general A makes a coup and both general B and C have decided to join the rebellion in case there is one. Third, there could be a failed coup. That happens when general A makes a coup and both general B and C have decided to remain loyal in case there is a coup. Finally, there coup be an undecided coup. That happens when general A makes a coup but one and only one of the other two generals, B or C, has decided to join the rebellion in case there is a coup.

The payoffs are as follows.

Table 1: Exercise 8Dg.

	no coup	failed coup	undecided coup	successful coup
General A	-2	-6	0	3
General B	1	2	0	1
General C	-1	1	0	2

(a) Find all pure strategy Nash equilibria of this game.

SOLUTION. In order to find all pure strategy Nash equilibria of this game, let us proceed by considering all possible combinations of strategies and studying for each of them whether there exists a profitable deviation for any of the players.

The possible combinations of strategies of this game are eight:

1. **(C,L,L), which results in a "failed coup":** Given the strategies played by generals B and C, general A has an incentive to deviate by playing "NC" and obtain a payoff of -2 instead of -6;

2. **(C,L,R), which results in an "undecided coup":** Given the strategies played by generals A and C, general B has an incentive to deviate by playing "R" and obtain a payoff of 1 instead of 0;

3. **(C,R,L), which results in an "undecided coup":** Given the strategies played by generals A and B, general C has an incentive to deviate by playing "R" and obtain a payoff of 1 instead of 0;

4. **(C,R,R), which results in a "successful coup":** Given the strategies played by generals B and C, general A has no incentive to deviate as by doing so he would obtain a payoff of -2 instead of 3; given the strategies played by generals A and C, general

B has no incentive to deviate as by doing so he would obtain a payoff of 0 instead of 1; given the strategies played by generals A and B, general C has no incentive to deviate as by doing so he would obtain a payoff of 0 instead of 2;

5. **(NC,L,L), whose outcome is "no coup":** Given the strategies played by generals B and C, general A has no incentive to deviate as by doing so he would obtain a payoff of -6 instead of -2; given the strategies played by generals A and C, general B has no incentive to deviate as by doing so he would not change his payoff of 1; given the strategies played by generals A and B, general C has no incentive to deviate as by doing so he would not change his payoff of -1;

6. **(NC,L,R), whose outcome is "no coup":** Given the strategies played by generals B and C, general A has an incentive to deviate by playing "C" and obtain a payoff of 0 instead of -2;

7. **(NC,R,L), whose outcome is "no coup":** Given the strategies played by generals B and C, general A has an incentive to deviate by playing "C" and obtain a payoff of 0 instead of -2;

8. **(NC,R,R), whose outcome is "no coup":** Given the strategies played by generals B and C, general A has an incentive to deviate by playing "C" and obtain a payoff of 3 instead of -2;

(b) Is the following list of strategies a mixed strategy Nash equilibrium of this game: A makes a coup, B remains loyal with probability $\frac{2}{3}$ and joins the rebellion with probability $\frac{1}{3}$, C remains loyal with probability $\frac{1}{3}$ and joins the rebellion with probability $\frac{2}{3}$.

SOLUTION. In order to check whether (C, ($\frac{2}{3}$L,$\frac{1}{3}$R), ($\frac{1}{3}$L,$\frac{2}{3}$R)) constitutes a Nash equilibrium we need to check whether each player is playing his best response to the strategies played by the other players or he has incentive to deviate.

Let us first consider general A. Let us compute his expected payoff associated to playing strategy C and compare it with the one associated to playing strategy NC.

$$EU_A(C) = \frac{2}{9}(-6) + \frac{1}{9}0 + \frac{4}{9}0 + \frac{2}{9}(3) = -\frac{2}{3} > -2 = EU_A(NC)$$

Indeed, C is general A's best response to (($\frac{2}{3}$L,$\frac{1}{3}$R), ($\frac{1}{3}$L,$\frac{2}{3}$R)).

Let us now consider general B and C and check whether the probability distributions under consideration respect the requirements of mixed strategy Nash equilibria. There exist two ways to check that: we could find the probabilities that maximize the expected payoffs of the players, or look for the probabilities that make the other player indifferent between his pure strategies. The two methods are equivalent.

(I) For each player, let us compute the expected payoff associated to playing strategy R and compare it with the one associated to playing strategy L.

$$EU_B(R) = \frac{1}{3}0 + \frac{2}{3}1 = \frac{2}{3} = \frac{1}{3}2 + \frac{2}{3}0 = EU_B(L)$$

$$EU_C(R) = \frac{2}{3}0 + \frac{1}{3}2 = \frac{2}{3} = \frac{2}{3}1 + \frac{1}{3}0 = EU_C(L)$$

The combination of strategies (C, ($\frac{2}{3}$L,$\frac{1}{3}$R), ($\frac{1}{3}$L,$\frac{2}{3}$R)) is a mixed strategy Nash equilibrium of this game.

(II) Alternatively, let us find the probabilities that maximize the expected payoffs of the players.

Let us call p (respectively q) the probability that general B (respectively C) remains loyal.

$$EU_B(p,q) = 2pq + o(1-p)q + 0(1-q)p + 1(1-p)(1-q) = 3pq + 1 - p - q$$

$$\frac{\partial EU_B}{\partial p} = 3q - 1 = 0$$

from which we get: $q = \frac{1}{3}$.

$$EU_C(p,q) = 1pq + o(1-p)q + 0(1-q)p + 1(1-p)(1-q) = 3pq + 2 - 2p - 2q$$

$$\frac{\partial EU_C}{\partial q} = 3p - 2 = 0$$

from which we get: $p = \frac{2}{3}$.

(c) Identify a Nash equilibrium in which the probability that B remains loyal is 0.9.

SOLUTION. Let us consider general B's mixed strategy (0.9L, 0.1R).

Suppose general A is playing C. In that case general C's best response to (C, (0.9L, 0.1R)) would be to play L with probability 1. Indeed, his expected payoff is

$$EU_C(q) = q * 0.9 * 1 + (1-q) * 0.1 * 2$$

where q represents the probability that general C assigns to playing L and player C maximizes his expected utility by setting $q = 1$.

Let us now consider general A and check whether playing C is his best response to ((0.9L, 0.1R), L). His expected payoff is:

$$EU_A(C) = 0.9 * (-6) + 0.1 * 0 = -5.4 < -2 = EU_A(NC)$$

Therefore, general A's best response to ((0.9L, 0.1R), L) is NC.

Knowing that general A will play NC, we need to verify that generals B and C have no incentive to deviate from playing ((0.9L, 0.1R), L). Indeed, since there is no coup, these two players cannot increase their payoff by changing strategy, therefore no profitable deviation exists.

The strategy profile (NC, (0.9L, 0.1R), L) is a mixed strategy Nash equilibrium of this game.

(d) Do agents have (strictly/weakly) dominated strategies in this game?

SOLUTION. Agents have no (strictly/weakly) dominated strategies in this game.

Let us consider general A. We have seen in the first point of this exercise that C is his best response to (R,R) and in this case it gives a strictly higher payoff than NC, hence C is not dominated. However, NC is his best response to (L,L) and in this case it gives a strictly higher payoff that C, hence NC is not dominated either.

Let us consider general B. Strategy L is his best response to (NC,L) and in this case it gives a strictly higher payoff than R, hence L is not dominated. However, R is his best response to (C,R) and in this case it gives a strictly higher payoff that L, hence R is not dominated either.

The same reasoning applies for general C.

8Dk. An employer would like to allocate a total amount of M to two deserving employees. The employer sets up the following game. Each employee is asked to announce the amount of money he or she thinks she deserves. Then, if the total amount of money claimed by the two employees does not exceed M, each employee receives what he or she claims. On the contrary, if the total amount of money claimed by the employees exceeds M, both employees are penalized: each gets the amount he or she claimed, minus a fine that is proportional to the total excess claim. Formally, there is a set $I = \{1, 2\}$ of players, with strategy sets $S_i = [0, M]$, $i \in I$. The payoff functions π_i are defined by

$$\pi_i(s_1, s_2) = \begin{cases} s_i & \text{if } s_1 + s_2 \leq M \\ s_i - \alpha\big((s_1 + s_2) - M\big) & \text{if } s_1 + s_2 > M \end{cases}$$

for some $\alpha > 0$. As a function of the value of α,

(a) what is the set of strictly dominated strategies?

(b) what is the set of weakly dominated strategies?

(c) what is the set of Nash equilibria?

SOLUTION. Let us start by rewriting player i's utility function in the following way:

$$\pi_i(s_1, s_2) = \begin{cases} s_i & \text{if } s_1 + s_2 \leq M \\ (1-\alpha)s_i + \alpha(M - s_j) & \text{if } s_1 + s_2 > M \end{cases}$$

This re-writing helps us to draw the utility function of agent i as a function of her strategy s_i, for a given amount s_j claimed by player j. The shape of the function depends on the value of α, as Figures 4, 5 and 6 show below.

Let us analyze the three different cases ($\alpha < 1$, $\alpha = 1$ and $\alpha > 1$) separately.

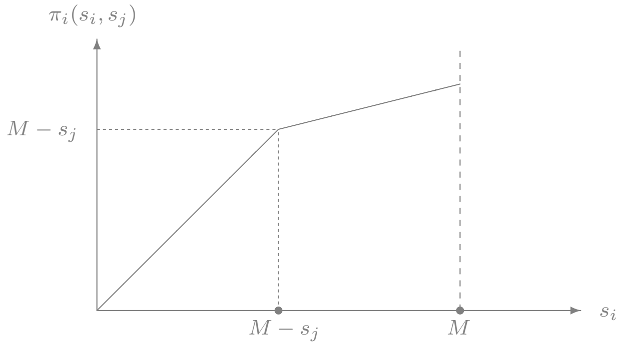

Figure 4: For $\underline{\alpha < 1}$, Player i's utility as a function of her own strategy s_i and for a given strategy s_j of player j.

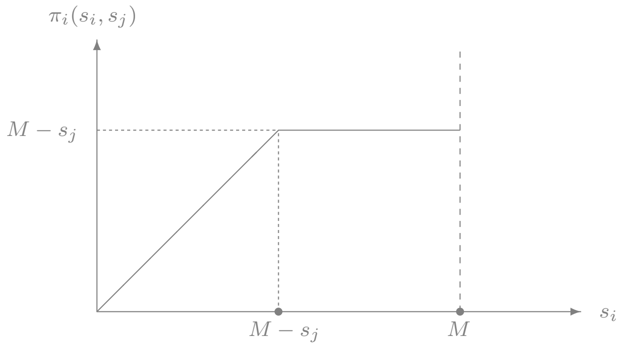

Figure 5: For $\underline{\alpha = 1}$, Player i's Utility as a function of her own strategy s_i and for a given strategy s_j of player j.

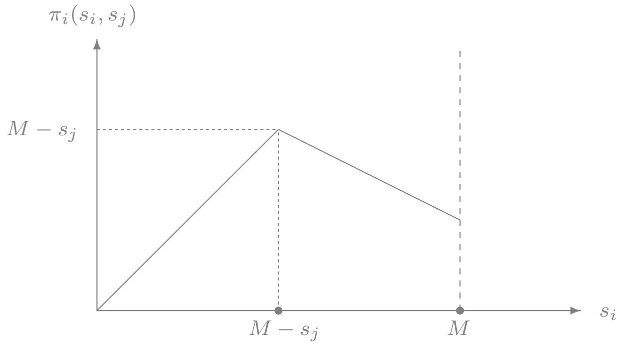

Figure 6: For $\alpha > 1$, Player i's utility as a function of her own strategy s_i and for a given strategy s_j of player j.

Case 1. $\alpha < 1$**:**

(a) Notice that, for any s_j, π_i is strictly increasing in s_i. Thus, $s_i = M$ is s strictly dominant strategy for player i. Another way to see it is to consider player i's utility as a function of j's strategy, as in Figure 7: for any s_j, strategy $s_i = M$ guarantees a strictly higher utility than any other strategy $s_i < M$. Thus, all strategies $s_i < M$ are strictly dominated for both players.

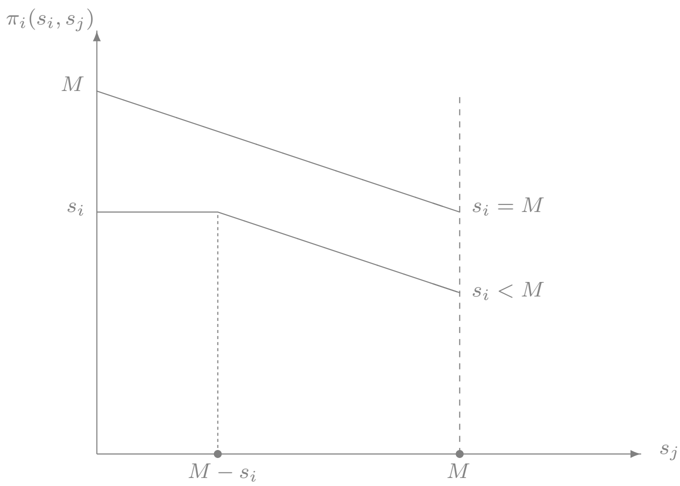

Figure 7: Utility of player i as a function player 2's strategy s_j of player j, $\alpha < 1$

(b) There is no weakly dominated strategy, as defined according to the standard definition. However, given that strictly dominated strategy are a particular case of weakly dominated ones, any $s_i < M$ is also weakly dominated for both players.

(c) The NE is clearly $(s_1^*, s_2^*) = (M, M)$.

Case 2. $\alpha = 1$**:**

(a) Notice that if $\alpha = 1$, player i's utility becomes

$$\pi_i(s_1, s_2) = \begin{cases} s_i & \text{if } s_1 + s_2 \leq M \\ M - s_j & \text{if } s_1 + s_2 > M \end{cases}$$

In other words, whenever $s_1 + s_2 > M$, i's payoff does not depend anymore on her strategy. This implies that, if she plays anything more than $M - s_j$ (so that $s_i + s_j > M$), she will anyway receive $M - s_j$.

Thus, playing $s_i = M$ guarantees a strictly higher utility than $s_i < M$ for all s_j such that $s_i + s_j \leq M$, and the same utility for all s_j such that $s_i + s_j > M$ (see Figure 8). In other words, $s_i = M$ is a weakly dominant strategy.

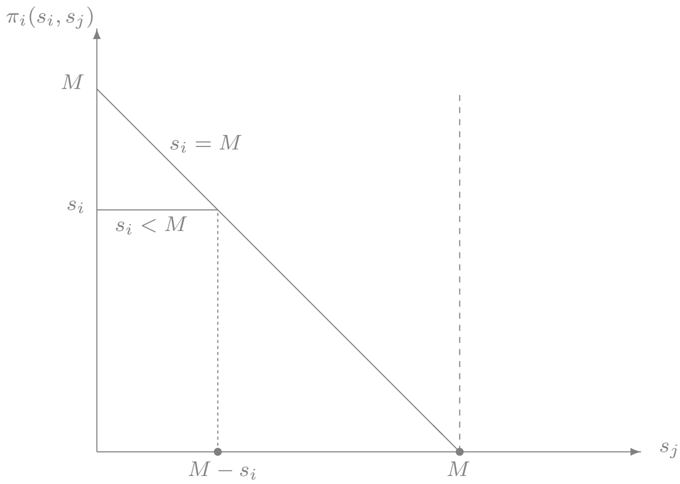

Figure 8: Utility of player i as a function player 2's strategy s_j of player j, $\underline{\alpha = 1}$

Thus, there is no strictly dominated strategy, as any strategy can perform at least as good as another one for at least one s_j.

(b) Given that $s_i = M$ is a weakly dominant strategy, all the strategies $s_i < M$ are weakly dominated.

(c) Let us fix a strategy $\hat{s}_j$ for player j. If player i plays s_i such that $s_i + \hat{s}_j < M$, then $\pi_i(s_i, \hat{s}_j) < M - \hat{s}_j$. Then, by playing $s'_i = M - s_j$, she will get $\pi_i(s'_i, \hat{s}_j) = M - \hat{s}_j > \pi_i(s_i, \hat{s}_j)$. Thus, no strategy profile such that $s_1 + s_2 < M$ can be a NE.

If player i plays $s_i = M - \hat{s}_j$ then there exists no profitable deviation: any $s'_i < M - \hat{s}_j$ gives lower payoff, while any $s'_i > M - \hat{s}_j$ gives the same payoff. Thus, all the strategy profiles such that $s_1 + s_2 = M$ are NEa.

Finally, assume player i plays $s_i > M - \hat{s}_j$, so that she receives $\pi_i(s_i, \hat{s}_j) = M - \hat{s}_j$. Playing any other strategy s'_i such that $s'_i + \hat{s}_j \geq M$ guarantees the same payoff, while any strategy s_i' such that $s_i' + \hat{s}_j < M$ gives something lower. Thus, also all the strategy profiles such that $s_i + s_j > M$ are NEa.

Summing up, the set of NEa when $\alpha = 1$ is given by

$$NE = \{(s_1, s_2) \in [0, M]^2, s_1 + s_2 \geq M\}$$

Case 3. $\alpha > 1$**:**

(a) Consider any two strategies s_i and s_i'. Figure 9 shows player i's utility function for these strategies, as a function of s_j. We immediately see that for some values of s_j, $s_i \succ s_i'$, while for some other values of s_j, $s_i' \succ s_i$. This holds for any two strategies of player i. Thus, there exists no strictly dominated strategy.

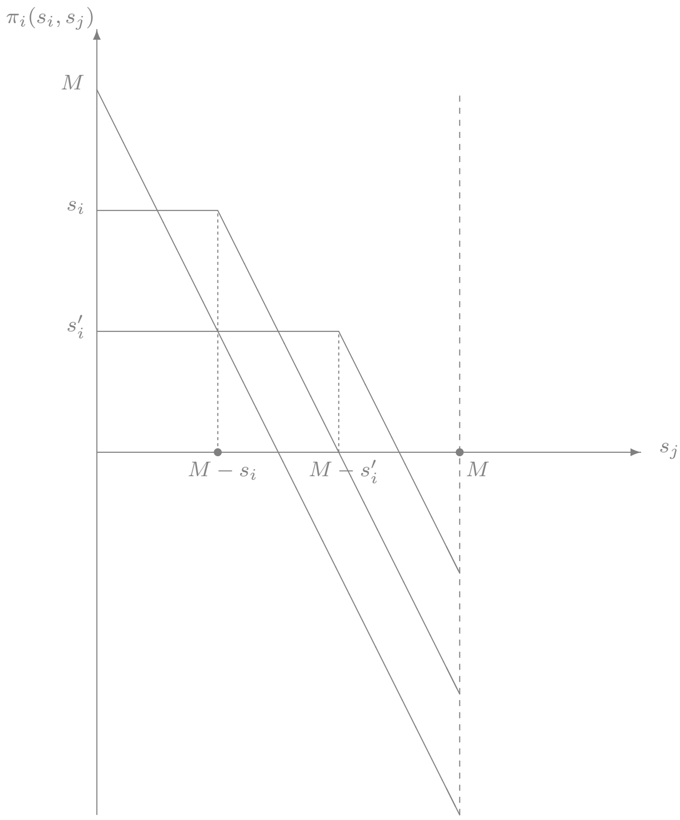

Figure 9: Utility of player i as a function player 2's strategy s_j of player j, $\alpha > 1$

(b) Since all strategies s_i are best responses to one strategy s_j, there is no weakly dominated strategy.

(c) The set of NEa is the set

$$NE = \{(s_1, s_2) \in [0, M]^2, s_1 + s_2 = M\}$$

Fix a strategy for player j, $\hat{s}_j$ and assume player i plays $s_i = M - \hat{s}_j$. Then her payoff will be $\pi_i(s_i, \hat{s}_j) = M - \hat{s}_j$. Let us check that there exists no profitable deviation.

If player i plays $s_i' = M - \hat{s}_j - \epsilon$ (for any $\epsilon \in (0, M - \hat{s}_j]$), then and

$$s_i' + \hat{s}_j = M - \epsilon \;\;\Rightarrow\;\; \pi_i(s_i', \hat{s}_j) = M - \hat{s}_j - \epsilon < M - \hat{s}_j$$

Thus, $s_i' = M - \hat{s}_j - \epsilon$ cannot be a profitable deviation.

If player i plays $s_i' = M - \hat{s}_j + \epsilon$ (for any $\epsilon > 0$), then then

$$s_i' + \hat{s}_j = M + \epsilon \;\;\Rightarrow\;\; \pi_i(s_i', \hat{s}_j) = M - \hat{s}_j + (1 - \alpha)\epsilon < M - \hat{s}_j$$

Thus, $s_i' = M - \hat{s}_j + \epsilon$ cannot be a profitable deviation.