LECON2112 Advanced Microeconomics II

– Assignment 5 –

(SOLUTIONS)

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Exercises[1]

8Ea. Let $I = \{1, 2\}$, $S_1 = (T, B)$, and $S_2 = (L, R)$. With probability $\frac{1}{3}$, agent 2 is of type a, and with probability $\frac{2}{3}$, she is of type b. Agent 1 does not know the type of agent 2 (but these probabilities are common knowledge). Depending on the type of agent 2, they play either of the matrices shown below. Compute the pure strategy Bayesian Nash equilibria of this game.

Type a

	L	R
T	6, 12	4, 6
B	3,6	9,0

Type b

	L	R
T	6, 0	4, 4
B	3,4	9,12

SOLUTION. Player 1 ignores player 2's type when choosing an action and can hence not condition her action on the type. She has two pure strategies:

$$S_1 = \{T, B\}.$$

Player 2 can condition her action on her type as she observes it before taking an action and has hence four pure strategies. For simplicity, we write $(L, R) = (L|a, R|b)$ the pure strategy for which she plays L if her type is a and R if her type is b. With this convention we have

$$S_2 = \{(L, L), (L, R), (R, L), (R, R)\}.$$

The normal form of the game is

	(L,L)	(L,R)	(R,L)	(R,R)
T	6 , 4	4.6 , 6.6	5.3 , 2	4 , 4.6
B	3 , 4.6	7 , 10	5 , 2.6	9 , 8

where the expected utility of players 1 and 2 for the particular pure strategy profile $(T, (L, R))$ is computed in the following way:

[1] Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

$$U_1(T, LR) = Prob(a)u_1(T, L|a) + Prob(b)u_1(T, R|b) = \frac{1}{3}6 + \frac{2}{3}4 = \frac{14}{3} = 4.6$$
$$U_2(T, LR) = Prob(a)u_2(T, L|a) + Prob(b)u_2(T, R|b) = \frac{1}{3}12 + \frac{2}{3}4 = \frac{20}{3} = 6.6$$

By analyzing the best responses (red for player 1 and blue for player 2), we can immediately see that the only Bayesian Nash Equilibrium of the game is $\big(B, (L, R)\big)$.

8Ee. Let $I = \{1, 2\}$, $S_1 = (T, B)$, and $S_2 = (L, R)$. With probability p_a, agent 2 is of type a, and with probability p_b, she is of type b ($p_a + p_b = 1$). Agent 1 does not know the type of agent 2 (but probabilities are common knowledge). Depending on the type of agent 2, they play either of the matrices shown below, which denote the game forms. If information were complete, then (T,L) and (B,R) would be two pure strategy Nash equilibria if agent 2 were of type a, and (T,R) and (B,L) two pure strategy Nash equilibria if she were of type b. Identify two pure strategy Bayesian Nash equilibria of this game.

Type a

	L	R
T	x	y
B	z	w

Type b

	L	R
T	r	s
B	t	v

SOLUTION. Using the information in the text, we know that

$$\left.\begin{array}{c} x \succ z \\ w \succ y \\ t \succ r \\ s \succ v \end{array}\right\} \text{for Player 1} \qquad \left.\begin{array}{c} x \succ y \\ w \succ z \\ s \succ r \\ t \succ v \end{array}\right\} \text{for Player 2}$$

The two Baysian NEa of the game can be easily derived using this information. They are

$$(T, \{LR\}) \quad \text{and} \quad (B, \{RL\})/$$

To see why they are Baysian NE, let us look for possible deviations.

FIRST EQUILIBRIUM. Consider the first equilibrium:
Player 1. Equilibrium utility for Player 1 is

$$u_i(T, \{LR\}) = p_a x + p_b s$$

deviating to B, would give her a utility

$$u_i(B, \{LR\}) = p_a z + p_b v$$

which, given the preferences we saw above (i.e. $x \succ z$ and $s \succ v$) , can never be profitable.
Player 2. When Player 1 plays T, we know that playing L is a best response if Player 2 is of type a, while playing R is a best response if she is of type b. Therefore, any strategy where Player 2 plays R if a or L if b can never be a profitable deviation.

SECOND EQUILIBRIUM. Let us consider the second equilibrium now:
Player 1. Equilibrium utility for Player 1 is

$$u_i(B, \{RL\}) = p_a w + p_b t$$

deviating to T, would give her a utility

$$u_i(T, \{RL\}) = p_a y + p_b r$$

which, given the preferences we saw above (i.e. $w \succ y$ and $t \succ r$) , can never be profitable.
Player 2. When Player 1 plays B, we know that playing R is a best response if Player 2 is of type a, while playing L is a best response if she is of type b. Therefore, any strategy where Player 2 plays L if a or R if b can never be a profitable deviation.

8Eb. Two agents, 1 and 2, have to share a dollar. Each agent can choose to enter into a conflict or not. If no agent chooses the conflict, then the dollar is divided evenly, and this is the payoffs. As soon as one agent chooses to enter into a conflict, the conflict follows. The agent who looses the conflict gets a payoff of zero. the winner of the conflict gets a payoff of δ. Each agent is either weak or strong. The probability that an agent is strong is π. Probabilities are independently distributed. In case of a conflict, if the two agents are of the same type, then they both have a probability of winning equal to 0.5. In case a strong agent is in conflict with a weak agent, the former wins the conflict with probability $q > 0.5$. Assume $\delta < 1$.

(a) What is a strategy in that game?
SOLUTION. See solutions from class.

(b) What is the payoff function?
SOLUTION. See solutions from class.

(c) What is, or what are, the Bayesian Nash equilibrium, or equilibria, of that game, as a function of the values of δ, π and q?
SOLUTION. See solutions from class.

(d) What is the probability of a conflict?
SOLUTION. See solutions from class.