LECON2112 Advanced Microeconomics II

– Assignment 9 –

(SOLUTIONS)

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Exercises[1]

11Ba. We have 2 agents (1,2). We have 2 goods, one private good in which both have an endowment (w_1 and w_2 respectively) and pollution. Their utility functions are $u_1 = x_1 + (-(h-5)^2)$ and $u_2 = x_2 - h$ where x is the amount of the private good they enjoy and h the amount of pollution they experience. 1 is the one choosing the level of h.

(a) What will happen if only the private good is transferable from one agent to the other? Is it Pareto efficient?

Solution. In this case, agent 1 maximizes its utility by solving

$$\max_h \quad u_1 = x_1 + (-(h-5)^2)$$

If we consider that $x_1 = w_1$ and $x_2 = w_2$, we get the following utility in this case

$$\frac{du_1}{dh} = -2(h-5) \overset{!}{=} 0 \quad \Leftrightarrow \quad h = 5 \quad \Longrightarrow \quad \begin{cases} u_1 = x_1 = w_1 \\ u_2 = w_2 - 5. \end{cases}$$

Then, Pareto efficiency in this context means that the marginal utility of h for one agent must be equal to the marginal cost for the other agent. Here we have

$$\Phi_1'(h) = 0 \qquad \text{and} \qquad \Phi_2'(h) = 1,$$

which means that the situation is not Pareto efficient.

(b) Let's now say that we have a market for h. To produce a unit of h, 1 must buy a permit from 2. What amount of h will be produced? Is it Pareto efficient?

Solution. Now if we have a market for h, the problem solved by the first agent become

$$\max_h \quad u_1 = w_1 - ph + (-(h-5)^2)$$
$$FOC: \quad -p - 2(h-5) = 0 \quad \Leftrightarrow \quad -2(h-5) = p,$$

where p is the price that 1 needs to pay 2 to produce one unit of h.

[1] Inspired by Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

The problem solved by 2 is

$$\begin{aligned} \max_{h} \quad & u_2 \;=\; w_2 + ph - h \\ FOC: \quad & p - 1 = 0 \quad \Leftrightarrow \quad p = 1. \end{aligned}$$

Now, if we put both FOCs together, we get that $h = 4.5$ in the presence of a market. To see if it is Pareto efficient, we need to compare the agents' marginal (dis)utility as before:

$$\Phi_1'(h) = -2(h-5) = -2(4.5-5) = 1 \qquad \text{and} \qquad \Phi_2'(h) = 1,$$

from which we can see that it is indeed Pareto efficient!