LECON2112 Advanced Microeconomics II

– Assignment 3 –

(SOLUTIONS)

Professor Benoît Decerf
TA: Thomas Eisfeld

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Exercises[1]

8D4. Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 dollars in profit, but they must agree on how to split the 100 dollars. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than 100 dollars, then they fail to agree, and each gets nothing. If their demands sum to less than 100 dollars, they do the project, each gets his demand, and the rest goes to charity.

(a) What are each player's strictly dominated strategies?

SOLUTION. If Player i demands $y \geq 100$, then any strategy of player j with $x \geq 0$ is payoff equivalent. Therefore, the exists no strictly dominated strategy.

(b) What are each player's weakly dominated strategies?

SOLUTION. Any strategy demanding more than 100 dollars is weakly dominated.

Case 1. Player 2 demands $y \geq 100$. Then any strategy of Player 1 with $x \geq 0$ is payoff equivalent.

Case 2. Player 2 demands $0 \geq y < 100$. Then, Player 1 could demand $x = 100 - y$, obtaining a payoff of $100 - y$. Demanding $x > 100$ will give Player 1 a payoff of zero.

Therefore, any strategy demanding more than 100 is weakly dominated.

(c) What are the pure strategy Nash equilibria of this game?

SOLUTION. Any pair $(x, 100 - x)$ with $x \in [0, 100]$ constitutes a pure strategy Nash equilibrium of this game. To see this, suppose that Player 1 demands $x \in [0, 100]$. If Plater 2 demand $y > 100 - x$, then their demands sum to more than 100 dollars and both get zero. If Player 2 demands $y \in [(]0, 100 - x)$, he will obtain his demand therefore be worse off than if he would have demanded $100 - x$. Thus, if Player 1 demands x, Player 2's best reply is to demand $y = 100 - x$. Similarly, if Player 2 demands $100 - x$, Player 1's best response is to demand x.

[1] Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

8D9. Consider the following game [based on an example from Kreps (1990)]:

		Player 2			
		LL	L	M	R
Player 1	U	$100, 2$	$-100, 1$	$0, 0$	$-100, -100$
	D	$-100, -100$	$100, -49$	$1, 0$	$100, 2$

(a) If you were Player 2 in this game and you were playing it once without the ability to engage in preplay communication with Player 1, what strategy would you choose?

SOLUTION. Playing L or R is quite risky, since we do not know what Player 1 will do. The risk of obtaining a payoff of -49 is very large compared to the payoff of 1 if Player 2 played L, and the risk of obtaining a payoff of -100 is very large to the payoff of 2 if Player 2 played LL or R. Therefore, it seams most reasonable to play M.

(b) What are all the Nash equilibria (pure and mixed) of this game?

SOLUTION. There exist two pure strategy NE in this game: (U, LL) and (D, R). To check for MSNE, Player 1 must mix between U (with probability p) and D (with probability $1 - p$). Player 2 then has 11 possible mixing combinations: $\{LL, L\}$, $\{LL, M\}$, $\{LL, R\}$, $\{L, M\}$, $\{L, R\}$, $\{M, R\}$, $\{M, L, R\}$, $\{LL, M, R\}$, $\{LL, L, R\}$, $\{LL, L, M\}$, and $\{LL, L, M, R\}$. For the sake of dispostion, we will show that only the first combination, $\{LL, L\}$, is part of a MSNE.

For Player 2 to mix between LL and L (with probabilities q and $q - 1$ respectively), we must have that

$$p\,(2) \;+\; (1-p)\,(-100) = p\,(1) + (1-p)\,(-49),$$

which yields $p = 51/52$. The utility of Player 2 from each strategy then is:

$$u_2(LL) = u_2(L) = \frac{1}{26}, \quad u_2(M) = 0, \quad \text{and} \quad u_2(R) < 0.$$

Then, for Player 1 to mix between U and D, we must have

$$q\,(100) \;+\; (1-q)\,(-100) \;=\; q\,(-100) \;+\; (1-q)\,(100),$$

which yields $q = 1/2$, and

$$u_1(U) = u_1(D) = 0.$$

Therefore, $p = 51/52$ and $q = 1/2$ is a MSNE.

We now show that no other mixing combination of Player 2 can be part of a MSNE.

(i) If Player 2 mixes with the combination $\{LL, M\}$, we must have $p = 50/51$, which gives utilities

$$u_2(LL) = u_2(M) = 0 \quad \text{and} \quad u_2(L) = \frac{1}{51} > 0,$$

so this cannot be part of a MSNE.

(ii) If Player 2 mixes with the combination $\{L, R\}$, we must have $p = 1/2$, which yields utilities

$$u_2(LL) = u_2(R) = -49 \quad \text{and} \quad u_2(M) = 0,$$

so this cannot be part of a MSNE either.

(iii) If Player 2 mixes with one of the combinations $\{L, M\}$, $\{L, R\}$, $\{M, R\}$, $\{L, M, R\}$, then Player 1 will have D as a strict best response, which in tern has R as Player 2's strict best response.

(iv) If Player 2 mixes with one of the combinations $\{LL, M, R\}$, $\{LL, L, R\}$, or $\{LL, L, M, R\}$, then the analysis of (ii) above implies that this cannot be part of a MSNE.

(v) If Player 2 mixes with the combination $\{LL, L, M\}$, then the analysis of (i) above implies that the cannot be part of a MSNE.

(c) Is your strategy choice in **(a)** a component of any Nash equilibrium strategy profile? Is it a rationalizable strategy?

SOLUTION. The choice in part **(a)** is not part of any NE described above. It is easy to see that strategy M is rationalizable: If Player 1 plays $p = 1/2$, then M is the Player 2's unique best reply.

(d) Suppose now that preplay communication were possible. Would you expect to play something different from your choice in **(a)**?

SOLUTION. If preplay communcation were possible, players could agree to play one of the pure strategy NE, which are payoff equivalent and Pareto dominant for both players. Therefore, Player 2 will play either LL or R, depending on the agreed upon equilibrium.

8Df. Consider the two-player game $\Gamma = \{N = \{1, 2\}, \Delta S = \Delta S_1 \times \Delta S_2, (u_1, u_2)\}$. Let us assume it is a zero sum game, that is, for all $s_1 \in S_1, s_2 \in S_2 : u_1(s_1, s_2) = -u_2(s_1, s_2)$.

- Assume there are two pure strategy Nash equilibria: (s_1^*, s_2^*) and (s_1^{**}, s_2^{**}). Prove that

$$u_1(s_1^*, s_2^*) = u_1(s_1^{**}, s_2^{**}) \quad \text{and} \quad u_2(s_1^*, s_2^*) = u_2(s_1^{**}, s_2^{**}).$$

- Show that the same equivalence property holds if the two equilibria are mixed strategy equilibria.

SOLUTION. This can be proved by contradiction. Suppose that one of the players is strictly better off in one of the two Nash equilibria. Say, for instance, that the following holds:

$$u_2(s_1^*, s_2^*) < u_2(s_1^{**}, s_2^{**}) \tag{1}$$

Since (s_1^{**}, s_2^{**}) is a Nash equilibrium, $\forall s_1' \in S_1$ it must hold that:

$$u_1(s_1', s_2^{**}) \leq u_1(s_1^{**}, s_2^{**})$$

Take $s_1' = s_1^*$, then

$$u_1(s_1^*, s_2^{**}) \leq u_1(s_1^{**}, s_2^{**})$$

Since $s_1 \in S_1, s_2 \in S_2 : u_1(s_1, s_2) = -u_2(s_1, s_2)$,

$$u_2(s_1^*, s_2^{**}) \geq u_2(s_1^{**}, s_2^{**}) \tag{2}$$

Combining 1 and 2 we get:

$$u_2(s_1^*, s_2^{**}) \geq u_2(s_1^{**}, s_2^{**}) > u_2(s_1^*, s_2^*)$$
$$\rightarrow u_2(s_1^*, s_2^{**}) > u_2(s_1^*, s_2^*)$$

which contradicts (s_1^*, s_2^*) being a Nash equilibrium since s_2^{**} would represent a profitable deviation for player 2.