LECON2112 Advanced Microeconomics II

– Assignment 6 –

(SOLUTIONS)

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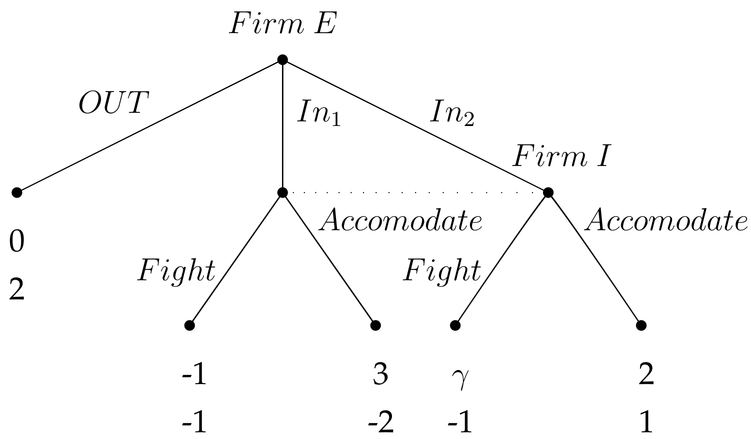
Exercises[1]

9C2. Consider the following game:



(a) What is the set of weak PBEs when $\gamma > 0$?

Solution. Let σ_F be the probability that I fights after entry, μ_1 be I's belief that In_1 was E's entry strategy if entry has occurred, and σ_0, σ_1, σ_2 be the probabilities with which E actually chooses OUT, In_1 and In_2 respectively.

Note first that I plays $Fight$ with positive probability if and only if its expected utility from doing so is at least as large as the expected utility of playing $Accomodate$:

$$\begin{aligned} EU_I(Fight) &\geq \; = EU_I(Accomodate) \\ \Leftrightarrow \quad -1(\mu) - 1(1-\mu) &\geq \; -2(\mu) + 1(1-\mu) \end{aligned}$$

which means $\mu_1 \geq \frac{2}{3}$.

Suppose, first, that $\mu_1 > \frac{2}{3}$ is the system of beliefs at the weak PBE. If that is the case, I should always play $Fight$ in case of entry, (so $\sigma_F = 1$). Therefore, it is always optimal for E to adopt entry strategy In_2 (so $\sigma_1 = 0$, $\sigma_2 = 1$). But the consistent beliefs condition requires that $\sigma_1 = 0$ implies $\mu_1 = 0$, a contradiction.

[1]Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

Suppose, instead, that $\mu_1 < \frac{2}{3}$ is the system of beliefs at the weak PBE. In this case, I should always play $Accomodate$ in case of entry (so $\sigma_F = 0$). Therefore, it is always optimal for E to adopt entry strategy In_1 (so $\sigma_1 = 1$, $\sigma_2 = 0$). But the consistent beliefs condition requires that $\sigma_1 = 1$ implies $\mu_1 = 1$, which is again a contradiction.

Finally, consider $\mu_1 = \frac{2}{3}$ is the system of beliefs at the weak PBE. In this case, E must randomize in the equilibrium with positive probabilities attached to both In_1 and In_2. I's probability of playing $Fight$ must make E indifferent between playing In_1 and In_2:

$$\begin{aligned} EU_E(In_1) &= EU_E(In_2) \\ \Leftrightarrow \quad -1(\sigma_F) + 3(1-\sigma_F) &= \gamma(\sigma_F) + 2(1-\sigma_F), \end{aligned}$$

which means $\sigma_F = \frac{1}{\gamma+2}$. Thus, E's payoff for entering (playing either In_1 or In_2) is $EU_E(In_1) = EU_E(In_2) = \frac{3\gamma+2}{\gamma+2}$. And since that is positive for any $\gamma > 0$, E never plays OUT. Note that as beliefs need to be consistent, it follows from $\mu_1 = \frac{2}{3}$ that $\sigma_1 = \frac{2}{3}$.

As a result, the unique weak PBE of this game when $\gamma > 0$ is $(\sigma_0, \sigma_1, \sigma_2) = (0, \frac{2}{3}, \frac{1}{3})$, $\sigma_F = \frac{1}{\gamma+2}$, and $\mu_1 = \frac{2}{3}$.

(b) What is the set of weak PBEs when $\gamma \in (-1, 0)$?

Solution. As in the previous case, we cannot have that $\mu_1 < \frac{2}{3}$ is the system of beliefs at the weak PBE. However, we can have $\mu_1 > \frac{2}{3}$ or $\mu_1 = \frac{2}{3}$.

For $\mu_1 > \frac{2}{3}$, I should always play $Fight$ in case of entry (so $\sigma_F = 1$). Given $\gamma \in (-1, 0)$, it is optimal for E to play OUT, which supports any beliefs in the information set of I. Therefore, one class of weak PBE is with $(\sigma_0, \sigma_1, \sigma_2) = (1, 0, 0)$, $\sigma_F = 1$, and $\mu_1 > \frac{2}{3}$.

When $\mu_1 = \frac{2}{3}$, and as before, E must randomize in the equilibrium with positive probabilities attached to both In_1 and In_2. I's probability of playing $Fight$ must make E indifferent between playing In_1 and In_2:

$$\begin{aligned} EU_E(In_1) &= EU_E(In_2) \\ \Leftrightarrow \quad -1(\sigma_F) + 3(1-\sigma_F) &= \gamma(\sigma_F) + 2(1-\sigma_F) \\ \Leftrightarrow \quad 3 - 2\sigma_F &= 2 - (2-\gamma)\sigma_F, \end{aligned}$$

which means $\sigma_F = \frac{1}{\gamma+2}$. Therefore, E's payoff for entering (by choosing In_1 or In_2), is $EU_E(In_1) = EU_E(In_2) = \frac{3\gamma+2}{\gamma+2}$.

With $\gamma \in (-1, 0)$, E will actually enter if and only if γ is large enough to guarantee that

$$\begin{aligned} EU_E(In_1) = EU_E(In_2) \quad &> \quad EU_E(OUT) \\ \Leftrightarrow \quad \frac{3\gamma+2}{\gamma+2} \quad &> \quad 0, \end{aligned}$$

which implies $\gamma > -\frac{2}{3}$.

So, if $\gamma \leq -\frac{2}{3}$, E will play OUT, meaning that a weak PBE with $\mu_1 = \frac{2}{3}$ does not exists. If, however, $\gamma > -\frac{2}{3}$, there exists a weak PBE such that $(\sigma_0, \sigma_1, \sigma_2) = (0, \frac{2}{3}, \frac{1}{3})$, $\sigma_F = \frac{1}{\gamma+2}$, and $\mu_1 = \frac{2}{3}$.