## **LECON2112 Advanced Microeconomics II**

- Assignment 8 -

(SOLUTIONS)

Professor Benoît Decerf Academic Year: 2023/2024

TA: Thomas Eisfeld Spring term

## Exercises<sup>1</sup>

13C5. Assume a single firm and a single consumer. The firm's product may be either high or low quality and is of high quality with probability  $\lambda$  The consumer cannot observe quality before purchase and is risk neutral. The consumer's valuation of a high-quality product is  $v_H$ ; her valuation of a low-quality product is  $v_L$ . The costs of production for high (H) and low (L) quality are  $c_H$  and  $c_L$ , respectively. The consumer desires at most one unit of the product. Finally, the firm's price is regulated and set at p. Assume that  $v_H > p > v_L > c_H > c_L$ .

(a) Given the level of p, under what conditions will the consumer buy the product?

**SOLUTION.** The consumer will buy the product if the expected value of the product is greater than the price, i.e., if

$$\lambda v_H + (1 - \lambda)v_L \ge p.$$

**(b)** Suppose that before the consumer decides to buy, the firm (which knows its type) can advertise. Advertising conveys no information directly, but consumers can observe the total amount of money that the firm is spending on advertising, denoted by *A*. Can there be a separating perfect Bayesian equilibrium, that is, an equilibrium in which the consumer rationally expects firms with different quality levels to pick different levels of advertising?

**SOLUTION.** Suppose there exists a separating equilibrium in which the high quality producer spends A on advertising and only the high quality product will be bought (in a separating equilibrium consumers know the quality of a product, so low quality products will not be bought since  $p > v_L$ ). This implies that the low quality producer makes no profit and the high quality producer makes a non-negative profit:

$$\pi_H = p - c_H - A \ge 0.$$

<sup>&</sup>lt;sup>1</sup>Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

However, a low quality producer can make a positive profit by spending *A* on advertising, since the consumer will then mistake him for a high quality producer and buy the good from him. Then, the low quality producer's profit will equal

$$\pi_L = p - c_L - A > p - c_H - A \ge 0.$$

Therefore, no separating equilibrium can exist.

**13Ca.** A firm has to decide whether or not to hire a worker. The productivity of the worker is unobservable. With probability  $\lambda$ , the worker is of high type and produces  $\overline{\theta}$ , and with probability  $1-\lambda$ , the worker is of low type and produces  $\underline{\theta}$ ,  $0<\underline{\theta}<\overline{\theta}$ . Expected productivity is written  $E(\theta)$ . If the firm decides to hire the worker, there is no flexibility on the wage w, which is equal to  $E(\theta)$ . Before the firm takes a decision, the worker, who knows her type, can choose a level of education e, which is observed by the firm. The worker's utility function is  $u(e,w)=w-c(e|\theta)$  where  $c(\cdot)$  is increasing, convex and satisfies  $c(0|\theta)=0$ ,  $c_e(e|\theta)>0$ ,  $c_{ee}(e|\theta)>0$ ,  $c_{e\theta}(e|\theta)<0$ . What are the separating weak perfect Bayesian equilibria of this game?

**SOLUTION.** See solutions from class.