LECON2112 Advanced Microeconomics II

– Assignment 2 –

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Deadline: Monday, February 19, 2024 at 5pm.

Instructions: To be submitted via Moodle as a single file (including your name and NOMA).

Exercises[1]

8Bd. Consider the following game. There are five players, three male singles 1, 2 and 3, and two female singles, a and b. The preferences $\succ_1$, $\succ_2$, $\succ_3$, $\succ_a$ and $\succ_b$ of those five players vis-à-vis a partner from the other gender and remaining single (which is denoted $\emptyset$) are as follows:

$$
\begin{array}{lll}
b \succ_1 a \succ_1 \emptyset & \qquad & 1 \succ_a 2 \succ_a 3 \succ_a \emptyset \\
a \succ_2 b \succ_2 \emptyset & \qquad & 2 \succ_b 1 \succ_b 3 \succ_b \emptyset \\
a \succ_3 b \succ_3 \emptyset & &
\end{array}
$$

In stage 1, players 1, 2 and 3 propose the name of one female player, a or b. In stage 2, each player among a and b, if she got two or three propositions in stage 1, chooses one of them. Then, couples are formed. In case one female player did not get any proposition, she remains single. If she gets one proposition, she is married to the player having made the proposition. If she gets several propositions, she is married to the proposition she has chosen in stage 2.

(e) Does there exist a Nash equilibrium whose outcome is such that 3 gets married with a? If yes, give one, otherwise prove it.

(f) Does there exist a Nash equilibrium in undominated strategies whose outcome is such that 3 gets married with a? If yes, give one, otherwise prove it.

8Ba. Two agents have to contribute to a common pot. They have preferences over the total amount in the pot, not over their own contributions. Each agent has an optimal amount, and moving further away from the optimal amount is worse. Formally let I =

[1]Source: Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

$\{1,2\}$, $S_1 = S_2 = \mathbb{R}_{++}$, and, for all $s_1 \in S_1$, $s_2 \in S_2$,

$$u_1(s_1, s_2) = -|A_1 - (s_1 + s_2)|$$
$$u_2(s_1, s_2) = -|A_2 - (s_1 + s_2)|$$

where A_1 (resp. A_2) is the optimal amount of agent 1 (resp. 2), and we assume $A_1 < A_2$. Which strategies survive iterative deletion of strictly dominated strategies?

8Dg. In a remote country, the national army is composed of three separate forces A, B and C, each of them controlled by a different general. One day, general A judges that the current civil government is taking decisions that are in contradiction with the country's constitution. Therefore, general A considers making a coup in order to replace the current civil government. Knowing that general A is likely to make a coup, generals B and C have to separately and simultaneously decide whether to be loyal and fight the coup in case there is one (L) or join the rebellion and support the coup in case there is one (R).

There are four possible outcomes of that game. First, there could be no coup. That happens if general A decides not to try and make a coup. Second, there could be a successful coup. That happens when general A makes a coup and both general B and C have decided to join the rebellion in case there is one. Third, there could be a failed coup. That happens when general A makes a coup and both general B and C have decided to remain loyal in case there is a coup. Finally, there coup be an undecided coup. That happens when general A makes a coup but one and only one of the other two generals, B or C, has decided to join the rebellion in case there is a coup.

The payoffs are as follows.

Table 1: Exercise 8Dg.

	no coup	failed coup	undecided coup	successful coup
General A	-2	-6	0	3
General B	1	2	0	1
General C	-1	1	0	2

1. Find all pure strategy Nash equilibria of this game.
2. Is the following list of strategies a mixed strategy Nash equilibrium of this game: A makes a coup, B remains loyal with probability $\frac{2}{3}$ and joins the rebellion with probability $\frac{1}{3}$, C remains loyal with probability $\frac{1}{3}$ and joins the rebellion with probability $\frac{2}{3}$.
3. Identify a Nash equilibrium in which the probability that B remains loyal is 0.9.
4. Do agents have (strictly/weakly) dominated strategies in this game?

8Dk. An employer would like to allocate a total amount of M to two deserving employees. The employer sets up the following game. Each employee is asked to announce the amount of money he or she thinks she deserves. Then, if the total amount of money claimed by the two employees does not exceed M, each employee receives what he or she claims. On the contrary, if the total amount of money claimed by the employees exceeds M, both employees are penalized: each gets the amount he or she claimed, minus a fine that is proportional to the total excess claim. Formally, there is a set $I = \{1, 2\}$ of players, with strategy sets $S_i = [0, M]$, $i \in I$. The payoff functions π_i are defined by

$$\pi_i(s_1, s_2) = \begin{cases} s_i & \text{if } s_1 + s_2 \leq M \\ s_i - \alpha\big((s_1 + s_2) - M\big) & \text{if } s_1 + s_2 > M \end{cases}$$

for some $\alpha > 0$. As a function of the value of α,

1. what is the set of strictly dominated strategies?
2. what is the set of weakly dominated strategies?
3. what is the set of Nash equilibria?