LECON2112 Advanced Microeconomics II

– Assignment 10 –

(SOLUTIONS)

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Exercises[1]

11Ba. We have two agents (1 and 2). We also have two goods, one private good in which both have an endowment (w_1 and w_2, respectively) and pollution. Their utility functions are $u_1 = x_1 + (-(h-5)^2)$ and $u_2 = x_2 - h$ where x is the amount of the private good they enjoy and h the amount of pollution they experience. 1 is the one choosing the level of h.

(b) Let's now say that we have a market for h. To produce a unit of h, 1 must buy a permit from 2. What amount of h will be produced? Is it Pareto efficient?

SOLUTION. Now if we have a market for h, the problem solved by the first agent become

$$\max_h \quad u_1 \;=\; w_1 - ph + (-(h-5)^2)$$
$$FOC: \quad -p - 2(h-5) = 0 \quad \Leftrightarrow \quad -2(h-5) = p,$$

where p is the price that 1 needs to pay 2 to produce one unit of h.

The problem solved by 2 is

$$\max_h \quad u_2 \;=\; w_2 + ph - h$$
$$FOC: \quad p - 1 = 0 \quad \Leftrightarrow \quad p = 1.$$

Now, if we put both FOCs together, we get that $h = 4.5$ in the presence of a market. To see if it is Pareto efficient, we need to compare the agents' marginal (dis)utility as before:

$$\Phi_1'(h) = -2(h-5) = -2(4.5-5) = 1 \qquad \text{and} \qquad \Phi_2'(h) = 1,$$

from which we can see that it is indeed Pareto efficient!

11Bb. We have two agents (1 and 2) and 2 goods (A and B). A is a normal good and B a public one. Each agent has an endowment in A of w. The utility functions of the agents are $u_1 = x_1 + \sqrt{B}$ and $u_2 = x_2 - B^2$. Agent 2 can decide the quantity of good B. x_1 and x_2 are the amount of the normal good A that agent 1 and 2 enjoy.

(a) Assume that there is no public intervention. What would be the equilibrium? Would it be Pareto efficient?

[1] Inspired by Mas-Colell, Whinston, & Green, 1995. "Microeconomic Theory," Oxford University Press.

SOLUTION. Agent 2 chooses B to maximize its utility. From the FOC, we see that the maximum is when 2 produces $B = 0$. Each agent only has their endowment. Pareto efficiency in this context means that the marginal cost of B equals the marginal benefit of B. Here we have:

$$\Phi_1' = \frac{1}{2\sqrt{B}} = \infty \qquad \text{and} \qquad \Phi_2' = -2B = 0.$$

As the marginal cost and marginal utility are not equal, the situation is not efficient.

(b) One way to get to the Pareto efficient level is to organize a market for B. Let's say that agent 1 can buy credits that will force agent 2 to produce a certain amount of good B. What would be the amount of B produced in this case? Would it still be Pareto efficient?

SOLUTION. Now if we add a market for B, agents' utility function become

$$u_1 = x_1 - pB + \sqrt{B} \qquad \text{and} \qquad u_2 = x_2 + pB - B^2.$$

If we compute the FOCs of each agent with respect to B, we then get that:

$$\begin{array}{lll} \text{Agent 1}: & -p + \dfrac{1}{2\sqrt{B}} = 0 \Rightarrow & p = \dfrac{1}{2\sqrt{B}} \\ \text{Agent 2}: & p - 2B = 0 \Rightarrow & p = 2B \\ \Rightarrow B = 0.3968 & & \end{array}$$

And, in this case, it is efficient, we can check it by computing the Φ' functions for the 2 agents at this value of B.

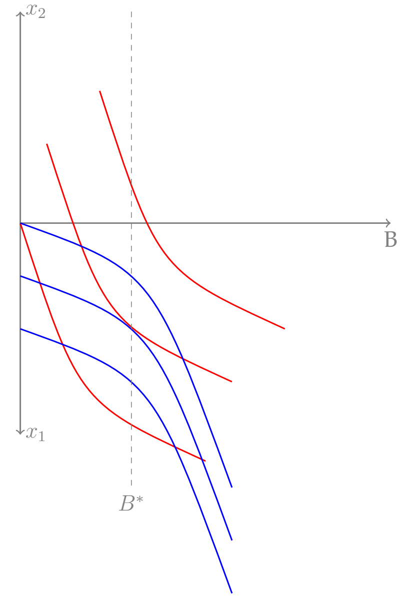
(c) Is it still the case if we assume a third agent has the same utility as agent 1? Why?

SOLUTION. If we add a third agent, we get the following utility functions:

$$u_1 = x_1 - pB + \sqrt{B}$$
$$u_2 = x_2 + pB - B^2$$
$$u_3 = x_3 - pB + \sqrt{B}$$

If we compute the FOCs and the optimal value of B, we will get the same value as before, $B = 0.3968$. To assess efficiency, we compute the Samuelson condition in this case, which does not hold: it is not efficient because the marginal cost is not equal to the sum of marginal utilities.

(d) Show the situation on a graph.



SOLUTION. In red, the indifference curves of agent 1, and in blue, the indifference curves of 2. B^* is the level of the public good at Pareto efficiency in our problem with 2 players ($B = 0.3968$).