STA404 Clinical Biostatistics

Exercise 01

Thomas Fischer
09930090

1 Problem 1

$$\begin{aligned} Sens &= Pr(Y=1|D=1) \\ Spec &= Pr(Y=0|D=0) \\ PPV &= Pr(D=1|Y=1) \\ NPV &= PR(D=0|Y=0) \end{aligned}$$

Show that

$$\begin{aligned} PPV &= \frac{SensPrev}{SensPrev + (1-Spec)(1-Prev)} \\ NPV &= \frac{Spec(1-Prev)}{Spec(1-Prev) + (1-Sens)Prev} \end{aligned}$$

$$\begin{aligned} PPV &= Pr(D=1|Y=1) \\ &= Pr(D=1, Y=1)/Pr(Y=1) \\ &= Pr(Y=1|D=1)\frac{Pr(D=1)}{Pr(Y=1)} \\ &= Sens\frac{Pr(D=1)}{Pr(Y=1)} \\ &= Sens\frac{Prev}{Pr(Y=1)} \\ &= Sens\frac{Prev}{Pr(Y=1|D=1)P(D=1) + Pr(Y=1|D=0)P(D=0)} \\ &= Sens\frac{Prev}{SensPrev + Pr(Y=1|D=0)(1-Prev)} \\ &= Sens\frac{Prev}{SensPrev + (1-Pr(Y=0|D=0))(1-Prev)} \\ &= Sens\frac{Prev}{SensPrev + (1-Spec)(1-Prev)} \end{aligned}$$

$$\begin{aligned} NPV &= Pr(D=0|Y=0) \\ &= Pr(D=0, Y=0)/Pr(Y=0) \end{aligned}$$

$$
\begin{aligned}
&= Pr(Y=0|D=0)\frac{Pr(D=0)}{Pr(Y=0)} \\
&= Spec\frac{1-Pr(D=1)}{Pr(Y=0)} \\
&= Spec\frac{1-Prev}{Pr(Y=0)} \\
&= Spec\frac{1-Prev}{Pr(Y=0|D=1)P(D=1)+Pr(Y=0|D=0)P(D=0)} \\
&= Spec\frac{1-Prev}{(1-Sens)Prev+Spec(1-Prev)} \\
&= Spec\frac{1-Prev}{(1-Sens)Prev+Spec(1-Prev)}
\end{aligned}
$$

2 Problem 2

$$
\begin{aligned}
se(\hat{\pi}) &= \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} \\
se(\log(\hat{\pi})) &= se(\hat{\pi})\frac{1}{\hat{\pi}} \\
&= \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}/\hat{\pi} \\
&= \sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n\hat{\pi}^2}} \\
&= \sqrt{\frac{1-\hat{\pi}}{n\hat{\pi}}} \\
&= \sqrt{\frac{1-x/n}{x}} \\
&= \sqrt{\frac{1}{x}-\frac{1}{n}}
\end{aligned}
$$