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**To be discussed on Thursday 26 September 2019**

**Exercise sheet 1 - STA404 Clinical Biostatistics**

1. Using Bayes' theorem, show that the positive predictive value (PPV) and the negative predictive value (NPV) can be rewritten as:

$$\text{PPV} = \frac{\text{Sens} \cdot \text{Prev}}{\text{Sens} \cdot \text{Prev} + (1 - \text{Spec}) \cdot (1 - \text{Prev})}$$

$$\text{NPV} = \frac{\text{Spec} \cdot (1 - \text{Prev})}{\text{Spec} \cdot (1 - \text{Prev}) + (1 - \text{Sens}) \cdot \text{Prev}}$$

2. Derive the formula for the estimated standard error of a log proportion:

$$\text{se}(\log(X/n)) = \sqrt{\frac{1}{x} - \frac{1}{n}}.$$

You have to use the delta method explained as follows on page 63 of the 'Applied Statistical Inference' Book:

**The delta method**

A standard error of $h(\hat{\theta})$ can be obtained by multiplying the standard error of $\hat{\theta}$ with the absolute value of the derivative $dh(\theta)/d\theta$ evaluated at $\hat{\theta}$:

$$\text{se}(h(\hat{\theta})) = \text{se}(\hat{\theta}) \cdot \left| \frac{dh(\hat{\theta})}{d\theta} \right|,$$

where $\hat{\theta}$ is a consistent estimator of $\theta$.

3. Using the result from (2) derive the standard error of the $\log(\text{LR}^+)$ (slide 26, first lecture). Give the 95% confidence interval of the $\text{LR}^+$. (Hint: if $X$ and $Y$ are independent then $\text{se}(X - Y) = \sqrt{\text{se}(X)^2 + \text{se}(Y)^2}$.)

4. To compare the likelihood ratios (see slides 15–17) from two different tests $A$ and $B$ we can use *relative likelihood ratios*. These are defined as

   - relative positive likelihood ratio $\text{rLR}^+ = \text{LR}^+_A/\text{LR}^+_B$
   - relative negative likelihood ratio $\text{rLR}^- = \text{LR}^-_A/\text{LR}^-_B$.

Show that $\text{rLR}^+$ is equivalent to the positive predictive (or posterior) odds ratio

$$\text{PPV}_A(1 - \text{PPV}_B)\text{PPV}_B^{-1}(1 - \text{PPV}_A)^{-1}$$

while $\text{rLR}^-$ is the *inverse* of the negative predictive odds ratio

$$\text{NPV}_A(1 - \text{NPV}_B)\text{NPV}_B^{-1}(1 - \text{NPV}_A)^{-1}.$$