



THE UNIVERSITY OF SYDNEY

SCHOOL OF AEROSPACE, MECHANICAL, MECHATRONIC, AND  
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# Aeroelastic Flutter Suppression

Honours Thesis Progress Report

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# 1 Introduction

When it comes to the design of supersonic airfoils, one of the most critical design cases is the aeroelastic flutter case. Aeroelastic flutter is defined as the unbalanced oscillation of body. This oscillation can quickly reach critical points and if left uncontrolled the structure will fail. On aircraft, this phenomenon usually occurs on thin bodies such as wings or tail plane foils.

As far as flutter suppression goes, the current methods used are largely passive. These range from increasing the thickness of the aerofoil, to limiting the flight envelope of the aircraft in order to avoid exceeding the critical velocity. This can and does at times effect the performance and all-round capabilities of the aircraft. In theory, the use of an active flutter suppression method should not only allow the aircraft to fly past the flutter speed, but also improve factors related to drag and weight as less structure would be required for a given flight envelope.

This report will outline the research and progress made on the development of a Proportional Integral Derivative (PID) controller for a flutter suppression method. The method used, is a flap positioned on the trailing edge of a fin. By rotating the flap about its hinge point, the structural properties, which in turns relates to the stiffness of the wing, can be changed. Thus, as the rocket approaches its flutter speeds, the fin can be "stiffened" to increase the critical velocity of the fin. The controller will be designed about three simulations. The first being a two-dimensional system based on the aeroelastic equations of motion. The second being a sim with the same degree of freedoms, however, the aeroelastics is replaced with Nastran's numerical methods. The final simulation will be a full three dimensional simulation using Nastran. A summary of the work is provided in this document with appendices to support in detail.

## 2 Literature Review

### 2.1 Aerodynamics

When searching for sources on the aerodynamics and general physics of aeroelasticity, the first sources to be examined were the introductory books into the topic. Works by Blisplinghoff [5], Scanlan [4], and Fung [7] provided a basic process to creating an aeroelastic model in two dimensions and given brief details into the solutions. By looking further into the references, the original NACA report written by T. Theodorsen were found [6]. This provided the full, detailed explanation of the 2D three degree of freedom model including the aerodynamic force and moments equations. In the appendices of this report, the tabulated values of the Bessel functions and solutions to  $C(k)$  were provided which allowed for verification of the solution process implemented into the code for the first model. From Scanlan's works, a section on solutions to the flutter stability determinant provided valuable information on the use of Theodorsen's method as well as outlining Wagner's function for the use of modelling the unsteady aerodynamics. supporting this with KC. Fung's work of a similar nature, the final process was able to be mapped out.

### 2.2 Flutter Suppression Method

Prior to the research into the aerodynamics of the system. A Method for the flutter analysis was found. The final method decided was to use a flap on the trailing edge of the wing to change the mass properties, and thus the stiffness of the wing to prevent flutter. When initially conducting research into the methods for active flutter suppression, the idea of controlling the stiffness of the wing was used. This lead to finding three main methods. The first method found, was presented in a thesis paper by J. Bamberger from Oregon State University [1]. This detailed the design and prototyping of a device that would produce electric propulsion in order to oppose the motion of flutter. Another idea that was proposed was to implement a variable stiffness spar into the trailing edge of the foil which would be adjusted to oppose the motion of the foil [2]. Finally a conference paper by P. Yang proposed the use of a trailing edge flap to offset the oscillation of the foil [3]. This would be implemented through the use of a Linear Quadratic Regulator (LQR) and provided a brief explanation of the aerodynamic model and some references. This method would eventually be the selected method for this project as it appeared most often in the theory mentioned above and was the most realistic option.

### 3 Summary of Completed Work

In terms of the current work being completed, at the time of writing, two of the 3 simulations have been started. The overall plan and methods have been decided, and the majority of the literature review has been completed. The final report has been begun, although the structure and content are still not fully decided. This like the literature review will be continued and improved right up until the completion of this project.

#### 3.1 2D 3DOF Theoretical Model

In terms of the initial model and controller, the mathematics of the aeroelastic system have been collated from the literature and are now being implemented into MATLAB and Simulink (see appendix A for detailed work). Following this, the vehicle's flight path will be generated and the testing can begin on the fin and controller design. There are currently two options to solve for the flutter velocity at each time step. The first is to use the aeroelastic system to generate the motion of the fin, and thus use the results to detect when the fin flutters. Another method and the most likely one to be used is to find the flutter velocity at each time step, is to solve using an eigenvalue solution to see the effect of the controller on the flutter speed without the need to detect flutter via the system [4].

#### 3.2 2D 3DOF Nastran Model

As the work on the first system continues, the code used to run the second and third simulations has been started. At the time of writing, the 2D trapezoidal model for the fin is in its later stage of development. The program which was written in python creates the grid points based on a set of dimensions and flap angle (See figure 1 and 2). Currently the work is being done in creating the panels based on the points generated. The future work on this section of the project includes the use of the pyNastran package to generate the input files and run the tests. The PK-method will be used to solve and either the CAERO1 or CAERO5 cards will be used to model the aerodynamics. The code will then be extended into a 3D model and the controller redesigned around that.

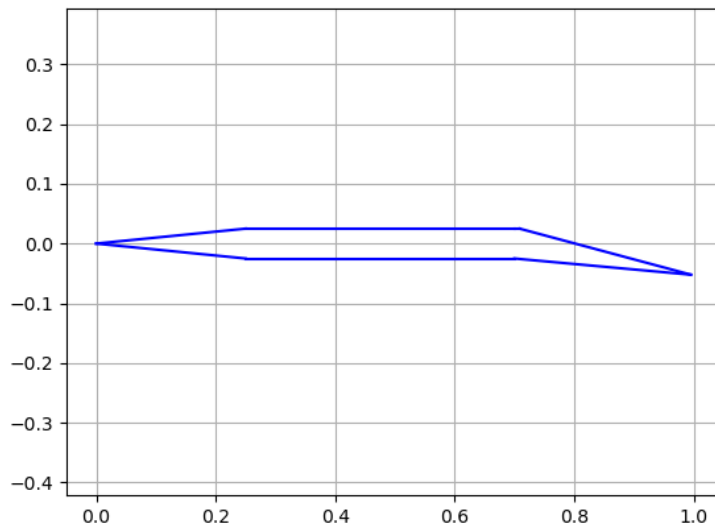


Figure 1: The shape of the trapezoidal fin based on inputs from the user.

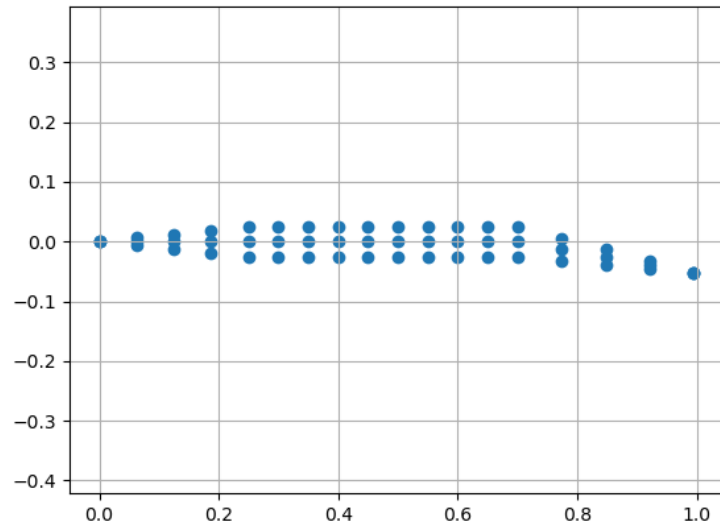


Figure 2: Points generated from the fin shape and a specified number of divisions.

## References

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## A 2D 3DOF Theoretical State Space Derivation

The focus of this appendix is to outline the derivation of the state space system which will be implemented via matlab to design and test the controller. The derivation begins with the aeroelastic equations of motion in two dimensions.

$$\ddot{\alpha}I_\alpha + \ddot{\beta}(I_\beta + b(c-a)S_\beta) + \ddot{h}S_\alpha + \alpha C_\alpha = M_\alpha \quad (1)$$

$$\ddot{\alpha}(I_\beta + b(c-a)S_\beta) + \ddot{\beta}I_\beta + \ddot{h}S_\beta + \beta C_\beta = M_\beta \quad (2)$$

$$\ddot{\alpha}S_\alpha + \ddot{\beta}S_\beta + \ddot{h}m + hC_h = P \quad (3)$$

This translated into matrix form gives

$$A\ddot{q} + B\dot{q} + Cq = F \quad (4)$$

Where

$$q = [\alpha \quad \beta \quad h]^T$$

$$A = \begin{bmatrix} I_\alpha & I_\beta + b(c-a)S_\beta & S_\alpha \\ I_\beta + b(c-a)S_\beta & I_\beta & S_\beta \\ S_\alpha & S_\beta & m \end{bmatrix}$$

$$B = [0]_{3 \times 3}$$

$$C = \begin{bmatrix} C_\alpha & 0 & 0 \\ 0 & C_\beta & 0 \\ 0 & 0 & C_h \end{bmatrix}$$

The aerodynamic force and moments as defined by Theodorsen in NACA Report 496 [6] are given by

$$M_\alpha = -\rho b^2(\pi(1/2 - a)vb\dot{\alpha} + (1/8 + a^2)\ddot{\alpha} + (T_4 + T_{10})v^2\beta + (T_1 - T_8 - (c-a)T_4 + \quad (5)$$

$$1/2T_{11})vb\dot{\beta} - (T_7 + (c-a)T_1)b^2\ddot{\beta} - a\pi b\ddot{h}) + 2\rho vb^2\pi(a + 1/2)C(k)(v\alpha + \dot{h} + b(1/2 - a)\dot{\alpha} \\ + 1/\pi T_{10}v\beta + b1/(2\pi)T_{11}\dot{\beta})$$

$$M_\beta = -\rho b^2((-2T_9 - T_1 + T_4(a - 1/2))vb\dot{\alpha} + 2T_{13}b^2\ddot{\alpha} + 1/\pi v^2\beta(T_5 - T_4T_{10}) \quad (6)$$

$$- 1/(2\pi)vb\dot{\beta}T_4T_{11} - 1/\pi T_3b^2\ddot{\beta} - T_1b\ddot{h}) - \rho vb^2T_{12}C(k)(v\alpha + \dot{h} + b(1/2 - a)\dot{\alpha} + 1/\pi T_{10}v\beta \\ + b1/(2\pi)T_{11}\dot{\beta})$$

$$P = -\rho b^2(v\pi\dot{\alpha} + \pi\ddot{h} - \pi ba\ddot{\alpha} - vT_4\dot{\beta} - T_1b\ddot{\beta}) - 2\pi\rho vbC(k)(v\alpha + \dot{h} + b(1/2 - a)\dot{\alpha} \quad (7)$$

$$+ 1/\pi T_{10}v\beta + b1/(2\pi)T_{11}\dot{\beta})$$

The values for the  $T$  constants as well as a method for solving for  $C(k)$  is given below in appendix Appendix B.

The above equations can be translated into the following matrix form

$$D\ddot{q} + E\dot{q} + Gq = F \quad (8)$$



Where

$$\begin{aligned}
D &= \begin{bmatrix} -\rho b^4 \pi (1/8 + a^2) & \rho b^4 (T_7 + (c - a)T_1) & \rho b^3 a \pi \\ \rho b^4 2T_{13} & \rho b^4 1/\pi T_3 & \rho b^3 T_1 \\ \rho b^3 \pi a & -\rho b^3 T_1 & -\rho b^2 \pi \end{bmatrix} \\
E_{\text{Col:1}} &= \begin{bmatrix} \rho b^3 \pi (1/2 - a)v + 2\rho v b^3 \pi (a^2 - 1/4)C(k) \\ -\rho b^3 (-2T_9 - T_1 + T_4(a - 1/2))v - \rho v b^3 T_{12}C(k)(1/2 - a) \\ -\rho b^2 v \pi - 2\pi \rho v b^2 C(k)(1/2 - a) \end{bmatrix} \\
E_{\text{Col:2}} &= \begin{bmatrix} -\rho b^3 (T_1 - T_8 - (c - a)T_4 + 1/2 T_{11})v + \rho v b^3 (a + 1/2)C(k)T_{11} \\ \rho b^3 1/(2\pi)v T_4 T_{11} - \rho v b^3 T_{12}C(k)1/(2\pi)T_{11} \\ \rho b^2 v T_4 - \rho v b^2 C(k)T_{11} \end{bmatrix} \\
E_{\text{Col:3}} &= \begin{bmatrix} 2\rho v b^2 \pi (a + 1/2)C(k) \\ -\rho v b^2 T_{12}C(k) \\ -2\pi \rho v b C(k) \end{bmatrix} \\
G &= \begin{bmatrix} 2\rho v^2 b^2 \pi (a + 1/2)C(k) & -\rho b^2 (T_4 + T_{10})v^2 + 2\rho v^2 b^2 (a + 1/2)C(k)T_{10} & 0 \\ -\rho v^2 b^2 T_{12}C(k) & -\rho b^2 1/\pi v^2 (T_5 - T_4 T_{10}) - \rho v^2 b^2 T_{12}C(k)1/\pi T_{10} & 0 \\ -2\pi \rho v^2 b C(k) & -2\rho v^2 b C(k)T_{10} & 0 \end{bmatrix}
\end{aligned}$$

By equating both equation 4 and 8, the following can be constructed

$$\ddot{q} = (A - D)^{-1}(E - B)\dot{q} + (A - D)^{-1}(G - C)q \quad (9)$$

which leads to the following state space equation

$$\begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} (A - D)^{-1}(E - B) & (A - D)^{-1}(G - C) \\ I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \begin{bmatrix} \dot{q} \\ q \end{bmatrix} \quad (10)$$

At the current time of writing, the author has not progressed passed this point in the model derivation. Following the above line, a PID controller will be implemented into the model.

## B Solutions to Constants and Smaller Functions

### B.1 Solution to the $T$ Constants

The  $T$  constants are given by the following equations written by Theodoresen in the same notation as NACA report 496 [6].

$$T_1 = -\frac{1}{3}\sqrt{1-c^2}(2+c^2+c\cos^{-1}c) \quad (11)$$

$$T_2 = c(1-c^2) - \sqrt{1-c^2}(1+c^2\cos^{-1}c + c(\cos^{-1}c)^2) \quad (12)$$

$$T_3 = -\left(\frac{1}{8}+c^2\right)(\cos^{-1}c)^2 + \frac{1}{4}c\sqrt{1-c^2}\cos^{-1}c(7+2c^2) - \frac{1}{8}(1-c^2)(5c^2+4) \quad (13)$$

$$T_4 = -\cos^{-1}c + c\sqrt{1-c^2} \quad (14)$$

$$T_5 = -(1-c^2) - (\cos^{-1}c)^2 + 2c\sqrt{1-c^2}\cos^{-1}c \quad (15)$$

$$T_6 = T_2 \quad (16)$$

$$T_7 = -\left(\frac{1}{8}+c^2\right)\cos^{-1}c + \frac{1}{8}c\sqrt{1-c^2}(7+2c^2) \quad (17)$$

$$T_8 = -\frac{1}{3}\sqrt{1-c^2}(2c^2+1) + c\cos^{-1}c \quad (18)$$

$$T_9 = \frac{1}{2}\left[\frac{1}{3}\sqrt{1-c^2} + aT_4\right] \quad (19)$$

$$T_{10} = \sqrt{1-c^2} + \cos^{-1}c \quad (20)$$

$$T_{11} = \cos^{-1}c(1-2c) + \sqrt{1-c^2}(2-c) \quad (21)$$

$$T_{12} = \sqrt{1-c^2}(2+c) - \cos^{-1}c(2c+1) \quad (22)$$

$$T_{13} = \frac{1}{2}[-T_7 - (c-a)T_1] \quad (23)$$

$$T_{14} = \frac{1}{16} + \frac{1}{2}ac \quad (24)$$

### B.2 Solution to $C(k)$

To solve for the value of  $C(k)$ , the following process is conducted [6]. The value of  $C$  at every time point is given by

$$C = F + Gi \quad (25)$$

Where

$$F = \frac{J_1(J_1 + Y_0) + Y_1(Y_1 - J_0)}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2} \quad (26)$$

$$G = -\frac{Y_1Y_0 + J_1J_0}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2} \quad (27)$$

The values of  $J$  and  $Y$  are given by the bessel functions of the corresponding orders. To solve for these, the built-in function in MATLAB was used to determine the value at each  $k$ .