Hydrogen atom

SphericalHarmonicY[l, m, θ, ϕ] gives the spherical harmonic $Y_l^m(\theta, \phi)$. It is the angular solution found in the Hydrogen atom.

Radial wave-function of the hydrogen atom:

In[1]:=
```
ψ[n_, l_, m_, r_, θ_, ϕ_] := Sqrt[(n - l - 1)! / (a^3 * (n + l)!)] E^(-r/(n*a)) (2 r/(n*a))^l
  2/n^2 LaguerreL[n - l - 1, 2 l + 1, 2 r/(n*a)] * SphericalHarmonicY[l, m, θ, ϕ]
```

$$\psi[n\_, l\_, m\_, r\_, \theta\_, \phi\_] := \sqrt{\frac{(n-l-1)!}{a^3 * (n+l)!}}\, e^{-\frac{r}{n*a}} \left(\frac{2\,r}{n*a}\right)^{l} \frac{2}{n^2}\, \text{LaguerreL}\left[n-l-1,\ 2\,l+1,\ \frac{2\,r}{n*a}\right] * \text{SphericalHarmonicY}[l, m, \theta, \phi]$$

In[2]:=
```
ψ[1, 0, 0, r, θ, ϕ]
```

Out[2]=

$$\frac{\sqrt{\frac{1}{a^3}}\, e^{-\frac{r}{a}}}{\sqrt{\pi}}$$

In[3]:=
```
ψ[2, 0, 0, r, θ, ϕ]
```

Out[3]=

$$\frac{\sqrt{\frac{1}{a^3}}\, e^{-\frac{r}{2\,a}} \left(2 - \frac{r}{a}\right)}{4\sqrt{2\,\pi}}$$

In[4]:=
```
For[n = 1, n <= 3, n++,
 For[l = 0, l < n, l++,
  For[m = 0, m <= l, m++, Print["(n,l,m)->", n, l, m, " : ψ=", ψ[n, l, m, r, θ, ϕ]]]]]
```

$(n,l,m)\text{->}100\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{a}}}{\sqrt{\pi}}$

$(n,l,m)\text{->}200\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{2a}}\left(2-\frac{r}{a}\right)}{4\sqrt{2\pi}}$

$(n,l,m)\text{->}210\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{2a}}\ r\,\mathrm{Cos}[\theta]}{4\,a\sqrt{2\pi}}$

$(n,l,m)\text{->}211\ :\ \psi=-\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{2a}+i\phi}\ r\,\mathrm{Sin}[\theta]}{8\,a\sqrt{\pi}}$

$(n,l,m)\text{->}300\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}}\left(27\,a^2-18\,a\,r+2\,r^2\right)}{81\,a^2\sqrt{3\pi}}$

$(n,l,m)\text{->}310\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}}\ r\left(4-\frac{2r}{3a}\right)\mathrm{Cos}[\theta]}{27\,a\sqrt{2\pi}}$

$(n,l,m)\text{->}311\ :\ \psi=-\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}+i\phi}\ r\left(4-\frac{2r}{3a}\right)\mathrm{Sin}[\theta]}{54\,a\sqrt{\pi}}$

$(n,l,m)\text{->}320\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}}\ r^2\left(-1+3\,\mathrm{Cos}[\theta]^2\right)}{81\,a^2\sqrt{6\pi}}$

$(n,l,m)\text{->}321\ :\ \psi=-\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}+i\phi}\ r^2\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta]}{81\,a^2\sqrt{\pi}}$

$(n,l,m)\text{->}322\ :\ \psi=\frac{\sqrt{\frac{1}{a^3}}\ e^{-\frac{r}{3a}+2i\phi}\ r^2\,\mathrm{Sin}[\theta]^2}{162\,a^2\sqrt{\pi}}$

Probability density

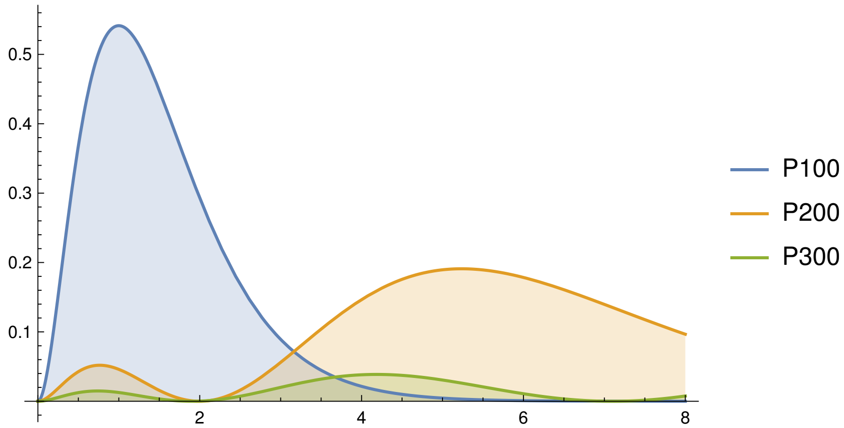
In[5]:=
```
P[n_, l_, m_, r_, θ_, ϕ_] :=
 Simplify[4 π * r^2 * ψ[n, l, m, r, θ, ϕ] * Conjugate[ψ[n, l, m, r, θ, ϕ]],
   Element[{a, r}, Reals]] /. {a → 1}
```

In[6]:=
```
P100 = P[1, 0, 0, r, θ, ϕ]
P200 = P[2, 0, 0, r, θ, ϕ];
P300 = P[3, 0, 0, r, θ, ϕ];
```

Out[6]= $4\, e^{-2 r}\, r^2$

In[9]:=
```
Plot[{P100, P200, P300}, {r, 0, 8}, PlotLegends → "Expressions", Filling → Bottom]
```

Out[9]=



In[10]:=
```
Plot[{P100, P200, P300}, {r, 0, 20}, PlotLegends → "Expressions"]
```

Out[10]=