Bessel function

BesselJ [n, z] gives the Bessel function of the first kind $J_n(z)$.

BesselY[n, z] gives the Bessel function of the second kind $Y_n(z)$ ... The Neumann function

It satisfies the differential equation $z^2 y'' + z y' + (z^2 - n^2) y = 0$ that we obtained for example in the solution of the Laplace equation in cilindrical coordenates.

Remenber that :

n = ν is a number (It could be complex number)
y = y (x) where x is the independent variable

Solution

This equation could be solve directly using Mathematica. It gives the geeral solution

```
DSolve[ x^2 * y''[x] + x * y'[x] + (x^2 - n^2) * y[x] == 0, y[x], x]
```

```
{{y[x] → BesselJ[n, x] C[1] + BesselY[n, x] C[2]}}
```

Examples :

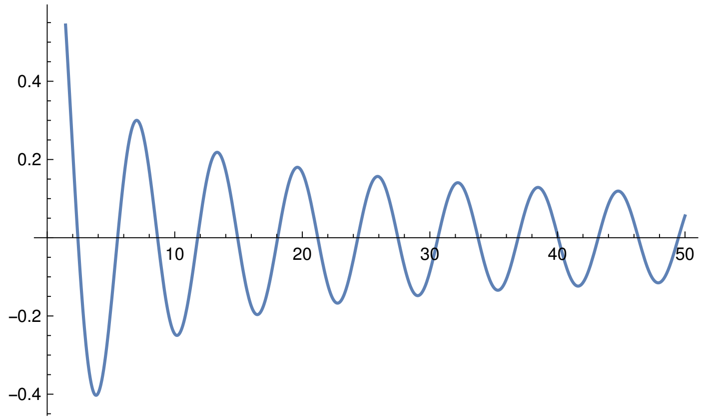
```
BesselJ[0, 5.2]
BesselY[0, 5.2]
```

```
-0.11029
```

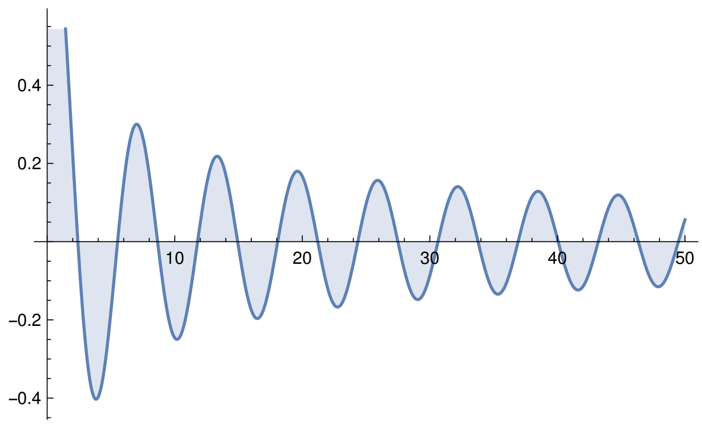
```
-0.331251
```

Plot the Bessel J_n

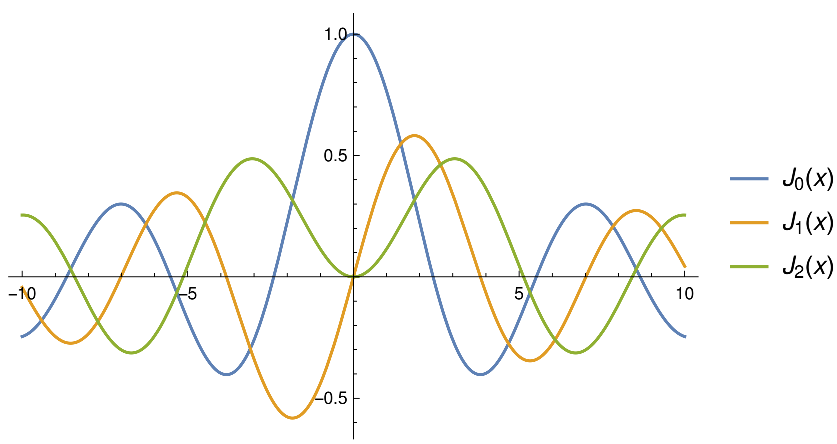
```
Plot[BesselJ[0, x], {x, 0, 50}]
```




```
Plot[BesselJ[0, x], {x, 0, 50}, Filling → Axis]
```

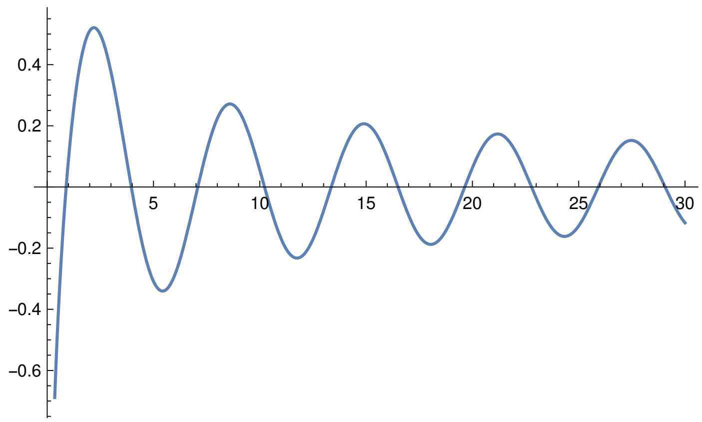



```
Plot[{BesselJ[0, x], BesselJ[1, x], BesselJ[2, x]},
 {x, -10, 10}, PlotLegends → "Expressions"]
```

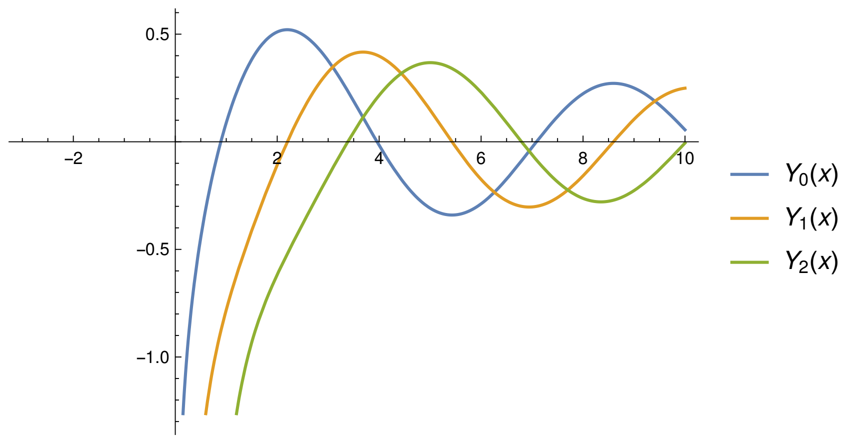


Plot the Bessel Y_n (The Newmann solution)

```
Plot[BesselY[0, x], {x, 0, 30}]
```




```
Plot[{BesselY[0, x], BesselY[1, x], BesselY[2, x]},
 {x, -3, 10}, PlotLegends → "Expressions"]
```



Series

```
Series[BesselJ[0, x], {x, 0, 10}]
```

$$1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147\,456} - \frac{x^{10}}{14\,745\,600} + O[x]^{11}$$

For half - integer indices, BesselJ and BesselY evaluates to elementary functions :

```
BesselJ[1/2, x]
BesselY[1/2, x]
```

$$\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Sin}[x]}{\sqrt{x}}$$

$$-\frac{\sqrt{\frac{2}{\pi}}\ \mathrm{Cos}[x]}{\sqrt{x}}$$

Traditional form

```
BesselJ[n, r] // TraditionalForm
BesselY[n, r] // TraditionalForm
```

$J_n(r)$

$Y_n(r)$

Applications: The Fraunhofer diffraction