Laguerre polinomials

LaguerreL[n, x] gives the Laguerre polynomial $L_n(x)$
which is the solution to EDO : $x\ddot{y} + (1 - x)\dot{y} + ny = 0$.
LaguerreL[n, k, x] gives the generalized Laguerre polynomial $L_n^k(x)$
which is the solution to EDO : $x\ddot{y} + (k + 1 - x)\dot{y} + ny = 0$.

Simple Laguerre $L_n(x)$

In[1]:=
```
LaguerreL[0, x]
```
Out[1]= 1

In[2]:=
```
For[i = 0, i < 7, i++, Print[LaguerreL[i, x]]]
```

1

$1 - x$

$\frac{1}{2}(2 - 4x + x^2)$

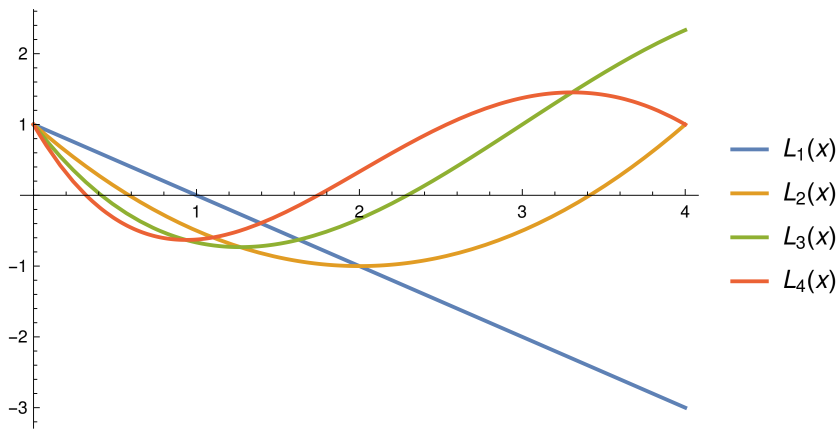
$\frac{1}{6}(6 - 18x + 9x^2 - x^3)$

$\frac{1}{24}(24 - 96x + 72x^2 - 16x^3 + x^4)$

$\frac{1}{120}(120 - 600x + 600x^2 - 200x^3 + 25x^4 - x^5)$

$\frac{1}{720}(720 - 4320x + 5400x^2 - 2400x^3 + 450x^4 - 36x^5 + x^6)$

In[3]:=
```
Plot[{LaguerreL[1, x], LaguerreL[2, x], LaguerreL[3, x], LaguerreL[4, x]},
 {x, 0, 4}, PlotStyle → Thick, PlotLegends → "Expressions"]
```

Out[3]=


Associated Laguerre $L_n^k(x)$

```
LaguerreL[n, μ, z] // TraditionalForm
```

$L_n^{\mu}(z)$

```
LaguerreL[1, 0, x]
```

1 – x

```
For[i = 0, i < 6, i++, Print[LaguerreL[i, i, x]]]
```

1

2 – x

$\frac{1}{2}\left(12-8x+x^2\right)$

$\frac{1}{6}\left(120-90x+18x^2-x^3\right)$

$\frac{1}{24}\left(1680-1344x+336x^2-32x^3+x^4\right)$

$\frac{1}{120}\left(30\,240-25\,200x+7200x^2-900x^3+50x^4-x^5\right)$

Print the Laguerre associated$L_n^k(x)$

```
For[i = 0, i < 3, i++,
 For[j = 0, j < 3, j++, Print["(n,k)->", "(", i, j, ")", "→", LaguerreL[i, j, x]]]]
```

(n,k)->(00)→1

(n,k)->(01)→1

(n,k)->(02)→1

(n,k)->(10)→1 – x

(n,k)->(11)→2 – x

(n,k)->(12)→3 – x

(n,k)->(20)→$\frac{1}{2}\left(2-4x+x^2\right)$

(n,k)->(21)→$\frac{1}{2}\left(6-6x+x^2\right)$

(n,k)->(22)→$\frac{1}{2}\left(12-8x+x^2\right)$

Radial wave-function of the hydrogen atom:

We used the Bohr radio equal to 1.

```
ψ[n_, l_, r_] := Sqrt[(n - l - 1)!/(n + l)!] E^(-r/n) (2 r/n)^l 2/n^2 LaguerreL[n - l - 1, 2 l + 1, 2 r/n]
```

```
ψ[1, 0, r]
```

$$2\, e^{-r}$$

```
ψ[2, 1, r]
```

$$\frac{e^{-r/2}\, r}{2\sqrt{6}}$$

```
ψ[3, 0, r]
```

$$\frac{2\, e^{-r/3}\left(27 - 18\, r + 2\, r^2\right)}{81\sqrt{3}}$$

Compute the energy eigenvalue from the differential equation:

```
energy[w_, r_, n_, l_] := Simplify[1/w (D[w, r, r] + 2/r D[w, r] - l (l + 1)/r^2 w + 2/r w)]
```

The energy is independent of the orbital quantum number l:

```
Solve[(energy[ψ[n, l, r], r, n, l] - ε // FullSimplify) == 0, ε]
```

$$\left\{\left\{\varepsilon \to \frac{1}{n^2}\right\}\right\}$$