

Bessel applications (Fraunhofer diffraction)

`BesselJ [n, z]` gives the Bessel function of the first kind $J_n(z)$.

`BesselY[n, z]` gives the Bessel function of the second kind $Y_n(z)$... The Neumann function

It satisfies the differential equation $z^2 y'' + zy' + (z^2 - n^2)y = 0$ that we obtained for example in the solution of the Laplace equation in cylindrical coordinates.

Remember that :

$n = \nu$ is a number (It could be complex number)
 $y = y(x)$ where x is the independent variable

Traditional form

`BesselJ[n, r]` // `TraditionalForm`

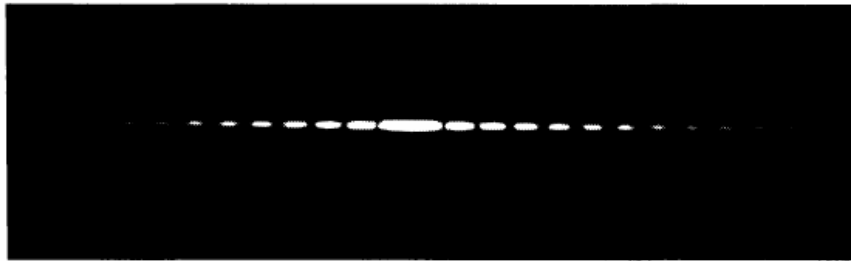
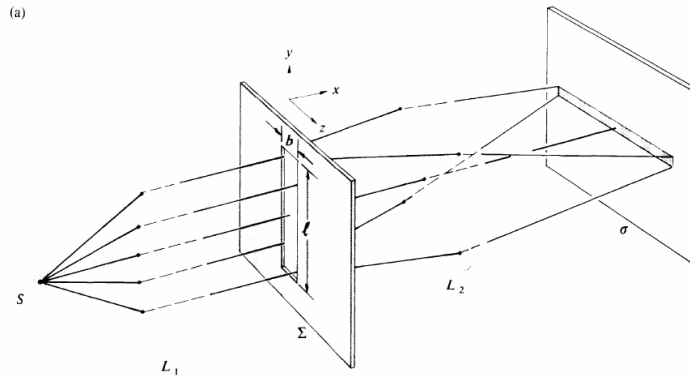
`BesselY[n, r]` // `TraditionalForm`

$J_n(r)$

$Y_n(r)$

The single slip (Optics, Eugene Hecht book)

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Import["Bessel-fig1.png"]
Import["Bessel-fig2.png"]
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(b)

Intensity of the Fraunhofer diffraction pattern of a circular aperture versus diffraction angle :

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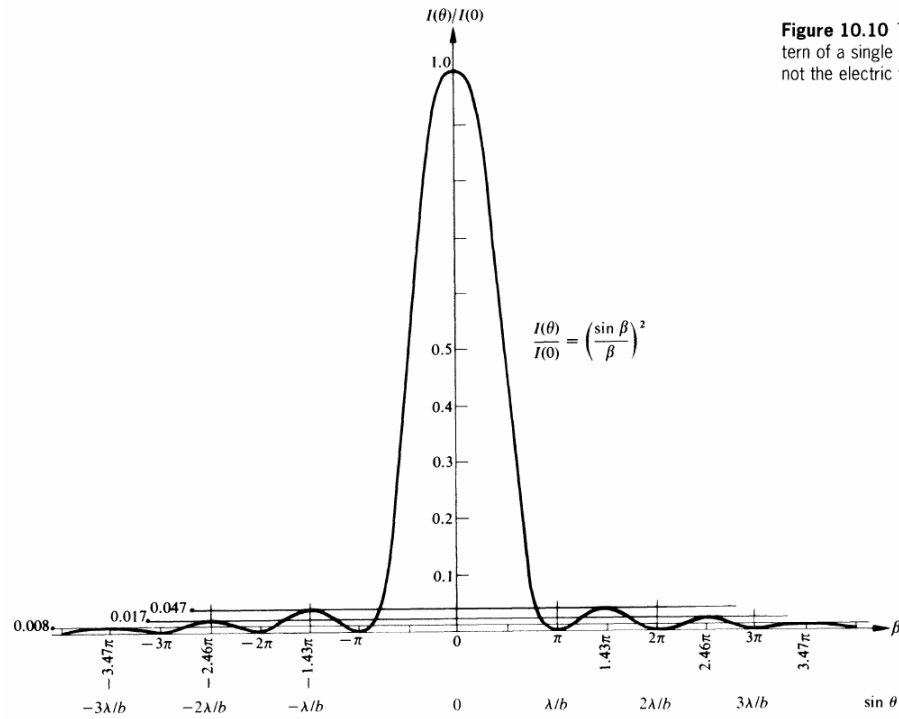


Figure 10.10 The Fraunhofer diffraction pattern of a single slit. This is the irradiance (and not the electric field) distribution

Multiple slips

Two slips

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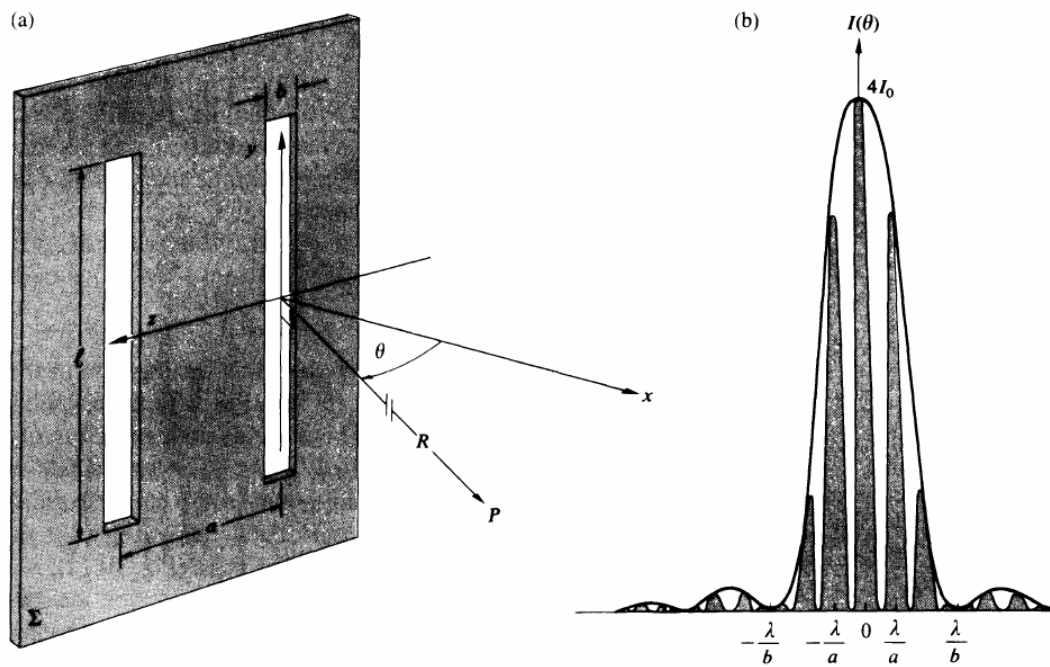
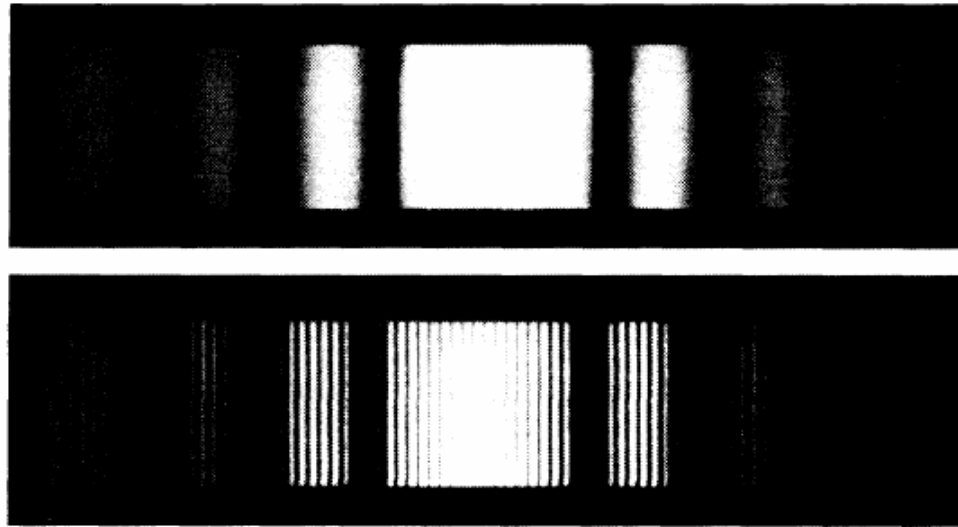
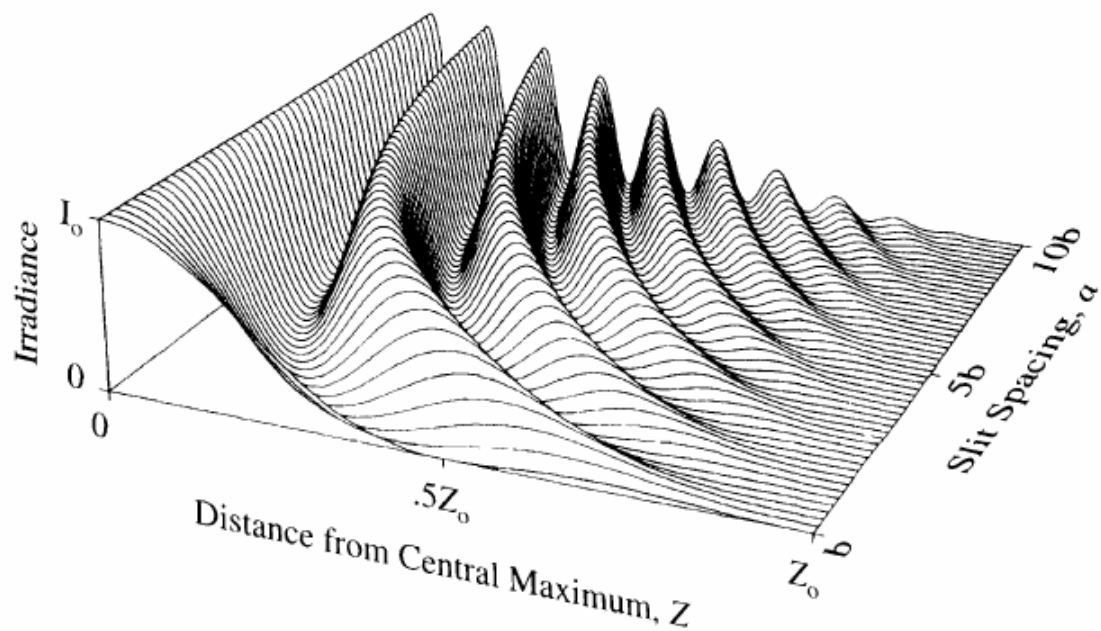


Figure 10.13 (a) Double-slit geometry. Point P on σ is essentially infinitely far away. (b) A double-slit pattern ($a = 3b$).

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(a)



(b)

Figure 10.14 Single- and double-slit Fraunhofer patterns. (a) Photographs taken with monochromatic light. The faint cross-hatching

Many slips

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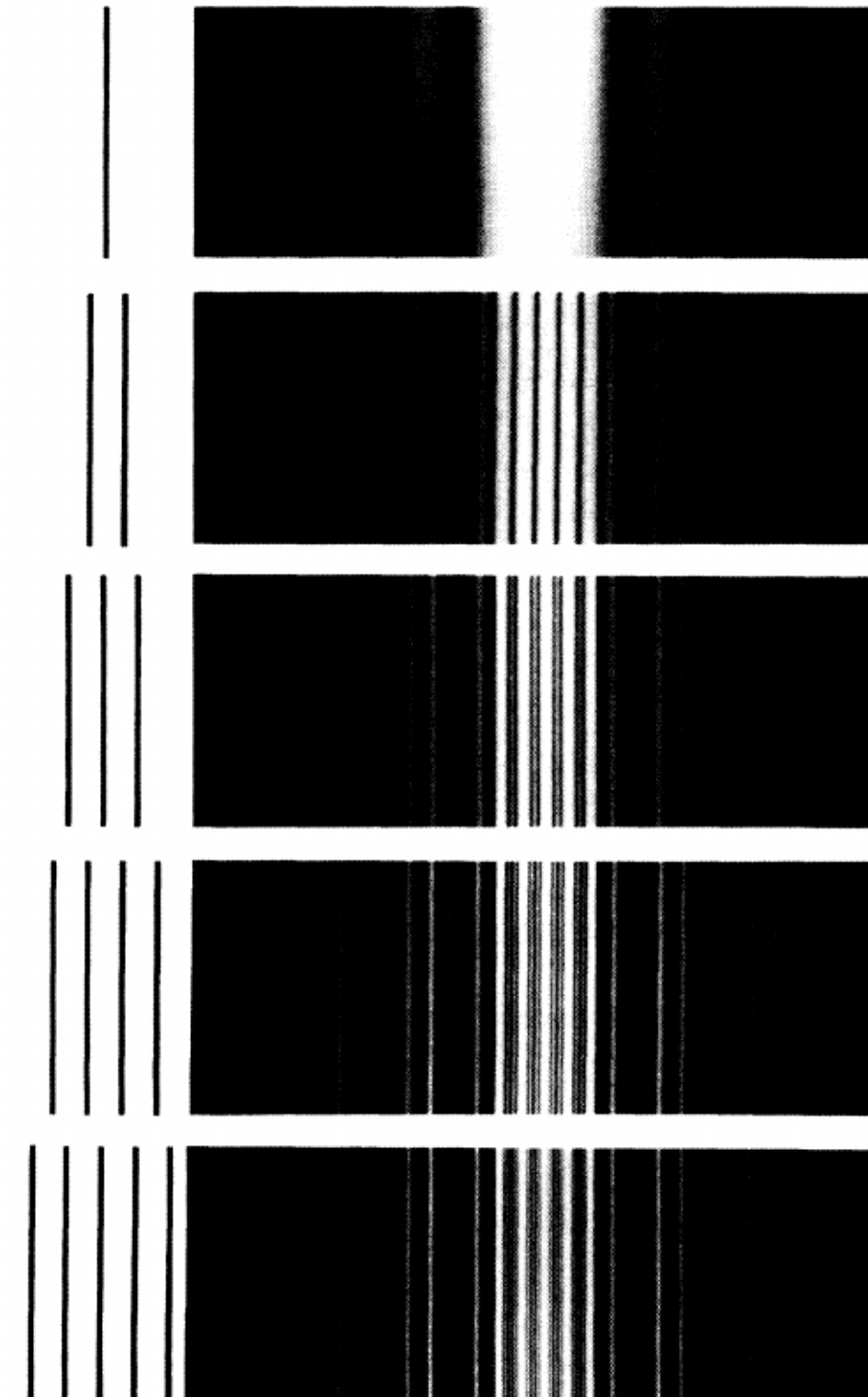


Figure 10.16 Diffraction patterns for slit systems shown at left. (Francis Weston Sears, *Optics*. Reprinted with permission of Addison Wesley Longman, Inc.)


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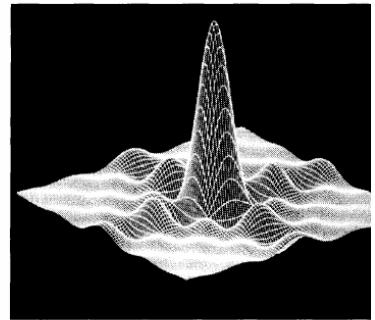
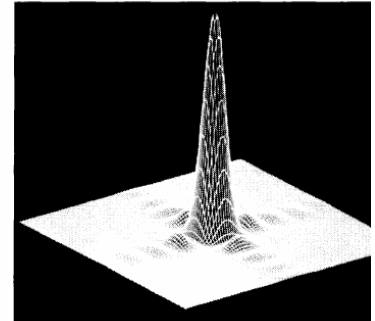
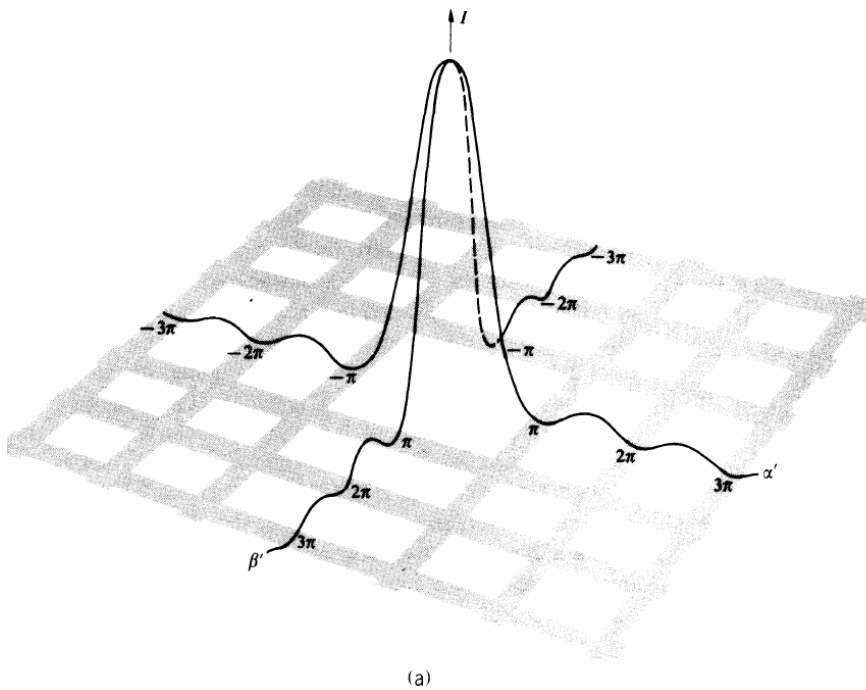
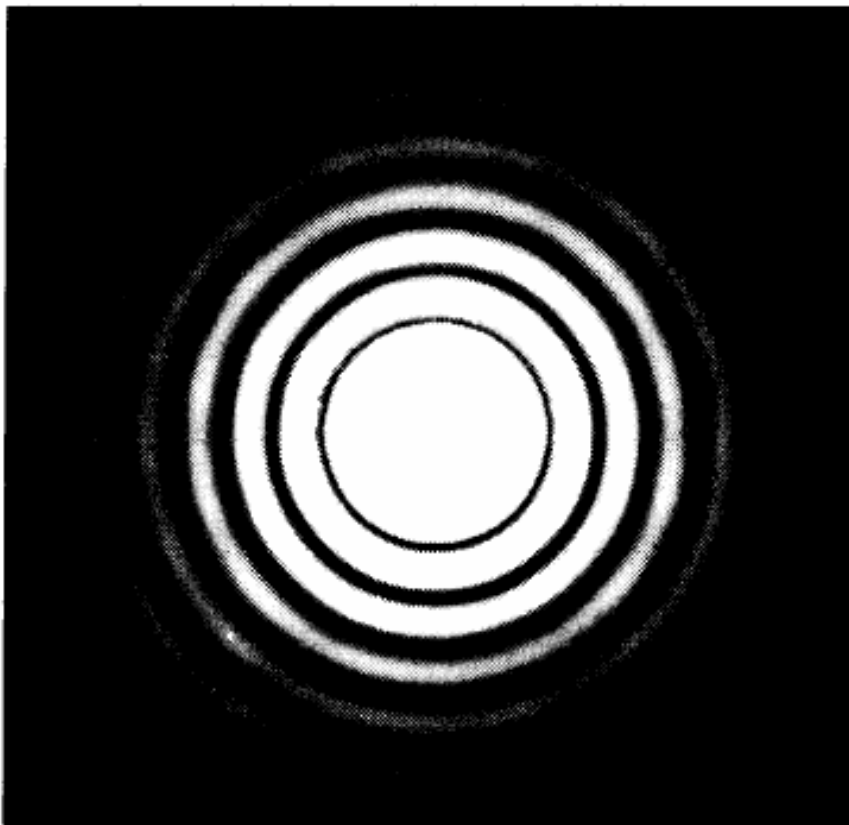
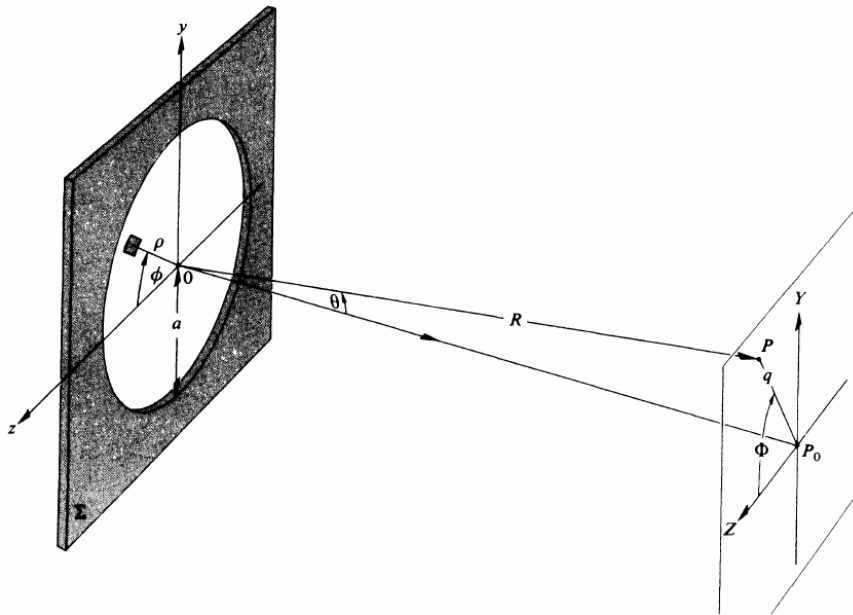
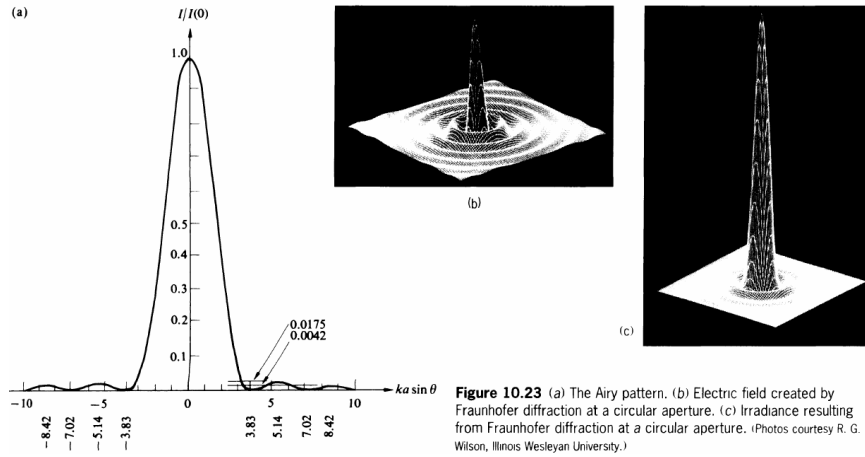


Figure 10.20 (a) The irradiance distribution for a square aperture. (b) The irradiance produced by Fraunhofer diffraction at a square aperture. (c) The electric-field distribution produced by Fraunhofer diffraction via a square aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

The circular aperture

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Import["Bessel-fig12.png"]
Import["Bessel-fig13.png"]
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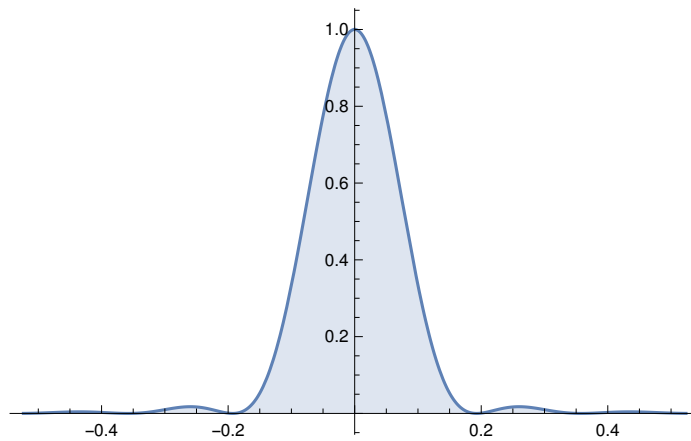





$$I(\theta) = I(0) \left[\frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2$$

Plot the function

`Plot[(2 * BesselJ[1, 20 Sin[θ]] / (20 Sin[θ]))^2, {θ, -π/6, π/6}, Filling → Axis]`



```
Plot[(2 * BesselJ[1, 20 Sin[θ]] / (20 Sin[θ]))^2, {θ, -π/3, π/3}, Filling → Axis]
```

