

Factorial function

In this notebook, we will study the factorial function. We see how to use Mathematica to compute and use the functions defined in classes... and so on ...

Factorial[3]

6

Alternatively

3!

6

(1/2)!

$$\frac{\sqrt{\pi}}{2}$$

N[(-1/4)!]

1.22542

Table[n!, {n, -2, 5}]

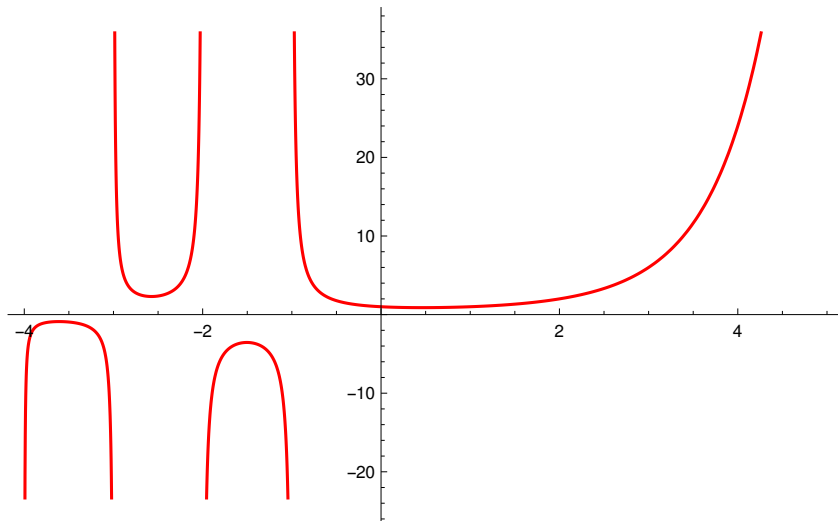
{ComplexInfinity, ComplexInfinity, 1, 1, 2, 6, 24, 120}

Table[n, {n, -2, 5}]!

{ComplexInfinity, ComplexInfinity, 1, 1, 2, 6, 24, 120}

Plot factorial:

Plot[n!, {n, -4, 5}, PlotStyle -> Red]

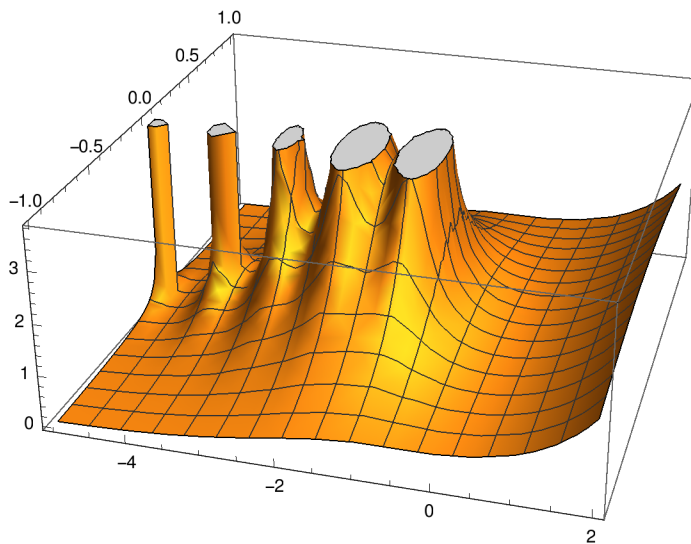


Evaluate for complex arguments (it was not studied in classes):

```
N[(2 + 3 * I) !]  
-0.440113 - 0.0636372 i
```

Plot of the absolute value of Factorial in the complex plane:

```
Plot3D[Abs[Factorial[x + I y]], {x, -5, 2}, {y, -1, 1}]
```



Gamma Function

Using is the Euler gamma function $\Gamma(z)$

```
Gamma[3]
```

2

```
Factorial[3]
```

6

```
3!
```

6

```
Gamma[1/2]
```

$\sqrt{\pi}$

Of course $\Gamma[z+1]=z!$ see the next table

```
Table[Gamma[n + 1], {n, 1, 5}]
```

```
Table[(n)!, {n, 1, 5}]
```

```
{1, 2, 6, 24, 120}
```

```
{1, 2, 6, 24, 120}
```

Double factorial of n

```
0!!
```

```
1
```

```
1!!!
```

```
1
```

```
(-1)!!
```

```
1
```

```
(-3)!!
```

```
-1
```

```
Table[n!!, {n, 10}]
```

```
{1, 2, 3, 8, 15, 48, 105, 384, 945, 3840}
```

Note the numerical evaluation

```
(1.2)!!
```

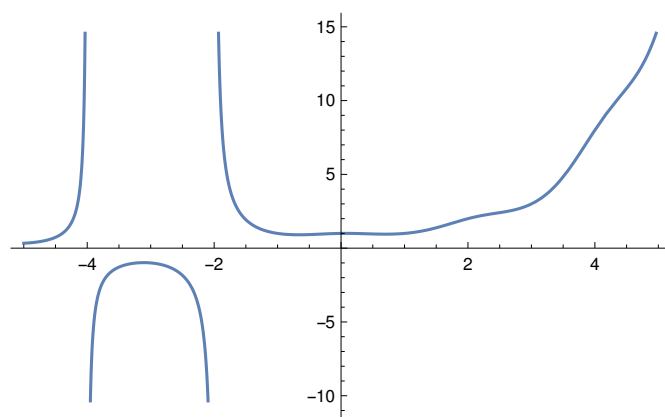
```
1.10414
```

```
N[(1/4)!]
```

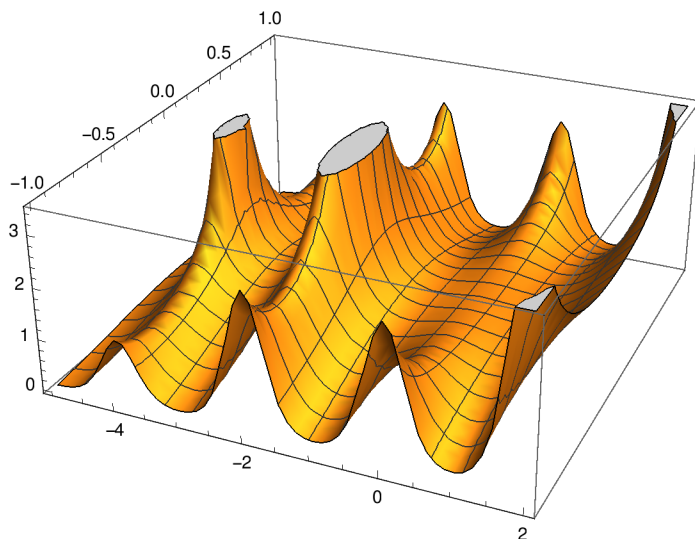
```
0.906402
```

Plot double factorial:

```
Plot[n!!, {n, -5, 5}]
```



```
Plot3D[Abs[Factorial2[x + I y]], {x, -5, 2}, {y, -1, 1}]
```



Beta function

For example : let see the next integral

$$\int_0^{\frac{\pi}{2}} \sqrt{\tan[x]} \, dx$$

$$N\left[\int_0^{\frac{\pi}{2}} \sqrt{\tan[x]} \, dx\right]$$

$$\frac{\pi}{\sqrt{2}}$$

2.22144

Now, using the beta function

$$N[0.5 * \text{Beta}[3/4, 1/4]]$$

2.22144

$$\int \sqrt{\tan[x]} \, dx$$

$$\frac{1}{2\sqrt{2}} \left(-2 \operatorname{ArcTan}\left[1 - \sqrt{2} \sqrt{\tan[x]}\right] + 2 \operatorname{ArcTan}\left[1 + \sqrt{2} \sqrt{\tan[x]}\right] + \right. \\ \left. \operatorname{Log}\left[1 - \sqrt{2} \sqrt{\tan[x]} + \tan[x]\right] - \operatorname{Log}\left[1 + \sqrt{2} \sqrt{\tan[x]} + \tan[x]\right] \right)$$

$$N\left[\int_0^{\frac{\pi}{2}} \sqrt{\cos[x]} \, dx\right]$$

1.19814

$$\int_0^{\frac{\pi}{2}} \cos[x]^2 * \sin[x]^2 \, dx$$

$$\frac{\pi}{16}$$

$$(1/2) * \text{Beta}[3/2, 3/2]$$

$$\frac{\pi}{16}$$

Quiz 3

$$\int_0^{\frac{\pi}{2}} \cos[x]^4 * \sin[x]^4 \, dx$$

$$\frac{3\pi}{256}$$

Error Function

$$\int e^{-x^2} \, dx$$

$$\frac{1}{2} \sqrt{\pi} \, \text{Erf}[x]$$

$$\int_0^{\infty} e^{-x^2} \, dx$$

$$\frac{\sqrt{\pi}}{2}$$