Spherical armonics

SphericalHarmonicY[l, m, θ, ϕ] gives the spherical harmonic $Y_l^m(\theta, \phi)$

Mathematical function, suitable for both symbolic and numerical manipulation.

The spherical harmonics are orthonormal with respect to integration over the surface of the unit sphere.

For $l \geq 0$, $Y_l^m(\theta, \phi) = \sqrt{(2\,l+1)/(4\,\pi)}\ \sqrt{(l-m)!/(l+m)!}\ P_l^m(\cos(\theta))\, e^{im\phi}$ where P_l^m is the associated Legendre function.

For $l \leq -1$, $Y_l^m(\theta, \phi) = Y_{-(l+1)}^m(\theta, \phi)$

Examples

In[18]:=
```
SphericalHarmonicY[0, 0, θ, ϕ]
SphericalHarmonicY[3, 1, θ, ϕ]
SphericalHarmonicY[1, 1, θ, ϕ]
SphericalHarmonicY[{0, 1, 2}, 0, θ, ϕ]
SphericalHarmonicY[3, (1.5 + I) / 3, Pi / 6, 1]
```

Out[18]= $\frac{1}{2\sqrt{\pi}}$

Out[19]= $-\frac{1}{8}\, e^{i\,\phi}\sqrt{\frac{21}{\pi}}\,\left(-1+5\,\mathrm{Cos}[\theta]^2\right)\mathrm{Sin}[\theta]$

Out[20]= $-\frac{1}{2}\, e^{i\,\phi}\sqrt{\frac{3}{2\,\pi}}\,\mathrm{Sin}[\theta]$

Out[21]= $\left\{\frac{1}{2\sqrt{\pi}}, \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\mathrm{Cos}[\theta], \frac{1}{4}\sqrt{\frac{5}{\pi}}\,\left(-1+3\,\mathrm{Cos}[\theta]^2\right)\right\}$

Out[22]= $0.0210562 - 0.215173\, i$

Traditional form

In[23]:=
```
SphericalHarmonicY[l, m, θ, ϕ] // TraditionalForm
```

Out[23]//TraditionalForm=

$Y_l^m(\theta, \phi)$

Eigenfunctions

SphericalHarmonicY[l, m, θ, ϕ] is an eigenfunction of the spherical part of the Laplace operator

In[24]:=
```
Simplify[
 ((1/Sin[θ] D[Sin[θ] D[#, θ], θ] + 1/Sin[θ]^2 D[#, ϕ, ϕ]) / #) &@SphericalHarmonicY[3, 1, θ, ϕ]]
```

Out[24]= -12