

# Legendre Polynomials

`LegendreP[n,x]` gives the Legendre polynomial  $P_n(x)$ .

`LegendreP[n, m, x]` gives the associated Legendre polynomial  $P_n^m(x)$ .

`BesselY[n, z]` gives the Bessel function of the second kind  $Y_n(z)$  ... The Neumann function

It satisfy the differential equation

$(1-x^2) \left( \frac{d^2 y}{dx^2} \right) - 2x \left( \frac{dy}{dx} \right) + n(n+1)y = 0$  that we obtained for example in the solution of the Laplace equation in spherical coordinates.

Remember that :

$n = l$  is a number

$y = y(x) = P(x)$  where  $x$  is the independent variable

## Traditional form

```
In[3]:= LegendreP[n, x] // TraditionalForm
```

```
LegendreP[n, m, x] // TraditionalForm
```

Out[3]//TraditionalForm=

$P_n(x)$

Out[4]//TraditionalForm=

$P_n^m(x)$

## Solve Equation

Solving the Legendre equation

```
In[29]:= DSolve[Sin[θ] D[Sin[θ] f'[θ], θ] - m^2 f[θ] == -λ (λ + 1) Sin[θ]^2 f[θ], f[θ], θ]
```

```
Out[29]= {{f[θ] → C[1] LegendreP[λ, m, Cos[θ]] + C[2] LegendreQ[λ, m, Cos[θ]]}}
```

## Examples

```
In[30]:= LegendreP[0, x]
LegendreP[1, x]
LegendreP[2, x]
LegendreP[3, x]
LegendreP[10, x]
```

```
LegendreP[3, 2, x]
```

```
Out[30]= 1
```

```
Out[31]= x
```

```
Out[32]=  $\frac{1}{2} (-1 + 3 x^2)$ 
```

```
Out[33]=  $\frac{1}{2} (-3 x + 5 x^3)$ 
```

```
Out[34]=  $\frac{1}{256} (-63 + 3465 x^2 - 30030 x^4 + 90090 x^6 - 109395 x^8 + 46189 x^{10})$ 
```

```
Out[35]=  $-15 x (-1 + x^2)$ 
```

```
In[36]:= Plot[{LegendreP[0, x], LegendreP[1, x], LegendreP[2, x], LegendreP[3, x],  
LegendreP[4, x]}, {x, -1.5, 1.5}, PlotLegends -> "Expressions"]
```

Out[36]=

