

# Math 240 Tutorial Questions

August 1

**Question 1.** Find the unit vector in the direction of the given vectors (a)  $\begin{pmatrix} -30 \\ 40 \end{pmatrix}$ , (b)  $\begin{pmatrix} 7/4 \\ 1/2 \\ 1 \end{pmatrix}$ , and (c)  $\begin{pmatrix} 8/3 \\ 2 \end{pmatrix}$ .

**Question 2.** (a) Let  $\vec{u}_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ,  $\vec{u}_2 = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ , and  $\vec{x} = \begin{pmatrix} 9 \\ -7 \end{pmatrix}$ . Does  $\{\vec{u}_1, \vec{u}_2\}$  form an orthogonal basis for  $\mathbf{R}^2$ ? If it does, write  $\vec{x}$  in terms of this basis. (b) Compute the orthogonal projection of  $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$  onto the line through  $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$  and the origin.

**Question 3.** Let  $\vec{y} \in \mathbf{R}^n$ . Prove  $\vec{x} \mapsto \langle \vec{x}, \vec{y} \rangle$  is a linear transformation  $\mathbf{R}^n \rightarrow \mathbf{R}$ .

**Question 4.** Let

$$\vec{y} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \quad \vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{u}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

Verify that  $\{\vec{u}_1, \vec{u}_2\}$  is an orthogonal set, and find the orthogonal projection of  $\vec{y}$  onto  $\text{span}\{\vec{u}_1, \vec{u}_2\}$ . Construct a nonzero vector  $\vec{z}$  that is orthogonal to  $\vec{u}_1$  and  $\vec{u}_2$ . Find the distance from  $\vec{y}$  to  $\text{span}\{\vec{u}_1, \vec{u}_2\}$ .

**Question 5.** Let  $W$  be a subspace of  $\mathbf{R}^n$  with an orthogonal basis  $\beta_1 = \{\vec{w}_1, \dots, \vec{w}_p\}$ , and let  $\beta_2 = \{\vec{v}_1, \dots, \vec{v}_q\}$  be an orthogonal basis for  $W^\perp$ .

- (a) Explain why  $\beta_1 \cup \beta_2$  is an orthogonal set.
- (b) Explain why the set in part (a) spans  $\mathbf{R}^n$ .
- (c) Show that  $\dim(W) + \dim(W^\perp) = n$ .

**Question 6.** Let  $A$  be an  $m \times n$  matrix with linearly independent columns, and let  $A = QR$  be its  $QR$ -factorization. Prove that  $R$  is invertible with positive eigenvalues.

**Question 7.** Recall that  $H = \text{span}\{x - 3, x^2 - 3x, x^3 - 3x^2\}$  is the subspace of  $\mathbf{P}_3(\mathbf{R})$  consisting of all those vectors divisible by  $x - 3$ . Do the following.

- (a) Verify that  $\langle p, q \rangle \equiv \int_{-1}^1 pq \, dx$  is an inner product on  $\mathbf{P}_n(\mathbf{R})$ .
- (b) Use the Gram-Schmidt Process to find an orthogonal basis  $\beta$  for  $H$ .
- (c) Let  $T$  be the linear map  $H \rightarrow \mathbf{P}_2(\mathbf{R})$  defined by  $p \mapsto \frac{dp}{dx}$ . Find the  $QR$ -factorization of  $[T]_\beta$ .