# Math 240 Tutorial Questions

**May 16**

**Question 1.** Place the following augmented matrices into an echelon form. Does the corresponding system of linear equations admit any solutions?

(a)
$$\left(\begin{array}{cccc|c} 4 & 8 & 12 & 4 & 7 \\ 2 & 5 & 6 & 6 & 11 \\ 0 & 5 & 1 & 26 & 13 \\ 0 & 5 & 0 & 21 & 17 \end{array}\right).$$

(b)
$$\left(\begin{array}{cccc|c} 4 & 8 & 12 & 4 & 0 \\ 2 & 5 & 6 & 6 & 0 \\ 0 & 5 & 1 & 25 & 0 \\ 0 & 5 & 0 & 20 & 0 \end{array}\right).$$

(c)
$$\left(\begin{array}{cccc|c} 4 & 8 & 12 & 4 & 7 \\ 2 & 5 & 6 & 6 & 11 \\ 0 & 5 & 1 & 25 & 13 \\ 0 & 5 & 0 & 20 & 17 \end{array}\right).$$

**Question 2.** Consider the following system of equations

$$\begin{aligned} w + x + y + z &= 6, \\ w + y + z &= 4, \\ w + y &= 2. \end{aligned}$$

(a) List the leading variables.

(b) List the free variables.

(c) Write the general solution to the equation (expressed in terms of the free variables).

(d) Suppose a fourth equation $-2w + y = 5$ is added to the system. What is the solution of the resulting system?

(e) Suppose the fourth equation is $-2w - 2y = -3$. What can we say about the solutions for the resulting system?

**Question 3.** Find the values of $k$ for which the system of equations

$$\begin{aligned} x + ky &= 1, \\ kx + y &= 1, \end{aligned}$$

has

(a) no solution,

(b) a unique solution, and

(c) infinitely many solutions.

(d) When there is exactly one solution, what are the values of $x$ and $y$.

**Question 4.** Consider the following system of linear equations

$$\begin{aligned} u + 2v - w - 2x + 3y &= b_1, \\ x - y + 2z &= b_2, \\ 2u + 4v - 2w - 4x + 7y - 4z &= b_3, \\ -x + y - 2z &= b_4, \\ 3u + 6v - 3w - 6x + 7y + 8z &= b_5, \end{aligned}$$

where $b_1,\ b_2,\ b_3,\ b_4,\ b_5 \in \mathbf{R}$.

(a) What are the leading and free variables?

(b) What conditions must the real constants $b_1,\ b_2,\ b_3,\ b_4,\ b_5$ satisfy in order that the system be consistent?

(c) Do the numbers $b_1 = 1$, $b_2 = -3$, $b_3 = 2$, $b_4 = b_5 = 3$ satisfy the conditions of part (b)? If so, find the general solution in terms of the free variables.