# Math 240 Tutorial
# Questions

**July 18**

**Question 1.** For the following matrices, give a basis for their null space.

(a)
$$A = \begin{pmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{pmatrix}.$$

(b)
$$A = \begin{pmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}.$$

(c)
$$A = \begin{pmatrix} 1 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

**Question 2.** Find a basis for the space spanned by

$$\begin{pmatrix} -8 \\ 7 \\ 6 \\ 5 \\ -7 \end{pmatrix}, \begin{pmatrix} 8 \\ -7 \\ -9 \\ -5 \\ 7 \end{pmatrix}, \begin{pmatrix} -8 \\ 7 \\ 4 \\ 5 \\ -7 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 9 \\ 6 \\ -7 \end{pmatrix}, \begin{pmatrix} -9 \\ 3 \\ -4 \\ -1 \\ 0 \end{pmatrix}.$$

**Question 3.** Given vectors $\vec{u}_1, \ldots, \vec{u}_p$ in a vector space $V$, show $\vec{x}$ is a linear combination of $\vec{u}_1, \ldots, \vec{u}_p$ if and only if $[\vec{x}]_B$ is a linear combination of $[\vec{u}_1]_B, \ldots, [\vec{u}_p]_B$.

**Question 4.** Find a basis for the vectors in $\mathbf{R}^4$ of the form

$$\begin{pmatrix} 3a + 6b - c \\ 6a - 2b - 2c \\ -9a + 5b + 3c \\ -3a + b + c \end{pmatrix}$$

where $a,\, b,\, c \in \mathbf{R}$.

**Question 5.** Find a basis for

$$H_1 = \{(a,\, b,\, c) : a - 3b + c = 0,\, b - 2c = 0,\, 2b - c = 0\}$$

and

$$H_2 = \{(a,\, b,\, c,\, d) : a - 3b + c = 0\}.$$

**Question 6.** The the space $C(\mathbf{R})$ of all continuous functions on the real line is an infinite dimensional vector space.

**Question 7.** For an $n \times n$ matrix $A$, we use

$$A\begin{pmatrix} i_1 & \cdots & i_k \\ i_1 & \cdots & i_k \end{pmatrix}$$

to denote the determinant of the submatrix formed by choosing the rows $i_1, \ldots, i_k$ and the columns $j_1, \ldots, j_k$. Let $\vartheta_1, \ldots, \vartheta_n$ be the not necessarily distinct and possibly complex eigenvalues of $A$. Prove that

$$\sum_{1 \leqq i_1 < \cdots < i_k \leqq n} \vartheta_{i_1} \cdots \vartheta_{i_k} = \sum_{1 \leqq i_1 < \cdots < i_k \leqq n} A \begin{pmatrix} i_1 & \cdots & i_k \\ j_1 & \cdots & j_k \end{pmatrix}.$$

Use this to prove $\text{tr}(A) = \sum_{i=1}^n A_{i,i} = \sum_{i=1}^n \vartheta_i$ and $\det(A) = \vartheta_1 \cdots \vartheta_n$. [Hint: You will need to consider the characteristic equation $\det(xI - A)$ and the multilinearity of the determinant.]