Math 240 Tutorial Questions

June 20

Question 1. Show the following for $\mathbf{R}^n$.

(a) Show that scalar multiplication is a linear transformation.

(b) When is this linear map invertible?

(c) Is its inverse a linear transformation?

(d) Fix an element $a \in \mathbf{R}^n$. What is the matrix corresponding to the linear transformation $\vec{v} \mapsto a\vec{v}$?

Question 2. Construct the standard matrix for the transformation that rotates the vectors of $\mathbf{R}^2$ by $-\pi/6$ radians.

Question 3. Consider the vector space $\mathbf{P}_3$. Define $p(x) = x^3 + x^2 + x + 1$. Show that the following.

(a) Multiplication by $p(x)$ is a linear transformation $\mathbf{P}_3 \to \mathbf{P}_3$.

(b) What is the matrix for multiplication by $p(x)$ with respect to the basis $\{1, x, x^2, x^3\}$.

(c) Show that $\{1, x, \frac{1}{2}(3x^2 - 1), \frac{1}{2}(5x^3 - 3x)\}$ is a basis for $\mathbf{P}_3$ by showing they are linearly independent and span $\mathbf{P}_3$. Write out the matrix for multiplication by $p(x)$ with respect to this basis.

Question 4. Calculate the following determinants.

(a)
$$\det \begin{pmatrix} 6 & 9 & 39 & 49 \\ 5 & 7 & 32 & 37 \\ 3 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

(b)
$$\det \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & -1 & 3 & 1 \\ 4 & 17 & 0 & -5 \end{pmatrix}.$$

(c)
$$\det \begin{pmatrix} 13 & 3 & -8 & 6 \\ 0 & 0 & -4 & 0 \\ 1 & 0 & 7 & -2 \\ 3 & 0 & 2 & 0 \end{pmatrix}.$$

Question 5. Solve the following equation for x.

$$\det \begin{pmatrix} 3 & -4 & 7 & 0 & 6 & -2 \\ 2 & 0 & 1 & 8 & 0 & 0 \\ 3 & 4 & -8 & 3 & 1 & 2 \\ 27 & 6 & 5 & 0 & 0 & 3 \\ 3 & x & 0 & 2 & 1 & -1 \\ 1 & 0 & -1 & 3 & 4 & 0 \end{pmatrix}.$$

Question 6. Let M be the matrix

$$\begin{pmatrix} 5 & 4 & -2 & 3 \\ 5 & 7 & -1 & 8 \\ 5 & 7 & 6 & 10 \\ 5 & 7 & 1 & 9 \end{pmatrix}.$$

The following hold.

(a) det M can be expressed as the constant 5 times the determinant of

$$\begin{pmatrix} 3 & 1 & 5 \\ 3 & & \\ 3 & & \end{pmatrix}.$$

(b) The determinant of the 3×3 is part (a) can be expressed as the constant 3 times the determinant of

$$\begin{pmatrix} 7 & 2 \\ 2 & \end{pmatrix}.$$

The determinant of the 2×2 matrix in part (b) is what? Thus the detmerinant of M is what?

Question 7. Consider again the vector space $\mathbf{P}_3$, and let $Q \subsetneqq \mathbf{P}_3$ be the subset of polynomials of degree at most 3 that vanish when $x = 3$. Is Q a subspace? If it is, give a spanning set of Q.