Math 240 Tutorial
Questions

August 1

Question 1. Find the unit vector in the direction of the given vectors (a) $\left(-30//40 \right)$, (b) $\begin{pmatrix} 7/4 \\ 1/2 \\ 1 \end{pmatrix}$, and (c) $\begin{pmatrix} 8/3 \\ 2 \end{pmatrix}$.

Question 2. 1. Let $\vec{u}_1 = \left(\begin{smallmatrix} 2 \\ -3 \end{smallmatrix} \right)$, $\vec{u}_2 = \left(\begin{smallmatrix} 6 \\ 4 \end{smallmatrix} \right)$, and $\vec{x} = \left(\begin{smallmatrix} 9 \\ -7 \end{smallmatrix} \right)$. Does $\{\vec{u}_1, \vec{u}_2\}$ form an orthogonal basis for $\mathbf{R}^2$? If it does, write $\vec{x}$ in terms of this basis. 2. Compute the orthogonal projection of $\left(\begin{smallmatrix} 1 \\ 7 \end{smallmatrix} \right)$ onto the line through $\left(\begin{smallmatrix} -4 \\ 2 \end{smallmatrix} \right)$ and the origin.

Question 3. Let $\vec{y} \in \mathbf{R}^n$. Prove $\vec{x} \mapsto \langle \vec{x}, \vec{y} \rangle$ is a linear transformation $\mathbf{R}^n \to \mathbf{R}$.

Question 4. Let

$$\vec{y} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \quad \vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{u}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

Verify that $\{\vec{u}_1, \vec{u}_2\}$ is an orthogonal set, and find the orthogonal projection of $\vec{y}$ onto $\operatorname{span}\{\vec{u}_1, \vec{u}_2\}$. Construct a nonzero vector $\vec{z}$ that is orthogonal to $\vec{u}_1$ and $\vec{u}_2$. Find the distance from $\vec{y}$ to $\operatorname{span}\{\vec{u}_1, \vec{u}_2\}$.

Question 5. Let W be a subspace of $\mathbf{R}^n$ with an orthogonal basis $\beta_1 = \{\vec{w}_1, \dots, \vec{w}_p\}$, and let $\beta_2 = \{\vec{v}_1, \dots, \vec{v}_q\}$ be an orthogonal basis for $W^\perp$.

(a) Explain why $\beta_1 \cup \beta_2$ is an orthogonal set.

(b) Explain why the set in part (a) spans $\mathbf{R}^n$.

(c) Show that $\dim(W) + \dim(W) = n$.

Question 6. Let A be an $m \times n$ matrix with linearly independent columns, and let $A = QR$ be its QR-factorization. Prove that R is invertible in to ways.

Question 7. Do the following.

(a) Verify that $\langle p, q \rangle \equiv \int_{-1}^{1} pq \, \mathrm{d}x$ is an inner product on $\mathbf{P}_n(\mathbf{R})$.

(b) Recall the standard basis $\beta = \{1, x, x^2\}$ for $\mathbf{P}_2(\mathbf{R})$. Use the Gram–Schmidt Process to find an orthogonal basis for $\mathbf{P}_2(\mathbf{R})$.

(c) Let T be the linear map $\mathbf{P}_2(\mathbf{R}) \to \mathbf{P}_1(\mathbf{R})$ defined by $p \mapsto \frac{\mathrm{d}p}{\mathrm{d}x}$. Find the QR-factorization of $[T]_\beta$.