0.1. Lemmata. In §2.2, we introduced the linear simplex code $\mathscr{S}_{q,n}$. There it was shown that the code had constant weight q^{n-1}; in particular, it follows that it is equidistant with constant Hamming distance q^{n-1} since the code is linear.

In **?**, and later reproduced in **?** using the language of BGW matrices, generalized Hadamard matrices $\mathrm{GH}(q, q^{n-1})$ were used recursively in conjunction with the classical parameter $\mathrm{BGW}((q^n-1)/(q-1), q^{n-1}, q^{n-1}-q^{n-2}; \mathrm{GF}(q)^*)$ in order to construct certain designs. It turns out out that the $\mathrm{GH}(q, q^{n-1})$ used in the construction can be replaced by $\mathscr{S}_{q,n}$, and so simplify the construction.

In order to apply the linear code $\mathscr{S}_{q,n}$, we will require the following lemma.

0.1. Lemma. Let $\mathrm{GF}(q) = \{a_0 = 0, a_1, \ldots, a_{q-1}\}$, and let $n > 1$. Then there exist disjoint $(0,1)$-matrices $A_{a_1}, \ldots, A_{a_{q-1}}$, and $A_{a_0} = J - \sum_i A_i$ of dimensions $q^n \times (q^n-1)/(q-1)$ such that

(0.1.a)

(0.1.b)

0.2. Construction. stuff here