A hybrid and reversible quantum language

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Introduction

In quantum computation, we can consider a fragment without measure that is purely reversible.

Existing reversible quantum languages

- **Lineal** [1] : Lambda calculus with an algebraic structure
- Restricted set of types and expressivity, deciding unitarity is hard
- **Symmetric pattern-matching** [2] : Reversible terms are defined with pattern-matching
- No normalization condition
- Only linear types allowed in both languages

Contributions

- Typed reversible language with only norm 1 terms
- Hybrid design: classical and quantum control flow
- Linear and non-linear terms through type discipline
- Predicate to characterize reversibility and unitarity

Types

$$T ::= \mathrm{Qbit} \mid B \mid T \multimap T \mid T \Rightarrow T \mid T \leftrightarrow T$$

- Base elements: $\mathrm{Qbit}, A \times B, \mathtt{nat}, [A], \dots$ and three functional arrows
- Disjoint between quantum and classical

Syntax

$$\begin{aligned} t ::= x &\mid |0\rangle \mid |1\rangle \mid \mathtt{qcase}\, t \left\{|0\rangle \to t_0 \mid |1\rangle \to t_1\right\} \\ &\mid c(t_1, \dots, t_n) \mid \mathtt{match}\, t : B \left\{c_1(\vec{x_1}) \to t_1 \mid \dots\right\} \\ &\mid \lambda x.t \mid \mathtt{letrec}\, f\, x = t \mid \mathtt{unit}(t) \mid t_1 t_2 \\ &\mid \sum_{i=1}^{n} \alpha_i \cdot t_i \mid \mathtt{shape}(t) \end{aligned}$$

- Pattern matching: $\mathtt{qcase}$ for quantum, $\mathtt{match}$ for classical
- $\mathtt{shape}$ extracts classical structure following [3]
- Equivalence relation $\equiv$ to gain linearity and simplify $s + 0 \cdot t \equiv s$
- $\mathtt{unit}(t)$: indicates unitary terms, verified through typing

Typing rules

A typing judgement $\Gamma; \Delta \vdash t : T$ has a linear context, Δ, and a non-linear context, Γ.

$$\frac{}{\Gamma; x : T \vdash x : T} \qquad \frac{}{\Gamma, x : C; \varnothing \vdash x : C}$$

$$\frac{\Gamma; \Delta, x : T \vdash t : T'}{\Gamma; \Delta \vdash \lambda x.t : T \multimap T'} \qquad \frac{\Gamma; \Delta \vdash t_1 : T \multimap T' \quad \Gamma; \Delta' \vdash t_2 : T}{\Gamma; \Delta, \Delta' \vdash t_1 t_2 : T'}$$

$$\frac{\Gamma; \Delta \vdash t_i : Q \quad \sum_{i=1}^{n} |\alpha_i|^2 = 1 \quad \forall i \neq j,\ t_i \perp t_j}{\Gamma; \Delta \vdash \sum_{i=1}^{n} \alpha_i \cdot t_i : Q}$$

Orthogonality

- $t \perp t'$ if same classical structure and their inner product is null
- $|0\rangle \perp |1\rangle$, $|+\rangle :: [\,] \perp |-\rangle :: [\,]$, $(0, |0\rangle) \not\perp (S\ 0, |1\rangle)$
- Deciding orthogonality is Π_2^0-complete
- Most use cases are decidable polynomially

Operational semantics

- Call by value, small-step semantics
- Superposition reduces each term if possible

$$(\lambda x.t)v \rightsquigarrow t[x \leftarrow v] \qquad \mathtt{qcase}\ |0\rangle \left\{|0\rangle \to t_0 \mid |1\rangle \to t_1\right\} \rightsquigarrow t_0$$

$$\frac{\sum_{i=1}^{n} \alpha_i \cdot t_i \in \mathtt{CAN} \setminus \mathbb{V} \qquad t_i \rightsquigarrow^{\leq 1} t_i'}{\sum_{i=1}^{n} \alpha_i \cdot t_i \rightsquigarrow \sum_{i=1}^{n} \alpha_i \cdot t_i'} \qquad \frac{}{\mathtt{shape}(|0\rangle) \rightsquigarrow ()}$$

Examples and properties

$$\mathtt{NOT} = \lambda x.\mathtt{qcase}\, x \left\{|0\rangle \to |1\rangle \mid |1\rangle \to |0\rangle\right\}$$

$$\mathtt{map} = \lambda \phi.\mathtt{letrec}\, f\, x = \mathtt{match}\, x : [A] \left\{[\,] \to [\,] \mid h :: t \to \phi h :: f t\right\}$$

$$\varnothing; \varnothing \vdash \lambda x.(x, \mathtt{length}(\mathtt{shape}(x))) : [\mathrm{Qbit}] \multimap [\mathrm{Qbit}] \times \mathtt{nat}$$

Subject reduction

Let $\Gamma; \Delta \vdash t : T$ and $t \rightsquigarrow t'$, then $\Gamma; \Delta \vdash t' : T$.

Progress

Let $; \vdash t : T$, then either $t \equiv v$ or $t \rightsquigarrow t'$.

Pattern matching

There exists a sound and complete predicate that derives an exhaustive and non-overlapping pattern matching.

Reversibility

Given $t : A \leftrightarrow B$, then t is unitary, and in particular reversible.

Work in progress

- Finalize the link between unitarity and reversiblity
- Translate our language into term rewriting systems:
  $$(\mathtt{map}\, \phi)[\,] \to [\,] \qquad (\mathtt{map}\, \phi)(h :: t) \to \phi\, h :: (\mathtt{map}\, \phi) t$$
- Existing results with interpretations to compute bounds [4]
- Use / adapt results to get polynomial bounds on the resources used

References

[1] Pablo Arrighi and Gilles Dowek. Lineal: A linear-algebraic Lambda-calculus. *Logical Methods in Computer Science*, Volume 13, Issue 1, March 2017.

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[3] Alexander S. Green, Peter LeFanu Lumsdaine, Neil J. Ross, Peter Selinger, and Benoît Valiron. Quipper: A scalable quantum programming language. *SIGPLAN Not.*, 48(6):333–342, June 2013.

[4] G. Bonfante, J. Y. Marion, and J. Y. Moyen. Quasi-interpretations a way to control resources. *Theoretical Computer Science*, 412(25):2776–2796, June 2011.