forall x notes

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1 Chapter 1 Practice Exercises

Part A Which of the following are 'sentences' in the logical sense?

1. **England is smaller than China.**
2. **Greenland is south of Jerusalem.**
3. Is New Jersey east of Wisconsin?
4. **The atomic number of helium is 2.**
5. **The atomic number of helium is π.**
6. **I hate overcooked noodles.**
7. Blech! Overcooked noodles!
8. **Overcooked noodles are disgusting.**
9. Take your time.
10. **This is the last question.**

Part B For each of the following: Is it a tautology, a contradiction, or a contingent sentence?

1. Caesar crossed the Rubicon. **(Contingent)**
2. Someone once crossed the Rubicon. **(Contingent)**
3. No one has ever crossed the Rubicon. **(Contingent)**
4. If Caesar crossed the Rubicon, then someone has. **(Tautology)**
5. Even though Caesar crossed the Rubicon, no one ever crossed the Rubicon. **(Contradiction)**
6. If anyone has ever crossed the Rubicon, it was Caesar. **(Contingent)**

Part C Look back at the sentences G1-G4 on p.11, and consider each of the following sets of sentences. Which are consistent? Which are inconsistent?

1. G2, G3, and G4 **(Consistent)**

2. G1, G3, and G4 **(Inconsistent)**

3. G1, G2, and G4 **(Consistent)**

4. G1, G2, and G3 **(Consistent)**

Part D Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument that has one false premise and one true premise

   This is possible. For example:

   All men are carrots.

   Socrates is a man.

   $\therefore$ Socrates is a carrot.

2. A valid argument that has a false conclusion

   This is possible. The previous example for instance.

3. A valid argument, the conclusion of which is a contradiction

   This is possible. For example:

   It is both raining and not raining.

   $\therefore$ It is both snowing and not snowing.

4. An invalid argument, the conclusion of which is a tautology

   This is not possible. All invalid arguments have true premises and a false conclusion; this means the conclusion cannot be tautology (which is always true).

5. A tautology that is contingent

   This is not possible since the definition of a contingent sentence requires that it not be a tautology.

6. Two logically equivalent sentences, both of which are tautologies

   This is possible. In fact, any two tautologies will always be logically equivalent as they are both always true.

7. Two logically equivalent sentences, one of which is a tautology and one of which is contingent

   This is not possible. Logical equivalence means that the sentences necessarily have the same truth-value. Since a contingent sentence may be false, it does not necessarily have the same truth value as a tautological sentence which is always true.

8. Two logically equivalent sentences that together are an inconsistent set

   This is possible. Consider two sentences which are both contradictions. They must be logically equivalent since contradictions are always false; this also means it is not logically possible for the set containing these two sentences to be true at the same time.

9. A consistent set of sentences that contains a contradiction

   This is not possible. Since this set contains a sentence which is always false, it is not logically possible for all the members of the set to be true at the same time.

10. An inconsistent set of sentences that contains a tautology

    This is possible. Any inconsistent set of sentences will remain inconsistent if you add a tautology to it.

2 Chapter 2 Practice Exercises

Part A Using the symbolization key given, translate each English-language sentence into SL.

M: Those creatures are men in suits.

C: Those creatures are chimpanzees.

G: Those creatures are gorillas.

1. Those creatures are not men in suits.

   $\neg M$

2. Those creatures are men in suits, or they are not.

   $M \vee \neg M$

3. Those creatures are either gorillas or chimpanzees.

   $G \vee C$

4. Those creatures are neither gorillas nor chimpanzees.

   $\neg(G \vee C)$

5. If those creatures are chimpanzees, then they are neither gorillas nor men in suits.

   $C \to \neg(G \vee M)$

6. Unless those creatures are men in suits, they are either chimpanzees or they are gorillas.

   $M \vee (C \vee G)$

Part B Using the symbolization key given, translate each English-language sentence into SL.

A: Mister Ace was murdered.

B: The butler did it.

C: The cook did it.

D: The Duchess is lying.

E: Mister Edge was murdered.

F: The murder weapon was a frying pan.

1. Either Mister Ace or Mister Edge was murdered.

   $A \vee E$

2. If Mister Ace was murdered, then the cook did it.

   $A \to C$

3. If Mister Edge was murdered, then the cook did not do it.

   $E \to \neg C$

4. Either the butler did it, or the Duchess is lying.

   $B \vee D$

5. The cook did it only if the Duchess is lying.

   $C \to D$

6. If the murder weapon was a frying pan, then the culprit must have been the cook.

   $F \to C$

7. If the murder weapon was not a frying pan, then the culprit was either the cook or the butler.

   $\neg F \to (C \vee B)$

8. Mister Ace was murdered if and only if Mister Edge was not murdered.

   $A \leftrightarrow \neg E$

9. The Duchess is lying, unless it was Mister Edge who was murdered.

   $D \vee E$

10. If Mister Ace was murdered, he was done in with a frying pan.

    $A \to F$

11. Since the cook did it, the butler did not.

    $C \wedge (C \to \neg B)$

12. Of course the Duchess is lying!

    D

Part C Using the symbolization key given, translate each English-language sentence into SL.

$\mathbf{E}_1$: Ava is an electrician.

$\mathbf{E}_2$: Harrison is an electrician.

$\mathbf{F}_1$: Ava is a firefighter.

$\mathbf{F}_2$: Harrison is a firefighter.

$\mathbf{S}_1$: Ava is satisfied with her career.

$\mathbf{S}_2$: Harrison is satisfied with his career.

1. Ava and Harrison are both electricians.

   $E_1 \wedge E_2$

2. If Ava is a firefighter, then she is satisfied with her career.

   $F_1 \rightarrow S_1$

3. Ava is a firefighter, unless she is an electrician.

   $F_1 \vee E_1$

4. Harrison is an unsatisfied electrician.

   $E_2 \wedge \neg S_2$

5. Neither Ava nor Harrison is an electrician.

   $\neg(E_1 \vee E_2)$

6. Both Ava and Harrison are electricians, but neither of them find it satisfying.

   $E_1 \wedge \neg S_1 \wedge E_2 \wedge \neg S_2$

7. Harrison is satisfied only if he is a firefighter.

   $S_2 \rightarrow F_2$

8. If Ava is not an electrician, then neither is Harrison, but if she is, then he is too.

   $E_1 \leftrightarrow E_2$

9. Ava is satisfied with her career if and only if Harrison is not satisfied with his.

   $S_1 \leftrightarrow \neg S_2$

10. If Harrison is both an electrician and a firefighter, then he must be satisfied with his work.

    $(E_2 \wedge F_2) \rightarrow S_2$

11. It cannot be that Harrison is both an electrician and a firefighter.

    $\neg(E_2 \wedge F_2)$

12. Harrison and Ava are both firefighters if and only if neither of them is an electrician.

    $(F_2 \wedge F_1) \leftrightarrow (\neg E_2 \wedge \neg E_1)$

Part D Give a symbolization key and symbolize the following sentences in SL.

A: Alice is a spy.

B: Bob is a spy.

C: The code has been broken.

G: The German embassy will be in an uproar.

1. Alice and Bob are both spies.

   $A \wedge B$

2. If either Alice or Bob is a spy, then the code has been broken.

   $(A \vee B) \rightarrow C$

3. If neither Alice nor Bob is a spy, then the code remains unbroken.

   $\neg(A \vee B) \rightarrow \neg C$

4. The German embassy will be in an uproar, unless someone has broken the code.

   $G \vee C$

5. Either the code has been broken or it has not, but the German embassy will be in an uproar regardless.

   $(C \vee \neg C) \wedge G$

6. Either Alice or Bob is a spy, but not both.

   $(A \vee B) \wedge \neg(A \wedge B)$

Part E Give a symbolization key and symbolize the following sentences in SL.

G: Gregor plays first base.

L: The team will lose.

C: Gregor's mom will bake cookies.

M: There is a miracle.

1. If Gregor plays first base, then the team will lose.

   $G \to L$

2. The team will lose unless there is a miracle.

   $L \vee M$

3. The team will either lose or it won't, but Gregor will play first base regardless.

   $(L \vee \neg L) \wedge G$

4. Gregor's mom will bake cookies if and only if Gregor plays first base.

   $C \leftrightarrow G$

5. If there is a miracle, then Gregor's mom will not bake cookies.

   $M \to \neg C$

Part F For each argument, write a symbolization key and translate the argument as well as possible into SL.

1. If Dorothy plays the piano in the morning, then Roger wakes up cranky. Dorothy plays piano in the morning unless she is distracted. So if Roger does not wake up cranky, then Dorothy must be distracted.

P: Dorothy plays piano in the morning.

R: Roger wakes up cranky.

D: Dorothy is distracted.

$$P \to C$$
$$P \vee D$$
$$\therefore \neg C \to D$$

2. It will either rain or snow on Tuesday. If it rains, Neville will be sad. If it snows, Neville will be cold. Therefore, Neville will either be sad or cold on Tuesday.

R: It will rain on Tuesday.

S: It will snow on Tuesday.

$\mathbf{N_1}$: Neville will be sad on Tuesday.

$\mathbf{N_2}$: Neville will be cold on Tuesday.

$$
\begin{array}{rl}
 & R \vee S \\
 & R \to N_1 \\
 & S \to N_2 \\
\therefore & N_1 \vee N_2
\end{array}
$$

3. If Zoog remembered to do his chores, then things are clean but not neat. If he forgot, then things are neat but not clean. Therefore, things are either neat or clean— but not both.

Z: Zoog remembered to do his chores.

C: Things are clean.

N Things are neat.

$$
\begin{array}{rl}
 & Z \to (C \wedge \neg N) \\
 & \neg Z \to (N \wedge \neg C) \\
\therefore & (N \vee C) \wedge \neg(N \wedge C)
\end{array}
$$

Part G For each of the following: (a) Is it a wff of SL? (b) Is it a sentence of SL, allowing for notational conventions?

1. (a) No. (b) No.
2. (a) No. (b) Yes.
3. (a) Yes. (b) Yes.

4. (a) No. (b) No.
5. (a) Yes. (b) Yes.
6. (a) No. (b) No.
7. (a) No. (b) Yes.
8. (a) No. (b) Yes.
9. (a) No. (b) No.

Part H

1. Are there any wffs of SL that contain no sentence letters? Why or why not?

   No, since the base case of a wff is always a sentence letter.

2. In the chapter, we symbolized an *exclusive or* using $\vee$, $\wedge$, and $\neg$. How could you translate an *exclusive or* using only two connectives? Is there any way to translate an *exclusive or* using only one connective?

   3 connectives: $(A \vee B) \wedge \neg(A \wedge B)$

   2 connectives: ?

   1 connective: ?

3 Chapter 3 Practice Exercises

Part A Determine whether each sentence is a tautology, a contradiction, or a contingent sentence. Justify your answer with a complete or partial truth table where appropriate.

1. $A \rightarrow A$ **(Tautology)**

A	$A \rightarrow A$
1	1 **1** 1
0	0 **1** 0

2. $\neg B \wedge B$ **(Contradiction)**

B	$\neg B \wedge B$
1	0 1 **0** 1
0	1 0 **0** 0

3. $C \rightarrow \neg C$ **(Contingent)**

C	$C \rightarrow \neg C$
1	1 **0** 0 1
0	0 **1** 1 0

4. $\neg D \vee D$ **(Tautology)**

D	$\neg D \vee D$
1	0 1 **1** 1
0	1 0 **1** 0

5. $(A \leftrightarrow B) \leftrightarrow \neg(A \leftrightarrow \neg B)$ **(Tautology)**

A	B	$(A \leftrightarrow B) \leftrightarrow \neg(A \leftrightarrow \neg B)$
1	1	1 1 1 **1** 1 1 0 0 1
1	0	1 0 0 **1** 0 1 1 1 0
0	1	0 0 1 **1** 0 0 1 0 1
0	0	0 1 0 **1** 1 0 0 1 0

6. $(A \wedge B) \vee (B \wedge A)$
7. $(A \rightarrow B) \vee (B \rightarrow A)$
8. $\neg[A \rightarrow (B \rightarrow A)]$
9. $(A \wedge B) \rightarrow (B \vee A)$
10. $A \leftrightarrow [A \rightarrow (B \wedge \neg B)]$
11. $\neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B)$
12. $\neg(A \wedge B) \leftrightarrow A$
13. $[(A \wedge B) \wedge \neg(A \wedge B)] \wedge C$

14. $A \to (B \lor C)$

15. $[(A \land B) \land C] \to B$

16. $(A \land \neg A) \to (B \lor C)$

17. $\neg[(C \lor A) \lor B]$

18. $(B \land D) \leftrightarrow [A \leftrightarrow (A \lor C)]$

Part B Determine whether each pair of sentences is logically equivalent. Justify your answer with a complete or partial truth table where appropriate.

1. A, $\neg A$
2. A, $A \lor A$
3. $A \to A$, $A \leftrightarrow A$
4. $A \lor \neg B$, $A \to B$
5. $A \land \neg A$, $\neg B \leftrightarrow B$
6. $\neg(A \land B)$, $\neg A \lor \neg B$
7. $\neg(A \to B)$, $\neg A \to \neg B$
8. $(A \to B)$, $(\neg B \to \neg A)$
9. $[(A \lor B) \lor C]$, $[A \lor (B \lor C)]$
10. $[(A \lor B) \land C]$, $[A \lor (B \land C)]$

Part E Answer each of the questions below and justify your answer.

1. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent. What can you say about $\mathcal{A} \leftrightarrow \mathcal{B}$?

   $\mathcal{A} \leftrightarrow \mathcal{B}$ is a tautology. Every line of the complete truth table will be true since $\leftrightarrow$ evaluates to true when its arguments have the same truth value.

2. Suppose that $(\mathcal{A} \wedge \mathcal{B}) \rightarrow \mathcal{C}$ is contingent. What can you say about the argument "$\mathcal{A}, \mathcal{B}, \therefore \mathcal{C}$"?

   The argument is invalid. Since $(\mathcal{A} \wedge \mathcal{B}) \rightarrow \mathcal{C}$ is contingent, there is some line where $(\mathcal{A} \wedge \mathcal{B})$ is true but $\mathcal{C}$ is false. This means there is some line where the premises $\mathcal{A}$ and $\mathcal{B}$ are true, and the conclusion $\mathcal{C}$ is false. Therefore, the argument is invalid.

3. Suppose that $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ is inconsistent. What can you say about $(\mathcal{A} \wedge \mathcal{B} \wedge \mathcal{C})$?

   $(\mathcal{A} \wedge \mathcal{B} \wedge \mathcal{C})$ is a contradiction. Since the set $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ is inconsistent, there is no line of a complete truth table where all of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ are true. This means that $(\mathcal{A} \wedge \mathcal{B} \wedge \mathcal{C})$ will be false on every line, or in other words, a contradiction.

4. Suppose that $\mathcal{A}$ is a contradiction. What can you say about the argument "$\mathcal{A}, \mathcal{B}, \therefore \mathcal{C}$"?

   The argument is valid. There cannot be a line where both premises $\mathcal{A}$ and $\mathcal{B}$ are true, yet $\mathcal{C}$ is false, which is required for an invalid argument. This is because $\mathcal{A}$ is a contradiction and not true on any line.

5. Suppose that $\mathcal{C}$ is a tautology. What can you say about the argument "$\mathcal{A}, \mathcal{B}, \therefore \mathcal{C}$"?

   The argument is valid. There cannot be a line where both premises $\mathcal{A}$ and $\mathcal{B}$ are true, yet $\mathcal{C}$ is false, which is required for an invalid argument. This is because $\mathcal{C}$ is a tautology and true on every line.

6. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent. What can you say about $(\mathcal{A} \vee \mathcal{B})$?

   Not much. $\mathcal{A}$ and $\mathcal{B}$ are either both true or both false on some line, so $(\mathcal{A} \vee \mathcal{B})$ could either be true or false on that line as well. $(\mathcal{A} \vee \mathcal{B})$ could be a tautology, a contradiction, or contingent.

7. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are *not* logically equivalent. What can you say about $(\mathcal{A} \vee \mathcal{B})$?

   $(\mathcal{A} \vee \mathcal{B})$ cannot be a contradiction, i.e. it is either a tautology or contingent. Since $\mathcal{A}$ is not logically equivalent to $\mathcal{B}$, there must be some line where $\mathcal{A}$ differs in truth value from $\mathcal{B}$, which means $(\mathcal{A} \vee \mathcal{B})$ must be true on some line.