480/905: Session 8

Online handout: plots of damped oscillations; *online listings:* filename_test.cpp, diffeq_pendulum.cpp, GnuplotPipe class

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Strings and Things

The filename_test.cpp code has examples of the use and manipulation of C++ strings, including building filenames the way we do stream output. **Be careful NOT to put << endlwhen creating filenames.**

1. Using make_filename_test, compile and link filename_test.cpp and run it. Look at the output files and the printout of the code to see how it works.
2. Modify the code so that there is a loop running from 0 to 3 with index variable j. For each j, open a file with a name that includes the current value of j. *Write "This is file j", where "j" here is the current value, into each file and then close it. Did you succeed?*

Yes, I was able to create four “filenum_j.out” files each containing the statement “This is file j,” with j being an integer between 0 and 3.

3. Modify the code to input a double named alpha and open a filename with 3 digits of alpha as part of the name. (E.g., something like pendulum_alpha5.22_plot.dat if alpha = 5.21934.) *Output something appropriate to the file. Did it work?*

Yes, I was about to output a double rounded off after three digits in the file name, and looped through four different values of alpha.

Upgrades from the diffeq_oscillation to diffeq_pendulum code

- There are three new menu items: plot_start, plot_end, and Gnuplot_delay. The equation is still solved from t_start to t_end, but results are only printed out from plot_start to plot_end. Initially these are the same time intervals, *but you can use plot_start to exclude a transient region.* So if the system settles down to periodic behavior at t=20, setting plot_start=20 means that 0 < t < 20 is not plotted, which makes the phase-space plots much easier to interpret.
- We've also incorporated code to do real-time plotting in gnuplot directly from C++ programs. We have made a class to do this but it is rather crude: the interface and documentation needs work, and it probably has bugs! *Look at the GnuplotPipe.h printout and the GnuplotPipe.cpp file to get an idea how it works.* Gnuplot_delay sets the time in milliseconds between plotted points.

Damped (Undriven) Pendulum

The pendulum modeled here has the analog of the viscous damping: $F_f = -b*v$, where v(t) is the velocity, that was used in session 7. The damping parameter is called alpha here.

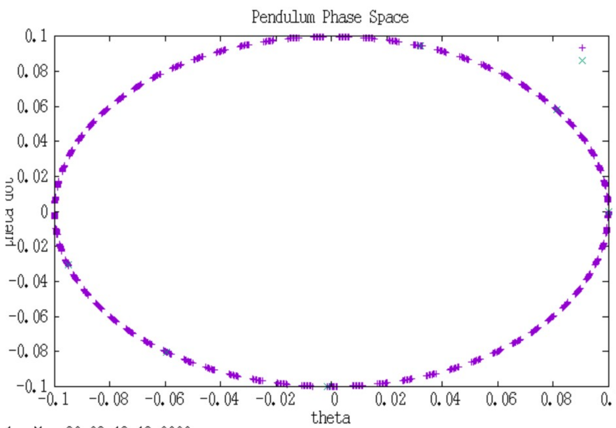
1. Use make_diffeq_pendulum to compile and link diffeq_pendulum.cpp. Run it while taking a look at the printout of the code. It should look a lot like diffeq_oscillations.cpp, with different parameter names. Run it with the default parameters, noting the real-time phase-space plot. There is also an output file diffeq_pendulum.dat.
2. *Modify the code so that the output file includes two digits of the variable alpha in the name. Did you succeed?*

The output file is now diffeq_pendulum0.2.dat and will change with different values of alpha.

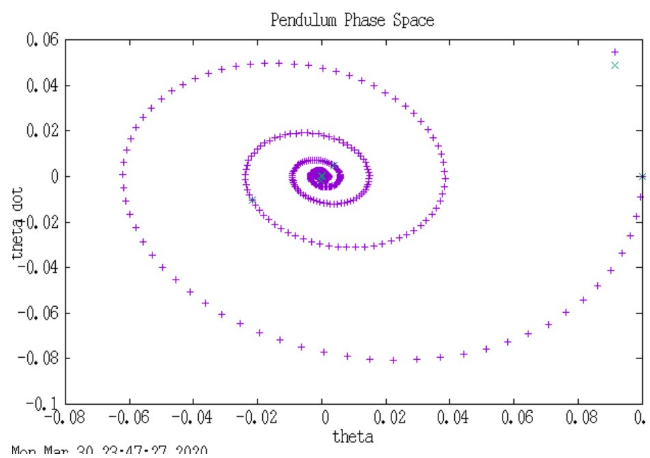
3. Generate the analogs of the four phase-space plots on the handout but with pendulum variables and

initial conditions theta_dot0=0 (at rest) and theta0 such that you are in the simple harmonic oscillator regime (note that theta is in radians). Set f_ext=0 (no external driving force) and then do four runs with four values of alpha corresponding to undamped, underdamped, critically damped, and overdamped (convert from the conditions on b discussed in the background notes). *What values of theta0 and alpha did you use?*

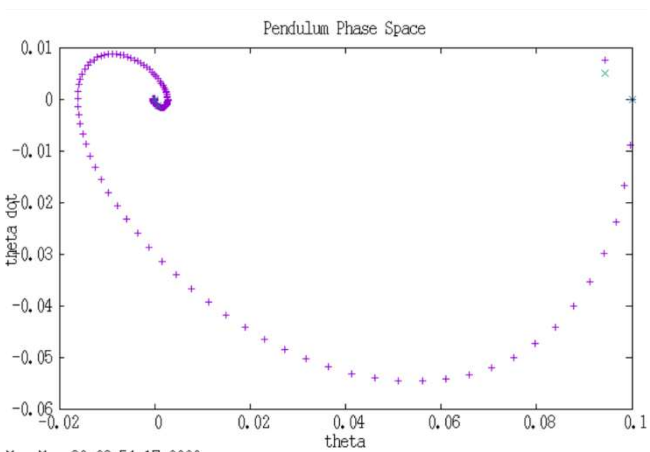
alpha = 0, theta0 = 0.1 (for a small oscillation).
This is the undamped case.



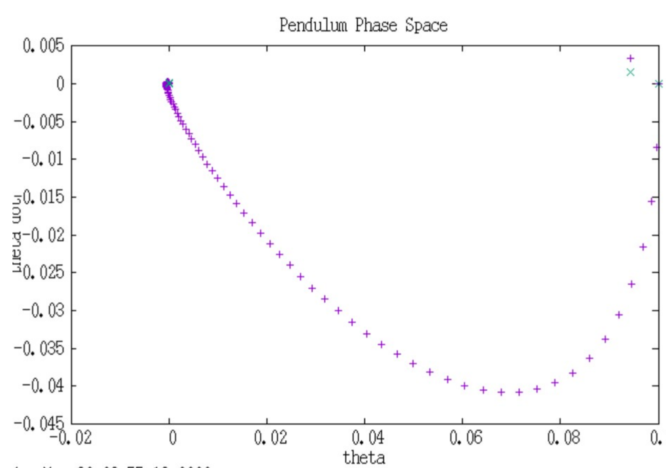
alpha = 0.3, theta0 = 0.1
This is the underdamped case



alpha = 1, theta0 = 0.1 This is the critically damped case.



alpha = 1.7, theta0 = 0.1.This is the overdamped case.



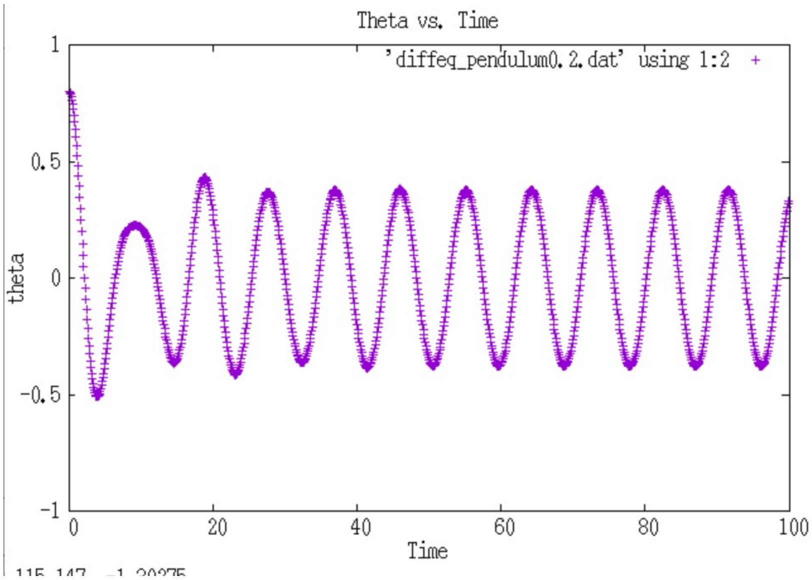
Damped, Driven Pendulum

This is a quick exercise to look at transients.

1. Restart the program so that we use the defaults. There is both damping and an external driving force, with frequency w_ext = 0.689. The initial plot is from t=0 to t=100. Run it. *The green points are plotted once every period of the external force. What good are they?*

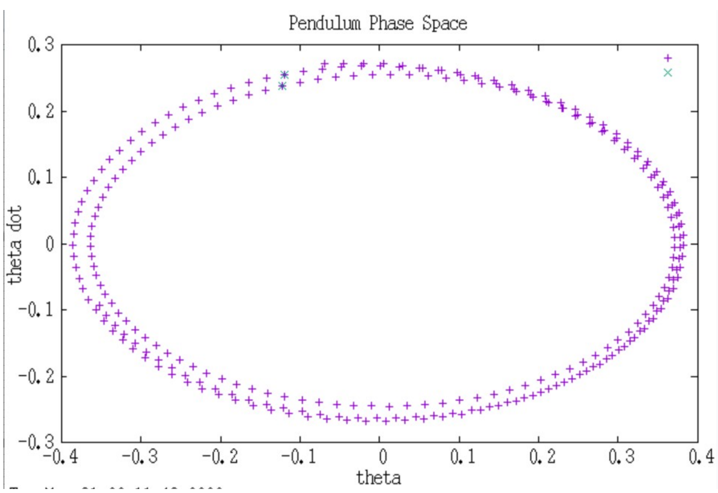
They show us if the motion of the system is mirroring the frequency of the driving force.

2. Note that it seems to settle down to a periodic orbit after a while. *Plot ("by hand" with gnuplot) theta vs. t from the output file* diffeq_pendulum.dat *and see how long it takes to become periodic.*



I would say it begins to look periodic after roughly 25 seconds

3. Run the code again with "plot_start" set to the time you just found. *Have you gotten rid of the transients? What is the frequency of the asymptotic theta(t)?*



This looks very close to purely periodic behavior. We appear to have gotten rid of our transients and the motion is entirely determined by the driving force.

The frequency is the same as the frequency of the driving force, w_ext=0.689

Looking for Chaos

Now we want to explore more of the parameter space and look at different structures. In Section f of the Session 7 notes there is a list of characteristic structures that can be found in phase space, with sample pictures in Figure 1.

1. In phase space, a fixed point is a (zero-dimensional) point that "attracts" the time-development of a system. By this we mean that many (or all) initial conditions end up at the same point in phase space. The clearest case is a damped, undriven system like a pendulum, which ends up at theta=0 and zero angular velocity no matter how it starts. If the steady-state trajectory in phase space is a closed (one-dimensional) curve, then we call it a limit cycle.

2. Try some prescribed values for the pendulum. You will need to adjust "plot_start" and extend the plot time (increase "t_end" and "plot_end"). *Try the first three combinations in this table:*

description	alpha	f_ext	w_ext	theta0	theta_dot0
period-1 limit cycle	0.0	0.0	0.689	0.8	0.0
Period -1 limit cycle	0.2	0.52	0.689	−0.8	0.1234
Not closed (?) Maybe mode locking (?)	0.2	0.52	0.694	0.8	0.8
Think this is a predictable attractor	0.2	0.52	0.689	0.8	0.8
chaotic pendulum	0.2	0.9	0.54	−0.8	0.1234

Can you tell how many "periods" the limit cycles have from the graphs? How might you identify whether a function of time f(t) is built from one, two, three, ... frequencies?

A function with one frequency will make an ellipse in phase space. If there are several “centers” of a phase space plot, that would imply the presence of multiple frequencies driving the motion of the system.

3. One characteristic of chaos is an "exponential sensitivity to initial conditions." *For the last combination, vary the initial conditions very slightly (e.g., change x0 by 0.01 or 0.001); what happens?*

The initial plot looked a lot like a peanut. I added 0.01 to the initial position and it took a hockey stick turn out of the peanut shape back the other way. I tried subtracting 0.01 instead and got no hockey stick turn, but the motion centered around a new point that wasn’t present in either of my other two solutions.
I understand why it is called a chaotic pendulum now.